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General Convex Integral Control

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Abstract: In this paper, a fire-new general integral control, named general convex integral control, is proposed. It is derived by defining a nonlinear function set to form the integral control action and educe a new convex function gain integrator, introducing the partial derivative of Lyapunov function into the integrator and resorting to a general strategy to transform ordinary control into general integral control. By using Lyapunov method along with the LaSalle′s invariance principle, the theorem to ensure regionally as well as semi-globally asymptotic stability is established only by some bounded information. Moreover, the lemma to ensure the integrator output to be bounded in the time domain is proposed. The highlight point of this integral control strategy is that the integral control action seems to be infinity, but it factually is finite in the time domain. Therefore, a simple and ingenious method to design the general integral control is founded. Simulation results showed that under the normal and perturbed cases, the optimum response in the whole control domain of interest can all be achieved by a set of control gains, even under the case that the payload is changed abruptly.

Keywords: General integral control, nonlinear integrator, convex integrator, nonlinear control, output regulation.

1 Introduction

Integral control[1] plays an important role in control system design because it ensures asymptotic tracking and disturbance rejection. In the presence of the parametric uncertainties and unknown constant disturbances, the integral control can still preserve the stability of the closed-loop system and create an equilibrium point at which the tracking error is zero. The main task of the integral controller is to stabilize this point, which is challenging because it depends on uncertain parameters and unknown disturbances.

1.1 Classical integral control

The simplest controllers that achieve integral action are of the proportional integral derivative (PID) form that introduces integral action by integrating the error. It is well known that integral-action controllers with this class of integrators often suffer a serious loss of performance due to integrator windup, which occurs when the actuators in the control loop saturate. Actuator saturation not only deteriorates the control performance, causing large overshoot and long settling time, but also leads to instability, since the feedback loop is broken for such saturation. To disguise this drawback, various anti-windup schemes have been proposed to deal with integrator windup or to improve transient performance. These schemes are classified into three different approaches: 1) conditional integration and/or integrator limiting[2−7], in which the integrator value is frozen or restricted when certain conditions are verified; 2) back-calculation[8−10], in which the difference between the controller output and the actual plant input is fed back to the integrator; and 3) a nonlinear integrator[11−15], whose output is shaped by a nonlinear error function before it enters the controller. Some conditional integration and/or integrator limiting may not guarantee a zero steady error and could result in an oscillatory system for the step-referent input when an estimated limitation is embedded in the controller. In the back-calculation approach, the compensation for integrators is active whenever actuators are saturated; integrator windup can not be completely avoided. For nonlinear integrators, the output still goes to infinity and integrator windup may occur. In addition, the universal integral continuous sliding mode control first reported in [1] has the same problem as a PID controller because it applies the same integrator. An improved version was proposed in [7], in which the integrator is modified to provide integral action only inside the boundary layer and the derivative of the error is introduced into the integrator. All these integrators, except for the one proposed in [7], were designed by using the error as the indispensable element. So, all of them is called classical integral control.

1.2 General integral control

In 2009, general integral control, which uses all available state variables to design the integrator, was proposed in [16], which presented a unified framework on general integral control, some general integrators and controllers, the necessary conditions and basic principles for designing a general integrator. However, their justification was not verified by strictly mathematical analysis. In 2012, based on linear system theory, we presented a systematic design method for the general integral control[17] with a linear integrator on the state of dynamics. The results, however, were local. The regionally as well as semi-globally results were proposed in [18], where a nonlinear integrator shaped by sliding mode manifold was presented, and the general integral control design was achieved by using sliding mode technique and linear system theory. In 2013, based on feedback linearization technique, a class of nonlinear integrators, which is shaped by a linear combination of the diffeomorphism, and a systematic method to design general integral control were presented in [19] and the conditions to ensure regionally as well as semi-globally asymptotic stability were provided. The general concave integral control was proposed in [20], in which the bounded integral control action and the concave function gain integrator were normalized, the partial derivative of Lyapunov function was introduced into the integrator design, a general strategy to

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transform ordinary control into general integral control was proposed, and the conditions on the control parameters to ensure regionally as well as semi-globally asymptotic stability were provided.

