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A High-order Internal Model Based Iterative Learning Control Scheme for Discrete Linear Time-varying Systems

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Abstract: In this paper, an iterative learning control algorithm is proposed for discrete linear time-varying systems to track iteration-varying desired trajectories. A high-order internal model (HOIM) is utilized to describe the variation of desired trajectories in the iteration domain. In the sequel, the HOIM is incorporated into the design of learning gains. The learning convergence in the iteration axis can be guaranteed with rigorous proof. The simulation results with permanent magnet linear motors (PMLM) demonstrate that the proposed HOIM based approach yields good performance and achieves perfect tracking.

Keywords: Iterative learning control, high-order internal model, discrete linear time-varying systems, iteration-varying desired trajectory, permanent magnet linear motors.

1 Introduction

The idea of iterative learning control (ILC) was generated firstly by Uchiyama in 1978[1]. The basic objective of ILC is to overcome the imperfect knowledge of the plant using previous tracking information and to achieve output tracking through repetition[2−5]. This makes ILC schemes particularly useful in applications with repetitive tasks, such as robot manipulator[6], tracking problem of the taking-off and landing planes[7], freeway traffic control[8], waste-water treatment[9], piezoelectric positioning stage system[10], heat flux boundary control[11], etc. The ILC algorithm has been developed widely from classical ILC[12], higher-order ILC[13,14], robust ILC[15,16], optimal ILC[17] to adaptive ILC[18−22] for the last two decades.

In the conventional ILC, the desired trajectories are assumed to be invariant from iteration to iteration. However, the tracking tasks can be different in different iterations sometimes. For instance, a robot arm is scheduled to move objects tracking one trajectory in the time interval $[0, T]$, then it may move objects tracking another trajectory in the next working process. And another example of a robot manipulator moving an object of mass m in the odd cycles and another object of mass $2m$ in the even cycle, as was explained in [23]. All these iteration varying problems are called non-repetitiveness in general. Thus, it is helpful that ILC could be designed to harness the non-repetitiveness. Considering the non-repetitiveness of the dynamic system, it can be divided into two circumstances: known variation pattern and unknown variation pattern of non-repetitiveness. To reflect known variation pattern of non-repetitiveness, an effective approach is to incorporate a high-order internal model (HOIM) to describe the non-repetitiveness in iteration domain. For instance, the iteration-varying desired trajectories in a robot manipulator can be formulated by an HOIM. In [24], a survey on HOIM applications of different ILC problems was given. According to the internal model principle[25], the generator of known variation pattern of non-repetitiveness must be involved in the ILC controller to improve tracking performance. To reflect iteration varying property of reference trajectories, a high-order internal model (HOIM) was introduced to describe the variations along the iteration axis in [26]. An HOIM-based ILC has been studied recently for iteration-varying reference trajectories for continuous-time linear time-varying systems, and both the initial resetting condition and extension to nonlinear cases are also explored[27]. And for continuous-time nonlinear systems, an HOIM-based P-type ILC scheme was presented for tracking iteration-varying trajectories[28] . In [27, 28], time-weighted norm method was used to prove the learning convergence in the iteration axis. Moreover, for nonlinear system with time-iteration-varying parameters, an HOIM-based ILC was proposed and composite energy function was used to derive convergence properties of the method[29].

In real implementation, it is necessary to discretize the system dynamics. Researchers have devoted a lot of efforts on ILC for discrete-time systems. A 2-D system theory based ILC has been studied for discrete linear time-invariant systems[30]. A unified learning scheme was considered for the initial shift problem of nonlinear systems with well-defined relative degree[31]. In [32], a feed-forward ILC method was designed and updated by past control data

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in the previous trials for a class of discrete-time nonlinear time-varying systems with initial state error, input disturbance and output measurement noise. Another ILC design was proposed to overcome the uncertainties and disturbances for discrete time nonlinear dynamic system[33]. And an adaptive ILC scheme was presented to deal with time-varying parametric uncertainties for discrete-time system and can achieve point-wise convergence over a finite interval under random initial states and iteration-varying reference trajectory[34]. Yu et al.[35] presented ILC for discrete-time systems with unknown control directions under the framework of adaptive ILC. An adaptive iterative learning control method combining with an n-step ahead predictor was employed for a class of nonlinear output feedback discrete-time systems with random initial conditions and iteration-varying desired trajectories[36].

