# Using Bayesian networks to discover relationships between bibliometric indices. A case study of computer science and artificial intelligence journals 

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#### Abstract

As they are used to evaluate the importance of research at different levels by funding agencies and promotion committees, bibliometric indices have received a lot of attention from the scientific community over the last few years. Many bibliometric indices have been developed in order to take into account aspects not previously covered. The result is that, nowadays, the scientific community faces the challenge of selecting which of this pool of indices meets the required quality standards. In view of the vast number of bibliometric indices, it is necessary to analyze how they relate to each other (irrelevant, dependent and so on). Our main purpose is to learn a Bayesian network model from data to analyze the relationships among bibliometric indices. The induced Bayesian network is then used to discover probabilistic conditional (in)dependencies among the indices and, also for probabilistic reasoning. We also run a case study of 14 well-known bibliometric indices on computer science and artificial intelligence journals.


Keywords Bibliometric indices • Bayesian networks - Conditional dependencies and conditional independencies • Computer science and artificial intelligence

## Introduction

Nowadays, many funding agencies and promotion committees use bibliometric indices to evaluate the impact of a researcher's work. These indices essentially involve counting the number of times scientific papers are cited. They are based on the assumption that influential researchers and important studies will be cited more frequently than others.

Bibliometric indices are used as a tool for journal evaluation (Garfield 1972a). The first bibliometric index to be calculated for journal assessment was the impact factor (Garfield 1972b). More recently, Braun et al. (2006) suggested that the well-known $h$-index could be usefully applied to evaluate the scientific impact of journals. Other indices like eigenfactor (Bergstrom et al. 2008), article influence (Bergstrom et al. 2008) and Scimago's journal rank index (SCImago 2007) have also been developed for the same purpose.

In view of the vast number of bibliometric indices, it is necessary to analyze how they relate to each other. We know that the degree of correlation among journal citation indices has been investigated in the past.

Over the last few years, some studies (Davis 2008; Bollen et al. 2009; Leydesdorff 2009) have examined correlations between a list of journal citation indices using the Pearson $\rho$ coefficient. Davis (2008) found a statistically significant correlation between the number of citations and eigenfactor $(\rho=0.95$ ). Bollen et al. (2009) also found statistically significant correlations between 39 measures of scholarly impact, although the exact values were not reported. Finally, Leydesdorff (2009) showed high correlations between indices, specially the 5 year impact factor and article influence $(\rho=0.956)$.

Similarly, three recent studies (Elkins et al. 2010; Franceschet 2010; Saad in press) have also examined the degree of correlation between some typical journal citation indices, tested using Spearman's $\rho$. Spearman's $\rho$ can assess how well two variables are related by any monotonic, not necessarily linear, function. Elkins et al. (2010) mentioned that all analyzed pairs of indices showed moderate to strong correlations. The strongest correlation was between Scimago's journal rank index and the 2year impact factor $(\rho=0.89)$. On the other hand, Franceschet (2010) also found strong correlations between the examined indices. He found that the strongest correlation was between the 2 year impact factor and the 5 year impact factor $(\rho=0.96)$. Finally, Saad (in press) noticed that the 2 year impact factor was more correlated with article influence than with eigenfactor.

To date there have not been many publications analyzing citations in computer science and artificial intelligence. Goodrum et al. (2001), though, did publish an article analyzing citations in computer science literature. The studies objective was to identify additional research areas dealing with information dissemination and citation practices in computer science. On the other hand, a recent article published by Serenko (2010) analyzed journals in the field of artificial intelligence, and calculated some bibliometric index' values ( $h$ index, g-index and hc-index) which correlated almost perfectly with each other (ranging from 0.97 to 0.99 ).

It should be mentioned that all analyzed indices are obviously correlated since they are all derived from the number of documents and citations, and these are highly correlated.

The interest and originality of our study is that it introduces a new Bayesian networkbased approach for analyzing the conditional (in)dependencies between journal citation indices. In this paper, we build some Bayesian networks (yearly and global models) to discover the relationships between 14 journal citation indices. These models are learned using journal publication and citation data (all the journals in the JCR Computer Science and Artificial Intelligence category) during the period 2000-2009, inclusive. Yearly and global models are developed to analyze index relationships within a 1-year publication and citation window. Finally, we measure how some indices influence others in probabilistic terms. Also, the network is able to perform all kinds of probabilistic reasoning, computing, say, the probability of a journal obtaining certain fixed index values given other known values.

The main advantage of our work over earlier studies, which analyze only bivariate correlations between indices, is that we calculate and analyze the joint probability
distribution over all analyzed indices, discovering probabilistic conditional (in)dependencies among triplets of indices. Journal citation indices have never been analyzed like this before.

Using our models, computer science and artificial intelligence journal editorial boards could answer some of the questions related to their journal citation indices, like, for example,

- What would happen to our journal's impact factor if articles, which are published by our journal, received more citations?
- What would happen to our journal's h-index if our journal published a lot of new documents?
- What would happen to our journal's g-index if our most cited studies, were the only ones to receive new citations?
- What would happen to our journal's immediacy-index if our journal accepted more documents, but received no new citations?

Obviously, this is a general-purpose methodology and can be used for other research areas, and, the editorial boards of any journals could find answers to the above questions.

The remainder of the study is organized as follows. The "Methods" section reviews some basic concepts about Bayesian networks and bibliometric indices on which our work is based. The "Results" section presents the dataset used, the Bayesian networks learned, the discovered probabilistic conditional (in)dependencies among the analyzed indices and examples of probabilistic reasoning. Finally, "Conclusions" contains some conclusions emphasizing the original contribution of the study and future research on the topic.

## Methods

## Bayesian networks

Bayesian networks are a kind of probabilistic graphical model with two main elements: the graphical component and the probabilistic component. The graphical component is a directed acyclic graph (DAG), which is used to capture the structure of the problem. The probabilistic component is the conditional probability distributions associated with the random variables (nodes in the graph) of the problem. Figure 1 illustrates an example of a Bayesian network where all the variables are binary.


Fig. 1 Example of a Bayesian network: graphical and probabilistic components

Formally, a Bayesian network (Pearl 1988) is defined as a pair ( $G, P$ ). The first element, $G$, is a DAG, $G=(V(G), A(G))$, with a set of nodes given by the random variables of the problem, i.e., $V(G)=\left\{X_{1}, \ldots, X_{n}\right\}$, and a set of $\operatorname{arcs} A(G) \subseteq V(G) \times V(G)$ representing the probabilistic conditional (in)dependencies among the nodes. A variable $X_{i}$ is conditionally independent of variable $X_{j}$ given variable $X_{k}$ iff for all $x_{i}, x_{j}, x_{k}, P\left(X_{i}=x_{i} \mid X_{j}=\right.$ $\left.x_{j}, X_{k}=x_{k}\right)=P\left(X_{i}=x_{i} \mid X_{k}=x_{k}\right)$. Let $I\left(X_{i}, X_{j} \mid X_{k}\right)$ denote this relationship. The second element, $P$, is the joint probability distribution over $\left(X_{1}, \ldots, X_{n}\right)$ associated with $G$, defined as:

$$
\begin{equation*}
P\left(X_{1}, \ldots, X_{n}\right)=\prod_{i=1}^{n} P\left(X_{i} \mid \Pi\left(X_{i}\right)\right) \tag{1}
\end{equation*}
$$

where $\Pi\left(X_{i}\right)$ represents the set of parents of $X_{i}$. A node $X_{i}$ is a parent of another node $X_{j}$ if there is an arc from $X_{i}$ to $X_{j}$.

## Learning a Bayesian network from data

One of the most popular approaches for learning Bayesian networks from data is based on the score and search methodology. This approach states the learning task as an optimization problem. This involves a search for the best network structure that maximizes a scoring function defined to represent how well a structure fits a given set of data. The K2 scoring metric (Cooper and Herskovits 1992) computes the marginal likelihood of the dataset given the structure, subject to a uniform prior assumption on each variable data distribution. This scoring metric is decomposable, which facilitates the search process.