In consideration of the twinning of the concave and convex concepts, this paper addresses general convex integral control. Compared with the general concave integral control proposed in [20], the main opposite points are as follows: 1) In terms of the integral control actions, the concave one is formed by the bounded nonlinear function and the convex one is shaped by the unbounded nonlinear function. 2) In terms of the gain functions of the integrators, the concave one is unbounded and the convex one is bounded. 3) In terms of the integrator outputs, the concave one could tend to infinity and the convex one is bounded in time domain. As a result, the main contributions of this paper are as follows: 1) An unbounded nonlinear function set, which is used to form the integral control action and educe the convex function gain integrator, is defined. 2) A fire-new convex function gain integrator, with output bound in the time domain, is proposed. 3) The lemma to ensure the integrator output to be bounded in the time domain is proposed. 4) By using Lyapunov method along with the LaSalle′s invariance principle, the theorem to ensure regionally as well as semi-globally asymptotic stability is established only by some bounded information. Moreover, the highlight point of this integral control strategy is that the integral control action seems to be infinity but it factually is finite in the time domain. Therefore, a simple and ingenious method to design general integral control is founded.

Throughout this paper, we use $\lambda_m(A)$ and $\lambda_M(A)$ to indicate the smallest and largest eigenvalues, respectively, of a symmetric positive define bounded matrix $A(x)$, for any $x \in \mathbf{R}^n$. The norm of vector x is defined as $\|x\| = \sqrt{x^{\mathrm{T}}x}$, and that of matrix A is defined as the corresponding induced norm $\|A\| = \sqrt{\lambda_M(A^{\mathrm{T}}A)}$.

The remainder of the paper is organized as follows: Section 2 describes the system under consideration, assumption, definition and lemma. Section 3 addresses the control design. Example and simulation are provided in Section 4. Conclusions are presented in Section 5.

2 Problem formulation

Consider the following nonlinear system

$$\begin{cases} \dot{x} = f(x,w) + g(x,w)u \\ y = h(x,w) \end{cases} \tag{1}$$

where $x \in \mathbf{R}^n$ is the state, $u \in \mathbf{R}^m$ is the control input, $y \in \mathbf{R}^m$ is the controlled output, and $w \in \mathbf{R}^l$ is a vector of unknown constant parameter and disturbance. Functions $f(x,\ w)$, $g(x,\ w)$ and $h(x,\ w)$ are continuous in $(x,\ w,\ u)$ on the control domain $D_x \times D_u \times D_w \subset \mathbf{R}^n \times \mathbf{R}^m \times \mathbf{R}^l$. In this study, function $f(x,\ w)$ does not necessarily vanish at the origin; i.e., $f(0,\ w) \neq 0$. Let $r \in D_r \subset \mathbf{R}^m$ be a vector of constant reference. Set $v \equiv (r,\ w) \in D_v$ and $D_v \equiv D_r \times D_w$. We want to design a feedback control law u such that $y(t) \to r$ as $t \to \infty$.

Assumption 1. For each $v \in D_v$, there is a unique pair (x_0, u_0) that depends continuously on v and satisfies the equations

$$\begin{cases} 0 = f(x_0,w) + g(x_0,w)u_0 \\ y = r = h(x_0,w) \end{cases} \tag{2}$$

so that x_0 is the desired equilibrium point, and u_0 is the steady-state control that is needed to maintain equilibrium at x_0, where $y = r$.

Without loss of generality, we state all definitions, assumptions and theorems for the case when the equilibrium point is at the origin of $\mathbf{R}^n$, that is, $x_0 = 0$.

Assumption 2. Without loss of generality, suppose that function $g(x,w)$ satisfies the following inequalities

$$g(x,w) > g_0 > 0 \qquad \forall\ w \in D_w, \quad \forall\ x \in D_x \tag{3}$$

$$\|g(x,w) - g(0,w)\| \leqslant l_g^x \|x\| \qquad \forall\ w \in D_w, \quad \forall\ x \in D_x \tag{4}$$

where l_g^x is a positive constant.

