



Archimedean Aggregation Operators Based on Complex Pythagorean Fuzzy Sets Using Confidence Levels and Their Application in Decision Making

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Abstract The diagnosed complex Pythagorean fuzzy (CPF) set is a more valuable and dominant tool than the Pythagorean and intuitionistic fuzzy sets to describe awkward and unreliable information more effectively. Further, Archimedean t-norm and t-conorm have a significant influence to depict the relation among aggregated values. To take advantage of the CPF set and Archimedean t-norm and t-conorm, and assume the relation among Archimedean norms and algebraic, Einstein, Hamacher, and frank norms at the same time, in this analysis, first, we proposed the fundamental Archimedean operational laws. Second, based on these laws, we developed confidence CPF Archimedean-weighted averaging (CCPFSAWA), confidence CPF Archimedean-ordered weighted averaging (CCPFSAWA), confidence CPF Archimedean-weighted geometric (CCPFSAWG), confidence CPF Archimedean-ordered weighted geometric (CCPFSAWOG) operators and implemented their valuable results and properties. We know that Archimedean t-norm and t-conorm are the general form of the all-aggregation operators, so by using different values of t-norm and t-conorm, we explored the confidence CPF-weighted averaging (CCPFWA), confidence CPF-ordered weighted averaging (CCPFOWA), confidence CPF Einstein-weighted averaging (CCPFEWA), confidence CPF Einstein-ordered weighted averaging (CCPFEOWA), confidence CPF Hamacher-weighted averaging (CCPFHWA),

confidence CPF Hamacher-ordered weighted averaging (CCPFHOWA), confidence CPF frank-weighted averaging (CCPFFWA), confidence CPF frank-ordered weighted averaging (CCPFFOWA), confidence CPF-weighted geometric (CCPFWG), confidence CPF-ordered weighted geometric (CCPFOWG), confidence CPF Einstein-weighted geometric (CCPFEWG), confidence CPF Einstein-ordered weighted geometric (CCPFEOWG), confidence CPF Hamacher-weighted geometric (CCPFHWG), confidence CPF Hamacher-ordered weighted geometric (CCPFHOWG), confidence CPF frank-weighted geometric (CCPFFWG), and confidence CPF frank-ordered weighted geometric (CCPFFOWG) operators. Then, we developed a multi-attribute decision-making (MADM) method based on the proposed operators. Finally, many examples are used to do comparative analysis among proposed and existing methods to show the validation of the new approaches.

Keywords Complex Pythagorean fuzzy sets · Archimedean · Algebraic · Einstein · Hamacher · Frank averaging/geometric aggregation operators · Decision-making methods

1 Introduction

A development environment is a group of techniques and procedures demonstrated to compute, test, and debug an application, program, or website. It is contained three major server tiers, for instance, the development server, which is the initial tier, is used to test pieces of code and see whether the considered application can successfully work in the presence of that code. Similarly, the staging server is used to create a way that mimics the final product or to check the reliability of the application, and the last

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one, called the production server, is used for observing that the under-consideration application will not fail, it then becomes a part of this server. multi-attribute decision-making (MADM) technique plays a valuable role in strategic decision-making sciences, and it has been more generally used in certain fields like economics, management sciences, computer science, road signals systems, and software engineering in the last few years. Its spirit is the procedure of ordering the decisions and considering a valuable decision between a family of decisions in the availability of their attributes. In every decision-making procedure, experts have faced many ambiguities when the procedure is how to feasibly aggregate the criteria values a very challenging task for every scholar/expert in the presence of classical information. On the other hand, because of the personal nature of every expert thinking in genuine decision-making dilemmas, experts' assessments of decisions are always ambiguous and fuzzy. To depict such sort of ambiguity, intuitionistic fuzzy (IF) set, pioneered by Atanassov [1, 2], is a modified and general shape of fuzzy information [3]. Two very well-known and dominant functions, called supporting $u(x)$ and supporting against $v(x)$ grades containing in IF set with an idealist mathematical situation/structure: $0 \leq u(x) + v(x) \leq 1$. Certain evaluations were done by different people, for instance, bipolar soft set [4], decision-making evaluation using variable weight-based hybrid approaches [5], divergence measures [6], construction of different distance measures [7], granular ambiguity with IFSs [8], analysis of COVID-19 pandemic performance [9], and correlation co-efficient using IFSs [10]. However, in several procedures of decision making, it is possible $u(x) + v(x) > 1$, but $0 \leq u^2(x) + v^2(x) \leq 1$. Alternatively, the main mathematical form of Pythagorean fuzzy (PF) set, pioneered by Yager [11] with a positive and effective tool: $0 \leq u^2(x) + v^2(x) \leq 1$, certain applications have been done by different peoples, for instance, weighted discrimination analysis based on PF sets [12], correlation measures [13], analysis of transport management [14], multi-criteria group decision-making analysis [15], ELECTRE-II technique [16], Yager aggregation operators [17], ELECTRE-I technique [18], and analysis of risk evaluation using PF information [19].

Study information in the last paragraph, we noticed that decision-making evaluations have been done based on IF and PF information. Once more noticed that these theories are not able to evaluate or manage time-periodic dilemmas and two-dimension theory is given in single sets. Therefore, Ramot et al. [20] employed the second term in the grade of supporting $u(x) = u_R(x)e^{i2\pi(u_I(x))}$ and given their shapes of complex numbers like polar coordinates, called complex fuzzy (CF) set whose real and imaginary parts are represented by the amplitude and phase term. The fuzzy set

is one of the most important cases of the CF set, because if we choose $u_I(x) = 0$, then the value of the supporting grade in the CF set is changed to the value of the supporting grade in a fuzzy set. Certain applications have been done by different people, for instance, cross-entropy measures [21], neighborhood operators [22], orthogonality [23], and complex fuzzy soft set theory [24]. But still, the grade of supporting against is missing in the CF information, due to these issues many scholars have faced a lot of ambiguity; therefore, to handle such kind of hurdles, Alkouri and Salleh [25] diagnosed the complex IF (CIF) set with two grades in the shape of polar coordinates in the shape: $0 \leq u_R(x) + v_R(x) \leq 1$ and $0 \leq u_I(x) + v_I(x) \leq 1$. Certain evaluations were done by different peoples, for instance, novel aggregation operators [26], robust averaging-geometric operators [27], generalized geometric operators [28], Hamacher aggregation operators [29], and prioritized aggregation operators [30]. However, in several procedures of decision making, it is possible $u_R(x) + v_R(x) > 1$ and $u_I(x) + v_I(x) > 1$, but $0 \leq u_R^2(x) + v_R^2(x) \leq 1$ and $0 \leq u_I^2(x) + v_I^2(x) \leq 1$. Alternatively, the main mathematical form of complex PF (CPF) set, pioneered by Ullah et al. [31] with a modified rule: $0 \leq u_R^2(x) + v_R^2(x) \leq 1$, $0 \leq u_I^2(x) + v_I^2(x) \leq 1$, and due to these modifications, certain people have utilized it in the region of different fields, for instance, Einstein geometric operators [32], Dombi operators [33], Einstein operators [34], and prioritized weighted operators [35].

Evaluating the relation between any number of attributes is a very challenging task for individuals. Therefore, to reduce the complications of the experts, certain researchers have evaluated different types of operators by using different types of norms. For instance, aggregation operators using t-norm and t-conorm for IF sets [36], generalized operators based on the above norms were diagnosed by Zhao et al. [37], Einstein hybrid aggregation operators were evaluated by Zhao and Wei [38], geometric operators for IF sets were implemented by Xu and Yager [39], Hamacher operators for IF sets were evaluated by Huang [40], Dombi operators for PF sets were utilized by Khan et al. [41], PF aggregation operators were evaluated by Peng and Yuan [42], Hamacher operators for PF sets were diagnosed by Wu and Wei [43] and generalized aggregation operators for PF sets was evaluated by Feng et al. [44]. Further, we noticed that the main and powerful decision-making procedure has three main stages:

1. Arrange the information based on some features to express the data.
2. Determine the preference information of the item by accumulating the distinct criteria values.
3. Rank the final aggregated values to obtain the beneficial decision.

Therefore, the theme of this theory is to evaluate a novel decision-making tool to manage the MADM techniques based on the CPF information with Archimedean-averaging/geometric aggregation operators based on the t-norm and t-conorm. The Archimedean aggregation operators are the modified version of many well-known aggregation operators, for instance, the simple weighted averaging/geometric aggregation operators, Einstein averaging/geometric aggregation operators, Hamacher averaging/geometric aggregation operators, and frank averaging/geometric aggregation operators are the special cases of the evaluated Archimedean aggregation operators. Further, by involving the confidence degree, the diagnosed operators are given their results more feasible compared to prevailing operators computed based on the different norms in the presence of confidence degree as well as without confidence degree. To propose the aggregation, Einstein, Hamacher, and frank operators, we computed the short and generalized shape of these all operators which can help many authors to find their results instead of implementing these all ideas.

