Direction-of-Arrival Estimation in Non-Linear Array with Gain-Phase Error Compensation.

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Abstract

Gain and phase error sensitivity of the array output is common to all high-resolution direction of arrival estimators. In this paper, we addressed the estimation of directions of arrival of multiple signals impinging on a non-uniform linear array in the presence of gain and phase perturbations. The effect of amplitude and phase distortion on a non-uniform array with defective or missing elements is considered. An iteration method is proposed to estimate and compensate for these errors to ensure maximum accuracy in DOA estimation. Computer simulations are shown to verify the efficacy of the proposed algorithm with the presence of array distortions.

Keywords-Direction-of-arrival (DOA) estimation, non-uniform linear array, error calibration, gain and phase errors, compensation

I. Introduction

Practical implementation of direction-finding algorithms has got a lot of significance in many areas such as wireless, mobile, vehicular, marine communications, RADAR. An accurate estimation of signal directions is critically important in most of these scenarios. The uniform linear array structures are the preferred choice in most of the research works. Non- uniform linear arrays are also getting attention for a few years due to their excellent performance in several scenarios. Also, there can be situations where missing sensors results in loss of data and non-uniform spacing between the elements. Other than this, most of the algorithms with high resolution undergo degradation in performance in the presence of array system errors. Such degradations in performance can be dealt with in the accurate knowledge of array parameters and proper calibration techniques. The robustness in the estimation of DOA can be achieved with active calibration techniques using calibration sources. The self-calibration methods use signal processing and optimization techniques for the calibration and compensation of these errors. An accurate calibration technique is adaptive to the varying physical conditions is required before estimating DOA in most of the applications [1,2,3].

Self-calibration algorithms are very well used for uniform linear structures for the joint estimation of DOA and unknown gain and phase errors. A joint calibration using the least square method to avoid distortion using one auxiliary source is proposed [4]. DOA and phase calibrations are performed jointly in[5] for a linear equally spaced array that uses the least squares-based algorithm for self-calibration; no prior information of direction is required. Algorithms based on Eigen decomposition along with optimization techniques are implemented iteratively and non-iteratively for calibration. In [6], Friedlander and Weiss use this method to find DOA and for calibration applicable to any array structures, but it faces the problem of convergence to local optima. For unknown phase perturbations, a phase retrieval method is suggested in [7]. Direction estimation of mixed signals of far-field and near field with array errors is implemented [8], which uses the matrix transformation method. The preprocessing techniques such as spatial smoothing can improve the effective aperture and direction estimation accuracy [9,10] even in the presence of array perturbations. Among all these techniques, Eigen structure-based methods promise to deal with array system errors. To deal with the problems faced in [6], Liu proposed a method that uses the Hadamard product of array output and its conjugate [11]. Cao proposed another method for DOA estimation that uses the Hadamard product of the signal covariance matrix after gain compensation [12]. Phase compensation is not required in both these methods, but still, they are computationally expensive. These methods are not suitable for uniform linear arrays and need no calibration sources.

There can be unequal spacing between elements due to space constraints when we operate with long uniform arrays or when there is a chance of some of the elements undergoing failure. A proper array output is not obtained due to the non-uniform spacing created by these elements. Since the last decade, the research carried out on the design, implementation, and effective usage of non-uniform array structures shows the importance of non-uniformly spaced arrays. Eigen structure methods work well with uniform and non-uniform array configurations. Spectral multiple signal classification (MUSIC), root MUSIC, expectation-maximization (EM) [13,14], etc., are some of the successful algorithms applied to direction estimation in non-uniform linear arrays. Several techniques, including array interpolation, polynomial approach, and adaptive filtering, are used to estimate DOAs for the arbitrary array structures [15,16,17,18]. But the effect of array model errors is rarely discussed in the case of non-uniform array structures.

This paper proposed to estimate gain-phase errors using a non-uniform array where the defective or missing elements create the unequal spacing. The effect of gain and phase errors and its calibration for an array with missing elements is not considered earlier according to the authors' knowledge. Here we consider performance impairments due to array perturbations and their compensation for the defective array. An iterative method is proposed to estimate the signal directions and array errors with the help of existing calibration and compensation techniques applied to a complex covariance matrix. The proposed method uses the calibration techniques used in [11] [12], but a complex error covariance matrix is used for DOA estimation, and a joint iterative method for calibration, compensation, and direction estimation is proposed.