From a dataset of $n$ variables, $\left\{X_{1}, \ldots, X_{n}\right\}$, and $N$ records, the K2 algorithm uses the marginal likelihood as score to greedily learn a Bayesian network. Starting from the empty graph and a fixed total order of variables, this algorithm adds a variable as a parent to a given variable (from the subset of variables that are before this variable in the ordering only) whenever its inclusion improves the marginal likelihood score. The algorithm stops the addition process when the marginal likelihood score decreases or the algorithm reaches the maximum admissible number of parents for each variable. This number is fixed beforehand. Given the decomposability of the score, the marginal likelihood is maximized by maximizing, for each variable $X_{i}$, the expression:

$$
\begin{equation*}
g\left(X_{i}, \Pi\left(X_{i}\right)\right)=\prod_{j=1}^{q_{i}} \frac{\left(r_{i}-1\right)!}{\left(N_{i j}+r_{i}-1\right)!} \prod_{k=1}^{r_{i}} N_{i j k}! \tag{2}
\end{equation*}
$$

where $r_{i}$ is the number of possible values of $X_{i} ; q_{i}$ is the number of possible values of $\Pi\left(X_{i}\right) ; N_{i j k}$ is the number of cases in the database in which variable $X_{i}$ takes its $k$-th value and $\Pi\left(X_{i}\right)$ its $j$-th value; and $N_{i j}$ is defined as $N_{i j}=\sum_{k=1}^{r_{i}} N_{i j k}$. Notice that the dataset contains all quantities in Eq. 2.

## Variables in the Bayesian networks

Our Bayesian networks represent relationships between journal citation indices. In particular, each node, $X_{i}$, in the network represents a specific index, while the arcs between indices represent the conditional (in)dependencies among these indices. By learning these

Bayesian networks from data, our aim is to discover probabilistic conditional (in)dependencies among the set of 14 bibliometric indices. The indices analyzed in this study are: documents, citations, h-index (Hirsch 2005), g-index (Egghe 2006), hg-index (Alonso et al. 2010), a-index (Jin 2006), m-index (Bornmann et al. 2008a), $q^{2}$-index (Cabrerizo et al. 2010), r-index (Jin et al. 2007), e-index (Zhang 2009), w-index (Woeginger 2008), rational h-index (Ruane and Tol 2008), impact factor (Garfield 1972b) and immediacy-index. All these indices were obtained from the information provided by the Institute for Scientific Information (http://isiwebofknowledge.com/).

Bibliometric indices
Bibliometric indices are quantitative metrics for evaluating and comparing the research activity of individual scientists according to their output. The main advantage of these indices is that they can summarize the scientific production of an author as a single number. At the same time, this advantage can be a limitation, because it removes many details of citation records.

We have evaluated the research activity of journals using bibliometric indices such as the ones listed in "Variables in the Bayesian networks". These indices, which were originally developed to evaluate the quality of a researcher's work, have been adapted to assess journals. In the following, we show how the indices were adapted.

## Documents

Documents is an index associated with the number of articles published by each analyzed journal in the entire period. This index represents the productivity of each specific journal.

## Citations

Citations is an index associated with the number of citations received by each journal analyzed in the entire period. This index represents the visibility of each specific journal.

## The h-index

Over the last few years, the $h$-index, proposed by Hirsch (2005), has received a lot of attention from the scientific community because it combines the productivity and visibility of a scientist in a single indicator. The $h$-index is defined as a number such that, for the all the articles published in a journal, $h$ papers received at least $h$ citations whereas the other studies received no more than $h$ citations. Braun et al. (2006) applied the $h$-index to journals:

A journal has index $h$ if $h$ of its published articles have at least $h$ citations each, and the other papers have no more than $h$ citations each.

The g-index

Since the h-index tends to underestimate the achievement of journals that have a "selective publication strategy", that is, journals that do not publish a lot of documents but have a major international impact (Egghe 2006). The g-index is defined as the highest rank such
that the cumulative sum of the number of citations received is larger than or equal to the square of this rank.

The highest number $g$ of papers that together received $g^{2}$ or more citations.
Unlike the h-index, the g-index takes into account the exact number of citations received by highly cited articles, favoring journals with a selective publication strategy.

## The hg-index

Alonso et al. (2010) presented a new index called the hg-index, which is based on the $h$-index and the $g$-index. It fuses both measures to obtain a more balanced view of the scientific production of journals to minimize some of their weaknesses. The hg-index of a journal is computed as the geometric mean of its $h$-index and $g$-index, that is,

$$
h g-\operatorname{index}=\sqrt{h \cdot g},
$$

where $h$ is the value of the $h$-index, and $g$ is the value of the $g$-index.

## The a-index

The $a$-index was proposed by Jin (2006). This index is calculated for papers that are in the Hirsch core (h-core) only, that is, the first $h$ papers. It is defined as the average number of citations received by the articles included in the $h$-core. This index measures the citation intensity in the $h$-core. The $a$-index can be very sensitive to just a very few studies receiving extremely high citation counts. Mathematically, this is:

$$
a-\text { index }=\frac{1}{h} \sum_{i=1}^{h} C i t(i),
$$

where $h$ is the value of the $h$-index; and Cit( $(i)$ is the number of citations received by article $i$ belonging to the $h$-core.

## The m-index

As the distribution of citation counts is usually skewed, the median and not the arithmetic mean should be used as the measure of central tendency. Therefore, Bornmann et al. (2008a) proposed a new index, called $m$-index, as a variation of the $a$-index. This index, which was designed to illustrate the impact of the papers in the $h$-core, is the median number of citations received by papers in the $h$-core.

The $q^{2}$-index
Cabrerizo et al. (2010) developed a new index, called $q^{2}$-index, to provide a more global view of scientific production. This index is based on the geometric mean of the h-index, describing the number of the papers (quantitative dimension), and the $m$-index, depicting the impact of the papers (qualitative dimension), that is,

$$
q^{2} \text {-index }=\sqrt{h \cdot m},
$$

where $h$ is the value of the $h$-index, and $m$ is the value of the $m$-index.

Some problems related to the $a$-index were overcome in Jin et al. (2007) by another index. Unlike the a-index, which involves a division by the $h$-index, this index does not punish journals for having a higher h-index value. Instead of dividing by the $h$-index, this index takes the square root of the sum of citations in the $h$-core to calculate the final value. As a mathematical formula the r-index is defined as

$$
r \text {-index }=\sqrt{\sum_{i=1}^{h} C i t(i)}
$$

where $h$ is the value of the $h$-index, and Cit $(i)$ is the number of citations received by article $i$ belonging to the $h$-core.

## The e-index

The $e$-index was developed by Zhang (2009) to solve a problem with the $h$-index: excess citations, which are not taken into account for calculating the $h$-index, are completely ignored. The $e$-index is a complement to the $h$-index and it represents the excess citations received by all studies in the $h$-core. As a mathematical formula the $e$-index is defined as:

$$
e \text {-index }=\sqrt{\sum_{i=1}^{h}(C i t(i)-h)}
$$

where $h$ is the value of the $h$-index, and $\operatorname{Cit}(i)$ is the number of citations received by article $i$ belonging to the $h$-core.

The w-index

The w-index was developed by Woeginger (2008) and is defined as follows:
A $w$-index of at least $k$ means that there are $k$ distinct publications that have at least
$1,2,3,4, \ldots, k$ citations, respectively.
Like the $h$-index, the w-index tends to cluster many journals into the same index value. However, its range could be up to twice the range of the $h$-index. Therefore, the $w$-index should lead to a somewhat finer ranking than the $h$-index.

## The rational h-index

Ruane and Tol (2008) proposed the rational h-index, which is an extension of the original $h$-index. This index takes into account the number of citations needed to increase the $h$-index by one unit. It measures the distance to the next value of the $h$-index. Mathematically, this is

$$
\text { rational } h \text {-index }=(h+1)-\frac{C i t(h+1)}{2 h+1},
$$

where $h$ is the value of the $h$-index, and $\operatorname{Cit}(h+1)$ is the number of citations received by article $h+1$.