CPF set is the modified version of the PF set by including the phase term in the grade of supporting and supporting against. The phase term and amplitude term in the CPF set represented the real and imaginary parts in complex numbers. If we remove the phase term from the grade of supporting and supporting against, the experts have lost a lot of data during the decision-making process. To moreover explain the significance of phase term, we discussed practical examples, such that, if a company wants to launch a novel type of laptop in the market, regarding each laptop, the owner of the laptop given information in the shape, called the name of the laptop and price of the laptop. The name of the laptop shows the amplitude term, and the price of the laptop shows the phase term. For the traditional PF set, it is very ambiguous to handle it because the prevailing PF set deals only with one-dimension information. The CPF information is much suitable and valuable for handling awkward and rational information. Under the consideration of the above worthwhile information, some important features are explained here:

1. To propose the fundamental Archimedean operational laws.
2. To develop CCPFSAWA, CCPFSAOWA, CCPFSAWG, and CCPFSAOWG operators and explore their valuable results and properties.
3. To propose the CCPFWA, CCPFOWA, CCPFEWA, CCPFEOWA, CCPFHOWA, CCPFFWA, CCPFFOWA, CCPFWG, CCPFOWG, CCPFEWA, CCPFEOWG, CCPFHOWG, CCPFFWG, and CCPFFOWG operators.

WG, CCPFEOWG, CCPFHOWG, CCPFFWG, CCPFFOWG operators.

4. To develop a MADM method based on the proposed operators.
5. To demonstrate many examples and illustrated comparative analysis among proposed and prevailing theories to show the validation and genuine worth of the new approaches. Figure 1 can easily describe the importance of the proposed work and implemented algorithm used in this manuscript.

Construction of this article is shown as: Sect. 2 gave some existing theories, such as CPF sets, and their application with t-norm and t-conorm. In Sect. 3, we firstly developed the fundamental Archimedean operational laws. Second, based on these laws, we proposed the CCPFSAWA, CCPFSAOWA, CCPFSAWG, and CCPFSAOWG operators and explored their valuable results and properties. We know that Archimedean t-norm and t-conorm are the general form of the all-aggregation operators, so by using different values of t-norm and t-conorm, we gave the CCPFWA, CCPFOWA, CCPFEWA, CCPFEOWA, CCPFHOWA, CCPFFWA, CCPFFOWA, CCPFWG, CCPFOWG, CCPFEWG, CCPFEOWG, CCPFHOWG, CCPFFWG, and CCPFFOWG operators. In Sect. 4, we proposed a MADM method based on the proposed operators. Finally, many examples are given to show the validation of the new approaches. Section 5 contained concluding remarks.

2 Preliminaries

This analysis talked about some existing theories, called CPF sets, and their application with t-norm and t-conorm. Different types of symbols and their meaning are illustrated in Table 1.

Definition 1 [31] Under the supposition of fixed set x , a structure:

$$\beta = \{(u(x), v(x)) : x \in x\} \quad (1)$$

with $0 \leq u_R^2(x) + v_R^2(x) \leq 1$, $0 \leq u_I^2(x) + v_I^2(x) \leq 1$, is called CPF set and the refusal grade: $R(x) = R_R(x)e^{i2\pi(R_I(x))} = (1 - (u_R^2(x) + v_R^2(x)))^{\frac{1}{2}}e^{i2\pi(1 - (u_I^2(x) + v_I^2(x)))^{\frac{1}{2}}}$. Finally, $\beta_s = (u_{R-s}e^{i2\pi(u_{I-s})}, v_{R-s}e^{i2\pi(v_{I-s})})$, $s = 1, 2, \dots, n$, used as a CPF number (CPFNs). Further, we proposed various algebraic laws for CPFNs as follows.

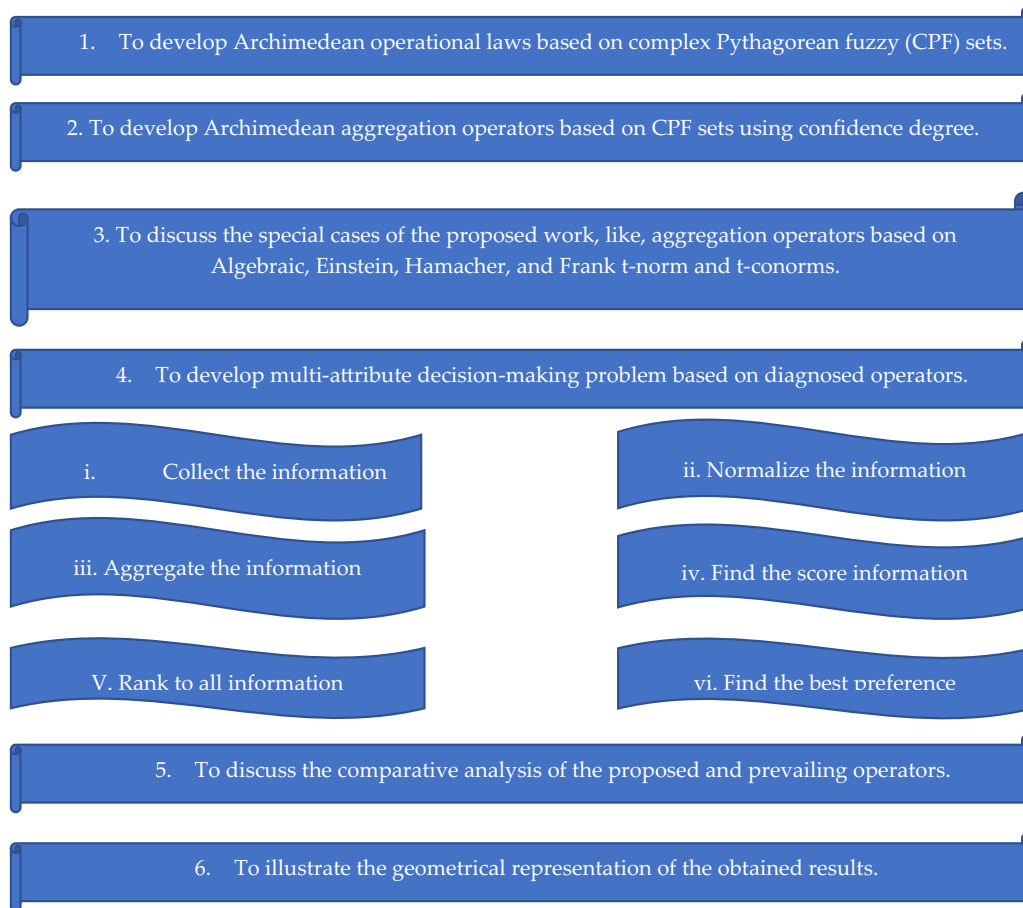


Fig. 1 Explanation of the proposed information

Table 1 Representation of different symbols and their meanings

Symbol	Meaning	Symbol	Meaning
$u(x)$	Membership function	u_R	The real part of the membership function
$v(x)$	Non-membership function	u_I	The imaginary part of the membership function
x	Universal set	v_R	The real part of the non-membership function
x	Element of universal sets	v_I	The imaginary part of the non-membership function
β	Complex Pythagorean fuzzy sets	$R_R(x)$	The real part of the neutral function
$R(x)$	Neutral function	$R_I(x)$	The imaginary part of the neutral function

$$\beta_1 \oplus \beta_2 = \left(\frac{(u_{R-1}^2(x) + u_{R-2}^2(x) - u_{R-1}^2(x)u_{R-2}^2(x))^{\frac{1}{2}} e^{i2\pi(u_{I-1}^2(x) + u_{I-2}^2(x) - u_{I-1}^2(x)u_{I-2}^2(x))^{\frac{1}{2}}}}{v_{R-1}v_{R-2}e^{i2\pi(v_{I-1}v_{I-2})}}, \beta_s^\rho = \left(u_{R-s}^\rho e^{i2\pi(u_{I-s}^\rho)}, (1 - (1 - v_{R-s}^2(x))^\rho)^{\frac{1}{2}} e^{i2\pi(1 - (1 - v_{I-s}^2(x))^\rho)^{\frac{1}{2}}} \right). \quad (2)$$

$$\beta_1 \otimes \beta_2 = \left(\frac{u_{R-1}u_{R-2}e^{i2\pi(u_{I-1}u_{I-2})}}{(v_{R-1}^2(x) + v_{R-2}^2(x) - v_{R-1}^2(x)v_{R-2}^2(x))^{\frac{1}{2}} e^{i2\pi(v_{I-1}^2(x) + v_{I-2}^2(x) - v_{I-1}^2(x)v_{I-2}^2(x))^{\frac{1}{2}}}}, \quad (3)$$

$$\rho \beta_s = \left((1 - (1 - u_{R-s}^2(x))^\rho)^{\frac{1}{2}} e^{i2\pi(1 - (1 - u_{I-s}^2(x))^\rho)^{\frac{1}{2}}}, v_{R-s}^\rho e^{i2\pi(v_{I-s}^\rho)} \right), \quad (4)$$

Definition 2 [31] Assume that $\beta_s = (u_{R-s}e^{i2\pi(u_{I-s})}, v_{R-s}e^{i2\pi(v_{I-s})})$, $s = 1, 2, \dots, n$ is a family of CPFNs. Then,

$$C(\beta_s) = \frac{1}{2} |u_{R-s} + u_{I-s} - v_{R-s} - v_{I-s}|, \quad C(\beta_s) \in [-1, 1], \quad (6)$$

$$A(\beta_s) = \frac{1}{2} |u_{R-s} + u_{I-s} + v_{R-s} + v_{I-s}|, \quad A(\beta_s) \in [0, 1]. \quad (7)$$

Used as a score and accuracy function with a rule: if $C(\beta_1) > C(\beta_2)$, then $\beta_1 > \beta_2$, if $C(\beta_1) = C(\beta_2)$, then if $A(\beta_1) > A(\beta_2)$, then $\beta_1 > \beta_2$.