The organization of the paper is as follows. The array signal and error model, calibration techniques, and compensating methods are discussed in section II. The simulation model and results that elucidate the performance of the proposed algorithm are given in Section III. Sections IV hold some conclusions.

Notations: Bold capital letters for matrices and small letters for vectors. E (.) denote expectation, and $(.)^H$ denotes Hermitian transpose and diag () for diagonal matrix.

II. Signal model and Problem formulation

A. Signal model

Consider an array antenna in the receiving mode with $\bar{Q}$ elements arranged linearly with half the wavelength among them. The array has defective elements, and the active elements count to Q. Then, J signals are impinging on this array from different directions. These signals are coming from far-field, which are narrowband and uncorrelated with directions $[\alpha_{s1}, \alpha_{s2} \ldots\ldots\ldots\ldots \alpha_{sJ}]$. α_{sJ} Corresponds to j^{th} direction. Then the received array signal can be represented as in (1)

$$\boldsymbol{Z}(t) = \boldsymbol{A}(\alpha_{sj})\boldsymbol{S}(t) + \boldsymbol{N}(t) \tag{1}$$

Where received array output signal is given as, $Q \times 1$ vector, $\boldsymbol{Z}(t) = [\boldsymbol{z_1}(t), \boldsymbol{z_2}(t), \ldots\ldots \boldsymbol{z_Q}(t)]$ the array steering vector is obtained as $\boldsymbol{A} = [\boldsymbol{a}(\alpha_{s1}), \boldsymbol{a}(\alpha_{s2}), \ldots\ldots \boldsymbol{a}(\alpha_{sJ})]$, and the j^{th} column is obtained as $\boldsymbol{a}(\alpha_{sJ}) = [1, \exp\left(-\frac{j2\pi\Delta_{2,J}}{\lambda}\right), \ldots\ldots, \exp\left(-\frac{j2\pi\Delta_{Q,J}}{\lambda}\right)]$, and $\Delta_{Q,J} = x sin((\alpha_{sJ}))$

the uncorrelated $J\times1$ complex signal vector is $\boldsymbol{S}(t)=[\boldsymbol{s_1}(t),\boldsymbol{s_2}(t)\dots\dots\boldsymbol{s_J}(t)]$ and the complex Gaussian noise $\boldsymbol{N}(t)=[\boldsymbol{n_1}(t),\boldsymbol{n_2}(t)\dots\dots\boldsymbol{n_Q}(t)]$ of $Q\times1$ dimension which is uncorrelated. K snapshots of signals can be considered for evaluation.

Considering the perturbations due to gain and phase, the array output matrix equation (1) can be modified accommodating the error as (2)

$$\boldsymbol{Z}_{err}(t)=\boldsymbol{\delta A}(\alpha_{sJ})\boldsymbol{S}(t)+\boldsymbol{N}(t) \tag{2}$$

where $\boldsymbol{\delta}=\boldsymbol{\rho}_Q e^{j\boldsymbol{\psi}_Q}$ is the $Q\times1$ error vector. where $\boldsymbol{\rho}_Q=diag(\rho_1,\rho_2,\dots\dots\rho_Q)$ and $\boldsymbol{\psi}_Q=diag(\psi_1,\psi_2,\dots\dots\psi_Q)$ are the $Q\times Q$ gain error and the phase error diagonal matrices.

DOA estimation

We use an Eigen decomposition-based algorithm for DOA estimation; the covariance matrix of the array signal output in the absence of amplitude or phase distortions is

$$\boldsymbol{R}_z=E[\boldsymbol{Z}(t)\boldsymbol{Z}(t)^H]=\boldsymbol{A}(\alpha_{sJ})\boldsymbol{R}_{sJ}\boldsymbol{A}(\alpha_{sJ})^H+\sigma_N^2\mathbf{I} \tag{3}$$

Here $\boldsymbol{R}_{sJ}$ is the $Q\times J$ matrix of signal covariance, 'I' is the Q×Q identity matrix, σ_N^2 is the noise variance

When we consider the error introduced due to gain and phase in the received signal, the covariance matrix of the array output $\boldsymbol{Z}_{err}(t)$can be expressed as $\boldsymbol{R}_{zerr}$.

