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PREFACE

Preface: Second *DGAA* Special Issue on Evolutionary Games

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Like its predecessor last year, this special issue of *Dynamic Games and Applications* is concerned with the theory and applications of evolutionary games. Evolutionary game theory initially arose in biology as a method to predict the outcome of natural selection acting on individual behaviors, with successful strategies having higher reproduction rates. One can trace the birth of the theory to the introduction of the notion of an evolutionarily stable strategy (ESS) by theoretical biologist John Maynard Smith. While the ESS concept can be viewed as a refinement of Nash equilibrium, which describes equilibrium behavior among rational agents in classical game theory, ESS has also proved to be a remarkably robust equilibrium notion for explicitly specified evolutionary dynamics. This is true whether this evolutionary dynamics emerges from biology or from one of the many other disciplines to which these ideas have spread, including computer science, economics, engineering, physics, and psychology. The contributions and author affiliations for these two special issues reflect this diversity. At the same time, there are common themes across the disciplines, evidenced by the following brief descriptions of the articles in this issue, that maintain evolutionary games as a unified field.

While many aspects of the theory of evolutionary game dynamics have been thoroughly explored, the development of applications of the theory, particularly to economic questions, is at an earlier stage of development. This issue contains two valuable contributions in this direction. Hahn uses evolutionary game techniques to study the possibility of price cycles

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in markets. The context of his analysis is Varian's [7] well-known model of sales, in which firms face both informed and uninformed consumers. He shows that when firms adjust their prices over time in the fashion of Gilboa and Matsui's [2] best response dynamics, their aggregate choices do not converge to an equilibrium, but instead lead to asymptotically stable cycles.

Auction theory is the basis for many of the most successful practical applications of game theoretic ideas. Nevertheless, little is known about how players can learn to play Nash equilibrium in these games. Louge and Riedel address this important question through an analysis of evolutionary dynamics in first price auctions, studying the evolution of the distribution of choices from parameterized sets of strategies under imitative dynamics. They show that while Nash equilibria of first price auctions are robust to many perturbations of the distribution of strategies, they nevertheless fail to be asymptotically stable.

An important role for models of disequilibrium game dynamics is to provide foundations for static analyses. Two papers in this special issue employ tools from stochastic approximation theory to address questions of this sort. Cho and Matsui introduce a dynamic model of the formation and dissolution of long-term relationships among randomly paired agents, where the benefit to each partner generated by the relationship is drawn once and for all from a fixed distribution. They show that if agents decide whether to initiate a relationship by employing a threshold rule that incorporates "sympathy"—that is, if they sometimes use their opponent's historical payoffs rather than their own as the relevant threshold for acceptance—then, in the long run, all agents in the economy will nearly always be in relationships that approximately generate the strongly Pareto efficient outcome that maximizes the agents' minimal payoff.

In models of reinforcement learning in games, players' choices in each period are random functions of statistics that describe the historical success of each strategy. An important new direction of research, inspired by techniques from control theory, enhances reinforcement learning models by allowing players to account for recent payoff trends. Chasparis and Shamma use techniques from stochastic approximation theory to show how such dynamic reinforcement can cause certain strict equilibria of coordination games to become unstable, thereby developing a new approach to equilibrium selection.

A topic in disequilibrium game dynamics closely related to equilibrium selection concerns the necessity of convergence to certain sets of strategy profiles. Saran and Serrano look at the process of regret matching introduced by Hart and Mas-Colell [3] but under the assumption that agents have memories of finite length. They show that this process must converge to a minimal product set that is closed under the same-or-better reply correspondence. They provide a variety of interesting examples of the behavior of their process, as well as a very useful survey of work on related learning processes for games.

The theory of evolution in games has been the source of interesting and subtle mathematical questions. For instance, in models of stochastic evolution based on repeated random matching of agents, it is often implicitly assumed that having large numbers of random matches will lead to essentially deterministic payoffs, so that the randomness in the matching process can be ignored. Molzon shows that this need not be the case. Looking at versions of the Kandori et al. [4] model of stochastic evolution, he proves that even when the number of agents is very large, deterministic matching (as in a round-robin tournament) and uniform random matching lead to very different long-run predictions of play: in simple coordination games, the former leads to selection of the risk-dominant equilibrium, while, surprisingly, the latter leads to selection of the Pareto-dominant equilibrium.

Brown's [1] fictitious play process is the original model of disequilibrium dynamics in games. Most analyses of this process, starting with that of Robinson [6], have been algebraic in nature. Berger instead investigates the convergence properties of fictitious play from a geometric point of view. Using tools from projective geometry, he offers an intuitively appealing analysis of the limit behavior of continuous-time fictitious play, and obtains a new selection result for 4×4 symmetric zero-sum games.

The remaining two articles in this special issue develop theoretical models and simulations that explore mechanisms to promote cooperative behavior in social dilemmas and to promote coordination between sender and receiver in signaling games, respectively. Signaling games are renowned for having multiple equilibrium outcomes and correspondingly complicated evolutionary dynamics (e.g. [5, 8]). Besides efficient equilibria that accurately match sender signals with desired actions by receivers, there are typically many (complete or partial) pooling equilibria with miscommunication between sender and receiver. In contrast to most signaling models in the evolutionary literature, in which senders choose among a fixed number of signals, Alexander, Skyrms and Zabell allow senders an additional option of inventing (and sending) a new signal. Assuming that reinforcement learning increases the probability of coordination when receivers by chance pick the correct action for this new signal but deletes this signal otherwise, they show through simulations that inefficient pooling equilibria do not arise.

In most work in the evolutionary game literature on social dilemmas, uncooperative individuals are punished by their peers. Isakov and Rand instead examine the level of cooperation that evolves when punishment is meted out by an institution. Their simulations of the dynamics show that the more accurately individuals learn through observing payoffs from others' actions, the closer the evolutionary outcome is to the predicted Nash equilibrium given the institution's punishment scheme. Moreover, when the institutional level of coercion also evolves according to its pay-off consequences, accurate population learning leads to high levels of punishment of noncooperative behavior, whereas less accurate learning results in institutions that punish little.

The original plan was to have a single special issue on evolutionary games in Dynamic Games and Applications. It soon became clear from the quantity and high quality of submissions that one issue would not suffice. We would like to thank wholeheartedly the contributors for this happy circumstance. We are also grateful to Georges Zaccour, the Editor-in-Chief, for agreeing to a second issue and for facilitating the process in so many other ways. Special thanks are due to all the reviewers for their timely criticisms and suggestions, without which these issues would not have been possible.

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