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Hankel Norm based Strict Passivity Performance of Digital Filter using Saturation Nonlinearities

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Abstract

The work proposes a new technique to examine and reduce the unusual memory effects in digital filters with saturation nonlinearity when external interference is present via strict passivity approach. Novel criterion is given to show the performance of Hankel norm in interfered digital filters by using strict passivity approach. The unusual behaviour of digital filters about previous excitations can be checked by using this proposed criterion. Given criterion ensures the asymptotic stability nature when there is no external input. The dominance of the proposed method is represented by a mathematical example.

Keywords: Digital filter, External interference, Hankel norm performance, Linear matrix inequality, Saturation nonlinearities, Strict passivity

1. Introduction

Since, last decades have shown the research importance in the field of signal processing digitally which leads to widespread investigation about the properties and behavior of digital filters [1]. Finite word length slows the performance while realizing the digital filters using any digital signal-based processors. Nonlinearities such as, quantization and overflow seem to happen when several signals are converted in discrete time systems using fixed point arithmetic, which may lead to limit cycles owing to self-sustained oscillations. Limit cycles results in instability of the filter [2-7].

While implementing a complex multi order digital filter on hardware (digital), it is usually made by combining numerous digital filters of low order. Due to this state in the system, there occurs external interference which results in instability and degrades the performance [8, 9]. Thus, several results have been reported based on stability of digital filters with overflow nonlinearities when external disturbance is present [10-24].

In general, undesired response of systems results in ringing due to past excitations. Ringing seems to appear in several electronic systems, mainly due to noise signals (e.g., oscillatory nature in digital filters, parasitic inductance and capacitance in electronics circuits). Regular monitoring of the systems should be done in the system in order to clear the ringing effects as it may result in performance degradation or malfunction. In electro-mechanical system, sudden change in parasitic components to resonate creates ringing effect. In audio systems, too much feedback oscillation results in ringing. Ringing in the system usually stores the energy and yields memory effects which is undesired when applied external inputs. Hence, ringing can be computed based on performance of Hankel norm in the system. Therefore, Hankel norm performance quantifies the unwanted memory effects which appears in the past with external inputs on future outputs [24, 25].

The basic concepts in the study of dynamical system are energy dissipation and consumption. Passivity symbolizes the energy consumption of a system and is used in various applications, e.g., electrical, chemical, mechanical, communication systems, etc. Passive systems are defined based on supply rate and storage function. The passivity theory provides a good method in analyzing the stability, design of complex energy-based systems [26, 27]. Passivity is related to the property of input-output stability that is, if bounded input energy to the system yields bounded output energy. Passivity in state-space digital filters via saturation nonlinearity with external interference has been discussed in [19-23].

This work proposes a new method to quantify the unwanted memory effects in digital filters along with saturation nonlinearity via passivity approach based on the performance of Hankel norm. A new criterion based on performance of Hankel norm in digital filters is attained with saturation nonlinearity via passivity approach. Given proposed criterion in digital filters can examine and reduce the unwanted memory effects which appears in the past with external inputs on future outputs. Along with the presented criterion, improvement of the given result can be done by using the diagonally dominant matrix approach based on Hankel norm performance, which are given as remarks. The attained criterion ensures the stability as well as asymptotic stability when there is no external input in digital filter. The result given in this paper are novel, which is based on memory effect and passivity approach.

The arrangement of the work is given as follows: Section 2 brought out the details about system taken for analysis trailed by criterion obtained for the stability of fixed-point digital filters with Hankel norm performance via passivity approach. Mathematical example is given in Section 3 to prove the dominance of the given proposed result. Section 4 presents conclusion of the paper.

