International Journal of Automation and Computing 12(6), December 2015, 588-599
DOI: 10.1007/s11633-015-0927-5

Finite Time Convergent Wavelet Neural Network Sliding Mode Control Guidance Law with Impact Angle Constraint

Qing-Chun Li Wen-Sheng Zhang Gang Han Ying-Hua Zhang
Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China

Abstract: This paper presents a novel guidance law to intercept non-maneuvering targets with impact angle and lateral acceleration command constraints. Firstly, we formulate the impact angle control to track the desired line-of-sight (LOS) angle, which is achieved by selecting the missile′s lateral acceleration to enforce the sliding mode on a sliding surface at impact time. Secondly, we use the Lyapunov stability theory to prove the stability and finite time convergence of the proposed nonlinear sliding surface. Thirdly, we introduce the wavelet neural network (WNN) to adaptively update the additional control command and reduce the high-frequency chattering of sliding mode control (SMC). The proposed guidance law, denoted WNNSMC guidance law with impact angle constraint, combines the SMC methodology with WNN to improve the robustness and reduce the chattering of the system. Finally, numerical simulations are performed to demonstrate the validity and effectiveness of the WNNSMC guidance law.

Keywords: Guidance law, sliding mode control, wavelet neural network, impact angle constraint, finite time convergence.

1 Introduction

Guidance laws that allow a missile to intercept targets at a certain impact angle to enhance the missile′s effectiveness[1−3]. A typical example is the guidance of a tactical ballistic missile towards ground targets, such as ground radar, tank, and gun turret. In this engagement, the terminal impact angle of the missile should have a preferred value of 90° in order to achieve a proper penetration and maximize warhead effectiveness. Though the proportional navigation guidance (PNG)[4] has been widely used in tactical guided weapon systems since the 1970s, it cannot satisfy some specified terminal constraints such as the terminal impact angle. Over the past several decades, many researchers have studied some advanced guidance laws with terminal impact angle. The main outcomes of guidance laws with impact angle constraint have been focused on optimal (OP) control biased PNG and sliding mode control (SMC).

As an application of OP theory, the optimal and suboptimal guidance laws with impact angle constraint for reentry vehicles in the vertical plane were put forward by Kim and Grider[5]. They were further improved by York and Pastrick[6] through matching the guidance law to a first-order autopilot. Furthermore, Song et al.[7] studied an optimal guidance law (OGL) for varying velocity missiles against a maneuvering target, where the guidance law is combined with a target-tracking filter to predict the intercept point. Ryoo et al.[8] proposed a generalized formulation of OGLs with impact angle and minimizing energy constraints for a constant-speed missile with an arbitrary systems order. Lee et al.[9] proposed the OGL with impact angle and terminal acceleration constrains, which minimizes the required acceleration to avoid the command saturation. Ryoo et al.[10] presented a time-to-go weighted optimal guidance with impact angle constraints for a constant missile against the stationary target. Jeon et al.[11, 12] presented a guidance law with impact angle and time constraints based on the minimum principle for a fight vehicle, which can be applied to an efficient salvo attack of antiship missiles with constant velocity. Most of these works were based on linear or near linear models of pursuit kinematics. In general, the linear models of pursuit kinematics are hard to obtain, so the analytic conditions could not be made possibly in real war. Therefore, it is hard to fulfill the guidance goal with impact angle constraint.

As mentioned above, most OGLs are based on minimizing predetermined performance indices with terminal constraints. However, Kim et al.[13] firstly studied a biased PNG law by adding a time-varying bias term in the conventional PNG to achieve the terminal impact angle. Considering small angle assumptions and a linear dynamics, Jeong et al.[14] proposed the guidance law with impact angle based on biased PNG. In their works, they used small angle approximation, resulting in narrow launch envelops and very restricted capture region. To improve this method, Akhil and Ghose[15] designed a guidance law which was capable of achieving a wide range of impact angle constraint. They put forward a modified angle constraint biased PNG, where the required bias term is derived in a closed form consider-

Regular paper
Manuscript received April 25, 2014; accepted September 25, 2014
This paper was supported by National Natural Science Foundation of China (Nos. U1135005 and 61304224), and General Equipment Department Advanced Research Project of China (No. 51301010206).
Recommended by Associate Editor Jyh-Horng Chou

ing non-linear equations. Furthermore, by adding a feedback control to biased PNG to eliminate the time-to-go error, Zhang et al.[16] proposed the guidance law with adjustable coefficients to control the impact time and impact angle simultaneously. Although PNG and its improved forms are widely used in application, there are drawbacks such as weak performances against large maneuvering targets, poor immunity, and low guided precision.

It is well known that sliding mode control (SMC) is an effective robust control scheme and has been widely used to control a variety of systems[17]. The design generally consists of two stages[18]. Firstly, it chooses an appropriate sliding surface, so that the sliding mode dynamics has desired performance. Secondly, a control law is designed, which ensures that the system state converges to the sliding surface in a finite time and remains in it thereafter. Using SMC theory, Shima[19] developed a guidance law that enabled imposing a predetermined interception angle relative to the target′s flight direction. Considering integrated guidance and control for homing missiles with terminal angular constraint, Wei et al.[20] formulated a time-varying integrated guidance and control model with unmatched uncertainties and developed an adaptive multiple sliding surface control in order to solve the multiple states regulation problem. Using the linear quadratic optimal theory and the variable structure control methodology, Hu et al.[21] developed a robust guidance law with impact angle constraint against the stationary target. To reduce the chattering and energy consumption, the radius basis function (RBF) neural network was applied to adaptive update of switching gains. Shi et al.[22] proposed to use RBF neural network to reduce the chattering of SMC, and Rao and Ghose[23] studied the SMC guidance laws with terminal impact angle constraint by using dual sliding surfaces so that the missile could intercept the targets at a desired impact angle. Harl and Balakrishnan[24] presented a sliding mode-based impact time and angle guidance law for engaging a modern warfare ship. In their works, they combined a line-of-sight shaping technique with a second-order sliding mode approach to satisfy the impact time and angle constraints. Adopting a backstepping scheme, Yan et al.[25] developed an adaptive nonlinear integrated guidance and control approach for interception of maneuvering targets (evaders). In addition, Hsueh et al.[26] combined the linear differential game theory with SMC method to design integrated guidance for a nonlinear autopilot system. Compared with OP and biased PNG guidance laws, SMC guidance laws have superior performances in the control of time-varying uncertain systems. However, the undesired chattering produced by the high frequency switching of the control may be considered as a problem in implementing the SMC methods for some real applications[27].

In this paper, we propose a novel guidance law to control the impact angle. The main idea of the proposed guidance law is that the nonlinear sliding surface is designed to introduce a smooth curve missile motion along the desired impact angle frame. By using the wavelet neural network (WNN), we can reduce the high-frequency chattering of acceleration control command. By imposing the lateral acceleration bound on the missile, the proposed guidance law can also meet impact angle and miss distance constraints at a finite time. The main contribution of our work is that the proposed guidance law, called WNNSMC guidance law, combines the SMC methodology with WNN to enhance the robustness of the system. Moreover, the chattering problem of SMC is solved by applying WNN. Therefore, the proposed guidance law not only has good robustness against uncertainty, but also hardly arises the chattering to the system.