As a summary of the above discussions, ILC for discrete-time systems tracking iteration-varying desired trajectories is definitely an important issue in practical applications. In this work, an HOIM-based P-type ILC law is proposed for a class of discrete linear time-varying systems with the relative degree of one, and a sufficient condition is derived under which the convergence of the learning process is guaranteed. The effectiveness of the proposed ILC algorithm is illustrated by means of simulation results with permanent magnet linear motors (PMLM).

2 Problem formulation

Consider a discrete linear time-varying system with the relative degree of one as

$$\begin{cases} x_k(t+1) = A(t)\,x_k(t) + B(t)\,u_k(t) \\ y_k(t) = C(t)\,x_k(t) \end{cases} \tag{1}$$

where the subscripts k and t denote the iteration number and discrete time index, matrix $A(t) \in \mathbf{R}^{n\times n}$, $B(t) \in \mathbf{R}^{n\times m}$, $C(t) \in \mathbf{R}^{m\times n}$ are uniformly bounded for all $t \in [0,T]$, state $x_k(t) \in \mathbf{R}^n$, $u_k(t)$ and $y_k(t)$ are input and output vectors of the k-th iteration, $u_k(t) \in \mathbf{R}^m$, $y_k(t) \in \mathbf{R}^m$, $C(t+1)\,B(t)$ is full column rank, and the system is minimum phased.

In this work, we consider the iteration-varying reference trajectories $y_{k+1}^r(t)$ which are related to the reference trajectories of the past iterations. The variations of desired trajectories along the iteration axis can be expressed by a high-order internal model as

$$y_{k+1}^r(t) = h_1 y_k^r(t) + h_2 y_{k-1}^r(t) + \cdots + h_m y_{k-(m-1)}^r(t) \tag{2}$$

where h_i, $i = 1, 2, \cdots, m$, are coefficients of a polynomial

$$S(z) = z^m - h_1 z^{m-1} - h_2 z^{m-2} - \cdots - h_m. \tag{3}$$

From (2), we can see that the trajectory of the $(k+1)$-th iteration is related to the trajectories of the past m iterations and the HOIM is essentially an auto-regression model in the iteration domain. Then, we consider a shift operator ω^{-1}[37] which was introduced with the property that $\omega^{-1} y_{k+1}^r(t) = y_k^r(t)$, $\forall t \in [0,T]$. Equality (2) can be rewritten as

$$y_{k+1}^r(t) = H\left(\omega^{-1}\right) y_k^r(t) \tag{4}$$

where polynomial $H\left(\omega^{-1}\right) = h_1 + \cdots + h_m \omega^{-m+1}$ is used to describe the high-order internal model.

Definition 1. The λ-norm[38] is defined for function $f(t)$ as $\|f(t)\|_\lambda = \sup\limits_{t\in[0,T]} \mathrm{e}^{-\lambda t}\|f(t)\|$. And the λ-norm for function $H\left(\omega^{-1}\right) f_k(t)$ is defined as

$$\left\|H\left(\omega^{-1}\right) f_k(t)\right\|_\lambda = |h_1|\,\|f_k(t)\|_\lambda + \cdots + |h_m|\,\|f_{k-m+1}(t)\|_\lambda. \tag{5}$$

Let $e_{k+1}(t) = y_{k+1}^r(t) - y_{k+1}(t)$ be the tracking error at the time instant $t \in [0,T]$ of the $(k+1)$-th iteration. The control objective is to find a control input sequence $u_{k+1}(t)$ for plant (1) such that the output errors $e_{k+1}(t)$ converge to zero as $k \to \infty$. For the known variation pattern of desired trajectories, the ILC law must embed the characteristic of variation according to internal model principle. To achieve this aim, a P-type ILC which employs the m-th order internal model (4) is given as