Assumption 3. Suppose that there is a control law $u_x(x)$ such that $x = 0$ is an exponentially stable equilibrium point of the system:

$$\dot{x} = f(x,w) - f(0,w) + g(x,w)u_x(x) \tag{5}$$

and there exists a Lyapunov function $V_x(x)$ that satisfies

$$c_1\|x\|^2 \leqslant V_x(x) \leqslant c_2\|x\|^2 \tag{6}$$

$$\frac{\partial V_x(x)}{\partial x}(f(x,w) - f(0,w) + g(x,w)u_x(x)) \leqslant -c_3\|x\|^2 \tag{7}$$

$$\left\|\frac{\partial V_x(x)}{\partial x}\right\| \leqslant c_4\|x\| \tag{8}$$

for all $x \in D_x$ and $w \in D_w$, where c_1, c_2, c_3 and c_4 are all positive constants.

Definition 1. $F_\beta(a_\beta, c_\beta, x)$ with $a_\beta > 0$, $c_\beta > 0$ and $x \in \mathbf{R}^n$ denotes the set of all continuous differential increasing functions:

$$\phi(x) = [\phi_1(x_1)\ \phi_2(x_2)\ \cdots\ \phi_n(x_n)]^{\mathrm{T}} \text{ such that}$$
$$\phi(0) = 0$$
$$0 < \left(\frac{\mathrm{d}\phi_i(x_i)}{\mathrm{d}x_i}\right)^{-1} < c_\beta, \quad i = 1, 2, \cdots, n$$

and given any $\varepsilon > 0$, there exists a positive constant a_β such that

$$\left(\frac{\mathrm{d}\phi_i(x_i)}{\mathrm{d}x_i}\right)^{-1} < \varepsilon, \quad \forall x_i \in \mathbf{R} : |x_i| > a_\beta$$

where $|\cdot|$ stands for the absolute value.

Fig. 1 describes an example curve (dashed line) and the region for the derivative reciprocal of one component of the functions belonging to the function set F_β. For instance, functions $x + x^3$, $\frac{x+x^5}{3.0}$, $\sinh(x)$ with $x \in \mathbf{R}$ and so on, all belong to the function set F_β.

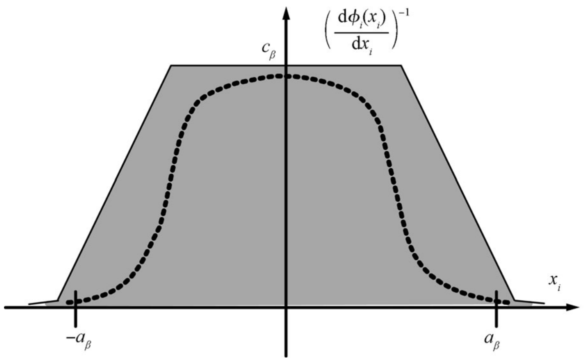

Fig. 1 Example curve and the region for the derivative reciprocal of the functions belonging to the function set F_β

Lemma 1. Let $\beta(y) = \left(\frac{\mathrm{d}\phi(y)}{\mathrm{d}y}\right)^{-1}$ with $y \in \mathbf{R}$ and $\phi(y) \in F_\beta$. Then the function

$$y(t) = \int_0^t \beta(y(\tau))\mathrm{d}\tau, \quad \forall t \in [0, \infty)$$

is a positive define bounded increasing function, that is, $0 < y(t) \leqslant c_\infty$ for all $t \in [0, \infty)$, where c_∞ is the limit of $y(t)$ as $t \to \infty$.

Proof. By Definition 1, it is easy to know that $y(t)$ is a strictly monotone positive define increasing function, and given any $\varepsilon > 0$, there exists T_0 which is large enough such that $\beta(y(t)) < \varepsilon$ holds for all $t > T_0$. Thus, given any $\varepsilon^* > 0$ ($\varepsilon^* < \varepsilon$), there exists $T_2 > T_1 > T_0$ such that

$$\int_{T_1}^{T_2} \beta(y(\tau))\mathrm{d}\tau \leqslant \varepsilon(T_2 - T_1) = \varepsilon^*$$

holds. By invoking Cauchy theorem and the monotony of $y(t)$, we conclude that the limit of $y(t)$ exists, that is, $c_\infty = \lim_{t\to\infty} y(t)$. Consequently, $0 < y(t) \leqslant c_\infty$ for all $t \in [0, \infty)$. □