This paper is organized as follows. In Section 2, we establish the equations of motion for the missile-target engagement and derive the relation between LOS angle and impact angle. In Section 3, a finite time convergent guidance law based on the SMC methodology is designed to eliminate the miss distance and achieve a desired impact angle. We propose the robust SMC guidance law based on WNN to reduce the high-frequency chattering of acceleration command in Section 4. In Section 5, a basic performance test and comparison simulation studies are carried out to investigate the feasibility of the WNNSMC guidance law. Finally, conclusions and possible future work are discussed in Section 6.

2 Problem formulation

Consider a two-dimensional homing scenario shown in Fig.1 where the missile has a constant velocity V_M and the target constant velocity is V_T. In order to facilitate the whole analysis of the proposed method, we assume $V_M > V_T$, or the target-to-missile ratio $\nu = \frac{V_T}{V_M} < 1$. The missile and the target positions are (x_M, y_M) and (x_T, y_T), respectively. The missile heading angle, velocity, and lateral acceleration are expressed by θ_M, V_M and A_M, respectively. Similarly, the target heading angle, velocity, and lateral acceleration are expressed by θ_T, V_T and A_T, respectively. The lateral acceleration command is normal to the velocity vector of the missile. Since only non-maneuvering targets are considered, the target lateral acceleration $A_T = 0$. q is the LOS angle and R is the relative distance between the missile and the target. η_M and η_T represent the heading error of the missile and the target, respectively. Hence, the non-linear kinematics based engagement dynamics can be described as

$$\dot{R} = V_T \cos \eta_T - V_M \cos \eta_M \tag{1}$$

$$R\dot{q} = V_M \sin \eta_M - V_T \sin \eta_T \tag{2}$$

$$\dot{\theta}_M = \frac{A_M}{V_M}, \dot{\theta}_T = 0 \tag{3}$$

$$\eta_M = q - \theta_M, \eta_T = q - \theta_T. \tag{4}$$

Here, the guidance law tries to force the LOS angle rate ($\dot{q}$) to 0.

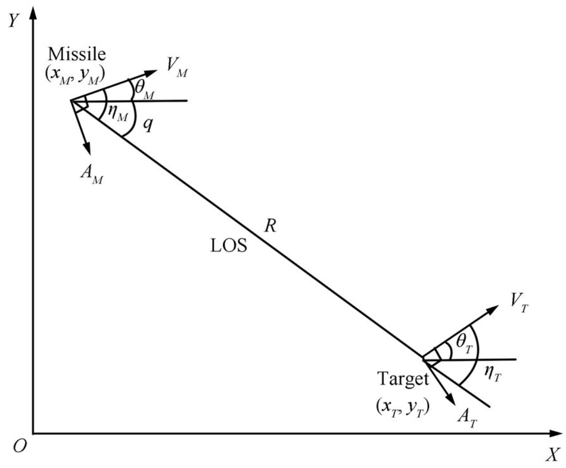


Fig. 1 Homing guidance geometry

With the heading angles of the missile and the target at the impact point as θ_{Mf} and θ_{Tf}, respectively, the impact angle, θ_{imp}, is defined as

$$\theta_{imp} = \theta_{Tf} - \theta_{Mf}. \tag{5}$$

Here, for the case of stationary targets, we define the heading angle of the target as $\theta_{Tf} = 0$, and for the slowly moving targets with definite heading angle $\theta_T \neq 0$, the terminal heading angle of the target satisfies $\theta_{Tf} \neq 0$.

To drive the relation between impact angle θ_{imp} and LOS angle (q), we assume that the missile and the target lie at the collision course. At the collision course, the LOS angle rate ($\dot{q}$) is 0. According to (2) and (4), we can derive the corresponding LOS angle, denoted as q_f, as follows:

$$V_M \sin \eta_{Mf} = V_T \sin \eta_{Tf} \tag{6}$$

$$\eta_{Mf} = q_f - \theta_{Mf}, \eta_{Tf} = q_f - \theta_{Tf}. \tag{7}$$

According to (6) and (7), we can derive that

$$\tan q_f - \tan \theta_{Tf} = -\frac{\sin \theta_{imp}(1 + \tan^2 \theta_{Tf})}{\cos \theta_{imp} + \tan \theta_{Tf} \sin \theta_{imp} - \nu}. \tag{8}$$

Equation (8) can also be written as

$$(1 + \tan q_f \tan \theta_{Tf}) \tan (q_f - \theta_{Tf}) = \\ -\frac{\sin \theta_{imp}(1 + \tan^2 \theta_{Tf})}{\cos \theta_{imp} + \tan \theta_{Tf} \sin \theta_{imp} - \nu}. \tag{9}$$

To a stationary or slowly moving object, we can derive θ_{Tf} is zero at the interception course. Thus, we can obtain the LOS angle at the interception course, q_f, which can be given by

$$q_f = \theta_{Tf} - \arctan\left(\frac{\sin \theta_{imp}}{\cos \theta_{imp} - \nu}\right) \tag{10}$$

where q_f and θ_{imp} are the LOS angle and impact angle at the time of interception, respectively. Based on this relation, we can conclude that there exists a unique relationship between q_f and θ_{imp}. Hence, we can transfer the impact angle constraint to the controlling LOS angle of the missile in the designing of guidance laws.

3 SMC guidance law with finite time convergence

A novel robust guidance law for a stationary or a slowly moving target will be covered in this section. In fact, the missile is affected in flight by the aerodynamic errors, the measure noise, the time-varying velocity, as well as the target maneuvering which would seriously worsen the guidance performance. To enhance the robustness of the guidance system, we derive a robust SMC guidance law based on WNN, which is used to reduce the high-frequency chattering introduced by the inherent discontinuous switching characteristics of SMC.

3.1 SMC guidance law design

To design the lateral acceleration A_M so that the desired LOS angle is achieved, we derive the nonlinear sliding surface as follows.

Differentiating (2) with respect to t, and according to (1) and (4), we can obtain

$$\dot{R}\dot{q} + R\ddot{q} = -\dot{R}\dot{q} + \dot{V}_M \sin(q - \theta_M) - V_M\dot{\theta}_M \cos(q - \theta_M) - \\ \dot{V}_T \sin(q - \theta_T) + V_T\dot{\theta}_T \cos(q - \theta_T). \tag{11}$$

From (11), we can get the second-order derivative of LOS angle, $\ddot{q}$, which can be written as

$$\ddot{q} = \frac{-2\dot{R}\dot{q}}{R} - \frac{A_c}{R} + \frac{A_d}{R} \tag{12}$$

where $A_c = V_M\dot{\theta}_M \cos(q - \theta_M) - \dot{V}_M \sin(q - \theta_M)$ and $A_d = V_T\dot{\theta}_T \cos(q - \theta_T) - \dot{V}_T \sin(q - \theta_T)$. A_c represents the control acceleration of the missile. Similarly, A_d represents the disturbances introduced by the target.