$$\begin{aligned} u_{k+1}(t) =& h_1 u_k(t) + h_2 u_{k-1}(t) + \cdots + h_m u_{k-(m-1)}(t) + \\ & \gamma_1 e_k(t+1) + \gamma_2 e_{k-1}(t+1) + \cdots + \\ & \gamma_m e_{k-(m-1)}(t+1) = \\ & H\left(\omega^{-1}\right) u_k(t) + \Gamma\left(\omega^{-1}\right) e_k(t+1) \end{aligned} \tag{6}$$

where $\Gamma\left(\omega^{-1}\right) = \gamma_1 + \gamma_2\omega^{-1} + \cdots + \gamma_m\omega^{-m+1}$ and γ_j is the learning gain. From (6), we can see that to harness the non-repetitiveness in the desired trajectories, the ILC law has to be high-order in the iteration direction.

With respect to the system dynamics (1) and desired trajectories (4), we have the following assumptions:

Assumption 1. Initialization is satisfied throughout repeated trainings, i.e.,

$$x_{k+1}(0) = H\left(\omega^{-1}\right) x_k(0). \tag{7}$$

Assumption 2. The matrix $Q(t)$ is defined with bound $b_Q = \sup_{t\in[0,T]} Q(t)$ ($Q \in \{A, B, C\}$).

Assumption 3. The polynomial $S(z)$ is stable or critically stable which means that all roots of $S(z) = 0$ are within the unit circle or at least one root is lying on the unit circle.

Remark 1. If all the roots of $S(z) = 0$ are within the unit circle, the polynomial is stable and the desired trajectories expressed by HOIM will converge to zero as the iteration number approaches infinity. If at least one root is lying on the unit circle, the polynomial is stable and the desired trajectories will vary periodically and never converge to zero. For example, there are two characteristic roots of the polynomial $S_1(z) = z^2 - 2\cos(0.1)z + 1$ lying on the unit circle. The desired trajectories generated by $S_1(z)$ are $y_{k+1}^r(t) = 2\cos(0.1)y_k^r(t) - y_{k-1}^r(t)$, $k = 2, 3, \cdots$. According to z-transform, we have $y_k^r(t) = A_1(t)\cos(0.1k) + A_2(t)\sin(0.1k)$, where $A_1(t)$ and $A_2(t)$ are time-varying coefficients which are iteration independent and only determined by initial values. It can be seen clearly that the second-order internal model constructed by $S_1(z)$ will change periodically and never converge to zero in iteration domain.

3 Learning convergence analysis

In this section, we will discuss the convergence property of the proposed HOIM-based ILC.

Theorem 1. For the discrete linear time-varying system (1), given the HOIM-based desired trajectories (4), we consider that the Assumptions 1 – 3 are satisfied. If the learning gain γ_j is chosen such that the asymptotic stability of the following polynomial is guaranteed

$$P(z) = z^m - \zeta_{t,k} z^{m-1} - \cdots - \zeta_{t,k-m+1} \tag{8}$$

where $\zeta_{t,j} = \|h_{k+1-j} - C(t+1)B(t)\gamma_{k+1-j}\|$, $t \in [0,T]$, $j \in [k, \cdots, k-m+1]$, then the output error $e_k(t)$ converges to zero in $[0,T]$ as $k \to \infty$ under the HOIM-based ILC law (6), i.e., $\lim_{k\to\infty} e_k(t) = 0$.

Proof. First substituting desired trajectories (4) to the $(k+1)$-th tracking error $e_{k+1}(t+1)$, we obtain

$$\begin{aligned} e_{k+1}(t+1) =& H(\omega^{-1}) y_k^r(t+1) - y_{k+1}(t+1) - \\ & H(\omega^{-1}) y_k(t+1) + H(\omega^{-1}) y_k(t+1) = \\ & H(\omega^{-1}) e_k(t+1) - y_{k+1}(t+1) + \\ & H(\omega^{-1}) y_k(t+1). \end{aligned} \tag{9}$$

Considering the system dynamics (1), we get

$$y_{k+1}(t+1) = C(t+1) x_{k+1}(t+1) \tag{10}$$

and

$$y_k(t+1) = C(t+1) x_k(t+1). \tag{11}$$

Then, substituting (10) and (11) to (9) yields

$$\begin{aligned} & e_{k+1}(t+1) = H(\omega^{-1}) e_k(t+1) + \\ & \quad H(\omega^{-1}) C(t+1) x_k(t+1) - C(t+1) x_{k+1}(t+1). \end{aligned} \tag{12}$$