Definition 3 [45] Assume $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ hold the below conditions:

1. $T(1, x) = x$ for all x
2. $T(x, y) = T(y, x)$ for all x and y
3. $T(x, T(y, z)) = T(T(x, y), z)$ for all x, y and z
4. If $x \leq x^*$ and $y \leq y^*$, then $T(x, y) \leq T(x^*, y^*)$

then T called t-norm. Similarly, if $S : [0, 1] \times [0, 1] \rightarrow [0, 1]$ hold the below conditions:

1. $S(0, x) = x$ for all x
2. $S(x, y) = S(y, x)$ for all x and y
3. $S(x, S(y, z)) = S(S(x, y), z)$ for all x, y and z
4. If $x \leq x^*$ and $y \leq y^*$, then $S(x, y) \leq S(x^*, y^*)$

then S called t-conorm. Noticed that $T(x, y) = \mathfrak{G}^{-1}(\mathfrak{G}(x) + \mathfrak{G}(y))$ and $S(x, y) = \mathfrak{H}^{-1}(\mathfrak{H}(x) + \mathfrak{H}(y))$, the additive modifier of \mathfrak{G} and \mathfrak{H} , where $\mathfrak{H}(t) = \mathfrak{G}(1 - t)$.

3 Archimedean Aggregation Operators Using Confidence Levels for CPF Information

Firstly, we propose the fundamental Archimedean operational laws. Further, based on these laws, we develop the CCPFSAWA, CCPFSAOWA, CCPFSAWG, and CCPFSAOWG operators and explore their valuable results and properties. We know that Archimedean t-norm and t-conorm are the general form of the all-aggregation operators, so by using different values of t-norm and t-conorm, we develop CCPFWA, CCPFOWA, CCPFEWA, CCPFEOWA, CCPFHWA, CCPFHOWA, CCPFFWA, CCPFFOWA, CCPFFWG, CCPFFOWG, CCPFEWG, CCPFEOWG, CCPFHWG, CCPFHOWG, CCPFFWG, and CCPFFOWG operators.

Definition 4 We propose various Archimedean laws for CPFNs $\beta_s = (u_{R-s}e^{i2\pi(u_{I-s})}, v_{R-s}e^{i2\pi(v_{I-s})})$, $s = 1, 2, \dots, n$, such that

$$\beta_1 \oplus \beta_2 = \left(\mathfrak{H}^{-1}(\mathfrak{H}(u_{R-1}) + \mathfrak{H}(u_{R-2}))e^{i2\pi(\mathfrak{H}^{-1}(\mathfrak{H}(u_{I-1}) + \mathfrak{H}(u_{I-2})))}, \mathfrak{G}^{-1}(\mathfrak{G}(v_{R-1}) + \mathfrak{G}(v_{R-2}))e^{i2\pi(\mathfrak{G}^{-1}(\mathfrak{G}(v_{I-1}) + \mathfrak{G}(v_{I-2})))} \right), \quad (8)$$

$$\beta_1 \otimes \beta_2 = \left(\mathfrak{G}^{-1}(\mathfrak{G}(u_{R-1}) + \mathfrak{G}(u_{R-2}))e^{i2\pi(\mathfrak{G}^{-1}(\mathfrak{G}(u_{I-1}) + \mathfrak{G}(u_{I-2})))}, \mathfrak{H}^{-1}(\mathfrak{H}(v_{R-1}) + \mathfrak{H}(v_{R-2}))e^{i2\pi(\mathfrak{H}^{-1}(\mathfrak{H}(v_{I-1}) + \mathfrak{H}(v_{I-2})))} \right), \quad (9)$$

$$\rho\beta_s = \left(\mathfrak{H}^{-1}(\rho\mathfrak{H}(u_{R-s}))e^{i2\pi(\mathfrak{H}^{-1}(\rho\mathfrak{H}(u_{I-s})))}, \mathfrak{G}^{-1}(\rho\mathfrak{G}(v_{R-s}))e^{i2\pi(\mathfrak{G}^{-1}(\rho\mathfrak{G}(v_{I-s})))} \right), \quad (10)$$

$$\beta_s^\rho = \left(\mathfrak{G}^{-1}(\rho\mathfrak{G}(u_{R-s}))e^{i2\pi(\mathfrak{G}^{-1}(\rho\mathfrak{G}(u_{I-s})))}, \mathfrak{H}^{-1}(\rho\mathfrak{H}(v_{R-s}))e^{i2\pi(\mathfrak{H}^{-1}(\rho\mathfrak{H}(v_{I-s})))} \right). \quad (11)$$

Definition 5 Under the supposition of fixed set x , a structure:

$$CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) = \oplus_{s=1}^n \bar{w}_s (\nabla_s \beta_s) \quad (12)$$

is called the CCPFAWA operator, where \bar{w}_s represents the weight vector with $\sum_{s=1}^n \bar{w}_s = 1$ and ∇_s , expresses the confidence level, where $0 \leq \nabla_s \leq 1$. When $\nabla_1 = \nabla_2 = \dots = \nabla_n = 1$, then the CCPFAWA operator is changed into a simple CPF-weighted averaging (CPFWA) operator, such that

$$CCPFAWA(\beta_1, \beta_2, \dots, \beta_n) = \oplus_{s=1}^n \bar{w}_s (\beta_s). \quad (13)$$

Theorem 1 The computed value of Eq. (12) is still CPF number, such that

$$CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) = \left(\mathfrak{H}^{-1}(\sum_{s=1}^n \bar{w}_s (\nabla_s \mathfrak{H}(u_{R-s})))e^{i2\pi(\mathfrak{H}^{-1}(\sum_{s=1}^n \bar{w}_s (\nabla_s \mathfrak{H}(u_{I-s})))}, \mathfrak{G}^{-1}(\sum_{s=1}^n \bar{w}_s (\nabla_s \mathfrak{G}(v_{R-s})))e^{i2\pi(\mathfrak{G}^{-1}(\sum_{s=1}^n \bar{w}_s (\nabla_s \mathfrak{G}(v_{I-s})))} \right), \quad (14)$$

where $0 \leq \nabla_s \leq 1$ and $\sum_{s=1}^n \bar{w}_s = 1$.

Proof Mathematical induction is used to prove Eq. (14).

Assume $n = 2$, then

$$\begin{aligned}
 &CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2)) \\
 &= \oplus_{s=1}^2 \overline{w}_s(\nabla_s \beta_s) = \overline{w}_1(\nabla_1 \beta_1) \oplus \overline{w}_2(\nabla_2 \beta_2) \\
 &= \left(\mathbb{h}^{-1}(\overline{w}_1(\nabla_1 \mathbb{h}(u_{R-1}))) e^{i2\pi(\mathbb{h}^{-1}(\overline{w}_1(\nabla_1 \mathbb{h}(u_{I-1})))}, \right. \\
 &\quad \left. \mathbb{g}^{-1}(\overline{w}_1(\nabla_1 \mathbb{g}(v_{R-1}))) e^{i2\pi(\mathbb{g}^{-1}(\overline{w}_1(\nabla_1 \mathbb{g}(v_{I-1})))} \right) \\
 &\oplus \left(\mathbb{h}^{-1}(\overline{w}_2(\nabla_2 \mathbb{h}(u_{R-2}))) e^{i2\pi(\mathbb{h}^{-1}(\overline{w}_2(\nabla_2 \mathbb{h}(u_{I-2})))}, \right. \\
 &\quad \left. \mathbb{g}^{-1}(\overline{w}_2(\nabla_2 \mathbb{g}(v_{R-2}))) e^{i2\pi(\mathbb{g}^{-1}(\overline{w}_2(\nabla_2 \mathbb{g}(v_{I-2})))} \right) \\
 &= \left(\mathbb{h}^{-1} \left(\sum_{s=1}^2 \overline{w}_s(\nabla_s \mathbb{h}(u_{R-s})) \right) e^{i2\pi \left(\mathbb{h}^{-1} \left(\sum_{s=1}^2 \overline{w}_s(\nabla_s \mathbb{h}(u_{I-s})) \right) \right)}, \right. \\
 &\quad \left. \mathbb{g}^{-1} \left(\sum_{s=1}^2 \overline{w}_s(\nabla_s \mathbb{g}(v_{R-s})) \right) e^{i2\pi \left(\mathbb{g}^{-1} \left(\sum_{s=1}^2 \overline{w}_s(\nabla_s \mathbb{g}(v_{I-s})) \right) \right)} \right).
 \end{aligned}$$

Thus, Eq. (14) kept when $n = 2$.