The error covariance matrix $\boldsymbol{R}_{zerr}$when calculated based on equation (3)

$$\boldsymbol{R}_{zerr}=E[\boldsymbol{Z}_{err}(t)\boldsymbol{Z}_{err}(t)^H]=\boldsymbol{\delta A}(\alpha_{sJ})\boldsymbol{R}_{sJ}\boldsymbol{A}(\alpha_{sJ})^H\boldsymbol{\delta^H}+\sigma_N^2\mathbf{I} \tag{4}$$

The Eigen decomposition of $\boldsymbol{R}_{zerr}$ gives large Eigenvalues corresponding to signal but with error and small Eigenvalues give noise information. The Eigenvectors corresponding to equation (4) spans the same subspace as $\boldsymbol{R}_z$. Spectral peak search will provide the directions corresponding to the error matrix.

Eigen decomposition of the matrix $\boldsymbol{R}_z$ yields the signal as well as noise Eigenvectors.

$$\boldsymbol{R}_{zerr}=\boldsymbol{U}_{qs}\boldsymbol{E}_s\boldsymbol{U}_{qs}^H+\boldsymbol{U}_{qn}\boldsymbol{E}_n\boldsymbol{U}_{qn}^H \tag{5}$$

$\boldsymbol{E}_s=diag[e_{1,}e_{2,}\dots\dots e_J]$ and $E_n=diag[e_{J+1,}e_{J+2,}\dots\dots e_Q]$ are diagonal matrices. U_{qs} and U_{qn} provide information about the signal and noise Eigen values. A spectral peak search will give actual signal directions.

B. Calibration and compensation

We construct the error covariance matrix of the array output given in Equation (4). Eigen decomposition gives the signal and noise Eigenvectors; the larger J values correspond to the signal, and J+1 to Q values correspond to the noise power. The noise power is removed for the estimation of gain error. If the Q^{th} diagonal element of $\boldsymbol{R}_{zerr}$ is given as $\boldsymbol{R}_{zerr}$(q,q) and the first element is $\boldsymbol{R}_{zerr}$(1,1), then gain error can be calculated from the diagonal elements of covariance matrix as

$$\boldsymbol{\rho}_q = sqrt\left(\frac{(\boldsymbol{R}_{zerr}(q,q)-\sigma_{NN}^2}{(\boldsymbol{R}_{zerr}(1,1)-\sigma_{NN}^2}\right) \tag{6}$$

and $\sigma_{NN}^2 = \frac{1}{Q-J}\sum_{q=J+1}^{Q}\boldsymbol{U}_{qn}$ (7)

$\boldsymbol{U}_{qn}$ are Eigenvalues corresponding to noise ρ_q is the gain error estimate, and σ_{NN}^2 is the estimate of σ_N^2 .

The phase error can be estimated by the following method.

$$\boldsymbol{F} = \sum_{j=1}^{J}\boldsymbol{M}_j^H\left(\alpha_{sJ}\right)\boldsymbol{U}_{Q-J}\boldsymbol{U}_{Q-J}^H\boldsymbol{M}_j\left(\alpha_{sJ}\right) \tag{8}$$

Where $\boldsymbol{M} = diag(\boldsymbol{a}\left(\alpha_{sJ}\right))$

$\boldsymbol{U} = [u_{J+1,}u_{J+2,}\dots\dots u_Q]$, corresponds to the J+1×Q noise Eigenvector

$\boldsymbol{w} = [1,0,0\dots\dots\dots 0]$

The solution for phase error is obtained from (8) as

$$p_e = \frac{\boldsymbol{F}^{-1}\boldsymbol{w}}{\boldsymbol{w}^H\boldsymbol{F}^{-1}\boldsymbol{w}} \tag{9}$$

ψ_e=angle(p_e)

The term ψ_e is the estimate of phase error.