3 Control design

For achieving asymptotic regulation and disturbance rejection, we need to include "integral action" in the control law. Therefore, the general integral controller can be given as

$$\begin{cases} u = u_x(x) - K_\sigma\phi(\sigma) \\ \dot{\sigma}^{\mathrm{T}} = \left(\dfrac{\mathrm{d}\phi(\sigma)}{\mathrm{d}\sigma}\right)^{-1} \dfrac{\partial V_x(x)}{\partial x} \end{cases} \tag{9}$$

where $\dot{\sigma}_i^{\mathrm{T}} = \left(\frac{\mathrm{d}\phi_i(\sigma_i)}{\mathrm{d}\sigma_i}\right)^{-1} \frac{\partial V_x(x)}{\partial x_i}$, $i = 1, 2, \cdots, m$, where K_σ is a positive define diagonal $m \times m$ matrix; $\phi(\cdot)$ belongs to the function set F_β.

Thus, substituting (9) into (1), we obtain

$$\begin{cases} \dot{x} = f(x, w) + g(x, w)u_x(x) - g(x, w)K_\sigma\phi(\sigma) \\ \dot{\phi}^{\mathrm{T}}(\sigma) = \dfrac{\partial V_x(x)}{\partial x}. \end{cases} \tag{10}$$

By Assumption 1 and Definition 1, and choosing K_σ to be nonsingular and large enough, and then setting $\dot{x} = 0$ and $x = 0$ of the closed-loop system (10), we obtain

$$g(0, w)K_\sigma\phi(\sigma_0) = f(0, w). \tag{11}$$

Therefore, we ensure that there is a unique solution σ_0, and then $(0, \sigma_0)$ is the unique equilibrium point of the closed-loop system (10) in the domain of interest. At the equilibrium point, $y = r$, irrespective of the value of w.

Now, the design task is to provide the conditions on the controller parameters so that $(0, \sigma_0)$ is an asymptotically stable equilibrium point of the closed-loop system (10) in the control domain of interest. This is established in the following theorem.

Theorem 1. Under Assumptions 1–3, if there exists a positive define diagonal matrix K_σ such that

$$\lambda_m(g_0 K_\sigma \phi(a_\beta)) \geqslant \|f(0, w)\| \tag{12}$$

$$c_3 > c_4 l_g^x c_\sigma \sqrt{m}\|K_\sigma\| \tag{13}$$

hold, then $(0, \sigma_0)$ is an exponentially stable equilibrium point of the closed-loop system (10). Moreover, if all assumptions hold globally, then it is globally exponentially stable.

Proof. To carry out the stability analysis, we consider the following Lyapunov function candidate:

$$V(x, \phi(\sigma) - \phi(\sigma_0)) = V_x(x) + (\phi(\sigma) - \phi(\sigma_0))^{\mathrm{T}} g(0, w) K_\sigma \frac{\phi(\sigma) - \phi(\sigma_0)}{2}. \tag{14}$$

Obviously, the Lyapunov function candidate (14) is positive define. Therefore, our task is to show that its time derivative along the trajectories of the closed-loop system (10) is negative define, which is given by

$$\dot{V}(x, \phi(\sigma) - \phi(\sigma_0)) = \frac{\partial V_x(x)}{\partial x}(f(x, w) + g(x, w)u_x(x) - g(x, w)K_\sigma\phi(\sigma)) + \frac{\partial V_x(x)}{\partial x} g(0, w)K_\sigma(\phi(\sigma) - \phi(\sigma_0)). \tag{15}$$

Substituting (11) into (15), we obtain

$$\dot{V}(x, \phi(\sigma) - \phi(\sigma_0)) = \frac{\partial V_x(x)}{\partial x}(f(x, w) - f(0, w) + g(x, w)u_x(x)) - \frac{\partial V_x(x)}{\partial x}(g(x, w) - g(0, w))K_\sigma\phi(\sigma). \tag{16}$$