Further, suppose that Eq. (14) also kept for $n = k$, then

$$\begin{aligned}
 &CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_k, \beta_k)) \\
 &= \left(\mathbb{h}^{-1} \left(\sum_{s=1}^k \overline{w}_s(\nabla_s \mathbb{h}(u_{R-s})) \right) e^{i2\pi \left(\mathbb{h}^{-1} \left(\sum_{s=1}^k \overline{w}_s(\nabla_s \mathbb{h}(u_{I-s})) \right) \right)}, \right. \\
 &\quad \left. \mathbb{g}^{-1} \left(\sum_{s=1}^k \overline{w}_s(\nabla_s \mathbb{g}(v_{R-s})) \right) e^{i2\pi \left(\mathbb{g}^{-1} \left(\sum_{s=1}^k \overline{w}_s(\nabla_s \mathbb{g}(v_{I-s})) \right) \right)} \right).
 \end{aligned}$$

Then, when $n = k + 1$, such that

$$\begin{aligned}
 &CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_k, \beta_k), (\nabla_{k+1}, \beta_{k+1})) \\
 &= \oplus_{s=1}^{k+1} \overline{w}_s(\nabla_s \beta_s) = \oplus_{s=1}^k \overline{w}_s(\nabla_s \beta_s) \oplus \overline{w}_{k+1}(\nabla_{k+1} \beta_{k+1}) \\
 &= \left(\mathbb{h}^{-1} \left(\sum_{s=1}^k \overline{w}_s(\nabla_s \mathbb{h}(u_{R-s})) \right) e^{i2\pi \left(\mathbb{h}^{-1} \left(\sum_{s=1}^k \overline{w}_s(\nabla_s \mathbb{h}(u_{I-s})) \right) \right)}, \right. \\
 &\quad \left. \mathbb{g}^{-1} \left(\sum_{s=1}^k \overline{w}_s(\nabla_s \mathbb{g}(v_{R-s})) \right) e^{i2\pi \left(\mathbb{g}^{-1} \left(\sum_{s=1}^k \overline{w}_s(\nabla_s \mathbb{g}(v_{I-s})) \right) \right)} \right) \\
 &\oplus \left(\mathbb{h}^{-1}(\overline{w}_{k+1}(\nabla_{k+1} \mathbb{h}(u_{R-(k+1)}))) e^{i2\pi(\mathbb{h}^{-1}(\overline{w}_{k+1}(\nabla_{k+1} \mathbb{h}(u_{I-(k+1)})))}, \right. \\
 &\quad \left. \mathbb{g}^{-1}(\overline{w}_{k+1}(\nabla_{k+1} \mathbb{g}(v_{R-(k+1)}))) e^{i2\pi(\mathbb{g}^{-1}(\overline{w}_{k+1}(\nabla_{k+1} \mathbb{g}(v_{I-(k+1)})))} \right) \\
 &= \left(\mathbb{h}^{-1} \left(\sum_{s=1}^{k+1} \overline{w}_s(\nabla_s \mathbb{h}(u_{R-s})) \right) e^{i2\pi \left(\mathbb{h}^{-1} \left(\sum_{s=1}^{k+1} \overline{w}_s(\nabla_s \mathbb{h}(u_{I-s})) \right) \right)}, \right. \\
 &\quad \left. \mathbb{g}^{-1} \left(\sum_{s=1}^{k+1} \overline{w}_s(\nabla_s \mathbb{g}(v_{R-s})) \right) e^{i2\pi \left(\mathbb{g}^{-1} \left(\sum_{s=1}^{k+1} \overline{w}_s(\nabla_s \mathbb{g}(v_{I-s})) \right) \right)} \right).
 \end{aligned}$$

Hence, Eq. (14) is right for all positive values of n . \square

In the following, we will explore some properties of $CCPFAWA_{operator}$ when $\nabla_1 = \nabla_2 = \dots = \nabla_n = 1$, i.e., $CCPFWA_{operator}$.

Property 1 (Idempotency) If $\beta_s = \beta = (u_R e^{i2\pi(u_I)}, v_R e^{i2\pi(v_I)})$, $s = 1, 2, \dots, n$, then $CCPFWA(\beta_1, \beta_2, \dots, \beta_n) = \beta$. (15)

Proof By hypothesis, we noticed that $\beta_s = \beta = (u_R e^{i2\pi(u_I)}, v_R e^{i2\pi(v_I)})$, then

$$\begin{aligned}
 &CCPFWA(\beta_1, \beta_2, \dots, \beta_n) = CCPFWA(\beta, \beta, \dots, \beta) \\
 &= \oplus_{s=1}^n \overline{w}_s \beta = \left(\mathbb{h}^{-1} \left(\sum_{s=1}^n \overline{w}_s(\mathbb{h}(u_R)) \right) e^{i2\pi \left(\mathbb{h}^{-1} \left(\sum_{s=1}^n \overline{w}_s(\mathbb{h}(u_I)) \right) \right)}, \right. \\
 &\quad \left. \mathbb{g}^{-1} \left(\sum_{s=1}^n \overline{w}_s(\mathbb{g}(v_R)) \right) e^{i2\pi \left(\mathbb{g}^{-1} \left(\sum_{s=1}^n \overline{w}_s(\mathbb{g}(v_I)) \right) \right)} \right) \\
 &= \left(\mathbb{h}^{-1}(\mathbb{h}(u_R)) e^{i2\pi(\mathbb{h}^{-1}(\mathbb{h}(u_I)))}, \right. \\
 &\quad \left. \mathbb{g}^{-1}(\mathbb{g}(v_R)) e^{i2\pi(\mathbb{g}^{-1}(\mathbb{g}(v_I)))} \right) = \beta.
 \end{aligned}$$

\square

Property 2 (Monotonicity) If $\beta_s = (u_{R-s} e^{i2\pi(u_{I-s})}, v_{R-s} e^{i2\pi(v_{I-s})})$, and $\beta_s^* = (u_{R-s}^* e^{i2\pi(u_{I-s}^*)}, v_{R-s}^* e^{i2\pi(v_{I-s}^*)})$, $s = 1, 2, \dots, n$, with $\beta_s \leq \beta_s^*$ then $CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \leq CCPFAWA((\nabla_1, \beta_1^*), (\nabla_2, \beta_2^*), \dots, (\nabla_n, \beta_n^*))$. (16)

Proof Noticed that $\beta_s \leq \beta_s^*$ it means that $u_{R-s} \leq u_{R-s}^*$, $u_{I-s} \leq u_{I-s}^*$ and $v_{R-s} \geq v_{R-s}^*$, $v_{I-s} \geq v_{I-s}^*$, then

$$\begin{aligned}
 &\mathbb{h}^{-1} \left(\sum_{s=1}^n \overline{w}_s(\nabla_s \mathbb{h}(u_{R-s})) \right) \leq \mathbb{h}^{-1} \left(\sum_{s=1}^n \overline{w}_s(\nabla_s \mathbb{h}(u_{R-s}^*)) \right) \\
 &\mathbb{h}^{-1} \left(\sum_{s=1}^n \overline{w}_s(\nabla_s \mathbb{h}(u_{I-s})) \right) \leq \mathbb{h}^{-1} \left(\sum_{s=1}^n \overline{w}_s(\nabla_s \mathbb{h}(u_{I-s}^*)) \right)
 \end{aligned}$$

and

$$\begin{aligned}
 &\mathbb{g}^{-1} \left(\sum_{s=1}^n \overline{w}_s(\nabla_s \mathbb{g}(v_{R-s})) \right) \geq \mathbb{g}^{-1} \left(\sum_{s=1}^n \overline{w}_s(\nabla_s \mathbb{g}(v_{R-s}^*)) \right) \\
 &\mathbb{g}^{-1} \left(\sum_{s=1}^n \overline{w}_s(\nabla_s \mathbb{g}(v_{I-s})) \right) \geq \mathbb{g}^{-1} \left(\sum_{s=1}^n \overline{w}_s(\nabla_s \mathbb{g}(v_{I-s}^*)) \right)
 \end{aligned}$$

Thus, we get

$$\begin{aligned}
 &CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\
 &\leq CCPFAWA((\nabla_1, \beta_1^*), (\nabla_2, \beta_2^*), \dots, (\nabla_n, \beta_n^*)).
 \end{aligned}$$

\square

Property 3 (Boundedness) If $\beta^- = (\min_u u_{R-s}, e^{i2\pi(\min_u u_{I-s})}, \max_v v_{R-s}, e^{i2\pi(\max_v v_{I-s})})$ and $\beta^+ = (\max_u u_{R-s}, e^{i2\pi(\max_u u_{I-s})}, \min_v v_{R-s}, e^{i2\pi(\min_v v_{I-s})})$, $s = 1, 2, \dots, n$, then $\beta^- \leq CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \leq \beta^+$. (17)

Proof Using property 1 and property 2, we can easily obtain the required result, such that

$$\beta^- \leq CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \leq \beta^+.$$

Further, by using different types of functions, we can obtain some new operators, such that

1. When $\mathbb{g}(t) = -\log(t^2)$, then

$$CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) = \left(\left(1 - \prod_{s=1}^n (1 - u_{R-s}^2)^{\bar{w}_s \nabla_s} \right)^{\frac{1}{2}} e^{i2\pi \left(1 - \prod_{s=1}^n (1 - u_{I-s}^2)^{\bar{w}_s \nabla_s} \right)^{\frac{1}{2}}}, \prod_{s=1}^n v_{R-s}^2 e^{i2\pi \left(\prod_{s=1}^n v_{I-s}^2 \right)^{\frac{1}{2}}} \right) \quad (18)$$

is called CCPFWA operator.