From the HOIM-based ILC law (6), we get

$$\begin{aligned} & e_{k+1}(t+1) = H(\omega^{-1}) e_k(t+1) - \\ & \quad C(t+1) B(t) \Gamma(\omega^{-1}) e_k(t+1) - C(t+1) A(t) \times \\ & \quad [x_{k+1}(t) - H(\omega^{-1}) x_k(t)]. \end{aligned} \tag{13}$$

Taking the norms of (13) and considering Assumption 2, it can be derived that

$$\begin{aligned} & \|e_{k+1}(t+1)\| \le \|H(\omega^{-1}) - C(t+1)B(t)\Gamma(\omega^{-1})\| \times \\ & \quad \|e_k(t+1)\| + b_C b_A \|x_{k+1}(t) - H(\omega^{-1}) x_k(t)\|. \end{aligned} \tag{14}$$

To evaluate $x_{k+1}(t) - H(\omega^{-1}) x_k(t)$, from plant (1), we have

$$\begin{aligned} & x_{k+1}(t+1) - H(\omega^{-1}) x_k(t+1) = \\ & \quad A(t) x_{k+1}(t) + B(t) u_{k+1}(t) - \\ & \quad H(\omega^{-1}) [A(t) x_k(t) + B(t) u_k(t)]. \end{aligned} \tag{15}$$

Taking the norms of (15) yields

$$\begin{aligned} & \|x_{k+1}(t+1) - H(\omega^{-1}) x_k(t+1)\| \le \\ & \quad \|A(t)[x_{k+1}(t) - H(\omega^{-1}) x_k(t)]\| + \\ & \quad \|B(t)[u_{k+1}(t) - H(\omega^{-1}) u_k(t)]\|. \end{aligned} \tag{16}$$

Applying Assumption 2 and substituting (6) into (16), we have

$$\begin{aligned} & \|x_{k+1}(t+1) - H(\omega^{-1}) x_k(t+1)\| \le \\ & \quad b_A \|x_{k+1}(t) - H(\omega^{-1}) x_k(t)\| + \\ & \quad b_B \|\Gamma(\omega^{-1}) e_k(t+1)\|. \end{aligned} \tag{17}$$

When $t=0$, the above inequality can be rewritten as

$$\begin{aligned} & \|x_{k+1}(1) - H(\omega^{-1}) x_k(1)\| \le \\ & \quad b_A \|x_{k+1}(0) - H(\omega^{-1}) x_k(0)\| + \\ & \quad b_B \|\Gamma(\omega^{-1}) e_k(1)\|. \end{aligned} \tag{18}$$

Taking Assumption 1 into account, we have

$$\|x_{k+1}(1) - H(\omega^{-1}) x_k(1)\| \leqslant b_B \|\Gamma(\omega^{-1}) e_k(1)\|. \tag{19}$$

Similarly, when $t=1$, we get

$$\begin{aligned} & \|x_{k+1}(2) - H(\omega^{-1}) x_k(2)\| \le \\ & \quad b_A \|x_{k+1}(1) - H(\omega^{-1}) x_k(1)\| + \\ & \quad b_B \|\Gamma(\omega^{-1}) e_k(2)\|. \end{aligned} \tag{20}$$

Substituting (19) into (20), it can be obtained that

$$\begin{aligned} & \|x_{k+1}(2) - H(\omega^{-1}) x_k(2)\| \le \\ & \quad b_A b_B \|\Gamma(\omega^{-1}) e_k(1)\| + b_B \|\Gamma(\omega^{-1}) e_k(2)\|. \end{aligned} \tag{21}$$

Following the same procedure, we can now conclude that for $t \in [0,T]$,

$$\begin{aligned} & \|x_{k+1}(t) - H(\omega^{-1}) x_k(t)\| \le \\ & \quad \sum_{j=0}^{t-1} b_A^{t-1-j} b_B \|\Gamma(\omega^{-1}) e_k(j+1)\|. \end{aligned} \tag{22}$$