Now, using (8) and Lemma 1, we have

$$\sigma_i(t) = \int_0^t \left(\frac{\mathrm{d}\phi_i(\sigma_i)}{\mathrm{d}\sigma_i}\right)^{-1} \frac{\partial V_x(x)}{\partial x_i}\mathrm{d}\tau \leqslant \gamma_x^i \int_0^t \left(\frac{\mathrm{d}\phi_i(\sigma_i)}{\mathrm{d}\sigma_i}\right)^{-1} \mathrm{d}\tau \leqslant c_\infty^i \gamma_x^i$$

and obtain

$$\|K_\sigma\phi(\sigma)\| \leqslant c_\sigma\sqrt{m}\|K_\sigma\| \tag{17}$$

where

$$\gamma_x^i = \max_{x\in D_x}\left(\left\|\frac{\partial V_x(x)}{\partial x_i}\right\|\right)$$

$$c_\infty^i = \lim_{t\to\infty}\int_0^t \left(\frac{\mathrm{d}\phi_i(\sigma_i)}{\mathrm{d}\sigma_i}\right)^{-1} \mathrm{d}\tau$$

$$c_\sigma = \max_{i\in m}(\phi_i(c_\infty^i\gamma_x^i)), i = 1, 2, \cdots, m.$$

Inserting (4), (7), (8) and (17) into (16), we obtain

$$\begin{aligned}&\dot{V}(x,\phi(\sigma)-\phi(\sigma_0)) \leqslant\\&\quad -c_3\|x\|^2+c_4 l_g^x c_\sigma\sqrt{m}\|K_\sigma\|\ \|x\|^2=\\&\quad -(c_3-c_4 l_g^x c_\sigma\sqrt{m}\|K_\sigma\|)\|x\|^2.\end{aligned} \tag{18}$$

Using the fact that the Lyapunov function (14) is a positive define function and its time derivative is a negative define function if inequalities (12) and (13) hold, we conclude that the closed-loop system (10) is stable. In fact, $\dot{V}=0$ means $x=0$ and $\sigma=\sigma_0$. By invoking the LaSalle′s invariance principle[21], it is easy to know that the closed-loop system (10) is exponentially stable.

Corollary 1. If function $g(x,w)$ is equal to a constant, then the integrator can be taken as $\dot{\sigma}^{\rm T}=\frac{\partial V_x(x)}{\partial x}$ or $\dot{\sigma}^{\rm T}=\left(\frac{{\rm d}\phi(\sigma)}{{\rm d}\sigma}\right)^{-1}\frac{\partial V_x(x)}{\partial x}$. Thus, under Assumptions 1 and 3, we only need to choose the gain matrix K_σ to be nonsingular and large enough such that inequality (12) holds, then $(0,\sigma_0)$ is an exponentially stable equilibrium point of the closed-loop system (10). Moreover, if all assumptions hold globally, then it is globally exponentially stable. The proof can follow the similar argument and procedure. It is omitted because of the limited space.

Discussion 1. Although Theorem 1 is proved by resorting to a stabilizing controller $u_x(x)$ along with a Lyapunov function $V_x(x)$, the rationality of the general convex integral control still can be verified because the stabilizing controller can be designed by using the linear system theory, feedback linearization technique, sliding mode technique and so on. As a result, there is great freedom in the choice of $u_x(x)$ and $V_x(x)$ such that the control engineers can choose the most appropriate control input $u_x(x)$ in hand to design their own general integral controller. This is one reason that why our integral control is called the "general" one. Moreover, it is clear that the conditions of (12) and (13) are mutual restrictions, therefore some compromises between them are needed in practice. Otherwise, we need to redesign the control law $u_x(x)$ such that the conditions of (12) and (13) all hold by adjusting the values of c_3 and c_4.

