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# Closed form expression for the inverse cumulative distribution function of Nakagami distribution

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## Abstract

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Quantile function or inverse cumulative distribution function (CDF) is heavily utilized in modelling, simulation, reliability analysis and random number generation. The use is often limited if the inversion method fails to estimate it from the cumulative distribution function. As a result, approximation becomes the other feasible option. The failure of the inversion method is often due to the intractable nature of the CDF of the distribution. Approximation may come in the form of series expansions, closed form or functional approximation, numerical algorithm and the closed form expression drafted in terms of the quantile function of another distribution. This work used the cubic spline interpolation to obtain the closed form

of the inverse cumulative distribution function of the Nakagami-m distribution. Consequently, the closed form of the quantile function obtained for the selected parameters of the distribution serves as an approximation which compares favourably with the R software values. The result obtained was a significant improvement over some results surveyed from literature for four reasons. Firstly, the approximates produced better results in simulation as evidenced by the some values of the root mean square error of this work when compared with others. Secondly, the result obtained at the extreme tail of the distribution is better than others selected from the literature. Thirdly, the closed form estimates are easy to compute and save computation time. The closed form of the quantile function obtained in this work can be used in simulating Nakagami random variables which are used in modelling attenuation and fading channels in wireless communications and ultrasonic tissue characterization.

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