2. When $\mathfrak{U}(t) = \log\left(\frac{2-t^2}{t^2}\right)$, then

$$CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) = \left(\left(\frac{\prod_{s=1}^n (1 + u_{R-s}^2)^{\bar{w}_s \nabla_s} - \prod_{s=1}^n (1 - u_{R-s}^2)^{\bar{w}_s \nabla_s}}{\prod_{s=1}^n (1 + u_{R-s}^2)^{\bar{w}_s \nabla_s} + \prod_{s=1}^n (1 - u_{R-s}^2)^{\bar{w}_s \nabla_s}} \right)^{\frac{1}{2}} e^{i2\pi \left(\frac{\prod_{s=1}^n (1 + u_{I-s}^2)^{\bar{w}_s \nabla_s} - \prod_{s=1}^n (1 - u_{I-s}^2)^{\bar{w}_s \nabla_s}}{\prod_{s=1}^n (1 + u_{I-s}^2)^{\bar{w}_s \nabla_s} + \prod_{s=1}^n (1 - u_{I-s}^2)^{\bar{w}_s \nabla_s}} \right)^{\frac{1}{2}}}, \left(\frac{2 \prod_{s=1}^n (v_{R-s}^2)^{\bar{w}_s \nabla_s}}{\prod_{s=1}^n (2 - v_{R-s}^2)^{\bar{w}_s \nabla_s} + \prod_{s=1}^n (v_{R-s}^2)^{\bar{w}_s \nabla_s}} \right)^{\frac{1}{2}} e^{i2\pi \left(\frac{2 \prod_{s=1}^n (v_{I-s}^2)^{\bar{w}_s \nabla_s}}{\prod_{s=1}^n (2 - v_{I-s}^2)^{\bar{w}_s \nabla_s} + \prod_{s=1}^n (v_{I-s}^2)^{\bar{w}_s \nabla_s}} \right)^{\frac{1}{2}}} \right) \quad (19)$$

is called CCPFEWA operator.

3. When $\mathfrak{U}(t) = \log\left(\frac{\gamma + (1-\gamma)t^2}{t^2}\right)$, then

$$CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) = \left(\left(\frac{\prod_{s=1}^n (1 + (\gamma - 1)u_{R-s}^2)^{\bar{w}_s \nabla_s} - \prod_{s=1}^n (1 - u_{R-s}^2)^{\bar{w}_s \nabla_s}}{\prod_{s=1}^n (1 + (\gamma - 1)u_{R-s}^2)^{\bar{w}_s \nabla_s} + (\gamma - 1) \prod_{s=1}^n (1 - u_{R-s}^2)^{\bar{w}_s \nabla_s}} \right)^{\frac{1}{2}} e^{i2\pi \left(\frac{\prod_{s=1}^n (1 + (\gamma - 1)u_{I-s}^2)^{\bar{w}_s \nabla_s} - \prod_{s=1}^n (1 - u_{I-s}^2)^{\bar{w}_s \nabla_s}}{\prod_{s=1}^n (1 + (\gamma - 1)u_{I-s}^2)^{\bar{w}_s \nabla_s} + (\gamma - 1) \prod_{s=1}^n (1 - u_{I-s}^2)^{\bar{w}_s \nabla_s}} \right)^{\frac{1}{2}}}, \left(\frac{2 \prod_{s=1}^n (v_{R-s}^2)^{\bar{w}_s \nabla_s}}{\prod_{s=1}^n (1 + (\gamma - 1)(1 - v_{R-s}^2)^{\bar{w}_s \nabla_s}) + (\gamma - 1) \prod_{s=1}^n (v_{R-s}^2)^{\bar{w}_s \nabla_s}} \right)^{\frac{1}{2}} e^{i2\pi \left(\frac{2 \prod_{s=1}^n (v_{I-s}^2)^{\bar{w}_s \nabla_s}}{\prod_{s=1}^n (1 + (\gamma - 1)(1 - v_{I-s}^2)^{\bar{w}_s \nabla_s}) + (\gamma - 1) \prod_{s=1}^n (v_{I-s}^2)^{\bar{w}_s \nabla_s}} \right)^{\frac{1}{2}}} \right) \quad (20)$$

is called CCPFHWa operator. If $\gamma = 1$ in Eq. (20), then we get Eq. (18); if we use $\gamma = 2$ in Eq. (20), then we get Eq. (19).

4. When $\mathfrak{U}(t) = \log\left(\frac{\gamma-1}{\gamma t^2 - 1}\right)$, then

$$CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) = \left(\left(1 - \log_{\gamma} \left(1 + \frac{\prod_{s=1}^n (\gamma^{1-u_{R-s}^2} - 1)}{\gamma - 1} \right) \right)^{\frac{1}{2}} e^{i2\pi \left(1 - \log_{\gamma} \left(1 + \frac{\prod_{s=1}^n (\gamma^{1-u_{I-s}^2} - 1)}{\gamma - 1} \right) \right)^{\frac{1}{2}}}, \left(\log_{\gamma} \left(1 + \frac{\prod_{s=1}^n (\gamma^{v_{R-s}^2} - 1)}{\gamma - 1} \right) \right)^{\frac{1}{2}} e^{i2\pi \left(\log_{\gamma} \left(1 + \frac{\prod_{s=1}^n (\gamma^{v_{I-s}^2} - 1)}{\gamma - 1} \right) \right)^{\frac{1}{2}}} \right) \quad (21)$$

is called CCPFFWA operator. \square

To explain the above aggregation operators, we give an example.

Example 1 We consider $\beta_1 = \{(0.7, 0.6e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.4)})\}$, $\beta_2 = \{(0.71, 0.61e^{i2\pi(0.51)}, 0.51e^{i2\pi(0.41)})\}$, $\beta_3 = \{(0.72, 0.62e^{i2\pi(0.52)}, 0.52e^{i2\pi(0.42)})\}$ and $\beta_4 =$

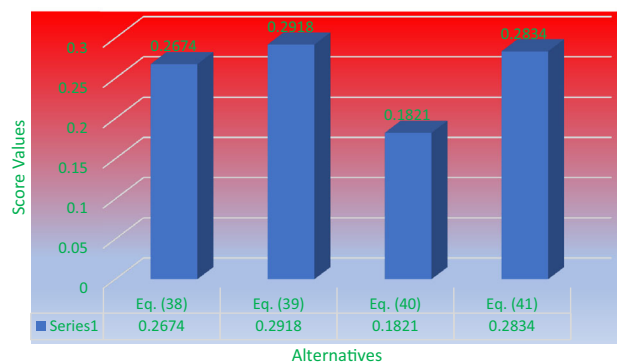


Fig. 2 Geometrical explanation for example 1

$\left\{ \left(0.73, 0.63e^{i2\pi(0.53)}, 0.53e^{i2\pi(0.43)} \right) \right\}$ with weight vectors 0.4, 0.3, 0.2, and 0.1, then by using Eqs. (18) to (21) ($\nabla_1 = \nabla_2 = \nabla_3 = \nabla_4 = 1$, we have

$$CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), (\nabla_3, \beta_3), (\nabla_4, \beta_4)) = \left(0.7041e^{i2\pi(0.62)}, 0.4391e^{i2\pi(0.3503)} \right) \dots \text{by Eq. (18)}$$

$$CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), (\nabla_3, \beta_3), (\nabla_4, \beta_4)) = \left(0.7207e^{i2\pi(0.6419)}, 0.4324e^{i2\pi(0.3465)} \right) \dots \text{by Eq. (19)}$$

$$CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), (\nabla_3, \beta_3), (\nabla_4, \beta_4)) = \left(0.598e^{i2\pi(0.5362)}, 0.4268e^{i2\pi(0.3432)} \right) \dots \text{by Eq. (20)}$$

$$CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), (\nabla_3, \beta_3), (\nabla_4, \beta_4)) = \left(0.4943e^{i2\pi(0.4324)}, 0.7613e^{i2\pi(0.7322)} \right) \dots \text{by Eq. (21)}$$

Then by using Eq. (6), we have

$$C(CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), (\nabla_3, \beta_3), (\nabla_4, \beta_4))) = 0.0906 \dots \text{by Eq. (18)}$$

$$C(CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), (\nabla_3, \beta_3), (\nabla_4, \beta_4))) = 0.1229 \dots \text{by Eq. (19)}$$

$$C(CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), (\nabla_3, \beta_3), (\nabla_4, \beta_4))) = 0.0329 \dots \text{by Eq. (20)}$$

$$C(CCPFAWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), (\nabla_3, \beta_3), (\nabla_4, \beta_4))) = 0.3778 \dots \text{by Eq. (21)}$$

Further, the geometrical interpretation of the above information is shown Fig. 2.