The impact factor (IF) for a given journal in the year $y$ is the average number of times the articles that it published in the past 2 years were cited in year $y$. The impact factor is calculated by dividing the number of citations during year $y$ by the total number of articles published by the journal in the previous 2 years. Mathematically, this is

$$
I F(y)=\frac{\text { Cites in year }(y) \text { to items published in years }(y-1) \text { and }(y-2)}{\text { Number of items published in years }(y-1) \text { and }(y-2)}
$$

## The immediacy-index

The immediacy-index is the average number of times an article is cited in the year that it is published. This index indicates how quickly articles in a journal are cited. It is calculated by dividing the number of citations to articles published in a given year by the number of articles published in that year. Mathematically, this is

$$
\text { immediacy }- \text { index }(y)=\frac{\text { Cites in year }(y) \text { to items published in year }(y)}{\text { Number of items published in year }(y)}
$$

## Results

Data collection
In this study, we have selected the Computer Science and Artificial Intelligence field as a case study. We have used Thomson Reuters' Web of Knowledge platform to download publication and citation data. In the following, we illustrate the different phases of dataset construction.

In the first step, we collected journal data from the JCR's Computer Science and Artificial Intelligence category. There are 94 journals in this category of the 2008 JCR Science Edition. In view of the objective of this study, we only took into account the 70 journals (Table 1) that published papers from January 1, 2000 to December 31, 2009. We stored this information in a database designed for this purpose.

The next step was to obtain the publication list and citation data for these journals. This information was downloaded from the Web of Science (WoS), hosted by the Web of Knowledge platform.

Finally, the last step was to use all the information stored in our database to calculate some scientific impact indices associated with the selected journals. These indices are: documents, citations, the $h$-index, the g-index, the hg-index, the $a$-index, the $m$-index, the $q^{2}$-index, the $r$-index, the $e$-index, the $w$-index, the rational $h$-index, the impact factor and the immediacy-index (all described in "Bibliometric indices" section). These index values have been calculated yearly for each of the 70 journals in the ten-year period from 2000 to 2009.

To illustrate some of the calculated index values, Table 2 lists some of the journals ranked top according to six selected indices: documents, citations, the $h$-index, the $g$-index, the impact factor and the immediacy-index. Table 2 shows some rankings obtained using data for a one-year publication window, specifically for 2009. These rankings reveal that some journals are always positioned near to the top. Taking IEEE Transactions on Pattern

Table 1 List of journals in JCR Computer Science and Artificial Intelligence category that have papers every year throughout the 2000-2009 period

## Journals

Adaptive Behavior
AI Communications
AI Edam-Artificial Intelligence for Engineering Design Analysis and Manufacturing AI Magazine
Annals of Mathematics and Artificial Intelligence
Applied Artificial Intelligence
Applied Intelligence
Artificial Intelligence
Artificial Intelligence in Medicine
Artificial Intelligence Review
Artificial Life
Autonomous Agents and Multi-Agent Systems
Autonomous Robots
Chemometrics and Intelligent Laboratory Systems
Computational Intelligence
Computer Speech and Language
Computer Vision and Image Understanding
Connection Science
Data and Knowledge Engineering
Data Mining and Knowledge Discovery
Decision Support Systems
Engineering Applications of Artificial Intelligence
Engineering Intelligent Systems for Electrical Engineering and Communications
Expert Systems
Expert Systems with Applications
IEEE Transactions on Evolutionary Computation
IEEE Transactions on Fuzzy Systems
IEEE Transactions on Image Processing
IEEE Transactions on Knowledge and Data Engineering
IEEE Transactions on Neural Networks
IEEE Transactions on Pattern Analysis and Machine Intelligence
IEEE Transactions on Systems, Man and Cybernetics Part B-Cybernetics
IEEE Transactions on Systems, Man and Cybernetics Part C-Applications and Reviews
Image and Vision Computing
International Journal of Approximate Reasoning
International Journal of Computer Vision
International Journal of Intelligent Systems
International Journal of Patter Recognition and Artificial Intelligence
International Journal of Software Engineering and Knowledge Engineering
International Journal of Uncertainty Fuzziness and Knowledge-Based Systems
Integrated Computer-Aided Engineering
Intelligent Automation and Soft Computing
Journal of Artificial Intelligent Research

Table 1 continued
Journals

Journal of Automated Reasoning<br>Journal of Chemometrics<br>Journal of Computer and Systems Sciences International<br>Journal of Experimental and Theoretical Artificial Intelligence<br>Journal of Heuristics<br>Journal of Intelligent and Fuzzy Systems<br>Journal of Intelligent Information Systems<br>Journal of Intelligent Manufacturing<br>Journal of Intelligent and Robotic Systems<br>Journal of Mathematical Imaging and Vision<br>Knowledge Engineering Review<br>Knowledge-Based Systems<br>Machine Learning<br>Machine Vision and Applications<br>Mechatronics<br>Medical Image Analysis<br>Minds and Machines<br>Network-Computation in Neural Systems<br>Neural Computation<br>Neural Computing and Applications<br>Neural Networks<br>Neural Processing Letters<br>Neurocomputing<br>Pattern Analysis and Applications<br>Pattern Recognition<br>Pattern Recognition Letters<br>Robotics and Autonomous Systems

Analysis and Machine Intelligence as an example, we find that this journal published 188 articles in 2009 , which received 54 citations in the same year. Furthermore, its $h$-index, value and g-index' value were 3 and 4, respectively. Finally, it had an impact factor value of 5.960 and an immediacy-index value of 0.669 . Remember that all these values were obtained using a 2009 -year publication window.

We also list some index values calculated in other years. Table 3 illustrates the range of index values in each analyzed year. Taking the documents value for the year 2000 as an example, we find that the minimum number of documents published by a specific journal in the year 2000 was 10 and the maximum was 219 .

Analyzing Table 3, we find that index values are higher for recent than older years. Specifically, most of the highest values for each index are obtained between 2007 and 2009. Although we note that the values of all indices tend to increase, there is no index whose value increases year by year.

We notice that the highest values differ greatly depending on the selected index and year. On the one hand, the values of indices like documents or citations have undergone a more significant increase than other indices over the analyzed years. The number of

Table 2 Top five positions of the journal rankings using a 2009-year publication and citation window according to six bibliometric indices

| Position | Journal | documents |
| :---: | :---: | :---: |
| 1 | Expert Systems with Applications | 1399 |
| 2 | Neurocomputing | 352 |
| 3 | Pattern Recognition | 312 |
| 4 | IEEE Transactions on Image Processing | 217 |
| 5 | IEEE Transactions on Pattern Analysis and Machine Intelligence | 188 |
| Position | Journal | citations |
| 1 | Expert Systems with Applications | 402 |
| 2 | Neurocomputing | 68 |
| 3 | IEEE Transactions on Pattern Analysis and Machine Intelligence | 54 |
| 4 | Neural Networks | 49 |
| 5 | Pattern Recognition | 45 |
| Position | Journal | $h$-index |
| 1 | Expert Systems with Applications | 5 |
| 2 | Neurocomputing | 4 |
| 3 | Neural Networks | 4 |
| 4 | IEEE Transactions on Pattern Analysis and Machine Intelligence | 3 |
| 5 | Image and Vision Computing | 2 |
| Position | Journal | g-index |
| 1 | Expert Systems with Applications | 6 |
| 2 | Neurocomputing | 5 |
| 3 | Neural Networks | 4 |
| 4 | IEEE Transactions on Pattern Analysis and Machine Intelligence | 4 |
| 5 | IEEE Transactions on Knowledge and Data Engineering | 3 |
| Position | Journal | impact factor |
| 1 | IEEE Transactions on Pattern Analysis and Machine Intelligence | 5.960 |
| 2 | International Journal of Computer Vision | 5.358 |
| 3 | IEEE Transactions on Evolutionary Computation | 3.736 |
| 4 | IEEE Transactions on Neural Networks | 3.726 |
| 5 | IEEE Transactions on Fuzzy Systems | 3.624 |
| Position | Journal | immediacy-index |
| 1 | Computational Intelligence | 1.091 |
| 2 | IEEE Transactions on Pattern Analysis and Machine Intelligence | 0.669 |
| 3 | Artificial Intelligence | 0.667 |
| 4 | International Journal of Computer Vision | 0.659 |
| 5 | Journal of Automated Reasoning | 0.600 |