Considering the impact angle constraint, we choose the state variables $x_1 = q(t) - q_f$ and $x_2 = \dot{q}(t)$. Hence, we can get the state equations as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{2\dot{R}}{R} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \\ \begin{bmatrix} 0 \\ -\dfrac{1}{R} \end{bmatrix} A_c + \begin{bmatrix} 0 \\ \dfrac{1}{R} \end{bmatrix} A_d. \tag{13}$$

In this guidance law, the LOS angle rate is required to be zero, which can be written as $\dot{q}_f = 0$, in order to guarantee the missile intercepts the target with the zero miss distance constraint. To satisfy the terminal angle constraint, we make the LOS angle error be zero, written as $q(t_f) - q_f = 0$, where t_f represents the terminal time and q_f represents the desired LOS angle. So consider the following switching function

$$s(t) = x_2(t) - \lambda\frac{\dot{R}(t)}{R(t)}x_1(t) \tag{14}$$

where λ is a positive constant.

To guarantee that the state of system (12) approaches the sliding mode $s = 0$ with a good dynamic performance, a general reaching law is selected as[28]

$$\dot{s}(t) = k\frac{\dot{R}(t)}{R(t)}s(t) - \frac{\varepsilon}{\dot{R}(t)}\mathrm{sgn}(s(t)) \tag{15}$$

where k is the positive constant, $\mathrm{sgn}(s(t))$ is the signum function of s, and ε is a time-varying constant, noted as a reaching law coefficient. At the terminal guidance phase, we consider the missile comes closer and closer to the target, so the $\dot{R}(t)$ is always smaller than 0. When the distance $R(t)$ between the missile and the target is large, the reaching law coefficient ε should properly lower its value in order to make the control acceleration no more than the peak normal load; when $R(t)$ is small, ε should rapidly increase its value to suppress divergence of LOS angle rate $\dot{q}$ so as to guarantee the intercepting accuracy.

According to (13) and (15), and differentiating (14), we can obtain the SMC guidance law as

$$A_c = -(2+\lambda+k)\dot{R}x_2 + \lambda(k+1)\frac{\dot{R}}{R}x_1 + \varepsilon\mathrm{sgn}s + A_d \tag{16}$$

where k and λ are the appropriately chosen positive constants, and A_d is the disturbance introduced by the uncertainty of the target.

Due to variable A_d is regarded as the disturbance, we can simplify the SMC guidance law as follows:

$$A_c = -(2+\lambda+k)\dot{R}x_2 + \lambda(k+1)\frac{\dot{R}}{R}x_1 + \varepsilon\mathrm{sgn}s. \tag{17}$$

In the above guidance law (17), there is a signum function variable $\varepsilon\mathrm{sgn}s$, which may introduce the high-frequency chattering of the system. The drawback of the classical SMC is the well-known chattering phenomenon, which may excite unmodeled high-frequency modes[29]. In order to reduce the high-frequency chatting, we use the WNN to solve this problem.

3.2 Analysis of finite time convergence

In order to design a finite time convergent guidance law, we introduce some results about finite time stability of nonlinear systems[30].

Definition 1. Consider a nonlinear system in the form of

$$\dot{s} = f(s,\ t),\ f(0,\ t) = 0,\ s \in \mathbf{R}^n \tag{18}$$

where $f:\ U_0 \times \mathbf{R} \rightarrow \mathbf{R}^n$ is continuous on $U_0 \times \mathbf{R}$, and U_0 is an open neighborhood of the origin $s = 0$.

The state of the system is said to converge to its local equilibrium $s = 0$ in a finite time, if for any given initial time t_0 and initial state $s(t_0) = s_0 \in U$, there exists a settling time $T \geq 0$, which is dependent on s_0, such that every solution of system (18), $s(t) = \vartheta(t;\ t_0,\ x_0) \in U/\{0\}$, satisfies

$$\begin{cases} \lim\limits_{t\rightarrow T(x_0)} \vartheta(t;\ t_0,\ x_0) = 0, & t \in [t_0,\ T(s_0)) \\ \vartheta(t;\ t_0,\ x_0) = 0, & t \in [T(s_0),\ +\infty). \end{cases} \tag{19}$$

Moreover, if the system equilibrium $s = 0$ (local) is Lyapunov stable and is finite time convergent in a neighborhood of the origin $U \in U_0$, then the system equilibrium is called finite time stable. If $U = \mathbf{R}^n$, the origin is a global finite time stable equilibrium.

Consider the nonlinear system described by (18), we introduce the following lemma to analyze the finite time convergence of the proposed sliding mode surface.

The following lemma will prove useful in the design of guidance law.

Lemma 1[31]**.** Consider the nonlinear system described by (16). Suppose that there is a C^1 (continuously differentiable) function $V(s,t)$ defined in a neighborhood $\widehat{U} \in \mathbf{R}^n$ of the origin, and that there are real numbers $c > 0$ and $0 < \alpha < 1$, such that $V(s,t)$ is positive-definite on $\widehat{U}$ and that $\dot{V}(s,\ t) + cV^{\alpha}(s,\ t) \leq 0$ on $\widehat{U}$. Then, the zero solution of system (18) is finite-time stable. The settling time is given by

$$T_s \leq \frac{V^{1-\alpha}(s_0)}{c(1-\alpha)} \tag{20}$$

where s_0 is a point in the neighborhood. If $\widehat{U} \in \mathbf{R}^n$ and $V(s,t)$ is radically unbounded (if $||s|| \rightarrow +\infty$, then $V(s,\ t) \rightarrow +\infty$), then the zero solution of system (18) is globally finite-time stable.

In order to analyze the stability of the proposed sliding surface, we choose a Lyapunov function V as

$$V = \frac{1}{2}s^2. \tag{21}$$

Taking the time derivative of (21) leads to

$$\dot{V} = s\dot{s} \tag{22}$$

Substituting (15) into (22) gives

$$\dot{V}(t) = k\frac{\dot{R}(t)}{R(t)}s^2(t) - \frac{\varepsilon}{R(t)}|s(t)|. \tag{23}$$

In the interception course, we define the missile is getting closer to the target. Thus, the distance between the missile and the target is becoming smaller with time, so $\dot{R}(t)$ is smaller than 0. When $s(t) \neq 0$, because of $\dot{R}(t) < 0$, we can derive $V < 0$. Hence, the system has a good performance of stability.

According to (23), we can obtain the inequality as

$$\dot{V} \leq \frac{\varepsilon}{R}|s|. \tag{24}$$

Substituting (21) into (24) gives

$$\dot{V}(s,\ t) \leq -\frac{\sqrt{2}\varepsilon}{R(t)}\sqrt{V(s,\ t)}. \tag{25}$$

Due to $0 \leq R(t) \leq R_0$, $\dot{V}(s,\ t)$ satisfies the following inequality:

$$\dot{V}(s,\ t) + \frac{\sqrt{2}\varepsilon}{R_0}\sqrt{V(s,\ t)} \leq 0. \tag{26}$$

Here, according to the Lemma 1, $\alpha = 0.5$ and $c = \frac{\sqrt{2}\varepsilon}{R_0}$, the finite convergent time satisfies

$$T \leq \frac{\sqrt{2}R_0\sqrt{V(s_0,\ 0)}}{\varepsilon} \tag{27}$$

where ε is a positive constant. Hence, the proposed sliding surface has the ability of convergence and can be got in a finite time.

Besides, the convergence of the state variables was analyzed in Section 3.1. Thus, we can derive the convergence of the state variables as

$$x_1(T) = q(T) - q_f \longrightarrow 0, x_2(T) = \dot{q}(T) \longrightarrow 0 \tag{28}$$

where T is the finite convergent time.

Therefore, the SMC guidance law has the ability of convergence and can be got in a finite time.