Definition 6 Under the supposition of fixed set x , a structure:

$$\begin{aligned} &CCPFAOWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ &= \bigoplus_{s=1}^n \bar{w}_s \left(\nabla_s \beta_{o(s)} \right) \end{aligned} \quad (22)$$

is called the CCPFAOWA operator with $\beta_{o(s)} \leq \beta_{o(s-1)}$, where \bar{w}_s represents the position weight vector with $\sum_{s=1}^n \bar{w}_s = 1$ and ∇_s expresses the confidence level, where $0 \leq \nabla_s \leq 1$. When $\nabla_1 = \nabla_2 = \dots = \nabla_n = 1$, then the CCPFAOWA operator is changed into a simple CPF-ordered weighted averaging (CPFOWA) operator, such that

$$CCPFAOWA(\beta_1, \beta_2, \dots, \beta_n) = \bigoplus_{s=1}^n \bar{w}_s \left(\beta_{o(s)} \right) \quad (23)$$

Theorem 2 The computed value of Eq. (22) is also CPF number, such that

$$\begin{aligned} &CCPFAOWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ &= \left(\mathbb{H}^{-1} \left(\sum_{s=1}^n \bar{w}_s \left(\nabla_{o(s)} \mathbb{H} \left(u_{R-o(s)} \right) \right) \right) e^{i2\pi \left(\mathbb{H}^{-1} \left(\sum_{s=1}^n \bar{w}_s \left(\nabla_{o(s)} \mathbb{H} \left(v_{I-o(s)} \right) \right) \right) \right) \right), \end{aligned} \quad (24)$$

where $0 \leq s \leq 1$ and $\sum_{s=1}^n \bar{w}_s = 1$.

Proof Omitted. \square

Property 4 (Idempotency) If $\beta_s = \beta = (u_R e^{i2\pi(u_I)}, v_R e^{i2\pi(v_I)})$, $s = 1, 2, \dots, n$, then

$$CCPFWA(\beta_1, \beta_2, \dots, \beta_n) = \beta \quad (25)$$

Property 5 (Monotonicity) If $\beta_s = (u_{R-s} e^{i2\pi(u_{I-s})}, v_{R-s} e^{i2\pi(v_{I-s})})$, and $\beta_s^* = (u_{R-s}^* e^{i2\pi(u_{I-s}^*)}, v_{R-s}^* e^{i2\pi(v_{I-s}^*)})$, $s = 1, 2, \dots, n$, with $\beta_s \leq \beta_s^*$ then

$$\begin{aligned} &CCPFAOWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ &\leq CCPFAOWA((\nabla_1, \beta_1^*), (\nabla_2, \beta_2^*), \dots, (\nabla_n, \beta_n^*)). \end{aligned} \quad (26)$$

Property 6 (Boundedness) If $\beta^- = (\min_s u_{R-s}, \min_s v_{R-s})$ and $\beta^+ = (\max_s u_{R-s}, \max_s v_{R-s})$, $s = 1, 2, \dots, n$, then

$$\begin{aligned} &e^{i2\pi \left(\min_s u_{I-s} \right)}, \max_s v_{R-s} e^{i2\pi \left(\max_s v_{I-s} \right)} \text{ and } \beta^+ = \left(\max_s u_{R-s}, \max_s v_{R-s} \right) e^{i2\pi \left(\max_s v_{I-s} \right)}, \\ &\beta^- \leq CCPFAOWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \leq \beta^+. \end{aligned} \quad (27)$$

Further, by using different types of functions, we can obtain some new operators, such that

1. When $\mathfrak{U}(t) = -\log(t^2)$, then

$$\begin{aligned} &CCPFAOWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ &= \left(\left(1 - \prod_{s=1}^n \left(1 - u_{R-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}} \right)^{\frac{1}{2}} e^{i2\pi \left(1 - \prod_{s=1}^n \left(1 - v_{I-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}} \right)^{\frac{1}{2}}}, \right. \\ &\quad \left. \prod_{s=1}^n v_{R-o(s)}^{\bar{w}_s \nabla_{o(s)}} e^{i2\pi \left(\prod_{s=1}^n v_{I-o(s)}^{\bar{w}_s \nabla_{o(s)}} \right)} \right) \end{aligned} \quad (28)$$

is called CCPFOWA operator.

2. When $\mathfrak{U}(t) = \log\left(\frac{2-t^2}{t^2}\right)$, then

$$\begin{aligned} &CCPFAOWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ &= \left(\left(\frac{\prod_{s=1}^n \left(1 + u_{R-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}} - \prod_{s=1}^n \left(1 - u_{R-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}}}{\prod_{s=1}^n \left(1 + u_{R-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}} + \prod_{s=1}^n \left(1 - u_{R-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}}} \right)^{\frac{1}{2}} e^{i2\pi \left(\frac{\prod_{s=1}^n \left(1 + v_{I-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}} - \prod_{s=1}^n \left(1 - v_{I-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}}}{\prod_{s=1}^n \left(1 + v_{I-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}} + \prod_{s=1}^n \left(1 - v_{I-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}}} \right)^{\frac{1}{2}}}, \right. \\ &\quad \left. \left(\frac{2 \prod_{s=1}^n \left(v_{R-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}}}{\prod_{s=1}^n \left(2 - v_{R-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}} + \prod_{s=1}^n \left(v_{R-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}}} \right)^{\frac{1}{2}} e^{i2\pi \left(\frac{2 \prod_{s=1}^n \left(v_{I-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}}}{\prod_{s=1}^n \left(2 - v_{I-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}} + \prod_{s=1}^n \left(v_{I-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}}} \right)^{\frac{1}{2}}} \right) \end{aligned} \quad (29)$$

is called CCPFEOWA operator.

3. When $\mathfrak{U}(t) = \log\left(\frac{\gamma + (1-\gamma)t^2}{t^2}\right)$, then

$$\begin{aligned} &CCPFAOWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ &= \left(\left(\frac{\prod_{s=1}^n \left(1 + (\gamma-1) u_{R-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}} - \prod_{s=1}^n \left(1 - u_{R-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}}}{\prod_{s=1}^n \left(1 + (\gamma-1) u_{R-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}} + (\gamma-1) \prod_{s=1}^n \left(1 - u_{R-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}}} \right)^{\frac{1}{2}} e^{i2\pi \left(\frac{\prod_{s=1}^n \left(1 + (\gamma-1) v_{I-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}} - \prod_{s=1}^n \left(1 - v_{I-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}}}{\prod_{s=1}^n \left(1 + (\gamma-1) v_{I-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}} + (\gamma-1) \prod_{s=1}^n \left(1 - v_{I-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}}} \right)^{\frac{1}{2}}}, \right. \\ &\quad \left(\frac{2 \prod_{s=1}^n \left(v_{R-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}}}{\prod_{s=1}^n \left(1 + (\gamma-1) \left(1 - v_{R-o(s)}^2 \right) \right)^{\bar{w}_s \nabla_{o(s)}} + (\gamma-1) \prod_{s=1}^n \left(v_{R-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}}} \right)^{\frac{1}{2}} e^{i2\pi \left(\frac{2 \prod_{s=1}^n \left(v_{I-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}}}{\prod_{s=1}^n \left(1 + (\gamma-1) \left(1 - v_{I-o(s)}^2 \right) \right)^{\bar{w}_s \nabla_{o(s)}} + (\gamma-1) \prod_{s=1}^n \left(v_{I-o(s)}^2 \right)^{\bar{w}_s \nabla_{o(s)}}} \right)^{\frac{1}{2}}} \right) \end{aligned} \quad (30)$$

is called CCPFHOWA operator. If $\gamma = 1$ in Eq. (30), then we get Eq. (28), if $\gamma = 2$ in Eq. (30), then we get Eq. (29).

4. When $\mathfrak{U}(t) = \log\left(\frac{\gamma-1}{\gamma t^2 - 1}\right)$, then

$$\begin{aligned} &CCPFAOWA((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ &= \left(\left(1 - \log_{\gamma} \left(1 + \frac{\prod_{s=1}^n \left(\gamma^2 - u_{R-o(s)}^2 - 1 \right)^{\bar{w}_s \nabla_{o(s)}}}{\gamma - 1} \right) \right)^{\frac{1}{2}} e^{i2\pi \left(1 - \log_{\gamma} \left(1 + \frac{\prod_{s=1}^n \left(\gamma^2 - v_{I-o(s)}^2 - 1 \right)^{\bar{w}_s \nabla_{o(s)}}}{\gamma - 1} \right) \right)^{\frac{1}{2}}}, \right. \\ &\quad \left(\log_{\gamma} \left(1 + \frac{\prod_{s=1}^n \left(\gamma^2 - u_{R-o(s)}^2 - 1 \right)^{\bar{w}_s \nabla_{o(s)}}}{\gamma - 1} \right) \right)^{\frac{1}{2}} e^{i2\pi \left(\log_{\gamma} \left(1 + \frac{\prod_{s=1}^n \left(\gamma^2 - v_{I-o(s)}^2 - 1 \right)^{\bar{w}_s \nabla_{o(s)}}}{\gamma - 1} \right) \right)^{\frac{1}{2}}} \right) \end{aligned} \quad (31)$$

is called CCPFFOWA operator.