2. System Description for Hankel norm analysis

The system under consideration of digital filter is

$$\begin{aligned}\boldsymbol{\zeta}(k+1) &= \boldsymbol{\psi}(\boldsymbol{\eta}(k)) + \boldsymbol{\lambda}(k) \\ &= \begin{bmatrix}\psi_1(\eta_1(k)) & \psi_2(\eta_2(k)) & \cdots & \psi_n(\eta_n(k))\end{bmatrix}^T\end{aligned}$$

$$+\begin{bmatrix}\lambda_1(k) & \lambda_2(k) & \cdots & \lambda_n(k)\end{bmatrix}^T, \tag{1}$$

$$\boldsymbol{\eta}(k) = \begin{bmatrix}\eta_1(k) & \eta_2(k) & \cdots & \eta_n(k)\end{bmatrix}^T = \boldsymbol{S}\,\boldsymbol{\zeta}(r), \tag{2}$$

$$\boldsymbol{\omega}(k) = \begin{bmatrix}\omega_1(k) & \omega_2(k) & \cdots & \omega_p(k)\end{bmatrix}^T = \boldsymbol{F}\,\boldsymbol{\zeta}(r), \tag{3}$$

where $\boldsymbol{\zeta}(k) \in \boldsymbol{R}^n$ represents the state vector, $\boldsymbol{\omega}(k) \in \boldsymbol{R}^p$ denotes the linear combination vector of the states, $\boldsymbol{\lambda}(k) \in \boldsymbol{R}^n$ indicates the external disturbance, a coefficient matrix $\boldsymbol{S} \in \boldsymbol{R}^{n\times n}$, and known matrix $\boldsymbol{F} \in \boldsymbol{R}^{p\times n}$. For, saturation nonlinearities $\psi_i(\eta_i(k))$ which are given as

$$\psi_i(\eta_i(k)) = \begin{cases} 1, & \eta_i(k) > 1 \\ \eta_i(k), & -1 \le \eta_i(k) \le 1, \qquad i=1, 2, \ldots, n. \\ -1, & \eta_i(k) < -1 \end{cases} \tag{4}$$

For a given $\alpha > 0$ and for time $K > 0$ the present work aims to find a stability criterion for the given system (1)-(3)

$$\sum_{k=K}^{\infty} \boldsymbol{\omega}^T(k)\,\boldsymbol{\omega}(k) \; < \; \alpha^2 \sum_{k=0}^{K-1} \boldsymbol{\lambda}^T(k)\,\boldsymbol{\lambda}(k), \tag{5}$$

with initial conditions as zero, when $\boldsymbol{w}(r) \neq \boldsymbol{0}$. The digital filter has performance based on Hankel norm for α if (5) is fulfilled. Condition given in (5) used to define effects which appears in the past with external inputs on the future output states for the system defined by (1)-(3).

For the strict passive system, the condition follows as

$$\sum_{k=0}^{K} \boldsymbol{\lambda}^T(k)\boldsymbol{\eta}(k) + \beta \ge \sum_{k=0}^{K} \Omega(\boldsymbol{\zeta}(k)), \qquad \forall K \ge 0, \tag{6}$$

where β is nonnegative constant and $\Omega(\boldsymbol{\zeta}(k))$ is a positive semi-definite storage function.

Theorem 1. *Given* $\alpha > 0$*, suppose if there exist* $n \times n$ *positive definite symmetric matrices* $\boldsymbol{J}$ *and* $\boldsymbol{G}$, *a positive definite diagonal matrix* $\mathbf{L}$ *and positive scalars* $\delta > 0$, $\delta_1 > 0$ *such that*

$$\boldsymbol{J} < \boldsymbol{K} \tag{7}$$

$$\begin{bmatrix} \delta \boldsymbol{S}^T \boldsymbol{S} - \boldsymbol{J} & \boldsymbol{S}^T \boldsymbol{L} & \left(\delta - \frac{1}{2}\right) \boldsymbol{S}^T \\ \boldsymbol{L}\boldsymbol{S} & \boldsymbol{P} - \delta \boldsymbol{I} - 2\boldsymbol{L} & \boldsymbol{J} \\ \left(\delta - \frac{1}{2}\right) \boldsymbol{S} & \boldsymbol{J} & \boldsymbol{J} - (\alpha^2 + \delta)\boldsymbol{I} - 1 \end{bmatrix} < \boldsymbol{0}, \tag{8}$$