4 WNN design for the SMC guidance law

Although the sliding mode correction term is utilized to improve the robustness of the guidance system, the high-frequency chattering would instead decrease the control precision in practical applications. From (17) it can be found that the performance of the robust guidance laws mainly lies on the signum function item, $\varepsilon \mathrm{sgn} s$. Because the signum function may easilly introduce high-frequency chattering and result in instability of the system, we should pay more attention to it when designing the guidance law. Therefore, the robust guidance law should be provided with the capability of adaptation to the signum function item.

In this work, a WNN is employed to adaptively update the additional lateral acceleration command since it has the advantages of faster convergence and higher accuracy over other neural networks. For this neural network, the inputs are $s(t)$ and $\dot{s}(t)$ and the output is the additional control acceleration A_{c2} to enhance the robustness of the system. Hence, the robust guidance law with the impact angle constraint can be written as

$$A_c = A_{c1} + A_{c2} \tag{29}$$

where $A_{c1} = -(2 + \lambda + k)\dot{R}x_2 + \lambda(k+1)\frac{\dot{R}}{R}x_1$.

4.1 WNN design

Wavelet neural network takes the topological structure of BP neural network as the foundation and selects the Morlet wavelet basis function[32, 33] as the transfer function of the hidden layer. As a feed-forward network, BP neural network includes the forward propagation of signal and the back propagation of error. While the network is in the training and learning process, the weight is continuously adjusted to obtain the minimum total error which is relevant to the appropriate output of the network. The wavelet neural network consists of three layers: the input layer which has 2 nodes, the hidden layer which has n nodes and the output layer which has 1 node. The structure is shown in Fig. 2.

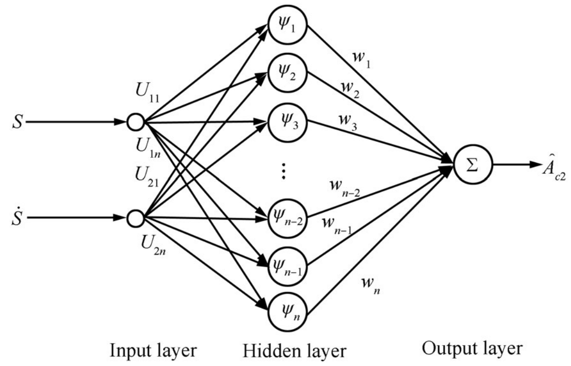


Fig. 2 The structure of WNN

The WNN model above can be expressed by the following mathematical formula[34]

$$\hat{y} = \hat{A}_{c2} = \sum_{j=1}^{n} w_j\psi_j\left(\sum_{k=1}^{2}\frac{x_kU_{kj} - b_j}{a_j}\right) \tag{30}$$

where $x = (s,\ \dot{s})^{\mathrm{T}} = (x_1, x_2)^{\mathrm{T}}$ is the input vector, $\hat{y}$ is the predicted output value, U_{kj} is the connection weight from the k-th node of the input layer to the j-th node of the hidden layer , ψ_j is the activation function of the j-th neuron of the hidden layer; w_j is the layer weight from the j-th node of the hidden layer to the output; a_j is the expansion parameter of the wavelet function; b_j is the translation parameter of the wavelet function.

The Morlet wavelet basis function can extract the amplitude and phase information of the analyzed signal. Here the Morlet wavelet basis function as the activation function is shown as follows:

$$\psi(z) = \cos(1.75z)\mathrm{e}^{-\frac{z^2}{2}}. \tag{31}$$

The wavelet neural network adopts the gradient descent algorithm[35] to correct the connection weight, so as to minimize the network error. In order to minimize the network error and guarantee a faster convergence of the WNN, we define the error function as follows:

$$E = (y - \hat{y})^2 \tag{32}$$

where $\hat{y}$ and y are the predicted output value and the desired output value, respectively of the WNN.

The back propagation of error corrects the weights of the wavelet neural network and the parameters of the wavelet function. The formulas are as follows:

$$U_{k,j}(i+1) = U_{k,j}(i) + \Delta U_{k,j}(i+1) + \beta_w(U_{k,j}(i) - U_{k,j}(i-1)) \tag{33}$$

$$w_j(i+1) = w_j(i) + \Delta w_j(i+1) + \beta_w(w_j(i) - w_j(i-1)) \tag{34}$$

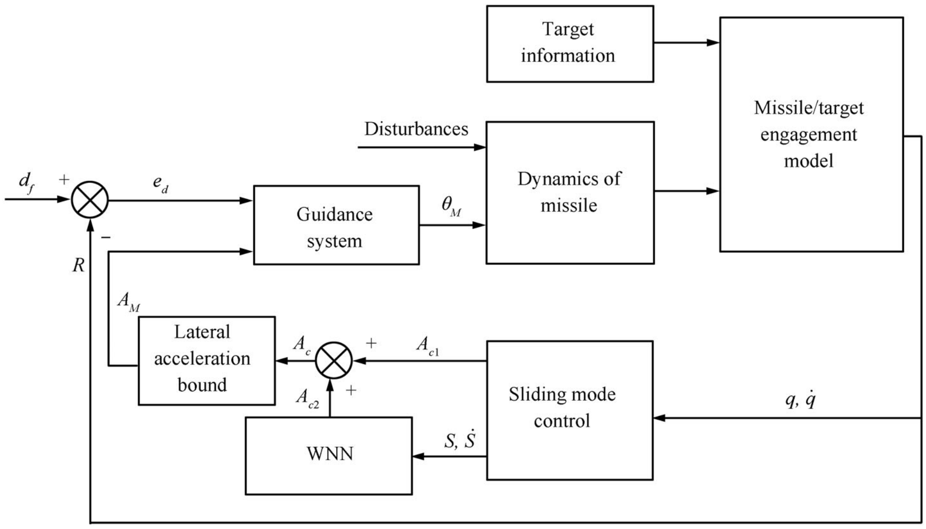


Fig. 3 The structure of robust guidance law

$$a_j(i+1) = a_j(i) + \Delta a_j(i+1) + \beta_w(a_j(i) - a_j(i-1)) \tag{35}$$

$$b_j(i+1) = b_j(i) + \Delta b_j(i+1) + \beta_w(b_j(i) - b_j(i-1)) \tag{36}$$

where

$$\Delta U_{k,j}(i+1) = -\alpha_w \left(\frac{\partial E}{\partial \Delta U_{k,j}(i)}\right)$$

$$\Delta w_j(i+1) = -\alpha_w \left(\frac{\partial E}{\partial \Delta w_j(i)}\right)$$

$$\Delta a_j(i+1) = -\alpha_w \left(\frac{\partial E}{\partial \Delta a_j(i)}\right)$$

$$\Delta b_j(i+1) = -\alpha_w \left(\frac{\partial E}{\partial \Delta b_j(i)}\right)$$

α_w is the learning rate of the momentum and β_w is the momentum factor.

When the targeted error E is less than the predetermined threshold ε_w ($\varepsilon_w > 0$) or the maximum number of iterations is exceeded, the WNN training is stopped; otherwise, the WNN training should be continued.

In order to make the guidance system have a faster response time, we train the WNN structure offline by substituting the saturation function for the signum function, and update the additional control demand adaptively in time. The saturation function can be written as

$$sat(s) = \frac{s}{|s| + \sigma} \tag{37}$$

where σ is a positive constant.