Table 3 Range of index values in each analyzed year. Numbers in boldface represent the maximum value for each index in the 2000-2009 period

| Index | 2000 |  | 2001 |  | 2002 |  | 2003 |  | 2004 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Min | Max | Min | Max | Min | Max | Min | Max | Min | Max |
| documents | 10 | 219 | 12 | 257 | 8 | 308 | 15 | 297 | 12 | 295 |
| citations | 0 | 45 | 0 | 47 | 0 | 39 | 0 | 78 | 0 | 48 |
| $h$-index | 0 | 3 | 0 | 2 | 0 | 3 | 0 | 4 | 0 | 3 |
| g-index | 0 | 3 | 0 | 4 | 0 | 3 | 0 | 4 | 0 | 4 |
| hg-index | 0.0 | 3.0 | 0.0 | 2.8 | 0.0 | 3.0 | 0.0 | 4.0 | 0.0 | 3.5 |
| a-index | 0.0 | 5.0 | 0.0 | 6.5 | 0.0 | 4.0 | 0.0 | 5.0 | 0.0 | 6.0 |
| $m$-index | 0 | 3 | 0 | 4 | 0 | 3 | 0 | 4 | 0 | 3 |
| $q^{2}$-index | 0.0 | 3.0 | 0.0 | 2.8 | 0.0 | 3.0 | 0.0 | 4.0 | 0.0 | 3.0 |
| $r$-index | 0.0 | 3.5 | 0.0 | 3.6 | 0.0 | 3.5 | 0.0 | 4.5 | 0.0 | 3.9 |
| $e$-index | 0.0 | 2.0 | 0.0 | 3.0 | 0.0 | 1.7 | 0.0 | 2.4 | 0.0 | 2.6 |
| w-index | 0 | 4 | 0 | 4 | 0 | 5 | 0 | 6 | 0 | 5 |
| rational h-index | 0.0 | 3.6 | 0.0 | 2.8 | 0.0 | 3.7 | 0.0 | 4.8 | 0.0 | 3.6 |
| impact factor | 0.0 | 2.8 | 0.0 | 2.7 | 0.0 | 2.7 | 0.0 | 2.9 | 0.0 | 4.4 |
| immediacy-index | 0.0 | 0.7 | 0.0 | 1.0 | 0.0 | 0.7 | 0.0 | 0.9 | 0.0 | 0.7 |
| Index | 2005 |  | 2006 |  | 2007 |  | 2008 |  | 2009 |  |
|  | Min | Max | Min | Max | Min | Max | Min | Max | Min | Max |
| documents | 15 | 255 | 18 | 335 | 16 | 342 | 3 | 523 | 3 | 1399 |
| citations | 0 | 86 | 0 | 81 | 0 | 73 | 0 | 164 | 0 | 402 |
| $h$-index | 0 | 4 | 0 | 4 | 0 | 3 | 0 | 4 | 0 | 5 |
| $g$-index | 0 | 5 | 0 | 4 | 0 | 5 | 0 | 5 | 0 | 6 |
| hg-index | 0.0 | 4.5 | 0.0 | 4.0 | 0.0 | 3.9 | 0.0 | 4.5 | 0.0 | 5.0 |
| a-index | 0.0 | 6.0 | 0.0 | 5.0 | 0.0 | 6.7 | 0.0 | 6.5 | 0.0 | 6.6 |
| $m$-index | 0 | $5 \mathrm{ePara>}$ | 0 | 4 | 0 | 4 | 0 | 5 | 0 | 6 |
| $q^{2}$-index | 0.0 | 4.5 | 0.0 | 4.0 | 0.0 | 3.5 | 0.0 | 4.5 | 0.0 | 5.5 |
| $r$-index | 0.0 | 4.9 | 0.0 | 4.2 | 0.0 | 4.5 | 0.0 | 4.8 | 0.0 | 5.7 |
| e-index | 0.0 | 2.8 | 0.0 | 2.4 | 0.0 | 3.3 | 0.0 | 3.0 | 0.0 | 2.8 |
| w-index | 0 | 7 | 0 | 5 | 0 | 6 | 0 | 7 | 0 | 8 |
| rational h-index | 0.0 | 4.9 | 0.0 | 4.4 | 0.0 | 3.9 | 0.0 | 4.9 | 0.0 | 5.8 |
| impact factor | 0.1 | 4.3 | 0.0 | 3.8 | 0.1 | 6.1 | 0.0 | 3.6 | 0.0 | 6.0 |
| immediacy-index | 0.0 | 1.0 | 0.0 | 1.0 | 0.0 | 1.2 | 0.0 | 0.8 | 0.0 | 1.1 |

documents and citations incremented sharply in the time period. In fact, they are 6 and 9 times greater, respectively, in the last year than in the first year. On the other hand, the values of other indices (h-index, g-index, m-index, w-index, rational h-index, impact factor) have not increased as significantly as documents and citations. In the most recent years we find them to be approximately 2 times greater than in the early years. For example, the g-index has a value of 3 in the year 2000 and a value of 6 in 2009. In the same way, the impact factor has a value of 2.8 in the year 2000 and a value of 6.0 in 2009. Finally, the values of the other indices (hg-index, a-index, $q^{2}$-index, r-index, e-index, immediacy-index) have also increased within the time period, but to a lesser extent than the other indices. For
example, the $a$-index has a value of 5.0 in the year 2000 and a value of 6.6 in 2009 . Similarly, the immediacy-index has a value of 0.7 in the year 2000 and a value of 1.1 in 2009.

## Yearly models

Ten yearly Bayesian networks models were learned to analyze the relationships among indices within the same 1-year publication window. For this reason, each yearly model is associated with one of the ten analyzed years and with one of the ten datasets. Each dataset contains 70 cases (journals), each one with its 14 index values.

Before running the K2 algorithm to learn a Bayesian network from each dataset, we had to make some decisions about three of K2's requirements. Since K2 needs the variables to be ordered, the first decision was to specify an order. Taking into account the index definitions, we placed indices that could be parents of the other indices first. The established order was: documents, citations, h-index, $g$-index, hg-index, $a$-index, m-index, $q^{2}$ index, r-index, e-index, w-index, rational h-index, impact factor and immediacy-index. We obtained high values of marginal likelihood using this order. The second requirement was to assign a value to the maximum number of parents. This was set at two due to the dataset characteristics. There was a third requirement: index values had to be discretized into intervals. Due to the number of dataset cases, we discretized the values into three intervals with equal frequency. In this way, the index values were assigned to one of the three possible values (low, medium and high).

In the following, we represent the structures of the learned yearly Bayesian networks and then explain some probabilistic conditional (in)dependencies between the indices that they discovered.

## Bayesian networks structure

In this section we present the structure of our yearly Bayesian networks, see Fig. 2. We observe that there are a lot of coincident arcs in our Bayesian networks as shown in Table 4. Taking the value of the first-row and second-column as an example, the value displayed is 14. This value indicates that the Bayesian network for the year 2000 and the Bayesian network for the year 2001 have 14 identical arcs. In other words, the Bayesian network of the year 2000 has 19 arcs, and 14 of these arcs are also represented in the Bayesian network of the year 2001. Examining Table 4, we find that the relationships between indices are very similar in each year of the analyzed period. They are described in further detail in the following section.

Analyzing the networks in Fig. 2, we notice that there are some specific arcs that are represented in most of the networks. For example, the arcs $h$-index $\rightarrow g$-index, documents $\rightarrow$ citations, citations $\rightarrow$ h-index, a-index $\rightarrow e$-index, among others, always appear in our 10 Bayesian networks. The number of times that an arc is shown in our Bayesian networks is reported in Table 5. Taking the value of the citations $\rightarrow m$-index arc as an example, the value displayed is 9 . This value means that the relationship between citations and m-index is present in 9 out of our 10 yearly Bayesian networks.

With the intention of representing the main relationships between indices in a single year-independent Bayesian network, we built an aggregated Bayesian network using only those arcs that had appeared at least 3 times. After applying the above filter, we obtained the aggregated Bayesian network shown in Fig. 3. The values above the arcs represent the number of times that the arc appeared in our 10 yearly Bayesian networks.