Discussion 2. The proof of Theorem 1 seems to be very simple, but the fact is there are two troubles that have been concealed in the stability analysis. One is that integral control action must be bounded, another is how to cancel the terms on $\phi(\sigma)-\phi(\sigma_0)$ in the time derivative of Lyapunov function. Therefore, an ingenious method is proposed as follows: The integrator is taken as $\dot{\sigma}^{\rm T}=\left(\frac{{\rm d}\phi(\sigma)}{{\rm d}\sigma}\right)^{-1}\frac{\partial V_x(x)}{\partial x}$, which is obtained by differentiating function $\phi(\sigma)$ and using the partial derivative of Lyapunov function $\frac{\partial V_x(x)}{\partial x}$ as the indispensable component of the integrator, and we get $\dot{\phi}^{\rm T}(\sigma)=\frac{\partial V_x(x)}{\partial x}$. Thus, we not only obtain a bounded integral control action $K_\sigma\phi(\sigma)$ but also cancel the terms on $\phi(\sigma)-\phi(\sigma_0)$ in the time derivative of Lyapunov function, then Theorem 1 can be established. Moreover, this results in a new integrator with a convex function gain $\left(\frac{{\rm d}\phi(\sigma)}{{\rm d}\sigma}\right)^{-1}$; see Fig. 2. This is why the control law (9) is called the general convex integral control.

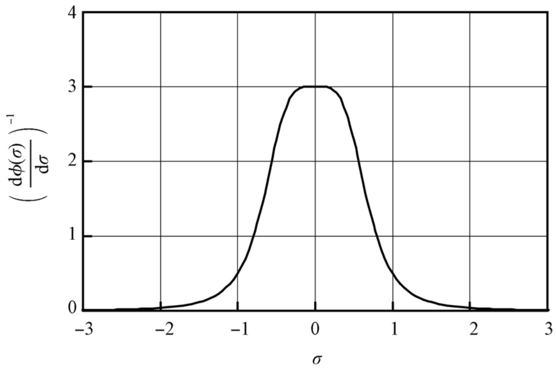

Fig. 2 The convex function gain curve

Discussion 3. Compared with the general concave integral control proposed in [20], the main differences are as follows: 1) In terms of the integral control actions, the concave one is formed by the bounded nonlinear function and the convex one is shaped by the unbounded nonlinear function. 2) In terms of the gain functions of the integrators, the concave one is unbounded and the convex one is bounded. 3) In terms of the integrator outputs, the concave one could tend to infinity and the convex one is bounded in the time domain.

Remark 1. From the control law (9) and the analysis procedure above, it is easy to see that the highlight point of this integral control strategy is that the integral control action seems to be infinity but it factually is finite in the time domain because the integrator output is bounded in the time domain. This means that this kind of integral control can devote its mind to counteract the unknown constant uncertainties and filter out the other action, and then the stability analysis is easy to be achieved in theory and actuator saturation is easy to be removed in practice.

Based on these statements above, it is not hard to know that all of them constitute a simple and ingenious method to design general integral control.

4 Example and simulation

Consider the pendulum system[21] described by

$$\ddot{\theta}=-a\sin\theta-b\dot{\theta}+cT$$

where $a=\frac{g}{l}>0$, $b=\frac{k}{m}>0$, $c=\frac{1}{ml^2}>0$, θ is the angle subtended by the rod and the vertical axis, and T is the torque applied to the pendulum. View T as the control input and suppose we want to regulate θ to δ. Taking $x_1=\theta-\delta$, $x_2=\dot{\theta}$ and $u=T$, the pendulum system can be written as

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=-a\sin(x_1+\delta)-bx_2+cu.\end{cases} \tag{19}$$

It can be verified that the desired equilibrium point is $x_0=[0\ 0]^{\rm T}$ and $u_0=\frac{a\sin(\delta)}{c}$ is the steady-state control that is needed to maintain equilibrium at x_0. Thus, the control law in Assumption 3 can be taken as

$$u_x(x)=-k_1x_1-k_2x_2$$

where k_1 and k_2 are all positive constants.

Substituting $u_x(x)$ into (19) and removing the constant term $-a\sin(\delta)$ and linearization of the system about the origin obtains

$$\dot{x} = Ax \tag{20}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -(a\cos(\delta) + ck_1) & -(b + ck_2) \end{bmatrix}.$$