4.2 The robust guidance law design

In this section, we design a robust guidance law which combines WNN with the SMC guidance law, denoted as WNNSMC guidance law. The structure of the robust guidance law is shown in Fig. 3.

In this guidance law, we consider the disturbances are introduced by the uncertainty of the target. The WNN is used to adaptively update the additional lateral acceleration command according to the different conditions. Hence, we can guarantee the robustness of the system against uncertain disturbances.

Considering the peak normal load of the missile, we establish the lateral acceleration bound block, as shown in Fig. 3. Hence, the absolute value of control acceleration should not exceed the peak normal load in order to guarantee the performance and the stability of the system. In this study, the lateral acceleration A_M is bounded according to the saturation function as

$$A_M = \begin{cases} A_{M\max}\mathrm{sgn}(A_c), & \text{if } |A_c| \geq A_{M\max} \\ A_c, & \text{if } |A_c| < A_{M\max} \end{cases} \tag{38}$$

where A_M is the lateral acceleration bound imposed on the missile.

In the engagement course, we can stop the guidance to the missile if the distance between the missile and the target is less than the defined miss distance. The error distance can be written as

$$e_d = d_f - R \tag{39}$$

where d_f is the defined miss distance, and R is the distance between the missile and the target in real time. Hence, if $e_d < 0$, stop the guidance so that the missile will intercept the target relying on inertia; if $e_d > 0$, then we should continue the guidance based on the WNNSMC guidance law.

5 Simulation results

To demonstrate and evaluate the performance of proposed guidance law, some simulation results are provided in this section. Firstly, we analyze the training process of WNN in order to combine it with SMC methodology to design the robust guidance law. Secondly, we present the simulation for showing the ability of impact angle control with various initial heading angles. Thirdly, we compare the WNNSMC guidance law with other guidance laws, such as SMC guidance law and BP neural network SMC guidance law. It is assumed that the missile has perfect measurements on q and R, and there are command limitations with lateral acceleration bound as (38).

5.1 WNN training process analysis

In this part of the study, we test the WNN performance in terms of convergence and training error. The structure of the WNN is 2-9-1, learning rate $\alpha_w = 0.001$, momentum factor $\beta_w = 0.95$, the threshold of error precision $\varepsilon_w = 0.50$, and the maximum iteration $Epochs_{\max} = 100$. The training process of WNN can be seen in Fig. 4. As can be seen from Fig. 4, the WNN reaches the convergent state with 54 training iterations. Moreover, except a few individual points, the WNN has a small training error and the main training error is 0.39. Therefore, the WNN has a superior performance in terms of training error and number of iterations. Hence, we can combine the WNN with SMC methodology to design the robust guidance law.

5.2 Basic performance test of the WNNSMC guidance law

Considering the vertical attack of air or land targets, we define the impact angle is 90°. The initial position of the missile is (0, 10 000) m, and the velocity is 300 m/s. A stationary target is placed on (15 000, 0) m. The SMC guidance law parameters are defined as follows: $k = 1.8$, $\lambda = 1.5$, and $\varepsilon = 50$. The saturation function parameter is defined as $\sigma = 0.01$. The lateral acceleration bound is chosen as $A_{M\max} = 40\,\mathrm{m/s^2}$. The defined error distance is assumed to be $d_f = 5\,\mathrm{m}$. In the design of WNN, the parameters are given in Section 5.1.

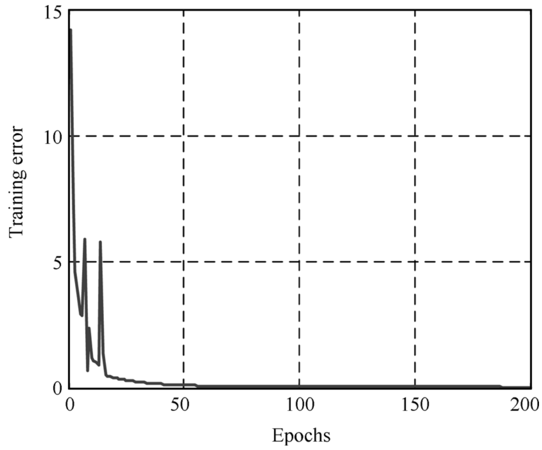


Fig. 4 The training process of WNN

In this engagement scenario studied, the missile initial heading angles are different (−90°, −60°, −30°, 0°, 30°, 60°, and 90°), and the impact angle is 90°. The results for this simulation are presented in Fig. 5. From Fig. 5, it can be clearly seen that the WNNSMC guidance law leads to a desired angle of 90° with a wide range of initial heading angles varying from −90° to 90°. The acceleration command profiles in Fig. 5 (c) show good performance in lateral acceleration bound. The missile heading angle and LOS angle are shown in Figs. 5 (b) and (d). The detailed simulation results are given in Table 1. At the final time, the missile LOS angle and heading angle are converged to −90° as the missile approaches the target, so that the desired impact angle is successfully achieved. By introducing the lateral acceleration bound, smooth acceleration commands are also be obtained and the miss distance at final time is smaller than 1 m in the simulation cases.

5.3 Performance comparison

In order to compare the robust performance of the WNNSMC guidance law with other guidance laws (SMC, and BPSMC), we conduct the following simulation experiments.

Table 1 The detailed results with various initial heading angles

Initial heading angle (deg)	Miss distance (m)	Impact angle error (deg)	Maximum lateral acceleration command ($\mathrm{m/s^2}$)	Flight time (s)
−90	0.72	0.00	40.00	70.12
−60	0.67	0.00	40.00	68.89
−30	0.69	0.00	40.00	69.30
0	0.99	0.00	14.12	70.60
30	0.50	0.00	−24.94	71.89
60	0.59	0.00	−32.18	74.66
90	0.54	0.00	−32.05	82.57