Fig. 2 Bayesian network structures learned for each analyzed year

Table 4 Number of coincident arcs in the 10 different networks

|  | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $2000(19 \mathrm{arcs})$ | - | 14 | 12 | 13 | 12 | 12 | 13 | 11 | 13 | 12 |
| $2001(18 \mathrm{arcs})$ | - | - | 10 | 11 | 12 | 11 | 11 | 9 | 11 | 10 |
| $2002(17 \mathrm{arcs})$ | - | - | - | 14 | 10 | 11 | 11 | 9 | 9 | 10 |
| $2003(18 \mathrm{arcs})$ | - | - | - | - | 13 | 14 | 14 | 11 | 11 | 14 |
| $2004(16 \mathrm{arcs})$ | - | - | - | - | - | 14 | 14 | 10 | 9 | 11 |
| $2005(19 \mathrm{arcs})$ | - | - | - | - | - | - | 13 | 11 | 11 | 13 |
| $2006(18 \mathrm{arcs})$ | - | - | - | - | - | - | - | 12 | 11 | 14 |
| $2007(17 \mathrm{arcs})$ | - | - | - | - | - | - | - | - | 12 | 13 |
| $2008(17 \mathrm{arcs})$ | - | - | - | - | - | - | - | - | - | 14 |
| $2009(18 \mathrm{arcs})$ | - | - | - | - | - | - | - | - | - | - |

## Dependencies among indices in the aggregated Bayesian network

Examining the index definitions, we note that some of them can be defined according to the values of other indices. For example, on the one hand, the hg-index can be expressed in terms of $h$ - and $g$-index's values ( $h g$-index $=\sqrt{h \cdot g}$ ) and, on the other hand, the $q^{2}$-index can be defined according to $h$ - and $m$-index's values $\left(q^{2}\right.$-index $\left.=\sqrt{h \cdot m}\right)$.

Although the deterministic aspect between index definitions is reduced after discretizing the index values into three intervals (low, medium and high), our aggregated Bayesian network discovers such dependencies between indices.

Looking at Fig. 3, we find that the dependencies that can be defined according to the values of other indices are represented in the aggregated Bayesian network. For example, the hg-index (see "The hg-index" section) is represented in the network since the $h$-index and the $g$-index are parents of the $h$-index in our network. Similarly, the $q^{2}$-index as a function of the $h$-index and the m-index (see "The $q^{2}$-index" section) is also represented in the network.

Some dependencies, like documents $\rightarrow$ citations and $h$-index $\rightarrow g$-index, among others, are not derived from the index definition, but were expected because many works showed their correlations (Costas and Bordons 2008; Schreiber 2008). Other dependencies, e.g., citations $\rightarrow$-index and citations $\rightarrow$ impact factor are also represented in the aggregated Bayesian network. We noticed that although the $h$-index and the impact factor cannot be defined in terms of citations values only, they do exhibit a high value correlation coefficient (Bornmann et al. 2008b).

Other dependencies, e.g., the arc between $a$-index and $e$-index, is an example of a dependency that was not initially expected. Remember that the $a$-index represents the average number of citations received by the articles included in the $h$-core, whereas the $e$-index represents the excess citations received by the articles in the $h$-core. Thus, both refer to citations of articles in the $h$-core. More examples of such dependencies are: m-index $\rightarrow$ rational $h$-index, a-index $\rightarrow$-index, and $w$-index $\rightarrow$ rational $h$-index among others.

## Conditional independencies among indices in the aggregated Bayesian network

Bayesian networks are powerful tools not only for capturing dependencies but also for encoding conditional independencies among our indices. In the following, we explain some
Table 5 Number of times that arcs appear in our 10 yearly Bayesian networks

|  | documents | citations | h-index | $g$-index | hg-index | -index | $m$-index | $q^{2}$-index | $r$-index | $e$-index | $w$-index | rational <br> $h$-index | impact <br> factor | immediacy- <br> index |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| documents | - | 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| citations | - | - | 10 | 2 | 0 | 1 | 9 | 0 | 0 | 0 | 1 | 4 | 3 | 2 |
| h-index | - | - | - | 10 | 3 | 0 | 7 | 8 | 4 | 1 | 0 | 1 | 1 | 0 |
| g-index | - | - | - | - | 7 | 9 | 0 | 0 | 5 | 7 | 1 | 0 | 2 | 2 |
| hg-index | - | - | - | - | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| a-index | - | - | - | - | - | - | 1 | 0 | 3 | 10 | 10 | 0 | 1 | 0 |
| m-index | - | - | - | - | - | - | - | 10 | 0 | 0 | 1 | 3 | 1 | 0 |
| $q^{2}$-index | - | - | - | - | - | - | - | - | 0 | 0 | 0 | 2 | 1 | 0 |
| r-index | - | - | - | - | - | - | - | - | - | 2 | 0 | 0 | 1 | 0 |
| e-index | - | - | - | - | - | - | - | - | - | - | 0 | 0 | 0 | 0 |
| w-index | - | - | - | - | - | - | - | - | - | - | - | 5 | 1 | 0 |
| rational h-index | - | - | - | - | - | - | - | - | - | - | - | - | 2 | 2 |
| impact factor | - | - | - | - | - | - | - | - | - | - | - | - | - | 10 |
| immediacy-index | - | - | - | - | - | - | - | - | - | - | - | - | - | - |



Fig. 3 Aggregated Bayesian network structure
properties for discovering conditional independencies among the analyzed indices. We choose Markov properties as the criteria for this purpose.

The local Markov property states that any node $X$ in any Bayesian network is conditionally independent of its non-descendants given its parents, that is, $I(X$, non-descendants $(X)$ $\Pi(X))$. The global Markov property is also used for discovering conditional independencies among the analyzed indices. This property states that any node $X$ is conditionally independent of any other node given its Markov blanket (MB). The MB of a node includes its parents, its children, and its children's parents, that is, $I(X$, non $-M B(X) \mid M B(X))$.

Table 6 illustrates such relationships. Although this table shows a specific list of relationships between indices, new relationships can be derived using some conditional independencies properties (Castillo et al. 1997):

- Symmetry: $I(X, Y \mid Z) \Leftrightarrow I(Y, X \mid Z)$
- Decomposition: $I(X,(Y \cup W) \mid Z) \Rightarrow I(X, Y \mid Z)$ and $I(X, W \mid Z)$
- Strong joint: $I(X, Y \mid Z) \Rightarrow I(X, Y \mid(Z \cup W))$

Taking the $q^{2}$-index as an example, we find in Table 6 that, given the $h$-index and the $m$-index together, the $q^{2}$-index is independent of most of the indices (documents, citations, the hg-index, the $a$-index, the r-index, the $e$-index, the $w$-index, the rational h-index, the impact factor and the immediacy-index). On the other hand, we observe that the g-index is independent of the $q^{2}$-index given the $h$-index. Taking into account the above independency relationships and the conditional independency properties, we state that given the $h$-index and the m-index together, the $q^{2}$-index is independent of any of the other indices.

Table 6 Conditional independencies among indices, derived using Markov properties in the aggregated Bayesian network

| Index | Is conditionally independent of | Given |
| :---: | :---: | :---: |
| documents | h-index, g-index, hg-index, a-index, $m$-index, $q^{2}$-index, $r$-index, $e$-index, w-index, rational h-index, impact factor, immediacy-index | citations |
| citations | hg-index, a-index, $r$-index, $e$-index, immediacy-index | documents, $h$-index, m-index, w-index, rational h-index, impact factor |
| $h$-index | immediacy-index | citations |
| $h$-index | rational h-index | citations, g-index, hg-index, $a$-index, m-index, $q^{2}$-index, r-index |
| g-index | documents, citations, <br> $m$-index, $q^{2}$-index, impact factor, immediacy-index | $h$-index |
| g-index | rational h-index | $h$-index, hg-index, <br> e-index, a-index, r-index |
| hg-index | documents, citations, $m$-index, $q^{2}$-index, $r$-index, $e$-index, $w$-index, rational h-index, impact factor, immediacy-index | $h$-index, g-index |
| a-index | documents, citations, $h$-index, hg-index, m-index, $q^{2}$-index, impact factor, immediacy-index | g-index |
| a-index | rational h-index | $h$-index, g-index, <br> r-index, e-index, w-index |
| $m$-index | hg-index, a-index, r-index, e-index, w-index, immediacy-index | citations, h-index |
| $q^{2}$-index | documents, citations, hg-index, a-index, r-index, $e$-index, w-index, rational h-index, impact factor, immediacy-index | $h$-index, m-index |
| $r$-index | documents, citations, $m$-index, $q^{2}$-index, rational h-index, impact factor, immediacy-index | h-index, g-index, a-index |
| e-index | documents, citations, $h$-index, hg-index, m-index, $q^{2}$-index, $r$-index, rational h-index, impact factor, immediacy-index | g-index, a-index |
| w-index | documents, citations, $h$-index, g-index, hg-index, m-index, $q^{2}$-index, $r$-index, $e$-index, impact factor, immediacy-index | a-index |
| rational h-index | $h$-index, g-index, hg-index, a-index, $q^{2}$-index, r-index, e-index, immediacy-index | citations, m-index, w-index |
| impact factor | $h$-index, g-index, <br> hg-index, a-index, m-index, <br> $q^{2}$-index, r-index, e-index, <br> w-index, rational h-index | citations |