Substituting (22) into (14), we have

$$\begin{aligned} & \|e_{k+1}(t+1)\| \le \\ & \quad \|H(\omega^{-1}) - C(t+1)B(t)\Gamma(\omega^{-1})\| \|e_k(t+1)\| + \\ & \quad b_C \sum_{j=0}^{t-1} b_A^{t-j} b_B \|\Gamma(\omega^{-1}) e_k(j+1)\|. \end{aligned} \tag{23}$$

Multiplying both sides of (23) by $\mathrm{e}^{-\lambda(t+1)}$, we can derive from the definition of λ-norm that

$$\begin{aligned} & \sup_{t\in[0,T]} \mathrm{e}^{-\lambda(t+1)} \|e_{k+1}(t+1)\| \le \\ & \quad \|H(\omega^{-1}) - C(t+1)B(t)\Gamma(\omega^{-1})\| \|e_k\|_\lambda + \\ & \quad \sup_{t\in[0,T]} \mathrm{e}^{-\lambda(t+1)} b_B b_C \sum_{j=0}^{t-1} b_A^{t-j} \|\Gamma(\omega^{-1}) e_k(j+1)\|. \end{aligned} \tag{24}$$

Since

$$\sup_{t\in[0,\,T]} \mathrm{e}^{-\lambda(t+1)} b_B b_C \sum_{j=0}^{t-1} b_A^{t-j} \left\| \Gamma\left(\omega^{-1}\right) e_k\left(j+1\right) \right\| \leq$$
$$a^2 \left\| \Gamma\left(\omega^{-1}\right) e_k \right\|_\lambda \sum_{j=0}^{t-1} a^{t(1-\lambda)} a^{(\lambda-1)j} \leq$$
$$a^2 \frac{1-a^{T(1-\lambda)}}{a^{(\lambda-1)}-1} \left\| \Gamma\left(\omega^{-1}\right) e_k \right\|_\lambda \tag{25}$$

where $a = \max\{e, b_A, b_B, b_C\}$, by expressing the HOIM in the above inequality, we can derive

$$a^2 \frac{1-a^{T(1-\lambda)}}{a^{(\lambda-1)}-1} \left\| \Gamma\left(\omega^{-1}\right) e_k \right\|_\lambda =$$
$$a^2\delta\left|\gamma_1\right| \left\|e_k\right\|_\lambda + \cdots + a^2\delta\left|\gamma_m\right| \left\|e_{k-m+1}\right\|_\lambda \tag{26}$$

where $\delta = \frac{1-a^{T(1-\lambda)}}{a^{(\lambda-1)}-1}$. Then, we have

$$\left\| H\left(\omega^{-1}\right) - C\left(t+1\right)B\left(t\right)\Gamma\left(\omega^{-1}\right) \right\| \left\|e_k\right\|_\lambda =$$
$$\left\|h_1 - C\left(t+1\right)B\left(t\right)\gamma_1\right\| \left\|e_k\right\|_\lambda + \cdots +$$
$$\left\|h_m - C\left(t+1\right)B\left(t\right)\gamma_m\right\| \left\|e_{k-m+1}\right\|_\lambda . \tag{27}$$

Substituting (26) and (27) into (24), we can obtain that

$$\left\|e_{k+1}\right\|_\lambda \leq \left\|h_1 - C\left(t+1\right)B\left(t\right)\gamma_1\right\| \left\|e_k\right\|_\lambda +$$
$$a^2\delta\left|\gamma_1\right| \left\|e_k\right\|_\lambda + \cdots +$$
$$\left\|h_m - C\left(t+1\right)B\left(t\right)\gamma_m\right\| \left\|e_{k-m+1}\right\|_\lambda +$$
$$a^2\delta\left|\gamma_m\right| \left\|e_{k-m+1}\right\|_\lambda . \tag{28}$$

Now, we can write the above inequalities from $t = 0$ to $t = T$. When $t = 0$, we have

$$\left\|e_{k+1}\left(1\right)\right\|_\lambda \leqslant \rho_{0,k} \left\|e_k\left(1\right)\right\|_\lambda + \cdots + \rho_{0,k-m+1} \left\|e_{k-m+1}\left(1\right)\right\|_\lambda . \tag{29}$$