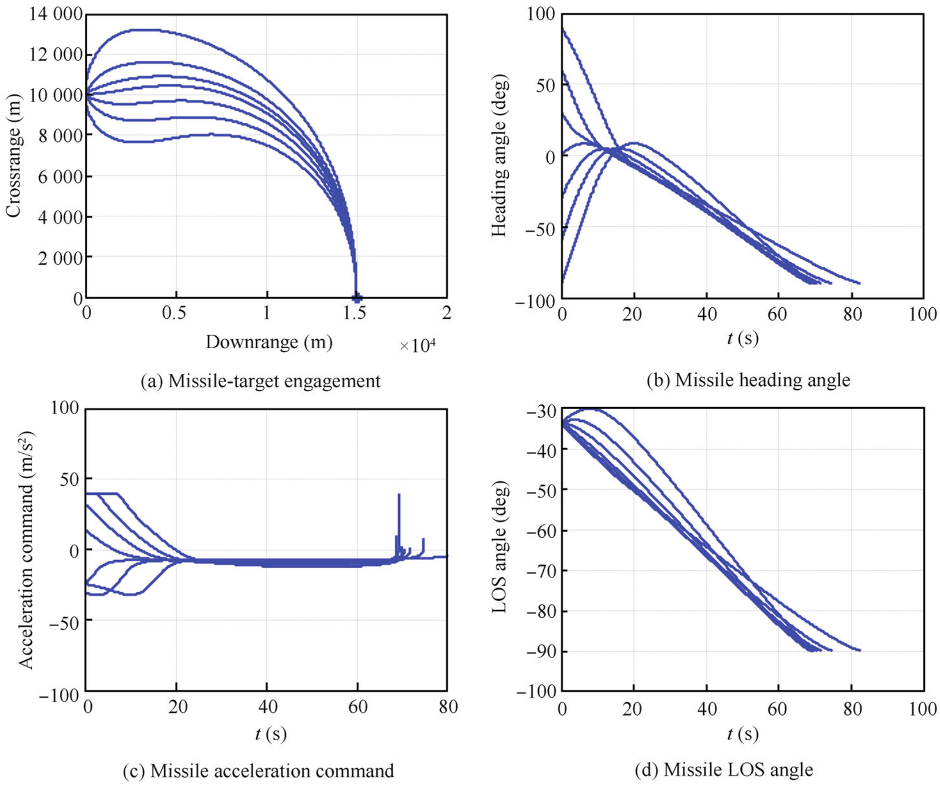


Fig. 5 The results with various initial heading angles

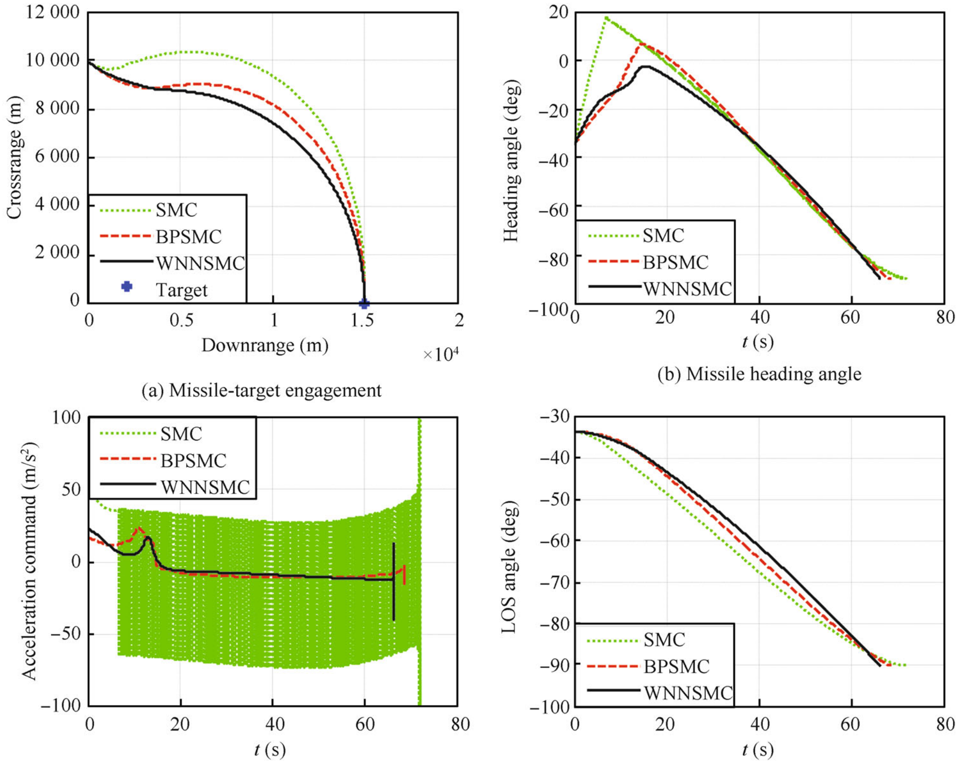
5.3.1 Stationary target

In this section, the results for the case of stationary targets are presented. The initial positions of the missile and the target are similar to the previous simulations. The initial heading angle of the missile is $-35°$. The impact angle is defined as $\theta_{imp} = 90°$ to achieve lateral attack against the stationary target. Hence, the terminal desired heading angle and LOS angle are $-90°$. We consider the missile is guided using the SMC guidance law, BPSMC guidance law, and WNNSMC guidance law, respectively. The results for this simulation are presented in Fig. 6.

From Fig. 6 (a), we can draw that the missile could approach the stationary target through applying different guidance laws. However, the acceleration command is significantly different among these guidance laws, which can be seen in Fig. 6 (c). The acceleration commands of the SMC guidance law presents the high-frequency chattering. Moreover, the acceleration command of the SMC, and BPSMC guidance laws excess the maximum allowable lateral acceleration $A_{M\max}$ at some times. In this aspect, the WNNSMC guidance law has a good performance for $| AM |\leq A_{M\max}$. From Figs. 6 (b) and (d), it can be seen that the SMC, BPSMC and WNNSMC guidance laws guide the missile to successively approach the target with impact angle at terminal time. The detailed results can be seen in Table 2. The missile can intercept the stationary target by using these guidance laws with the miss distance smaller than 1m in the finite time. The SMC, BPSMC and WNNSMC guidance laws satisfy the impact angle constraint with impact angle error smaller than $3°$. However, the BPSMC and WNNSMC guidance laws meet the lateral acceleration bound constraint. As the flight time of the WNNSMC guidance law is shorter than SMC and BPSMC ones, we can conclude that the WNNSMC guidance law shows superior convergent performance. Hence, the WNNSMC guidance law presents better performances than other guidance laws with the impact angle constraint against stationary target.

Table 2 Performance comparison for stationary target

Guidance law	Miss distance (m)	Impact angle error (deg)	Maximum lateral acceleration command (m/s^2)	Flight time (s)
SMC	0.91	−0.31	791.42	71.93
BPSMC	0.88	0.05	23.43	68.48
WNNSMC	0.24	−0.03	−40.00	66.21



(a) Missile-target engagement

(b) Missile heading angle

(c) Missile acceleration command

(d) Missile LOS angle

Fig. 6 Performance comparison for stationary target

5.3.2 Slowly moving target

The proposed guidance law can also be applied to engaging moving targets. In order to demonstrate the effectiveness of the WNNSMC guidance law, simulation is performed for a moving target. The initial positions in the previous simulations are used here too, only now the target is assumed to be moving at a velocity of $V_T = 60\,\mathrm{m/s}$ with a heading angle of $\theta_T = 0°$. The initial heading angle of the missile is $-45°$. The target velocity is smaller than the missile velocity, so the assumption of target-to-missile ratio $\nu < 1$ is satisfied. For slowly moving target tests, we consider the desired impact angle $\theta_{imp} = 90°$.

Fig. 7 presents the comparison results for slowly moving target: missile-target engagement trajectories, missile heading angles, missile acceleration commands, and missile LOS angles. As shown in Fig. 7 (a), under various guidance laws, the missile can approach the slowly moving target with desired angle in a finite time. From Figs. 7 (b) and (d), we can observe that the SMC, BPSMC and WNNSMC guidance laws can guide the missile to intercept the slowly moving target with small desired heading angle errors and LOS angle errors. From Fig. 7 (c), we can see that the missile acceleration command of the WNNSMC guidance law satisfies the lateral acceleration bound, while the missile acceleration commands of the SMC, and BPSMC guidance law are not within the lateral acceleration bound at some times. What′s more, the missile acceleration command of the SMC guidance law exhibits high-frequency chattering which we do not expect. The detailed comparison results can be seen in Table 3. From Table 3, we can see that the WNNSMC presents better performances than the SMC and BPSMC in terms of miss distance, impact angle error, latex command as well as flight time.