$$\begin{bmatrix} \delta_1 \boldsymbol{S}^T \boldsymbol{S} + \boldsymbol{F}^T \boldsymbol{F} - \boldsymbol{Q} & \boldsymbol{S}^T \boldsymbol{L} \\ \boldsymbol{L}\boldsymbol{S} & \boldsymbol{G} - \delta_1 \boldsymbol{I} - 2\boldsymbol{L} \end{bmatrix} > \boldsymbol{0}, \tag{9}$$

then the digital filter (1)-(3) *is strictly passive with Hankel norm performance* α .

Proof. By choosing a Lyapunov function of quadratic as

$$\boldsymbol{B}(\boldsymbol{\zeta}(k)) = \boldsymbol{\zeta}^T(k) \boldsymbol{J} \boldsymbol{\zeta}(k). \tag{10}$$

The first difference of (10) is specified by

$$\begin{aligned} \Delta \boldsymbol{B}(\boldsymbol{\zeta}(k)) &= \boldsymbol{B}(\boldsymbol{\zeta}(k+1)) - \boldsymbol{B}(\boldsymbol{\zeta}(k)) \\ &= [\boldsymbol{\psi}(\boldsymbol{\eta}(k)) + \boldsymbol{\lambda}(k)]^T \boldsymbol{J} [\boldsymbol{\psi}(\boldsymbol{\eta}(k)) + \boldsymbol{\lambda}(k))] - \boldsymbol{\zeta}^T(k) \boldsymbol{J} \boldsymbol{\zeta}(k) \\ &= \boldsymbol{\psi}^T(\boldsymbol{\eta}(k)) \boldsymbol{J} \boldsymbol{\psi}(\boldsymbol{\eta}(k)) + \boldsymbol{\psi}^T(\boldsymbol{\eta}(k)) \boldsymbol{J} \boldsymbol{\lambda}(k) + \boldsymbol{\lambda}^T(k) \boldsymbol{J} \boldsymbol{\psi}(\boldsymbol{\eta}(k)) \\ &\quad + \boldsymbol{\lambda}^T(k) \boldsymbol{J} \boldsymbol{\lambda}(k) - \boldsymbol{\zeta}^T(k) \boldsymbol{J} \boldsymbol{\zeta}(k). \end{aligned} \tag{11}$$

Then, for a positive scalar δ , we have

$$\delta[\boldsymbol{\eta}^T(k)\boldsymbol{\eta}(k) - \boldsymbol{\psi}^T(\boldsymbol{\eta}(k))\boldsymbol{\psi}(\boldsymbol{\eta}(k))] \geq 0 \tag{12}$$

$$\delta[\boldsymbol{\zeta}^T(k)\boldsymbol{S}^T\boldsymbol{S}\boldsymbol{\zeta}(k) + \boldsymbol{\lambda}^T(k)\boldsymbol{S}\boldsymbol{\zeta}(k) + \boldsymbol{\zeta}^T(k)\boldsymbol{S}^T\boldsymbol{\lambda}(k) + \boldsymbol{\lambda}^T(k)\boldsymbol{\lambda}(k) - \boldsymbol{\psi}^T(\boldsymbol{\eta}(k))\boldsymbol{\psi}(\boldsymbol{\eta}(k))] \geq 0. \tag{13}$$