Definition 7 Under the supposition of fixed set x , a structure:

$$CCPFAWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ = \otimes_{s=1}^n (\beta_s^{\nabla_s})^{\overline{w}_s} \quad (32)$$

is called the CCPFAWG operator, where \overline{w}_s represents the weight vector with $\sum_{s=1}^n \overline{w}_s = 1$ and ∇_s expresses the confidence level, where $0 \leq \nabla_s \leq 1$. When $\nabla_1 = \nabla_2 = \dots = \nabla_n = 1$, then the CCPFAWG operator is changed into a simple CPF-weighted geometric (CPFWDG) operator, such that

$$CCPFAWG(\beta_1, \beta_2, \dots, \beta_n) = \otimes_{s=1}^n (\beta_s)^{\overline{w}_s} \quad (33)$$

Theorem 3 The computed value of Eq. (32) is also CPF number, such that

$$CCPFAWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ = \left(\mathbb{G}^{-1} \left(\sum_{s=1}^n \overline{w}_s (\nabla_s \mathbb{G}(u_{R-s})) \right) e^{i2\pi \left(\mathbb{G}^{-1} \left(\sum_{s=1}^n \overline{w}_s (\nabla_s \mathbb{G}(u_{I-s})) \right) \right)}, \right. \\ \left. \mathbb{H}^{-1} \left(\sum_{s=1}^n \overline{w}_s (\nabla_s \mathbb{H}(v_{R-s})) \right) e^{i2\pi \left(\mathbb{H}^{-1} \left(\sum_{s=1}^n \overline{w}_s (\nabla_s \mathbb{H}(v_{I-s})) \right) \right)} \right) \quad (34)$$

where $0 \leq \nabla_s \leq 1$ and $\sum_{s=1}^n \overline{w}_s = 1$.

Property 7 (Idempotency) If $\beta_s = \beta = (u_R e^{i2\pi(u_I)}, v_R e^{i2\pi(v_I)})$, $s = 1, 2, \dots, n$, then

$$CCPFWG(\beta_1, \beta_2, \dots, \beta_n) = \beta. \quad (35)$$

Property 8 (Monotonicity) If $\beta_s = (u_{R-s} e^{i2\pi(u_{I-s})}, v_{R-s} e^{i2\pi(v_{I-s})})$, and $\beta_s^* = (u_{R-s}^* e^{i2\pi(u_{I-s}^*)}, v_{R-s}^* e^{i2\pi(v_{I-s}^*)})$, $s = 1, 2, \dots, n$, with $\beta_s \leq \beta_s^*$ then

$$CCPFAWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ \leq CCPFAWG((\nabla_1, \beta_1^*), (\nabla_2, \beta_2^*), \dots, (\nabla_n, \beta_n^*)). \quad (36)$$

Property 9 (Boundedness) If $\beta^- = \left(\min_s u_{R-s} e^{i2\pi \left(\min_s u_{I-s} \right)}, \max_s v_{R-s} e^{i2\pi \left(\max_s v_{I-s} \right)} \right)$ and $\beta^+ = \left(\max_s u_{R-s} e^{i2\pi \left(\max_s u_{I-s} \right)}, \min_s v_{R-s} e^{i2\pi \left(\min_s v_{I-s} \right)} \right)$, $s = 1, 2, \dots, n$, then

$$\beta^- \leq CCPFAWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \leq \beta^+ \quad (37)$$

Further, by using different types of functions, we can obtain some new operators, such that.

1. When $\mathbb{G}(t) = -\log(t^2)$, then

$$CCPFAWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ = \left(\prod_{s=1}^n u_{R-s}^{\overline{w}_s \nabla_s} e^{i2\pi \left(\prod_{s=1}^n u_{I-s}^{\overline{w}_s \nabla_s} \right)}, \right. \\ \left. \left(1 - \prod_{s=1}^n (1 - v_{R-s}^2)^{\overline{w}_s \nabla_s} \right)^{\frac{1}{2}} e^{i2\pi \left(1 - \prod_{s=1}^n (1 - v_{I-s}^2)^{\overline{w}_s \nabla_s} \right)^{\frac{1}{2}}} \right) \quad (38)$$

is called CCPFWG operator.

2. When $\mathbb{G}(t) = \log\left(\frac{2-t^2}{t^2}\right)$, then

$$CCPFAWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ = \left(\left(\frac{2 \prod_{s=1}^n (u_{R-s}^2)^{\overline{w}_s \nabla_s}}{\prod_{s=1}^n (2 - u_{R-s}^2)^{\overline{w}_s \nabla_s} + \prod_{s=1}^n (u_{R-s}^2)^{\overline{w}_s \nabla_s}} \right)^{\frac{1}{2}} e^{i2\pi \left(\frac{2 \prod_{s=1}^n (u_{I-s}^2)^{\overline{w}_s \nabla_s}}{\prod_{s=1}^n (2 - u_{I-s}^2)^{\overline{w}_s \nabla_s} + \prod_{s=1}^n (u_{I-s}^2)^{\overline{w}_s \nabla_s}} \right)^{\frac{1}{2}}}, \right. \\ \left. \left(\frac{\prod_{s=1}^n (1 + v_{R-s}^2)^{\overline{w}_s \nabla_s} - \prod_{s=1}^n (1 - v_{R-s}^2)^{\overline{w}_s \nabla_s}}{\prod_{s=1}^n (1 + v_{R-s}^2)^{\overline{w}_s \nabla_s} + \prod_{s=1}^n (1 - v_{R-s}^2)^{\overline{w}_s \nabla_s}} \right)^{\frac{1}{2}} e^{i2\pi \left(\frac{\prod_{s=1}^n (1 + v_{I-s}^2)^{\overline{w}_s \nabla_s} - \prod_{s=1}^n (1 - v_{I-s}^2)^{\overline{w}_s \nabla_s}}{\prod_{s=1}^n (1 + v_{I-s}^2)^{\overline{w}_s \nabla_s} + \prod_{s=1}^n (1 - v_{I-s}^2)^{\overline{w}_s \nabla_s}} \right)^{\frac{1}{2}}} \right) \quad (39)$$

is called CCPFEWG operator.

3. When $\mathbb{G}(t) = \log\left(\frac{\gamma + (1-\gamma)t^2}{t^2}\right)$, then

$$CCPFAWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ = \left(\left(\frac{2 \prod_{s=1}^n (u_{R-s}^2)^{\overline{w}_s \nabla_s}}{\prod_{s=1}^n (1 + (\gamma - 1)(1 - u_{R-s}^2)^{\overline{w}_s \nabla_s}) + (\gamma - 1) \prod_{s=1}^n (u_{R-s}^2)^{\overline{w}_s \nabla_s}} \right)^{\frac{1}{2}} e^{i2\pi \left(\frac{2 \prod_{s=1}^n (u_{I-s}^2)^{\overline{w}_s \nabla_s}}{\prod_{s=1}^n (1 + (\gamma - 1)(1 - u_{I-s}^2)^{\overline{w}_s \nabla_s}) + (\gamma - 1) \prod_{s=1}^n (u_{I-s}^2)^{\overline{w}_s \nabla_s}} \right)^{\frac{1}{2}}}, \right. \\ \left. \left(\frac{\prod_{s=1}^n (1 + (\gamma - 1)v_{R-s}^2)^{\overline{w}_s \nabla_s} - \prod_{s=1}^n (1 - v_{R-s}^2)^{\overline{w}_s \nabla_s}}{\prod_{s=1}^n (1 + (\gamma - 1)v_{R-s}^2)^{\overline{w}_s \nabla_s} + (\gamma - 1) \prod_{s=1}^n (1 - v_{R-s}^2)^{\overline{w}_s \nabla_s}} \right)^{\frac{1}{2}} e^{i2\pi \left(\frac{\prod_{s=1}^n (1 + (\gamma - 1)v_{I-s}^2)^{\overline{w}_s \nabla_s} - \prod_{s=1}^n (1 - v_{I-s}^2)^{\overline{w}_s \nabla_s}}{\prod_{s=1}^n (1 + (\gamma - 1)v_{I-s}^2)^{\overline{w}_s \nabla_s} + (\gamma - 1) \prod_{s=1}^n (1 - v_{I-s}^2)^{\overline{w}_s \nabla_s}} \right)^{\frac{1}{2}}} \right) \quad (40)$$

is called CCPFHWG operator. If $\gamma = 1$ in Eq. (40), then we get Eq. (38), if $\gamma = 2$ in Eq. (40), then we get Eq. (39).

4. When $\mathbb{G}(t) = \log\left(\frac{\gamma-1}{\gamma t^2 - 1}\right)$, then

$$CCPFAWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ = \left(\left(\log_{\gamma} \left(1 + \frac{\prod_{s=1}^n (\gamma^{\overline{w}_s \nabla_s} - 1)}{\gamma - 1} \right) \right)^{\frac{1}{2}} e^{i2\pi \left(\log_{\gamma} \left(1 + \frac{\prod_{s=1}^n (\gamma^{\overline{w}_s \nabla_s} - 1)}{\gamma - 1} \right) \right)^{\frac{1}{2}}}, \right. \\ \left. \left(1 - \log_{\gamma} \left(1 + \frac{\prod_{s=1}^n (\gamma^{1 - \overline{w}_s \nabla_s} - 1)}{\gamma - 1} \right) \right)^{\frac{1}{2}} e^{i2\pi \left(1 - \log_{\gamma} \left(1 + \frac{\prod_{s=1}^n (\gamma^{1 - \overline{w}_s \nabla_s} - 1)}{\gamma - 1} \right) \right)^{\frac{1}{2}}} \right) \quad (41)$$

is called CCPFFWG operator.