Table 6 continued

| Index | Is conditionally independent of | Given |
| :--- | :--- | :--- |
| immediacy-index | documents, citations, $h$-index, <br> g-index, $h g$-index, $a$-index, <br> m-index, $q^{2}$-index, r-index, <br> $e$-index, w-index, rational $h$-index | impact factor |
|  |  |  |

This means that when we know the h-index and the m-index values, knowledge of the $e$-index, for example, provides no information on the occurrence of the $q^{2}$-index. Similarly, we note that given the $h$-index and the g-index together, the $h g$-index is independent of any of the other indices.

The above conditional independencies between indices can be checked by analyzing the index definitions. Our aggregated Bayesian network is able to encode the conditional independencies that are derived from the index definition, but also discovers other new conditional independencies that are not strictly derived from definitions.

On the one hand, we expected the $m$-index, which is the median number of citations received by papers in the $h$-core, to be conditionally independent of some variables given citations and $h$-index. Table 6 shows that given citations and $h$-index, the m-index is conditionally independent of hg-index, a-index, r-index, e-index, $w$-index and immediacyindex.

On the other hand, some other conditional independencies were not so obvious. For example, the immediacy-index, which is defined by means of documents' values and citations' values, is independent of documents and citations given the impact factor. The presence of impact factors values has a significant influence on immediacy-index values. This means that when we know the impact factor, knowledge of documents and citations does not provide any information on the occurrence of the immediacy-index.

Remember that the conditional independencies between indices encoded in our Bayesian networks do not represent a causality relationship, but refer to a probability relationship between indices.

## Global model

The global model's objective is to analyze the relationships between indices within a oneyear target window. It has the same as the yearly models, but, unlike them, this global model is built using a different dataset.

We select and merge the previous datasets into a single dataset to build the dataset for the global model. In this way, the global model dataset contains 700 cases. Each case in the dataset was referred to index values calculated within a 1 -year target window, and, consequently, they all cases can be easily merged into a single dataset.

We have also used the K2 scoring metric to learn our Bayesian network. Although we have established the same variables order, we have slightly modified some decisions about K2's requirements. As there are more cases we can increase the maximum number of parents for any node up to 3 and the number of intervals for the index discrete domain to 4 (low, medium-low, medium-high and high).


Fig. 4 Global Bayesian network structure

## Bayesian network structure

Figure 4 shows our global Bayesian network structure.
Comparing the arcs in Fig. 3 (aggregated Bayesian network structure) and Fig. 4 (global Bayesian network structure), we find that the network structures are similar. There are many arcs that appear in both networks, although this new model includes some specific arcs not represented before: citations $\rightarrow$ a-index, $h$-index $\rightarrow$ a-index, h-index $\rightarrow$ $e$-index, $h$-index $\rightarrow$ rational h-index, among others. For this reason, we believe that the global model more accurately represents the index definition than the aggregated model. We explain these new dependencies in the following.

We examined some centrality measures in order to analyze some of the index characteristics in our global Bayesian network. These values are shown in Table 7.

Degree centrality is defined as the number of arcs incident upon an index. Degree is often interpreted in terms of the opportunity for influencing any other index. We define two separate measures of degree centrality (indegree and outdegree). A node's indegree is the number of arcs directed to the node, and outdegree is the number of arcs that the node directs to others. Therefore, indegree is the number of parents, whereas outdegree is the number of children.

Examining Table 7, we note that the $h$-index has a lot of influence on other indices. It has the highest degree centrality value $(1+7=8)$. On the other hand, indices like documents or hg-index, which have a degree centrality of 1 , do not influence the other indices so much.

Finally, Table 8 displays the range of index values and the values assigned to each interval after the discretization to illustrate the global model dataset. We observe that the interval width is not the same for the four categories since the dataset has been discretized

Table 7 Measures of centrality (indegree and outdegree) for the 14 analyzed indices in the global Bayesian network

| Index | Indegree | Outdegree |
| :--- | :--- | :--- |
| documents | 0 | 1 |
| citations | 1 | 4 |
| h-index | 1 | 7 |
| g-index | 1 | 3 |
| hg-index | 1 | 0 |
| a-index | 3 | 2 |
| m-index | 2 | 1 |
| $q^{2}$-index | 2 | 3 |
| r-index | 2 | 2 |
| e-index | 3 | 1 |
| w-index | 3 | 0 |
| rational h-index | 3 | 0 |
| impact factor | 1 | 1 |
| immediacy-index | 2 | 0 |

Table 8 Range of index values related to each interval of the global model dataset

| Indices | Values' range | low | medium-low | medium-high | high |
| :--- | :--- | :--- | :--- | :--- | :--- |
| documents | $[3,1399]$ | $[3,29)$ | $[29,47)$ | $[47,79)$ | $[79,1399]$ |
| citations | $[0,402]$ | $[0,1)$ | $[1,3)$ | $[3,12)$ | $[12,402]$ |
| h-index | $[0,5]$ | $[0,1)$ | $[1,2)$ | $[2,3)$ | $[3,5]$ |
| g-index | $[0,6]$ | $[0,1)$ | $[1,2)$ | $[2,3)$ | $[3,6]$ |
| hg-index | $[0.0,5.5]$ | $[0.0,1.0)$ | $[1.0,1.4)$ | $[1.4,2.4)$ | $[2.4,5.5]$ |
| a-index | $[0.0,6.7]$ | $[0.0,1.0)$ | $[1.0,2.0)$ | $[2.0,3.0)$ | $[3.0,6.7]$ |
| $m$-index | $[0,6]$ | $[0.0,1.0)$ | $[1.0,2.0)$ | $[2.0,3.0)$ | $[3.0,6]$ |
| $q^{2}$-index | $[0.0,5.5]$ | $[0.0,1.0)$ | $[1.0,1.4)$ | $[1.4,2.0)$ | $[2.0,5.5]$ |
| $r$-index | $[0.0,5.7]$ | $[0.0,1.0)$ | $[1.0,1.4)$ | $[1.4,2.4)$ | $[2.4,5.7]$ |
| e-index | $[0.0,3.3]$ | $[0.0,1.0)$ | $[1.0,1.4)$ | $[1.4,1.7)$ | $[1.7,3.3]$ |
| w-index | $[0,8]$ | $[0.0,1.0)$ | $[1,2)$ | $[2,3)$ | $[3,8]$ |
| rational $h$-index | $[0.0,5.8]$ | $[0.0,1.0)$ | $[1.0,1.3)$ | $[1.3,2.0)$ | $[2.0,5.8]$ |
| impact factor | $[0.0,6.1]$ | $[0.0,0.4)$ | $[0.4,0.8)$ | $[0.8,1.5)$ | $[1.5,6.1]$ |
| immediacy-index | $[0.0,1.2]$ | $[0.0,0.0]$ | $(0.0,0.1]$ | $(0.1,0.2]$ | $(0.2,1.2]$ |

in intervals of equal frequency. A journal publishes a number of between 3 and 1399 documents per year, and these journals can then be categorized according to the index values. For example, a journal that has published 43 documents per year is placed in the medium-low category.

Dependencies among indices in the global Bayesian network
After examining the dependency relationships between indices shown in Fig. 4, we find that the aggregated model identified some, but not all, the index dependencies.