Adding and subtracting $\left[\boldsymbol{\lambda}^T(k)\, \boldsymbol{\eta}(k)\right]$ and by using (13), (11) can be rewritten as

$$\begin{aligned} \Delta \boldsymbol{B}(\boldsymbol{\zeta}(k)) \leq\ & \boldsymbol{\psi}^T(\boldsymbol{\eta}(k)) \boldsymbol{J} \boldsymbol{\psi}(\boldsymbol{\eta}(k)) + \boldsymbol{\psi}^T(\boldsymbol{\eta}(k)) \boldsymbol{J} \boldsymbol{\lambda}(k) + \boldsymbol{\lambda}^T(k) \boldsymbol{J} \boldsymbol{\psi}(\boldsymbol{\eta}(k)) + \boldsymbol{\lambda}^T(k) \boldsymbol{J} \boldsymbol{\lambda}(k) \\ & - \boldsymbol{\zeta}^T(k) \boldsymbol{J} \boldsymbol{\zeta}(k) - \boldsymbol{\lambda}^T(k)\boldsymbol{S}\boldsymbol{\zeta}(k) - \boldsymbol{\lambda}^T(k)\boldsymbol{\lambda}(k) + 2\boldsymbol{\psi}^T(\boldsymbol{\eta}(k))\boldsymbol{L}\boldsymbol{S}\,\boldsymbol{\zeta}(k) \end{aligned}$$

$$+ 2\boldsymbol{\psi}^T(\boldsymbol{\eta}(k))\boldsymbol{L}\,\boldsymbol{\lambda}(k) - 2\boldsymbol{\psi}^T(\boldsymbol{\eta}(k))\,\boldsymbol{L}\,\boldsymbol{\psi}(\boldsymbol{\eta}(k)) + \delta\,\boldsymbol{\zeta}^T(k)\boldsymbol{S}^T\boldsymbol{S}\boldsymbol{\zeta}(k)$$

$$+\delta\boldsymbol{\zeta}^T(k)\boldsymbol{S}^T\boldsymbol{\lambda}(k) + \delta\boldsymbol{\lambda}^T(k)\boldsymbol{S}\boldsymbol{\zeta}(k) + \delta\boldsymbol{\lambda}^T(k)\boldsymbol{\lambda}(k)$$

$$-\boldsymbol{\psi}^T(\boldsymbol{\eta}(k))\,\boldsymbol{J}\,\boldsymbol{\psi}(\boldsymbol{\eta}(k)) - \phi(k)$$

$$= \begin{bmatrix} \boldsymbol{\zeta}(k) \\ \boldsymbol{\psi}(\boldsymbol{\eta}(k)) \\ \boldsymbol{\lambda}(k) \end{bmatrix}^T \begin{bmatrix} \delta\boldsymbol{S}^T\boldsymbol{S} - \boldsymbol{J} & \boldsymbol{S}^T\boldsymbol{L} & \left(\delta - \frac{1}{2}\right)\boldsymbol{S}^T \\ \boldsymbol{L}\boldsymbol{S} & \boldsymbol{P} - \delta\boldsymbol{I} - 2\boldsymbol{L} & \boldsymbol{J} \\ \left(\delta - \frac{1}{2}\right)\boldsymbol{S} & \boldsymbol{J} & \boldsymbol{J} - (\alpha^2 + \delta)\boldsymbol{I} - 1 \end{bmatrix}$$

$$\times \begin{bmatrix} \boldsymbol{\zeta}(k) \\ \boldsymbol{\psi}(\boldsymbol{\eta}(k)) \\ \boldsymbol{\lambda}(k) \end{bmatrix} + \boldsymbol{\lambda}^T(k)\,\boldsymbol{\eta}(k) + \alpha^2\boldsymbol{\lambda}^T(k)\boldsymbol{\lambda}(k) - \phi(k), \tag{14}$$

where $\phi(r) = 2\boldsymbol{\psi}^T(\boldsymbol{\eta}(k))\boldsymbol{L}\left[\boldsymbol{\eta}(k) - \boldsymbol{\psi}(\boldsymbol{\eta}(k))\right]$. It may be observed that $\phi(r)$ is nonnegative with respect to (4). Let us split the available storage function as $\Delta\boldsymbol{B}(\boldsymbol{x}(r))$ and $\Delta\boldsymbol{B}_1(\boldsymbol{x}(r))$ which represents past and future output-based energy for the state $\boldsymbol{B}(\boldsymbol{\zeta}(0))$ [25].