C. Compensation of gain-phase error

After estimating the error in gain and phase present (6),(8), and (9), the gain –phase error vector is reconstructed as $\boldsymbol{\delta_{est}} = \boldsymbol{\rho}_{Qest} e^{j\boldsymbol{\psi}_{Qest}}$.The diagonal matrix of which is

$\boldsymbol{\delta}'_{est} = diag(\boldsymbol{\delta}_{est})$

The combined gain–phase error matrix of the estimated values is compensated.

$$\boldsymbol{R}'_{zerr} = \boldsymbol{\delta}'_{est}\boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^H{\boldsymbol{\delta}'_{est}}^H + \sigma^2_{NN}\boldsymbol{I} \tag{10}$$

$$\boldsymbol{R}'_{zerr} - \sigma^2_{NN}\boldsymbol{I} = \boldsymbol{\delta}'_{est}\boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^H{\boldsymbol{\delta}'_{est}}^H \tag{11}$$

Using (11) compensated matrix $\boldsymbol{R}_{zcomp}$ can be obtained as

$$\boldsymbol{R}_{zcomp} = (\boldsymbol{\delta}'_{est})\boldsymbol{P}({\boldsymbol{\delta}'_{est}}^{-1})^H \tag{12}$$

Where $\boldsymbol{P} = \boldsymbol{R}'_{zerr} - \sigma^2_{NN}\boldsymbol{I}$

For the compensated covariance matrix $\boldsymbol{R}_{zcomp}$the Eigenvalue decomposition is performed by including the estimated noise. The directions after compensating the errors are calculated by searching the peaks near the expected direction of signals.

D. Summary of the Proposed Algorithm

In summary, the proposed algorithm consists of the following steps:

Step1: Construct the error covariance matrix of complex signals using (4) assuming initial DOAs

Step2: Perform Eigen decomposition of (4)

Step3: Calculate noise power from (5) using (7)

Step4: Estimate the gain and phase error vectors in the absence of noise using (6),(8), and (9)

Step5: Reconstruct the covariance matrix with estimated error values and noise with (10)

Step5: Errors are compensated using (12)

Step6: Estimate the DOAs using compensated error covariance matrix using a peak search method

Discussion

This paper deals with gain-phase calibration for an array of non-uniform spacing created by the missing or defective elements. The missing elements result in the loss of sufficient data for the estimation process. Other than this, the adverse effect caused by the gain phase perturbations also causes signal imperfections. The Eigen structure methods can be effective in the case of minimum perturbations. But as the error increases, these algorithms fail and show inferior

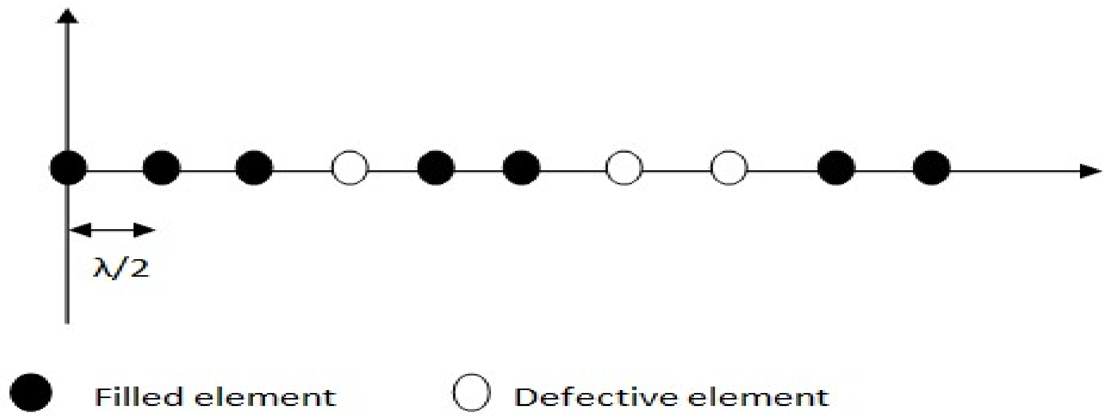

Fig.1. A 7 element non-uniform linear array with three defective elements

estimation performance. Hence in the proposed method, we combine the Eigen structure method and existing calibration and compensation techniques for DOA estimation. The proposed method is not dealt with this way in any earlier works for a non-uniform linear array structure. The Eigen structure methods in [11] and [12] also deal with non-uniform spacing, but the array structure and signal model are different. When applied to the missing elements array, the comparison of these methods provide the proposed method's efficiency. The computational time taken for the proposed method is less than half the time taken by the other two methods.