In the following, we analyze some index dependencies encoded in our global Bayesian network. We find that the citations and the $h$-index are the parent nodes of the $a$-index in the Bayesian network. These nodes are represented in this index's definition (see "The a-index" section). Similarly, the $m$-index definition is also represented in the network, since its parent nodes are the citations and the $h$-index. On the other hand, the $q^{2}$-index, which is dependent on the $h$-index and the $m$-index (see "The $q^{2}$-index" section), has these nodes as parents in the network. The parents of the $r$-index are the $h$-index and the $a$-index. Initially, these indices do not appear in the r-index definition (see "The r-index" section), but, after a transformation of the original definition, we obtain that the r-index is also obviously defined as $\sqrt{a \cdot h}$. Finally, the $e$-index is dependent on the $h$-index and the a-index. Like the $r$-index, the above indices are not part of the $e$-index definition (see "The e-index" section), but, after few transformations of this definition, we conclude that the $e$-index is defined as $\sqrt{ } \overline{a \cdot h-h^{2}}$.
Besides discovering dependencies between indices, which can be checked against the index definitions, the Bayesian network is also able to discover other kinds of probabilistic dependencies. One example is $h$-index $\rightarrow$ rational $h$ - index, not directly derived from, but related to index definitions. In this case, we note that the information about $h$-index influences the probability of the rational h-index. Another example (see Fig. 4) is the probabilistic dependency between r-index and rational h-index. The objective of the $r$-index is to measure the citation intensity in the $h$-core, whereas the rational $h$-index measures the distance to the next value of the $h$-index. We note that these indices measure different things, but they are probabilistically dependent. More examples of such dependencies are: $q^{2}$-index $\rightarrow$ w-index, r-index $\rightarrow w$-index and $q^{2}$-index $\rightarrow$ immediacy-index.

Both the aggregated and the global models have been learned using Elvira software (Elvira-Consortium 2002). One of the most useful features of Elvira is the automatic coloring of arcs, which offers qualitative insight about the conditional probability tables attached to each node. This coloring is based on the sign of influence (Wellman 1990) and the magnitude of influence (Lacave 2003).

In order to understand the dependencies between the indices represented in Fig. 4, we explain some concepts about the influence and color of the arcs. For example, an arc from $X$ to $Y$ is said to have a positive influence if higher values of $X$ lead to higher probabilities of $Y$ taking higher values for any configuration of its other parents. The definition of negative influence and null influence are analogous. When the influence is neither positive nor negative nor null, then it is said to be undefined. Positive, negative, undefined, and null influence is colored in red, blue, purple, and black, respectively.

Taking into account the above concepts, we find that documents has a positive influence on citations. Likewise, citations influences the $h$-index and impact factor positively. High values of citations are associated with high values of $h$-index and impact factor. Furthermore, the $h$-index has a positive influence on the $g$-index, which also has a positive influence on the $h g$-index. On the other hand, the other arcs in Fig. 4 represent an undefined influence between the parent and child nodes. Finally, the thickness of the arc is proportional to the magnitude of the influence. Thus, we find that the relationships that have a grater influence are: citations $\rightarrow$ h-index, $h$-index $\rightarrow g$-index and g-index $\rightarrow$ hg-index.

## Conditional independencies among indices in the global Bayesian network

Table 9 lists conditional independencies between indices, derived using the Markov properties in the global Bayesian network.

Remember that new conditional independencies can be derived using some conditional independency properties, such as, symmetry, factorization or strong joint.

Analyzing Table 9, we note that some conditional independency relationships are represented in both the aggregated Bayesian network and the global Bayesian network. Some of these conditional independencies are:

$$
\begin{array}{cl}
-I(\text { documents, } g \text {-index } \mid \text { citations }) & -I(g \text {-index, immediacy-index } \mid h \text {-index }) \\
-I \text { (impactfactor, } h \text {-index } \mid \text { citations }) & -I\left(q^{2} \text {-index, } e \text {-index } \mid h \text {-index, } m \text {-index }\right)
\end{array}
$$

We also observe that other conditional independencies were not shown before because they had a slightly different Bayesian network structure. We note that the $a$-index, the $w$-index and the rational h-index are indices whose parents have undergone an important change. Taking the a-index independency relationships as an example, we note that citations and $h$-index are new parents of $a$-index in the global model, and this determines new a-index conditional independencies, such as, $\mathbf{I}$ (a-index, documents $\mid$ citations, $h$-index, $g$-index).

Our global Bayesian network finds conditional independencies of which some are justified by index definitions. On the other hand, thought, it discovers other conditional independencies that were not derived from such definitions.

Looking at the $e$-index, it represents the excess citations received by all papers in the $h$-core. According to this definition, it is reasonable to expect that $e$-index and citations would be dependent, but our global model shows that the above indices are independent given $h$-index, g-index and a-index.

Similarly, we analyze the relationships between $a$-index and $m$-index. The $a$-index is the average number of citations received by the articles included in the $h$-core, whereas the $m$-index is the median number of citations received by papers in the $h$-core. Initially, one might expect there to be a dependency relationship between $a$-index and $m$-index, but our global model suggests that the relationship is of conditional independency, given citations and $h$-index.

Finally, a $w$-index of at least $k$ means that there are $k$ distinct publications that have at least $1,2,3,4, \ldots, k$ citations, respectively. According to its definition, the $w$-index should depend on documents and citations, but the global model shows that they are conditionally independent given $q^{2}$-index, r-index and e-index. Other conditional independencies between indices that are not derived from index definitions are:

- I (w-index, $h$-index $\mid q^{2}$-index, $r$-index, $e$-index)
- I (rational h-index, g-index $\mid h$-index, $q^{2}$-index, r-index)
- I (e-index, impact factor $\mid$ h-index, $g$-index, a-index)

Exploiting the global Bayesian network model
We believe that our best model is the global Bayesian network because its structure reflects more index definitions than the aggregated model and discovers new interesting conditional independencies between indices. For this reason, we apply evidence propagation and abduction to our global model.

So far, we have used the graphical component of global Bayesian network to discover conditional (in)dependencies only. In this section, however, we also use the probabilistic component of the global Bayesian network to precisely quantify, the effect of knowing some fixed variables on the occurrence of other variables.

Table 9 Conditional independencies among indices, derived using Markov properties in the global Bayesian network

| Index | Is conditionally independent of | Given |
| :---: | :---: | :---: |
| documents | h-index, g-index, hg-index, a-index, $m$-index, $q^{2}$-index, r-index, $e$-index, w-index, rational h-index, impact factor, immediacy-index | citations |
| citations | hg-index, e-index,w-index, rational h-index, immediacy-index | documents, $h$-index, $a$-index, $m$-index, impact factor |
| $h$-index | immediacy-index | citations, g-index, a-index, $m$-index, $q^{2}$-index, r-index, e-index, rational h-index |
| $g$-index | $w$-index, rational $h$-index | $h$-index, hg-index, a-index, e-index |
| g-index | documents, citations, $m$-index, $q^{2}$-index, impact factor, immediacy-index | $h$-index |
| hg-index | documents, citations, $h$-index, a-index, m-index, $q^{2}$-index, $r$-index, $e$-index, w-index, rational h-index, impact factor, immediacy-index | g-index |
| a-index | $w$-index, rational $h$-index | citations, $h$-index, g-index, r-index, e-index |
| a-index | $q^{2}$-index, immediacy-index | citations, $h$-index, g-index |
| $m$-index | $w$-index, rational $h$-index, immediacy-index | citations, $h$-index, $q^{2}$-index |
| $m$-index | hg-index, a-index, r-index, e-index | citations, $h$-index |
| $q^{2}$-index | documents, citations, hg-index, a-index, r-index, e-index, impact factor | $h$-index, m-index |
| $r$-index | documents, citations, g-index, $h g$-index, m-index, $q^{2}$-index, e-index, impact factor, immediacy-index | h-index, a-index |
| $e$-index | documents, citations, m-index, $q^{2}$-index, rational h-index, impact factor, immediacy-index | h-index, g-index, a-index |
| w-index | documents, citations, $h$-index, g-index, hg-index, a-index, m-index, rational h-index, impact factor, immediacy-index | $q^{2}$-index, r-index, e-index |
| rational h-index | documents, citations, g-index, hg-index, a-index, m-index, e-index, w-index, impact factor, immediacy-index | $h$-index, $q^{2}$-index, r-index |
| impact factor | h-index, g-index, <br> hg-index, a-index, m-index, <br> $q^{2}$-index, r-index, e-index, <br> w-index, rational h-index | citations |
| immediacy-index | documents, citations, $h$-index, g-index, hg-index, a-index, $m$-index, $q^{2}$-index, r-index, $e$-index, w-index, rational h-index | impact factor |

In the context of Bayesian networks, evidence propagation usually refers to computing the posterior probability of each single variable given the available evidence (i.e., some fixed variables), while abduction consists of finding the most probable configuration of a set of variables of interest given the evidence.