And when $t = T$, we get

$$\left\|e_{k+1}\left(T+1\right)\right\|_\lambda \leq \rho_{T,k} \left\|e_k\left(T+1\right)\right\|_\lambda + \cdots +$$
$$\rho_{T,k-m+1} \left\|e_{k-m+1}\left(T+1\right)\right\|_\lambda \tag{30}$$

where $\rho_{i,j} = \left\|h_{k+1-j} - C\left(i+1\right)B\left(i\right)\gamma_{k+1-j}\right\| + a^2\delta\left|\gamma_{k+1-j}\right|$, $i \in [0, 1, \cdots, T]$, $j \in [k, k-1, \cdots, k-m+1]$. Thus, we can deduce that

$$\begin{bmatrix} \left\|e_{k+1}\left(1\right)\right\|_\lambda \\ \cdots \\ \left\|e_{k+1}\left(T+1\right)\right\|_\lambda \end{bmatrix} \leq F_k \begin{bmatrix} \left\|e_k\left(1\right)\right\|_\lambda \\ \cdots \\ \left\|e_k\left(T+1\right)\right\|_\lambda \end{bmatrix} + \cdots +$$
$$F_{k-m+1} \begin{bmatrix} \left\|e_{k-m+1}\left(1\right)\right\|_\lambda \\ \cdots \\ \left\|e_{k-m+1}\left(T+1\right)\right\|_\lambda \end{bmatrix} \tag{31}$$

where $F_j = \begin{bmatrix} \rho_{0,j} & 0 & \cdots & 0 \\ 0 & \rho_{1,j} & \cdots & 0 \\ \cdots & \cdots & & \cdots \\ 0 & 0 & \cdots & \rho_{T,j} \end{bmatrix}$.

From (31), we can see that the convergence of $\left\|e_{k+1}\right\|_\lambda$ is determined by F_s, $s \in [k, k-1, \cdots, k-m+1]$. Consider δ in (26), we can see that δ is arbitrarily small with sufficiently large λ. Thus, we can obtain that the convergence of $\left\|e_{k+1}\right\|_\lambda$ is determined by $P\left(z\right) = z^m - \zeta_{t,k}z^{m-1} - \cdots - \zeta_{t,k-m+1}$, where $\zeta_{t,j} = \left\|h_{k+1-j} - C\left(t+1\right)B\left(t\right)\gamma_{k+1-j}\right\|$, $t \in [0, T]$, $j \in [k, \cdots, k-m+1]$. If all eigenvalues of $P\left(z\right)$ are inside the unit circle, we have $\lim_{k\to\infty} \left\|e_k\right\|_\lambda = 0$, and from the definition of λ-norm, we get that $\sup_{t\in[0,T]} \left\|e_k\left(t\right)\right\| \to 0$ as $k \to \infty$. Hence, the convergence of $e_k\left(t\right) \to 0$ is obtained as $k \to \infty$. □

4 Simulation example

Consider a direct current motor control problem for velocity tracking. The dynamics of a permanent magnet linear motor (PMLM)[39,40] can be described by

$$\begin{cases} \dot{x}\left(t\right) = v\left(t\right) \\ u\left(t\right) = k_1\psi_f\dot{x}\left(t\right) + Ri\left(t\right) + L\dot{i}\left(t\right) \\ f_l\left(t\right) = m\dot{v}\left(t\right) + f_{fri}\left(t\right) + f_{rip}\left(t\right) + f_{loa}\left(t\right) + f_w\left(t\right) \end{cases} \tag{32}$$

where $x\left(t\right)$ and $f_l\left(t\right)$ are the motor position and the developed force, $v\left(t\right)$ is rotor velocity, $u\left(t\right)$, $i\left(t\right)$, R and L are the voltage, current, resistance and inductance of stator, $k_1 = \frac{\pi}{\tau}$, τ is pole pitch, ψ_f is the flux linkage, m is the rotor mass, $f_{fri}\left(t\right)$, $f_{rip}\left(t\right)$ and $f_{loa}\left(t\right)$ denote the frictional, ripple and applied load forces respectively, the term $f_w\left(t\right)$ includes other uncertainties and disturbances.