Table 3 Performance comparison for slowly moving target

Guidance law	Miss distance (m)	Impact angle error (deg)	Maximum lateral acceleration command ($\mathrm{m/s^2}$)	Flight time (s)
SMC	1.05	3.36	724.56	75.54
BPSMC	0.39	3.54	−455.47	73.44
WNNSMC	0.02	0.82	−40.00	71.90

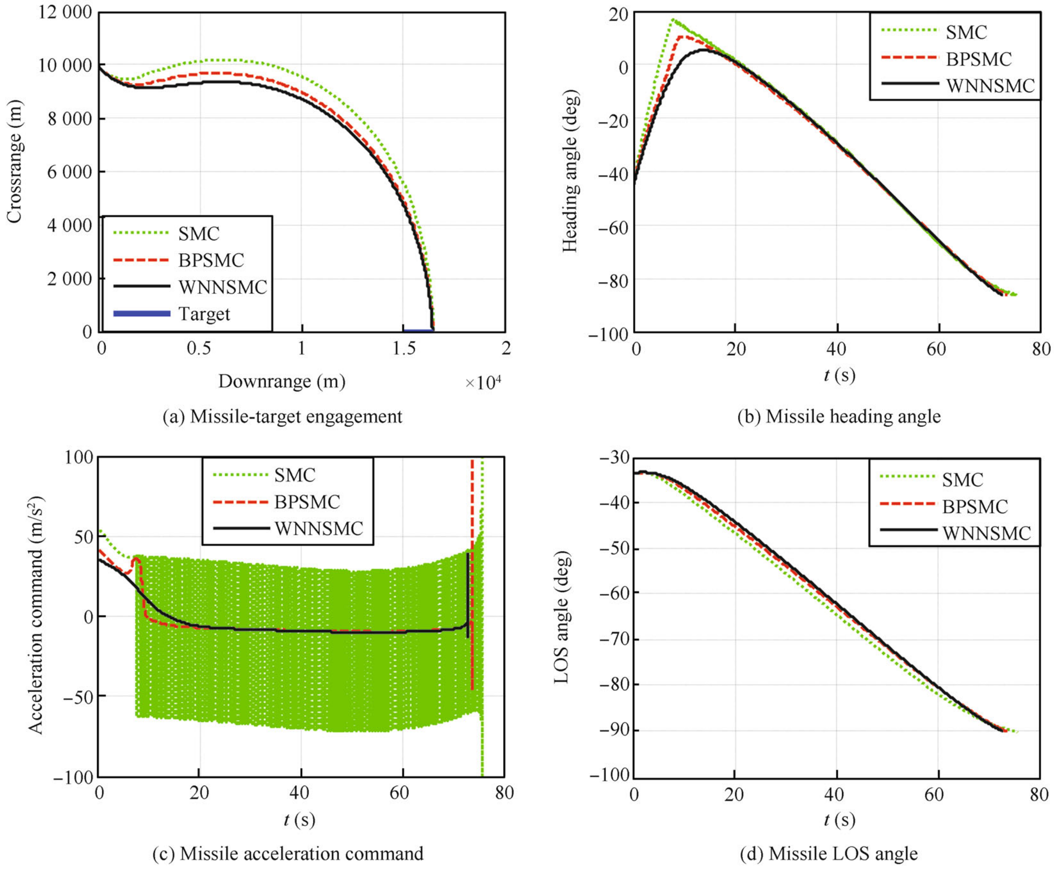


Fig. 7 Performance comparison for slowly moving target

6 Conclusions

In this paper, a novel robust guidance law, WNNSMC guidance law, is proposed for the missile to intercept the target with impact angle in a finite time. Through using Lyapunov stability theory, we prove that the proposed sliding surface has the ability of finite time convergence. It is shown that the structure of the WNNSMC guidance law is characterized as the combination of SMC with WNN, which has a good performance on limiting the high-frequency chattering of acceleration command. Some numerical simulations show that the proposed guidance law is capable of guiding the missile to intercept the target with impact angle and lateral acceleration bound constraints. Moreover, the proposed guidance law provides better performances than the SMC and BPSMC guidance laws in terms of miss distance, impact angle error, maximum lateral acceleration command as well as flight time.

In future work, we will consider several factors such as the autopilot lag or aerodynamic model of the missile for the practical implementation. Furthermore, the WNNSMC guidance law could be extended to the general case of a maneuvering target. Although the maneuvering target is not considered in the derivation of the theory, the proposed guidance law can also be utilized to intercept the maneuvering target with impact angle constraint.

References

[1] L. Dou, J. Dou. Three-dimensional large landing angle guidance based on two-dimensional guidance laws. *Chinese Journal of Aeronautics*, vol. 24, no. 6, pp. 756–761, 2011.

[2] M. G. Yoon. Relative circular navigation guidance for the impact angle control problem. *IEEE Transactions on Aerospace and Electronic Systems*, vol. 44, no. 4, pp. 1449–1463, 2008.

[3] Q. Z. Song, X. Y. Meng. Design and simulation of guidance law with angular constraint based on non-singular terminal sliding mode. *Physics Procedia*, vol. 25, pp. 1197–1204, 2012.

[4] P. Zarchan. *Tactical and Strategic Missile Guidance*, 2nd ed., Reston, USA: AIAA, 1994.

[5] M. Kim, K. V. Grider. Terminal guidance for impact attitude angle constrained flight trajectories. *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-9, no. 6, pp. 852–859, 1973.

[6] R. J. York, H. L. Pastrick. Optimal terminal guidance with constraints at final time. *Journal of Spacecraft and Rockets*, vol. 14, no. 6, pp. 381–382, 1977.

[7] T. L. Song, S. J. Shin. Time-optimal impact angle control for vertical plane engagements. *IEEE Transactions on*

Aerospace and Electronic Systems, vol. 35, no. 2, pp. 738–742, 1999.

[8] C. K. Ryoo, H. Cho, M. J. Tahk. Closed-form solutions of optimal guidance with terminal impact angle constraint. In *Proceedings of IEEE Conference on Control Applications*, IEEE, Istanbul, Turkey, vol. 1, pp. 504–509, 2003.

[9] Y. I. Lee, C. K. Ryoo, E. Kim. Optimal guidance with constraints on impact angle and terminal acceleration. In *Proceedings of AIAA Guidance, Navigation, and Control Conference and Exhibit*, AIAA, Austin, USA, pp. 1–7, 2003.

[10] C. K. Ryoo, H. Cho, M. J. Tahk. Time-to-go weighted optimal guidance with impact angle constraints. *IEEE Transactions on Control Systems Technology*, vol. 14, no. 3, pp. 483–492, 2006.

[11] I. S. Jeon, J. I. Lee, M. J. Tahk. Guidance law to control impact time and angle. In *Proceedings of 2005 International Conference on Control and Automation*, IEEE, Budapest, Hungary, pp. 852–857, 2005.

[12] J. I. Lee, I. S. Jeon, M. J. Tahk. Guidance law to control impact time and angle. *IEEE Transactions on Aerospace and Electronic Systems*, vol. 43, no. 1, pp. 301–310, 2007.