III. Simulation and results

Some representative simulations are implemented for a linear array of 10-elements with missing elements. For the following simulations, an array configuration [0, 1, 2, 4, 5, 8, 9]*λ/2is considered as shown in Fig.1, where λ is the wavelength corresponding to the center frequency. The array has one by third of the elements present at the ends and at the center keeping the aperture same [13], [16]. Three signal sources are assumed to be sending signals from directions -30, -10, and 20 degrees received by the array. The signals are Gaussian and are uncorrelated with narrow bandwidth arriving from far-field. At the array output, these signals are affected by noise and perturbations caused by array elements. The noise is assumed as white and un-correlated. The error introduced by gain is generated as $\rho_Q = 1 + 3.4641\sigma_\rho\eta_\rho$and the error in phase as $\psi_Q = 3.4641\sigma_\psi\eta_\psi$. The random numbers η_ρ and $\eta_\psi$uniformly distributed in the range between -0.5 and +0.5. The standard deviations in gain and phase are respectively $\sigma_\rho$and σ_ψ.

For all simulations, gain error σ_ρ and phase error σ_ψ are fixed as 0.4 and 40 degrees. The simulations are demonstrated for the array set up shown in Fig.1.Comparing the suggested method with Eigen structure methods proposed in [11] and [12] is illustrated. These methods use the array covariance matrix's complex conjugate matrix with gain error, and phase error compensated separately and did not follow a joint iterative method. And in our method, we follow joint calibration and estimation for a joint gain phase error matrix.

The performance metric chosen for accuracy is by calculating the Average Root Mean Square Error (ARMSE) given in Equation (13) with 100 iterations. The ARMSE is estimated in different experiments for various signals to noise ratio(SNR), the number of signal snapshots, and varying standard deviation of a phase error value. The average ARMSE is calculated as

$$\mathrm{A}RMSE = \sqrt{\sum_{i=1}^{n}\sum_{l=1}^{K}(|(\alpha_{sJ}) - (\alpha_{sJest})|)^2/(nK))} \qquad (13)$$

Whereα_{sJ} corresponds to actual values, and α_{sJest} are the estimated values.

A. Performance with SNR and Snapshots

Experiment-1 is conducted to evaluate the performance of various SNR. To know the SNR effect, the snapshots to be fixed as 500and vary the SNR from -5dB to +20dB in steps of 5dB for the signals mentioned. The deviations in amplitude and phase fixed at σ_ρ=0.4 and σ_ψ=40degrees. For 100 iterations, the direction estimation accuracy is found for different SNR and plotted in Fig.2. This experiment is performed with a large error to know the proposed methods' efficiency in a worse scenario. And it is obvious that compared to other methods for the proposed method, the accuracy is excellent.

Experiment-2 evaluates the performance of algorithms with more sample points. The out-turn of the number of snapshots in the estimation accuracy is evaluated, for which the SNR to be 10 dB, as shown in Fig.3. The standard deviations in gain and phase error are fixed as σ_ρ= 0.4 andσ_ψ=40degrees as in the previous case. The snapshots are varied from 200 to 1200 in steps of 200 and estimated RMSE with the same number of iterations. However, the performance is improved with the increase in snapshots for the proposed and methods in [11] & [12]. So, an increase in SNR and snapshots helps improve the accuracy to a great extent when unknown array errors are present.

B. Performance with deviations in gain and phase errors

Experiment-3 is conducted with the standard deviation of phase error σ_ψ varying from 10 to 90 degrees for the same setup. Stable performance is obtained from all methods, including the proposed method with phase error variations.Fig.4.illustrates independence from the error after compensation for all methods. The phase error independence of algorithms in [11] & [12] are already proven.

The spatial spectrum showing the effect of perturbations in gain and phase is demonstrated in Fig.5. It shows the effectiveness of the proposed method in eliminating array errors to an extent. The spectrum is plotted for 1000 snapshots and SNR 20dB with the standard deviations in gain and phase 0.4 and 40 degrees.

Experiments 4 & 5 are performed to know the influence of gain and phase error variations on arrays with different missing elements for a particular SNR. These experiments are conducted for 500 snapshots considering gain error and phase error separately. It is concluded from Fig.6. and Fig.7. that the proposed method gives fewer error variations across the elements.