Given LMI (8) is fulfilled, we have

$$\Delta\boldsymbol{B}(\boldsymbol{x}(r)) < \alpha^2\boldsymbol{\lambda}^T(k)\boldsymbol{\lambda}(k). \tag{14}$$

Sum on both the sides of (14) from 0 to $K-1$ gives

$$\sum_{k=0}^{K-1}\Delta\boldsymbol{B}(\boldsymbol{\zeta}(k)) = \boldsymbol{B}(\boldsymbol{\zeta}(K)) - \boldsymbol{B}(\boldsymbol{\zeta}(0)) = \boldsymbol{B}(\boldsymbol{\zeta}(K)) < \alpha^2\sum_{r=0}^{K-1}\boldsymbol{\lambda}^T(k)\boldsymbol{\lambda}(k). \tag{15}$$

If LMI (8) is satisfied, we have

$$\Delta\boldsymbol{B}_1(\boldsymbol{\zeta}(k)) < \boldsymbol{\lambda}^T(k)\,\boldsymbol{\eta}(k)$$

Sum on both the sides of (15) from 0 to K gives

$$\sum_{K=0}^{K}\Delta\boldsymbol{B}_1(\boldsymbol{\zeta}(k)) = \boldsymbol{B}_1(\boldsymbol{\zeta}(K)) - \boldsymbol{B}_1(\boldsymbol{\zeta}(0)) < \sum_{k=0}^{K}\boldsymbol{\lambda}^T(k)\boldsymbol{\eta}(k). \tag{16}$$

Let us assume $\beta = \boldsymbol{B}_1(\boldsymbol{\zeta}(0))$. Then,

$$\sum_{k=0}^{K} \boldsymbol{w}^T(k)\ \boldsymbol{y}(k) + \beta \geq \boldsymbol{B}_1(\boldsymbol{x}(k)) \tag{17}$$

holds true since $\boldsymbol{B}_1(\boldsymbol{\zeta}(k)) \geq 0$. The given relation (17) satisfies (6) and, hence the digital filter (1)-(3) is supposed to be strict passive with external interference $\boldsymbol{w}(r)$ to the output $\boldsymbol{y}(r)$. On the other hand, consider $\boldsymbol{w}(r) = \boldsymbol{0}$, $\forall r \geq K$.

Choose

$$\boldsymbol{W}(\boldsymbol{\zeta}(k)) = \boldsymbol{\zeta}^T(k)\boldsymbol{G}\,\boldsymbol{\zeta}(k). \tag{18}$$

The first difference of (18) is specified by

$$\begin{aligned}
\Delta \boldsymbol{W}(\boldsymbol{\zeta}(k)) &= \boldsymbol{W}(\boldsymbol{\zeta}(k+1)) - \boldsymbol{W}(\boldsymbol{\zeta}(k)) \\
&= [\boldsymbol{\psi}(\boldsymbol{\eta}(k)) + \boldsymbol{\lambda}(k)]^T\ \boldsymbol{G}\,[\boldsymbol{\psi}(\boldsymbol{\eta}(k)) + \boldsymbol{\lambda}(k)] - \boldsymbol{\zeta}^T(k)\boldsymbol{G}\,\boldsymbol{\zeta}(k) \\
&= \boldsymbol{\psi}^T(\boldsymbol{\eta}(k))\,\boldsymbol{G}\,\boldsymbol{\psi}(\boldsymbol{\eta}(k)) - \boldsymbol{\zeta}^T(k)\boldsymbol{G}\,\boldsymbol{\zeta}(k).
\end{aligned} \tag{19}$$

Then, for a positive scalar δ_1, we have

$$\delta_1[\boldsymbol{\lambda}^T(k)\boldsymbol{\lambda}(k) - \boldsymbol{\psi}^T(\boldsymbol{\eta}(k))\boldsymbol{\psi}(\boldsymbol{\eta}(k))] \geq 0 \tag{20}$$

$$\delta_1[\boldsymbol{\zeta}^T(k)\boldsymbol{S}^T\boldsymbol{S}\,\boldsymbol{\zeta}(k) - \boldsymbol{\psi}^T(\boldsymbol{\eta}(k))\boldsymbol{\psi}^T(\boldsymbol{\eta}(k))] \geq 0. \tag{21}$$