Now, using the linear system theory, the choice of $k_1 > -\frac{a\cos(\delta)}{c}$ and $k_2 > 0$ ensures that matrix A is Hurwitz for all the parameter perturbations on $a > 0$, $b > 0$, $c > 0$ and $\delta \in [-\pi, \pi]$, and then $x = 0$ is an exponentially stable equilibrium point of system (20). Consequently, for any given positive define symmetric matrix Q there is a positive define symmetric matrix P that satisfies Lyapunov equation $PA + A^{\mathrm{T}}P = -Q$, and then the Lyapunov function in Assumption 3 can be taken as $V_x(x) = x^{\mathrm{T}}Px$. Thus, taking $\phi(\sigma) = \sigma + \sigma^5$, $a_\beta = 2$, and choosing k_σ such that $ck_\sigma\phi(a_\beta) > a\sin(\delta)$ holds for all $a > 0$, $c > 0$ and $\delta \in [-\pi, \pi]$, we have a globally exponentially stable controller

$$\begin{cases} u(x,\sigma) = -k_1x_1 - k_2x_2 - k_\sigma(\sigma + \sigma^5) \\ \dot{\sigma} = \dfrac{2(p_{12}x_1 + p_{22}x_2)}{1 + 5\sigma^4}. \end{cases}$$

Now, taking $k_1 = 8$, $k_2 = 3$, $k_\sigma = 10$, $a = 10$, $b = 1$ and $c = 10$, and solving the Lyapunov equation $PA + A^{\mathrm{T}}P = -Q$, we obtain

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 38 & 1 \\ 1 & 0.1 \end{bmatrix}$$

where

$$Q = \begin{bmatrix} 140 & 0 \\ 0 & 4.2 \end{bmatrix}, \quad A = -\begin{bmatrix} 0 & -1 \\ 70 & 31 \end{bmatrix}.$$

In the simulation, the normal parameters are $a = c = 10$ and $b = 1$. In the perturbed case, b and c are reduced to 0.5 and 5, respectively, corresponding to doubling the mass. Moreover, we consider an additive impulse-like disturbance $d(t)$ of magnitude 60 acting on the system input between 18 s and 19 s.

Fig. 3 shows the simulation results under the normal (solid line) and perturbed (dashed line) parameters. The following observations can be made: Under the normal and perturbed cases, the optimum response in the whole domain of interest can all be achieved by a set of control gains, even under the case that the payload is changed abruptly. This demonstrates that general convex integral control has strong robustness, fast convergence, and good flexibility and can more effectively deal with unknown exogenous disturbances, nonlinearity, and uncertainties of dynamics.

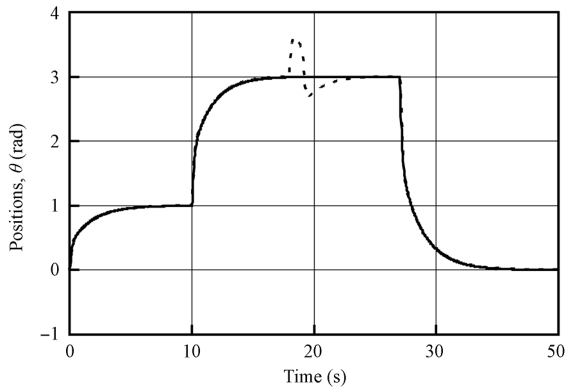

Fig. 3 System output under the normal (solid line) and perturbed (dashed line) cases

5 Conclusions

This paper proposed a new general integral control, named general convex integral control. The main contributions are as follows: 1) An unbounded nonlinear function set is defined, which is used to form the integral control action and educe the convex function gain integrator. 2) A fire-new convex function gain integrator, whose output is bounded in the time domain, is proposed. 3) The lemma to ensure the integrator output to be bounded in the time domain is proposed. 4) By using Lyapunov method along with the LaSalle′s invariance principle, the theorem to ensure regionally as well as semi-globally asymptotic stability is established only by some bounded information. Moreover, the highlight point of this integral control strategy is that the integral control action seems to be infinity but it factually is finite in the time domain. Therefore, a simple and ingenious method to design the general integral control is founded.

Simulation results not only confirmed the effectiveness of the general convex integral control but also showed that it has strong robustness, fast convergence, and good flexibility and can more effectively deal with unknown exogenous disturbances, nonlinearity, and uncertainties of dynamics.

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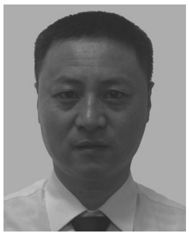

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