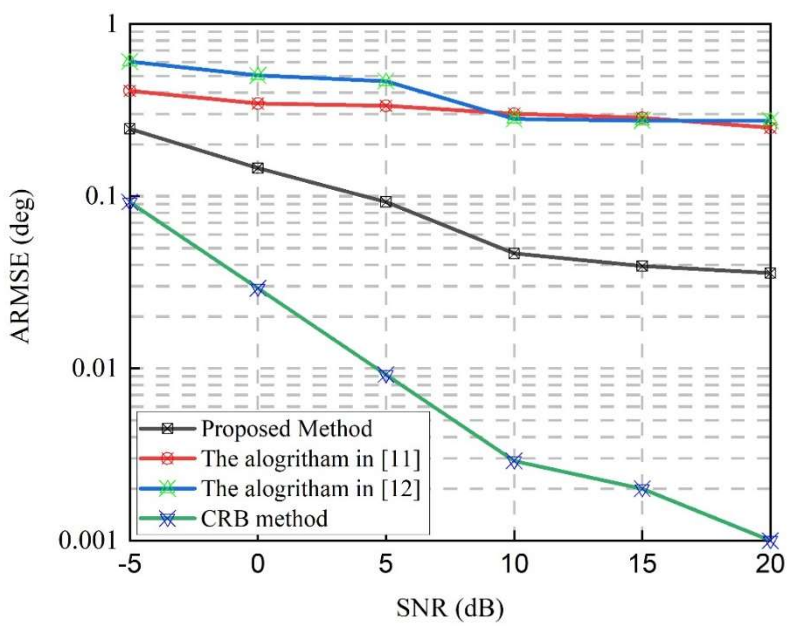

Fig.2. ARMSE vs. SNR curves for 500 snapshots, the standard deviation of gain and phase error $\sigma\rho=0.4$, $\sigma_\psi=40°$.

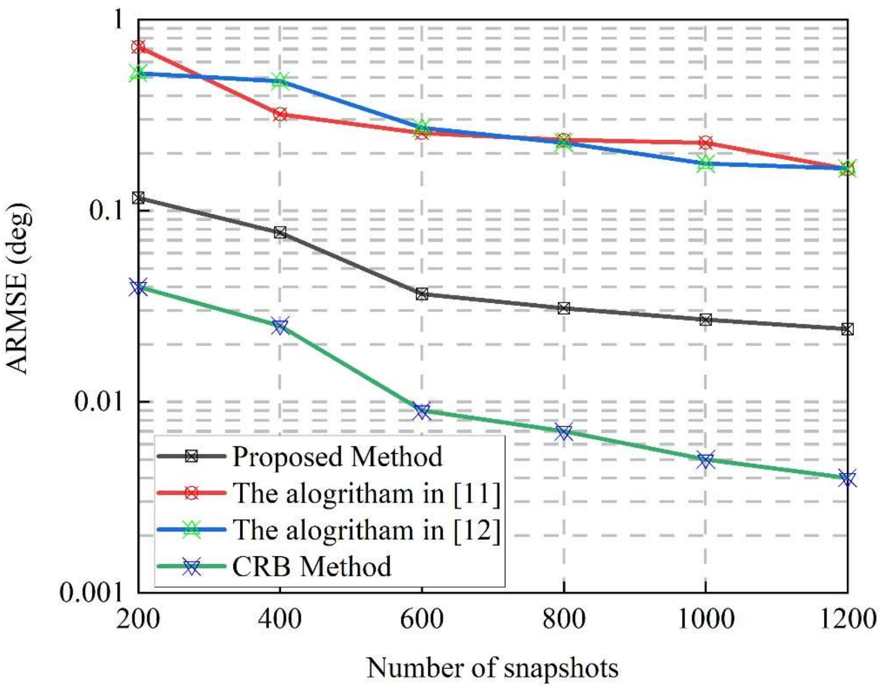

Fig.3. ARMSE vs. the number of snapshots curves for SNR=10dB, the standard deviation in gain and phase errors σρ=0.4, σψ=40°