Using (21), (19) can be rewritten as

$$\begin{aligned}
\Delta \boldsymbol{W}(\boldsymbol{\zeta}(k)) \leq\ & \boldsymbol{\psi}^T(\boldsymbol{\eta}(k))\,\boldsymbol{G}\,\boldsymbol{\psi}(\boldsymbol{\eta}(k)) - \boldsymbol{x}^T(r)\,\boldsymbol{G}\ \boldsymbol{x}(r) - \boldsymbol{\lambda}^T(k)\boldsymbol{S}\ \boldsymbol{\zeta}(k) - \boldsymbol{\lambda}^T(k)\,\boldsymbol{\lambda}(k) \\
& + \delta_1\,\boldsymbol{\zeta}^T(k)\boldsymbol{S}^T\boldsymbol{S}\ \boldsymbol{\zeta}(k) - \delta_1\boldsymbol{\psi}^T(\boldsymbol{\eta}(k))\,\boldsymbol{\psi}^T(\boldsymbol{\eta}(k)) + \boldsymbol{\zeta}^T(k)\boldsymbol{F}^T\boldsymbol{F}\,\boldsymbol{\zeta}(k)
\end{aligned}$$

$$\Delta \boldsymbol{W}(\boldsymbol{\zeta}(k)) = \begin{bmatrix} \boldsymbol{\zeta}(k) \\ \boldsymbol{\psi}(\boldsymbol{\eta}(k)) \end{bmatrix}^T \begin{bmatrix} \delta_1\boldsymbol{S}^T\boldsymbol{S} - \boldsymbol{G} + \boldsymbol{F}^T\boldsymbol{F} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{G} - \delta_1\boldsymbol{I} \end{bmatrix} \times \begin{bmatrix} \boldsymbol{\zeta}(k) \\ \boldsymbol{\psi}(\boldsymbol{\eta}(k)) \end{bmatrix} - \boldsymbol{\omega}^T(k)\,\boldsymbol{\omega}(k), \tag{22}$$

Given LMI (9) is fulfilled, we have

$$\Delta \boldsymbol{W}(\boldsymbol{\zeta}(k)) < -\boldsymbol{\omega}^T(k)\,\boldsymbol{\omega}(k)\,. \tag{23}$$

Sum on both the sides of (23) from K to ∞ gives

$$\sum_{r=k}^{\infty} \boldsymbol{\omega}^T(k)\,\boldsymbol{\omega}(k) < \Delta \boldsymbol{W}(\boldsymbol{\zeta}(k)) = \boldsymbol{W}(\boldsymbol{\zeta}(K)) - \boldsymbol{W}(\boldsymbol{\zeta}(\infty)) = \boldsymbol{W}(\boldsymbol{\zeta}(K)) \tag{24}$$

$$\sum_{r=k}^{\infty} \boldsymbol{\omega}^T(k)\,\boldsymbol{\omega}(k) < \boldsymbol{W}(\boldsymbol{\zeta}(K)) < \boldsymbol{V}(\boldsymbol{\zeta}(K)) < \boldsymbol{V}_1(\boldsymbol{\zeta}(K)) < \alpha^2 \sum_{r=0}^{K-1} \boldsymbol{\lambda}^T(k)\,\boldsymbol{\lambda}(k) + \sum_{r=0}^{K} \boldsymbol{\lambda}^T(k)\,\boldsymbol{\eta}(k) + \beta\,, \tag{25}$$

which satisfies (5) and (6). Therefore, the digital filter from (1)-(3) is said to be as Hankel norm performance and strictly passive. This completes Theorem 1 proof.