Using DQ decomposition theory, we transform the PMLM model (32) by neglecting all the uncertainties and nonlinearities $f_w\left(t\right)$. We assume that $f_{fri}\left(t\right) + f_{rip}\left(t\right) + f_{loa}\left(t\right) = 0$ and $i_d \equiv 0$ to simplify the system. Considering $f_l = k_2\psi_f i_q\left(t\right)$, where $k_2 = 1.5\frac{\pi}{\tau}$, we have

$$\begin{cases} u_q\left(t\right) = k_1\psi_f\dot{x}\left(t\right) + Ri_q\left(t\right) + L_q\dot{i}_q\left(t\right) \\ k_2\psi_f i_q\left(t\right) = m\ddot{x}\left(t\right). \end{cases} \tag{33}$$

Since $\dot{i}_q\left(t\right) = \frac{m\dddot{x}\left(t\right)}{\left(k_2\psi_f\right)}$, substituting it into (33), we get

$$u_q\left(t\right) = \frac{m\dddot{x}\left(t\right)}{k_2\psi_f} + \frac{Rm}{k_2\psi_f}\ddot{x}\left(t\right) + k_1\psi_f\dot{x}\left(t\right). \tag{34}$$

Neglecting the third order differential part of (34), we obtain

$$u_q\left(t\right) = \frac{Rm}{k_2\psi_f}\ddot{x}\left(t\right) + k_1\psi_f\dot{x}\left(t\right). \tag{35}$$

Considering $i_d \equiv 0$ and replacing u_q by u, we have

$$\begin{cases} \dot{x}\left(t\right) = v\left(t\right) \\ \dot{v}\left(t\right) = -\frac{k_1k_2\psi_f^2}{Rm}v\left(t\right) + \frac{k_2\psi_f}{Rm}u\left(t\right) \\ y\left(t\right) = v\left(t\right). \end{cases} \tag{36}$$

With the discrete time interval $\Delta = 10\,\mathrm{ms}$, the operation cycle is $N = \{0, 1, \cdots, 100\}$. Discretizing the system by the

Euler method yields

$$\begin{cases} x(t+1) = v(t)\Delta + x(t) \\ v(t+1) = -\Delta\frac{k_1 k_2 \psi_f^2}{Rm} v(t) + v(t) + \Delta\frac{k_2\psi_f}{Rm} u(t) \\ y(t) = v(t) \end{cases} \quad (37)$$

where $\tau = 0.031\,\mathrm{m}$, $R = 8.6\,\Omega$, $m = 1.635\,\mathrm{kg}$ and $\psi_f = 0.35\,\mathrm{Wb}$, respectively. It is easy to find that (37) can be described as a kind of system as (1) with $A = \begin{bmatrix} 1 & \Delta \\ 0 & 1-\Delta\frac{k_1k_2\psi_f^2}{Rm} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \Delta\frac{k_2\psi_f}{Rm} \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$. From (37), we can see that the product of the output/input coupling matrix $CB = 0.0378$ is full column rank.

The iteration-varying trajectory is given by

$$y_{k+1}^r(t) = 2\cos(10\Delta)\, y_k^r(t) - y_{k-1}^r(t). \quad (38)$$

And the desired trajectories of the first iteration and second iteration are

$$\begin{aligned} y_1^r(t) &= -0.2\Delta^2\left(60\Delta t^3 - 30\Delta^2 t^4 - 30t^2\right) \\ y_2^r(t) &= -0.2\Delta^2\left(60\Delta t^3 - 31\Delta^2 t^4 - 28t^2\right). \end{aligned} \quad (39)$$

The HOIM-based iterative learning control is designed as

$$\begin{aligned} u_{k+1}(t) =& 2\cos(10\Delta)\, u_k(t) - u_{k-1}(t) + \\ & 35e_k(t+1) - 28e_{k-1}(t+1). \end{aligned} \quad (40)$$

We choose the learning control gain as $\gamma_1 = 35$ and $\gamma_2 = -28$ so that $\|2\cos(10\Delta) - CB\gamma_1\| = 0.667 < 1$ and $\|-1 - CB\gamma_2\| = 0.058 < 1$. The corresponding characteristic polynomial is $z^2 - 0.667z - 0.058$ with two eigenvalues inside the unit circle.