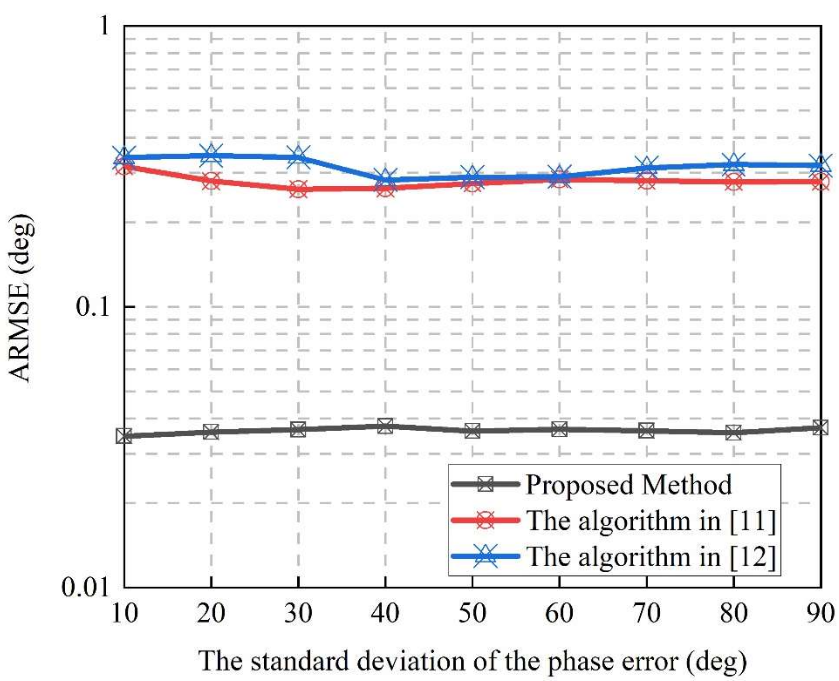

Fig.4. ARMSE vs. the number of snapshots curves for SNR=10dB, the standard deviation in gain and phase errors σρ=0.4, σψ=40°

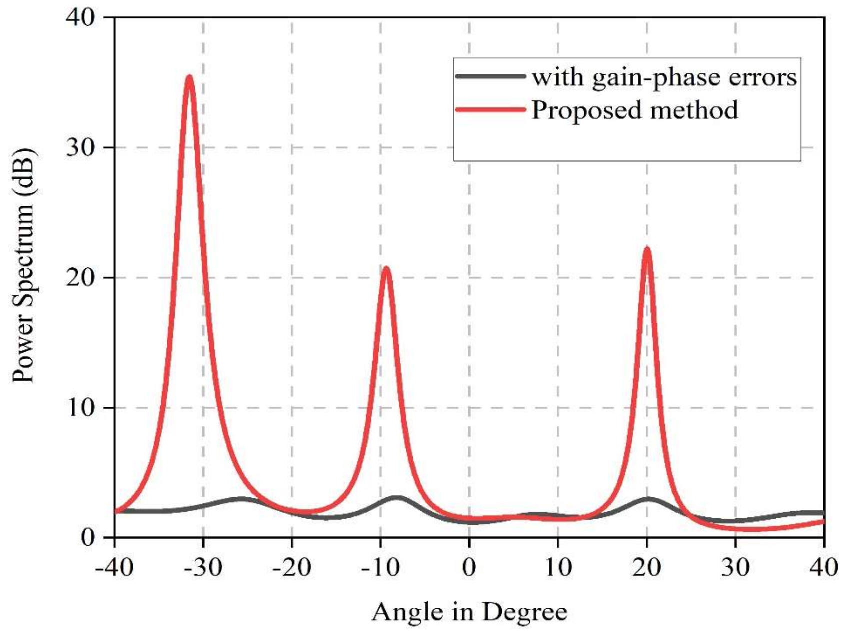

Fig.5. Spatial spectrum with error and after compensation for a non-uniform linear array