Remark 1. Criterion in Theorem 1 is given in the form of Linear Matrix inequality (LMI)s and, hence, convex optimization procedures [28, 29] are used to fix a feasible solution to the obtained LMIs.

Remark 2. To reduce the potential conservatism of Theorem 1, the diagonally dominant matrix approach used in [11] can be applied.

Remark 3. Theorem 1 ensures absence of overflow oscillations when interference is present in digital filters with Hankel norm performance via strict passive approach. Further, it also shows the asymptotic stability nature without external interference.

Remark 4. In the realization of a digital filter, it is generally needed to choose a filter structure which does not exhibit limit cycles and delivers acceptable performance. Further, the work can be extended for multidimensional [30] systems which seems to be near future effort.

3. Illustrative examples

Consider the subsequent example to confirm the superiority of the proposed result.

Example 1. For simulation analysis, a digital filter (1)-(3) is considered with

$$\boldsymbol{S}=\begin{bmatrix} 0.4 & -0.6 \\ 0.12 & 0.2 \end{bmatrix}, \boldsymbol{F}=\begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \boldsymbol{\lambda}(k)=0.1\begin{bmatrix} \sin(0.5k) \\ \cos(k) \end{bmatrix},\ \alpha=0.45. \tag{26}$$

With the assistance of LMI toolbox in MATLAB [19, 20] one will be able to verify that given example for (8) and (9) has feasible values of unknown parameters.

$$\boldsymbol{J}=\begin{bmatrix} 0.2895 & -0.0328 \\ -0.0328 & 0.3805 \end{bmatrix},\ \boldsymbol{L}=\begin{bmatrix} 0.0634 & 0 \\ 0 & 0.1952 \end{bmatrix}, \tag{27}$$

$$\boldsymbol{G}=\begin{bmatrix} 10.2583 & -1.2651 \\ -1.2651 & 11.6411 \end{bmatrix}, \delta=0.5920,\ \delta_1=16.4203\ . \tag{28}$$

Therefore, the it is stable system with $\alpha=0.45$ in view of Theorem 1. Let K = 50, apply the external input for the time bound [0, 99] and do not exceed the bound. First and second components of $\omega(k)$ are given in Figures 1 and 2 respectively, when $\boldsymbol{\lambda}(k)$ for $0\le k<100$ and $\boldsymbol{\lambda}(k)=\begin{bmatrix}0 & 0\end{bmatrix}^T$ $k\ge 100$ is applied with external input when initial condition as zero. $\omega(k)$ for $k\ge 100$ denotes the memory effect in digital filter. Then, we attain

$$\sqrt{\frac{\sum_{k=100}^{200}\omega^T(k)\omega(k)}{\sum_{k=0}^{99}\lambda^T(k)\lambda(k)}}=0.0907, \tag{29}$$

which is under the given level. Thus, the performance based on Hankel norm is simulated and verified. Asymptotically stable nature of the digital filter without the external input is shown in Figures 1 and 2.