Fig. 1 shows that the desired trajectories are changing in the iteration domain continuously. Let $e_{k,\max} = \max_{t\in[0,T]} |e_k(t)|$ be maximum absolute tracking error of time interval $t \in [0, T]$ of the k-th iteration. The tracking results of HOIM-based ILC are illustrated in Fig. 2. The effectiveness of HOIM can be clearly seen.

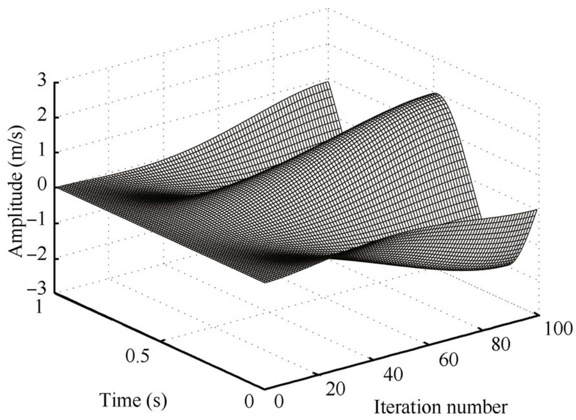

Fig. 1 Iteratively varying reference trajectory

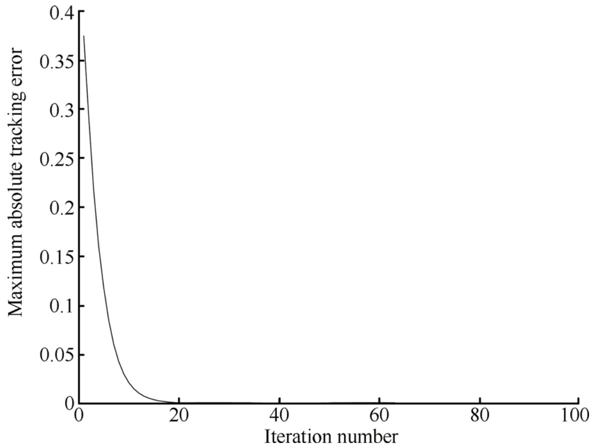

Fig. 2 The maximum absolute tracking error along the iteration axis

To show the time-domain behavior, Fig. 3 gives the tracking profiles at the 4th, 7th and 20th iterations. It can be seen from Fig. 3 that the 7th tracking profile is close to the reference trajectory and the 20th output curve coincides with tracking trajectory. The HOIM based control scheme shows the perfect tracking performance for the discretized PMLM systems (32) to track iteration-varying desired trajectories shown in Fig. 1.

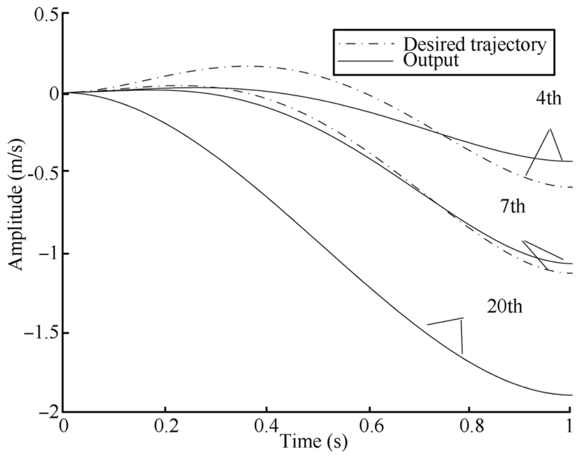

Fig. 3 Tracking profiles of the HOIM-based ILC for the 4th, 7th and 20th iterations

5 Conclusions

An HOIM-based ILC scheme is proposed for a class of discrete-time linear time-varying (LTV) systems with relative degree of one to track iteration-varying desired trajectories. It is shown that under some sufficient conditions on the learning operators, the convergence of the learning system can be guaranteed. The proposed ILC algorithm is applied to the tracking control of PMLM. Simulation results confirm the efficacy of the proposed ILC method.

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