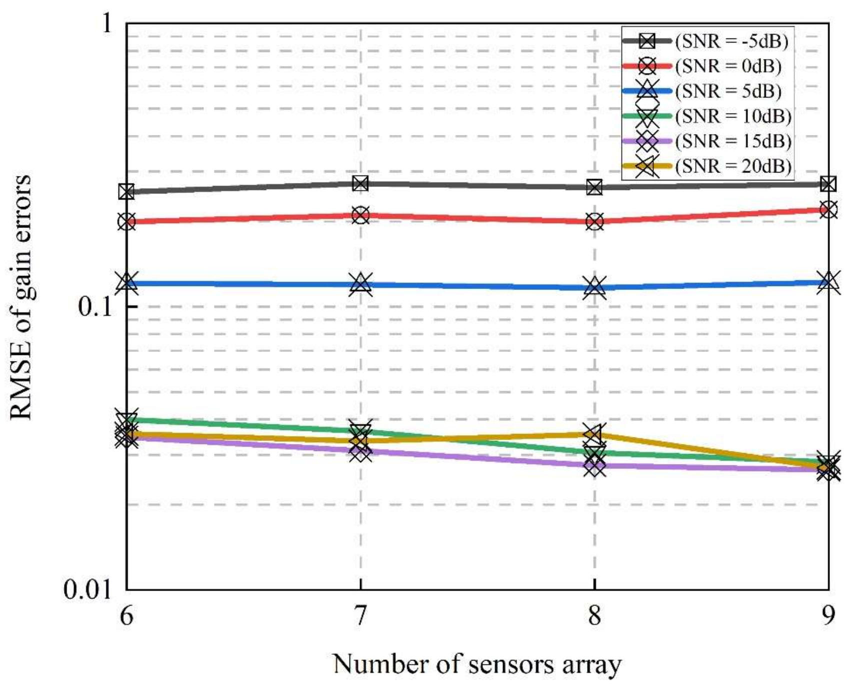

Fig.6. ARMSE curves for the number of sensors vs. gain error estimates for different SNR from -5dB to 20dB in steps of 5dB.

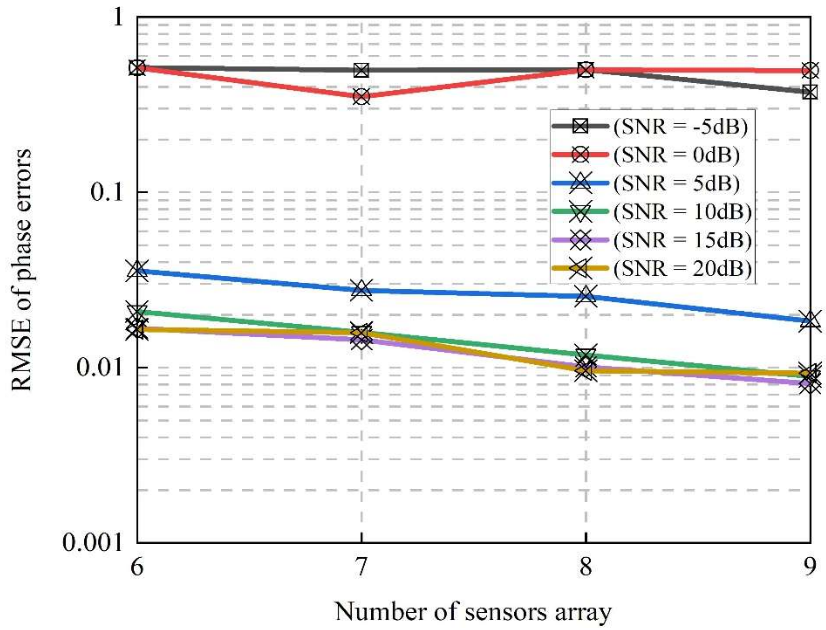

Fig.7. ARMSE curves for the number of sensors vs. phase error estimates for different SNR from -5dB to 20dB in steps of 5dB.

C. Performance with Number of sensors

In experiment 6, comparing the performance of non-uniform linear array with error and with compensation by removing the elements one by one is given in Table.1. The experiment is performed for 500 snapshots and SNR 10dB with the same error specifications as in other experiments. From the ARMSE values given in the table, the proposed method works effectively with a greater number of missing elements.

Table.1. RMSE values for different configurations of NLA with gain phase error and with compensation

Number of missing elements	Array configuration	ARMSE without compensation (deg)	ARMSE with compensation (deg)
1	[0 1 2 3 4 6 7 8 9] *λ/2	0.9567	0.0283
2	[0 1 2 4 5 7 8 9] *λ/2	1.0993	0.0306
3	[0 1 2 4 5 8 9] *λ/2	1.0771	0.0580
4	[0 24 5 8 9] *λ/2	1.1355	0.1319
5	[0 1 3 6 9] *λ/2	1.1372	0.5547

IV. Conclusion

An algorithm to reduce the effect of array perturbations caused due to amplitude and phase changes is demonstrated in this paper. Several experiments are conducted to evaluate the proposed methods' performance to determine the accuracy of DOA estimation in a defective array. The proposed algorithm could significantly eliminate the perturbations caused by gain and phase errors. Comparing the existing Eigen structure methods shows the improved performance of the proposed method with SNR and snapshots. However, the proposed method is good at compensating for the performance deterioration caused due to gain and phase variations. The effect of coherent signals in such a scenario can be considered for future research.

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Conflicts of interest/Competing interests: NO

Availability of data and material: YES

Code availability: YES

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