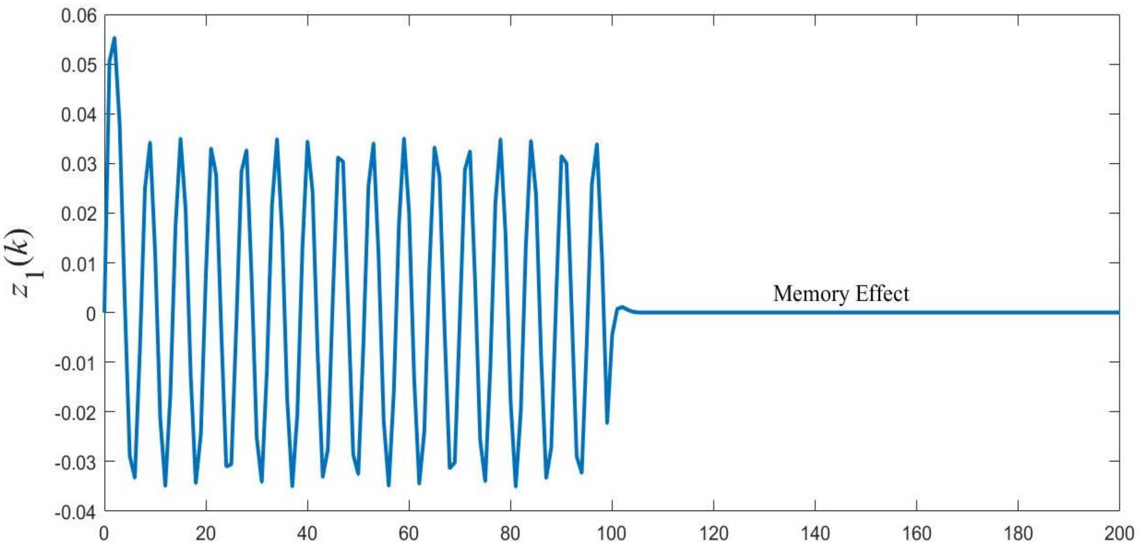

Figure 1. Memory effect of $z_1(k)$ for Example 1

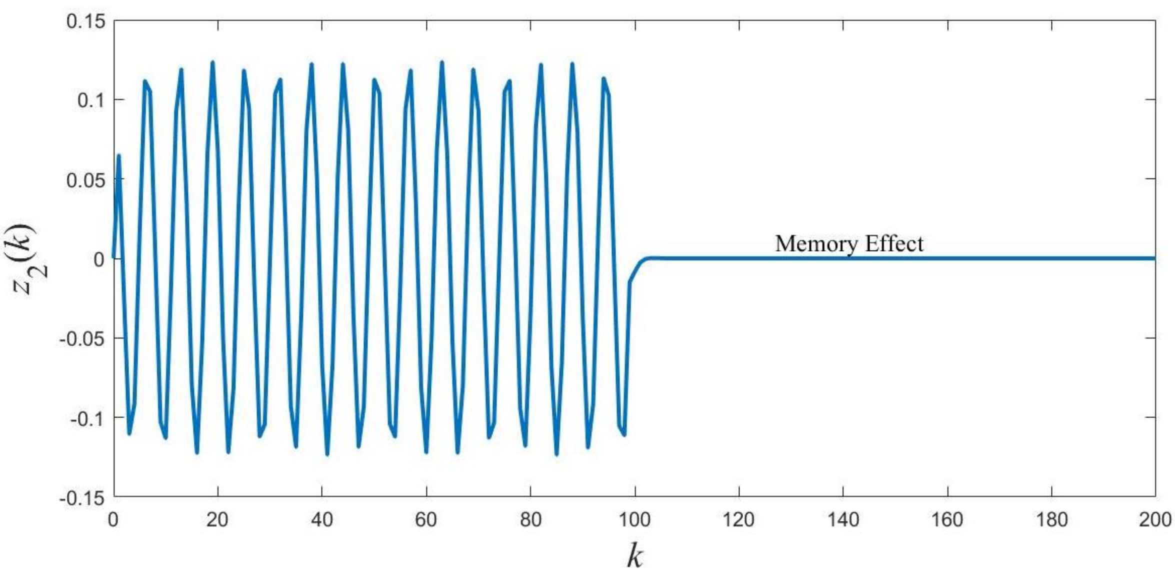

Figure 2. Memory effect of $z_2(k)$ for Example 1

4. Conclusion

The proposed work discusses the performance based on Hankel norm in digital filters via strict passivity approach. Novel LMI criterion describing the performance of Hankel norm in interfered digital filters via strict passivity approach are presented. By means of the obtained criterion in digital filters, it is easy to reduce the unwanted memory effects. The asymptotic stability property for digital filter when there is no external input are discussed. The worthful ness of the attained result is shown via mathematical example.

Ethics Declarations

Conflict of interest

The authors declare that there is no conflict of interest.

Consent to Participate

Not applicable

Authors Contributions

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Competing Interests

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