As regards evidence propagation, we would like to know the effect on index probabilities of introducing some specific values for other indices as evidence.

Our first inference is to fix the citations value to medium-low. After setting this evidence level, we calculate the posterior probabilities of each index in Fig. 5, top. Note that the mode of all indices is low or medium-low. Taking the impact factor as an example, we observe the following probabilities:

- $\mathrm{P}($ impact factor $=$ low $\mid$ citations $=$ medium-low $)=0.37$
- $\mathrm{P}($ impact factor $=$ medium-low $\mid$ citations $=$ medium-low $)=0.30$
- $\mathrm{P}($ impact factor $=$ medium-high $\mid$ citations $=$ medium-low $)=0.22$
- $\mathbf{P}($ impact factor $=$ high $\mid$ citations $=$ medium-low $)=0.11$

These conditional probabilities are reasonable since fixing citations=medium-low as evidence, impact factor, which depends on citations (positive influence), the value of the mode should be low or medium-low. Analyzing the above conditional probabilities, we observe that low and medium-low are the most probable values, at 0.37 and 0.30 , respectively.

On the other hand, the second inference is to assign a high value to citations, see Fig. 5, bottom. In this case, the value of the mode of most of the indices is high. Now, the different probabilities of the impact factor are:

- $\mathbf{P}($ impact factor $=$ low $\mid$ citations $=$ high $)=0.02$
- $\mathrm{P}($ impact factor $=$ medium-low $\mid$ citations $=$ high $)=0.10$
- $\mathbf{P}($ impact factor $=$ medium-high $\mid$ citations $=$ high $)=0.31$
- $\mathbf{P}($ impact factor $=$ high $\mid$ citations $=$ high $)=0.56$

These conditional probabilities are also reasonable.
The probabilities of the above inferences answer a question raised in the introduction, namely, What would happen to a specific journal impact factor if the papers, that it published received more citations?. The answer lies in the total distribution of the different impact factor values. Similarly, setting citations=low, we answer the question, What would happen to a specific journal impact factor if the papers that it published received fewer citations?

Analyzing Fig. 5, we notice that our Bayesian network model rules out many situations with the above index values. These situations are indicated by a probability equal to zero, like, e.g.,

- $\quad P($ h-index $=$ high $\mid$ citations $=$ medium-low $)=0.00$
- $\quad P($ g-index $=$ high $\mid$ citations $=$ medium-low $)=0.00$
- $\quad P(a$-index $=$ low $\mid$ citations $=$ high $)=0.00$
- $\quad P($ m-index $=$ low $\mid$ citations $=$ high $)=0.00$.

To get the most likely plausible explanation $P$ (configuration $\mid$ evidence), we should search the configuration of values of the non-observed indices (called explanation set) that maximizes the above probability. This is possible using abductive inference (Pearl 1988).

Table 10 shows three examples of abductive inference. We set three different evidence levels at $h$-index=medium-low (like, e.g., International Journal of Pattern Recognition and


Fig. 5 Index probabilities after setting the citations value at medium-low (top) and high (bottom)

Table 10 Most likely configurations of indices for a given evidence level

| Explanation set <br> Index $\left(X_{i}\right)$ | Evidence 1 <br> h-index <br> medium-low | Evidence 2 <br> impact factor= <br> medium-high | Evidence 3 <br> immediacy- <br> index=high |
| :--- | :--- | :--- | :--- |
| documents | low | medium-high | high |
| citations | medium-low | medium-high | high |
| h-index | - | medium-low | medium-high |
| g-index | medium-low | medium-low | medium-high |
| hg-index | medium-low | medium-low | medium-high |
| a-index | medium-low | medium-low | medium-high |
| m-index | low | medium-low | medium-high |
| $q^{2}$-index | low | medium-low | high |
| r-index | medium-low | medium-low | medium-high |
| e-index | low | low | low |
| w-index | medium-low | medium-low | medium-high |
| rational h-index | medium-low | medium-high | high |
| impact factor | low | - | high |
| immediacy-index | low | medium-high | - |
| $P$ (Explanation set 1 evidence) | 0.000245 | 0.000147 | 0.000839 |

Artificial Intelligence), impact factor=medium-high (like, e.g., International Journal of Intelligent Systems) and immediacy-index=high (like, e.g., Machine Learning) in Table 10, columns 2,3 , and 4 , respectively.

Taking the third inference as an example, we show that the most probable configuration of index values when immediacy-index=high is documents=high, citations=high, h-index= medium-high, g-index=medium-high, hg-index=medium-high, a-index=medium-high, m-index=medium-high, $q^{2}$-index=high, r-index=medium-high, e-index=low, w-index $=$ medium-high, rational $h$-index $=$ high and impact factor $=$ high.

The above configuration of index values answers the question, What kind of actions should journal editorial boards take to get a higher immediacy-index value in a specific year?. The answer is publish a lot of documents ( $\geq 79$ ), receive a lot of citations ( $\geq 12$ ), get high $q^{2}$-index $(\geq 2.0)$, rational $h$-index $(\geq 2.0)$ and impact factor $(\geq 1.5)$ index values. Moreover, journal editorial boards should aspire to the following index values: $h$-index $=$ $[2,3), \quad$ g-index $=[2,3)$, hg-index $=[1.4,2.4), \quad$-index $=[2.0,3.0), \quad$ m-index $=[2.0,3.0)$, $r$-index $=[1.4,2.4), e$-index $=[0.0,3.3)$ and $w$-index $=[2,3)$.

Let us examine the joint probability ( $P=0.000839$ ) of these index values in the last row of Table 10 . Although it seems very low, the number of different configurations of index values is $4^{13}$. Therefore, the joint probability obtained via abduction is considerably greater than would be expected purely by chance $\left(\frac{1}{4^{13}}=1.5 \cdot 10^{-8}\right)$.

## Conclusions

The advantages of Bayesian networks justify their choice as a tool for building graphical models and representing relationships among bibliometric indices. On the one hand, Bayesian networks are a graph-based model of joint multivariate probability distributions
that capture properties of conditional independency among variables. On the other hand, the statistical foundations and computational algorithms for learning Bayesian networks from observations, are well understood and have been used successfully in many applications. These models have been widely used to solve different kinds of problems (classification, regression, simulation...), and they can perform different types of reasoning (predictive, diagnostic, abductive...)

We believe that Bayesian networks are a promising tool for modeling and analyzing the dependencies and independencies among bibliometric indices. This approach places no restriction on the number of indices analyzed, and many other indices besides the 14 wellknown indices covered in this study could be added.

In this study, several graphical models (yearly and global models) were developed to discover the relationships among bibliometric indices. The aim of both models is to represent relationships between index values using a within 1-year publication and citation window.

Analyzing our best Bayesian network (global model), we notice that its structure matches many index definitions. In addition, this model learns new knowledge derived from index definitions and discovers new interesting conditional (in)dependencies between analyzed indices. These conditional (in)dependency relationships have been analyzed using Markov properties.

Using our models, editorial boards of journals could find the answer to questions related to their journal citation indices. Evidence propagation and abduction inference in Bayesian networks are very useful for answering bibliometric questions.

In the future, our target will be to build new models that incorporate other journal citation indices like eigenfactor, article influence and Scimago's journal rank index, among others. These models could also be induced using different Bayesian network learning algorithms. The way index values are handled influences the results. They could be modeled as continuous variables instead of discretizing the values. Finally, the number of citations could vary depending on the consulted source (Google Scholar, Scopus, WoS, etc.), see Bar-Ilan (2008), which is a point to be taken into account.

Acknowledgments Research supported by Spanish Ministry of Science and Innovation, Project TIN2008-04528-E. The study has also been partially supported by Spanish Ministry of Science and Innovation, grants TIN2010-20900-C04-04, Cajal Blue Brain and Consolider Ingenio 2010-CSD2007-00018.

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