[13] B. S. Kim, J. G. Lee, H. S. Han. Biased PNG law for impact with angular constraint. *IEEE Transactions on Aerospace and Electronic Systems*, vol. 34, no. 1, pp. 277-288, 1998.

[14] S. K. Jeong, S. J. Cho, E. G. Kim. Angle constraint biased PNG. In *Proceedings of the 5th Asian Control Conference*, IEEE, Melbourne, Victoria, Australia, pp. 1849–1854, 2004.

[15] G. Akhil, D. Ghose. Biased PN based impact angle constrained guidance using a nonlinear engagement model. In *Proceedings of 2012 American Control Conference*, IEEE, Montreal, Canada, pp. 950–955, 2012.

[16] Y. A. Zhang, G. X. Ma, A. L. Liu. Guidance law with impact time and impact angle constraints. *Chinese Journal of Aeronautics*, vol. 26, no. 4, pp. 960–966, 2013.

[17] M. U. Salamci, M. K. Özgören, S. P. Banks. Sliding mode control with optimal sliding surfaces for missile autopilot design. *Journal of Guidance, Control, and Dynamics*, vol. 23, no. 4, pp. 719–727, 2000.

[18] L. P. Liu, Z. M. Fu, X. N. Song. Sliding mode control with disturbance observer for a class of nonlinear systems. *International Journal of Automation and Computing*, vol. 9, no. 5, pp. 487–491, 2012.

[19] T. Shima. Deviated velocity pursuit. In *Proceedings of AIAA Guidance, Navigation and Control Conference and Exhibit*, AIAA, Hilton Head, USA, pp. 4364–4379, 2007.

[20] Y. Y. Wei, M. Z. Hou, G. R. Duan. Adaptive multiple sliding surface control for integrated missile guidance and autopilot with terminal angular constraint. In *Proceedings of the 29th Chinese Control Conference*, IEEE, Beijing, China, pp. 2162–2166, 2010.

[21] Z. D. Hu, X. M. Tang, Y. P. Wang. A 3-dimensional robust guidance law with impact angle constraint. In *Proceedings of 2011 Chinese Control and Decision Conference*, IEEE, Mianyang, China, pp. 999–1006, 2011.

[22] Y. F. Shi, C. J. Zhou, X. F. Huang, Q. Yin. Intelligent fault-tolerant control for swing-arm system in the spaceborne spectrograph. *Acta Astronautica*, vol. 73, pp. 67–75, 2012.

[23] S. Rao, D. Ghose. Sliding mode control based terminal impact angle constrained guidance laws using dual sliding surface. In *Proceedings of the 12th IEEE Workshop on Variable Structure Systems*, IEEE, Mumbai, India, pp. 325–330, 2012.

[24] N. Harl, S. N. Balakrishnan. Impact time and angle guidance with sliding mode control. *IEEE Transactions on Control Systems Technology*, vol. 20, no. 6, pp. 1436–1449, 2012.

[25] H. Yan, X. H. Wang, B. F. Yu, H. B. Ji. Adaptive integrated guidance and control based on backstepping and input-to-state stability. *Asian Journal of Control*, vol. 16, no. 2, pp. 602–608, 2014.

[26] M. H. Hsueh, T. K. Wang, L. C. Fu. Integrated game based guidance with nonlinear autopilot design for maneuvering target interception. *Asian Journal of Control*, vol. 16, no. 2, pp. 431–440, 2014.

[27] A. Nizar, B. M. Houda, N. A. Said. A new sliding function for discrete predictive sliding mode control of time delay systems. *International Journal of Automation and Computing*, vol. 10, no. 4, pp. 288–295, 2013.

[28] H. Zhao, W. J. Liu, J. H. Yang, Y. B. Xuan. Design of stochastic sliding mode variable structure guidance law based on adaptive EKF. *Procedia Engineering*, vol. 23, pp. 276–283, 2011.

[29] K. Jamoussi, M. Ouali, L. Chrifi-Alaoui, H. Benderradji, A. El Hajjaji. Robust sliding mode control using adaptive switching gain for induction motors. *International Journal of Automation and Computing*, vol. 10, no. 4, pp. 303–311, 2013.

[30] S. Sun, D. Zhou, W. T. Hou. A guidance law with finite time convergence accounting for autopilot lag. *Aerospace Science and Technology*, vol. 25, no. 1, pp. 132–137, 2013.

[31] D. Zhou, S. Sun, K. L. Teo. Guidance laws with finite time convergence. *Journal of Guidance, Control, and Dynamics*, vol. 32 , no. 6, pp. 1838–1846, 2009.

[32] H. Z. Yang, W. G. Zhao, W. Chen, X. Q. Chen. The study of prediction model based on Morlet wavelet neural network. *Applied Mechanics and Materials*, vol. 121–126, pp. 4847–4851, 2011.

[33] Y. S. Lu, B. X. Wu, S. F. Lien. An improved sliding-mode repetitive learning control scheme using wavelet transform. *Asian Journal of Control*, vol. 14, no. 4, pp. 991–1001, 2012.

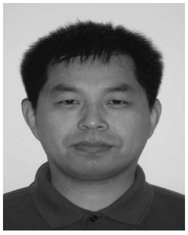
[34] L. Q. Xu, S. Y. Liu. Study of short-term water quality prediction model based on wavelet neural network. *Mathematical and Computer Modelling*, vol. 58 , no. 3–4, pp. 807–813, 2013.

[35] Y. Y. Yan, B. L. Guo. Application of wavelet neural network (WNN) and gradient descent method (GDM) in natural image denoising. *Journal of Computational Information Systems*, vol. 2, no. 2, pp. 625–631, 2006.

Qing-Chun Li received the M. Sc. degree in control theory and control engineering from the College of Automation at Qingdao University, Qingdao, China in 2012. He is currently a Ph. D. candidate in Institute of Automation, Chinese Academy of Sciences, China.

His major research interests include modeling complex systems, machine learning, and intelligence control theory and application.

E-mail: qingchun.li@ia.ac.cn (Corresponding author)
ORCID iD: 0000-0002-9771-6819



Wen-Sheng Zhang received the Ph. D. degree in pattern recognition and intelligent systems from the Institute of Automation, Chinese Academy of Sciences (CAS), Beijing, China in 2000. He joined the Institute of Software, CAS in 2001. He is a professor of machine learning and data mining and the director of Research and Development Department, Institute of Automation, CAS. He has published over 100 papers in the area of modeling complex systems, statistical machine learning, data mining, and intelligence control.

His research interests include computer vision, pattern recognition, intelligent control, artificial intelligence and computer human interaction.

E-mail: wensheng.zhang@ia.ac.cn

Gang Han received the Ph. D. degree in control theory and control engineering from Institute of Automation, Chinese Academy of Science, Beijing, China in 2011. He is currently an associate professor of Research Center of Precision Sensing and Control, Institute of Automation, Chinese Academy of Sciences.

His research interests include aircraft and robot control, complex system modeling and simulation, and embedded computing theory and systems.

E-mail: gang.han@ia.ac.cn

Ying-Hua Zhang received the Ph. D. degree in pattern recognition and intelligent system from the University of Chinese Academy of Sciences, Beijing, China in 2013. She is currently an assistant professor of Institute of Automation, Chinese Academy of Sciences.

Her research interests include artificial intelligence, machine learning, and complex system modeling and simulation.

E-mail: yinghua.zhang@ia.ac.cn