To explain the above aggregation operators, we illustrate an example.

Example 2 We consider $\beta_1 = \{(0.7, 0.6e^{i2\pi(0.51)}, (0.5), 0.5e^{i2\pi(0.4)})\}$, $\beta_2 = \left\{ \left(0.71, 0.61e^{i2\pi(0.51)}, 0.51e^{i2\pi(0.41)} \right) \right\}$, $\beta_3 =$

$\left\{ \left(\begin{array}{c} 0.72, 0.62e^{i2\pi(0.52)} \\ 0.52e^{i2\pi(0.42)} \end{array} \right) \right\}$ and $\beta_4 = \left\{ \left(\begin{array}{c} 0.73, 0.63e^{i2\pi(0.53)} \\ 0.53e^{i2\pi(0.43)} \end{array} \right) \right\}$ with weight vectors 0.4, 0.3, 0.2, and 0.1, then by using Eq. (38) to Eq. (41), we have

$$CCPFAWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), (\nabla_3, \beta_3), (\nabla_4, \beta_4)) = \left(\begin{array}{c} 0.7041e^{i2\pi(0.62)} \\ 0.4391e^{i2\pi(0.3503)} \end{array} \right) \dots \text{by Eq. (38)}$$

$$CCPFAWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), (\nabla_3, \beta_3), (\nabla_4, \beta_4)) = \left(\begin{array}{c} 0.7207e^{i2\pi(0.6419)} \\ 0.4324e^{i2\pi(0.3465)} \end{array} \right) \dots \text{by Eq. (39)}$$

$$CCPFAWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), (\nabla_3, \beta_3), (\nabla_4, \beta_4)) = \left(\begin{array}{c} 0.598e^{i2\pi(0.5362)} \\ 0.4268e^{i2\pi(0.3432)} \end{array} \right) \dots \text{by Eq. (40)}$$

$$CCPFAWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), (\nabla_3, \beta_3), (\nabla_4, \beta_4)) = \left(\begin{array}{c} 0.4943e^{i2\pi(0.4324)} \\ 0.7613e^{i2\pi(0.7322)} \end{array} \right) \dots \text{by Eq. (41)}$$

Then by using Eq. (6), we have

$$C(CCPFAWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), (\nabla_3, \beta_3), (\nabla_4, \beta_4))) = 0.2674 \dots \text{by Eq. (38)}$$

$$C(CCPFAWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), (\nabla_3, \beta_3), (\nabla_4, \beta_4))) = 0.2918 \dots \text{by Eq. (39)}$$

$$C(CCPFAWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), (\nabla_3, \beta_3), (\nabla_4, \beta_4))) = 0.1821 \dots \text{by Eq. (40)}$$

$$C(CCPFAWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), (\nabla_3, \beta_3), (\nabla_4, \beta_4))) = 0.2834 \dots \text{by Eq. (41)}$$

Further, the geometrical interpretation of the above information is described in Fig. 3.

Definition 8 Under the supposition of fixed set x , a structure:

$$CCPFAOWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) = \otimes_{s=1}^n \left(\beta_{o(s)}^{\nabla_{o(s)}} \right)^{\overline{w}_s} \quad (42)$$

is called the CCPFAOWG operator with $\beta_{o(s)} \leq \beta_{o(s-1)}$, where \overline{w}_s represents the position weight vector with $\sum_{s=1}^n \overline{w}_s = 1$ and ∇_s expresses the confidence level, where $0 \leq \nabla_s \leq 1$. When $\nabla_1 = \nabla_2 = \dots = \nabla_n = 1$, then the CCPFAOWG operator is changed into a simple CPF-ordered weighted geometric (CPFOWG) operator, such that

$$CCPFAOWG(\beta_1, \beta_2, \dots, \beta_n) = \otimes_{s=1}^n \left(\beta_{o(s)} \right)^{\overline{w}_s} \quad (43)$$

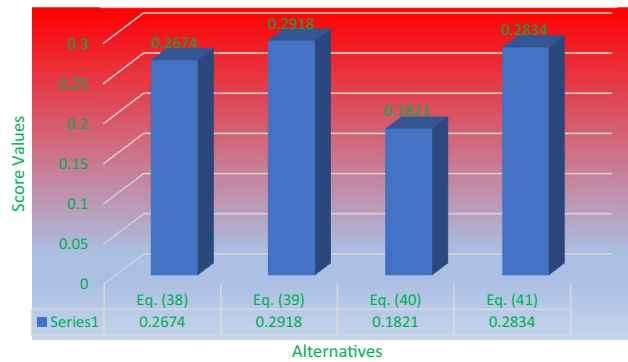


Fig. 3 Geometrical explanation of example 2

Theorem 4 The computed value of Eq. (42) is also CPF number, such that

$$CCPFAOWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) = \left(\begin{array}{c} \mathfrak{g}^{-1} \left(\sum_{s=1}^n \overline{w}_s (\nabla_{o(s)} \mathfrak{g}(u_{R-o(s)})) \right) e^{i2\pi \left(\mathfrak{g}^{-1} \left(\sum_{s=1}^n \overline{w}_s (\nabla_{o(s)} \mathfrak{g}(u_{I-o(s)})) \right) \right)} \\ \mathfrak{h}^{-1} \left(\sum_{s=1}^n \overline{w}_s (\nabla_{o(s)} \mathfrak{h}(v_{R-o(s)})) \right) e^{i2\pi \left(\mathfrak{h}^{-1} \left(\sum_{s=1}^n \overline{w}_s (\nabla_{o(s)} \mathfrak{h}(v_{I-o(s)})) \right) \right)} \end{array} \right), \quad (44)$$

where $0 \leq \nabla_s \leq 1$ and $\sum_{s=1}^n \overline{w}_s = 1$.

Proof Omitted. \square

Property 10 (Idempotency) If $\beta_s = \beta = (u_R e^{i2\pi(u_I)}, v_R e^{i2\pi(v_I)})$, $s = 1, 2, \dots, n$, then

$$CCPFAOWG(\beta_1, \beta_2, \dots, \beta_n) = \beta. \quad (45)$$

Property 11 (Monotonicity) If $\beta_s = (u_{R-s} e^{i2\pi(u_{I-s})}, v_{R-s} e^{i2\pi(v_{I-s})})$, and $\beta_s^* = (u_{R-s}^* e^{i2\pi(u_{I-s}^*)}, v_{R-s}^* e^{i2\pi(v_{I-s}^*)})$, $s = 1, 2, \dots, n$, with $\beta_s \leq \beta_s^*$ then

$$CCPFAOWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \leq CCPFAOWG((\nabla_1, \beta_1^*), (\nabla_2, \beta_2^*), \dots, (\nabla_n, \beta_n^*)). \quad (46)$$

Property 12 (Boundedness) If $\beta^- =$

$$\left(\min_s u_{R-s} e^{i2\pi \left(\min_s u_{I-s} \right)}, \max_s v_{R-s} e^{i2\pi \left(\max_s v_{I-s} \right)} \right) \text{ and } \beta^+ = \left(\max_s u_{R-s} e^{i2\pi \left(\max_s u_{I-s} \right)}, \min_s v_{R-s} e^{i2\pi \left(\min_s v_{I-s} \right)} \right), s = 1, 2, \dots, n, \text{ then}$$

$$\beta^- \leq CCPFAOWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \leq \beta^+. \quad (47)$$

Further, by using different types of functions, we can obtain some new operators, such that.

1. When $\mathfrak{U}(t) = -\log(t^2)$, then

$$CCPFAOWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ = \left(\prod_{s=1}^n u_{R-o(s)}^{\overline{w}_s \nabla_{o(s)}} e^{i2\pi \left(\prod_{s=1}^n u_{I-o(s)}^{\overline{w}_s \nabla_{o(s)}} \right)}, \right. \\ \left. \left(1 - \prod_{s=1}^n (1 - v_{R-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}} \right)^{\frac{1}{2}} e^{i2\pi \left(1 - \prod_{s=1}^n (1 - v_{I-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}} \right)^{\frac{1}{2}}} \right) \quad (48)$$

is called CCPFOWG operator.

2. When $\mathfrak{U}(t) = \log\left(\frac{2-t^2}{t^2}\right)$, then

$$CCPFAOWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ = \left(\left(\frac{2 \prod_{s=1}^n (u_{R-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}}{\prod_{s=1}^n (2 - u_{R-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}} + \prod_{s=1}^n (u_{R-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}} \right)^{\frac{1}{2}} e^{i2\pi \left(\frac{\prod_{s=1}^n (u_{I-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}}{\prod_{s=1}^n (2 - u_{I-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}} + \prod_{s=1}^n (u_{I-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}} \right)}, \right. \\ \left. \left(\frac{\prod_{s=1}^n (1 + v_{R-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}} - \prod_{s=1}^n (1 - v_{R-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}}{\prod_{s=1}^n (1 + v_{R-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}} + \prod_{s=1}^n (1 - v_{R-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}} \right)^{\frac{1}{2}} e^{i2\pi \left(\frac{\prod_{s=1}^n (1 + v_{I-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}} - \prod_{s=1}^n (1 - v_{I-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}}{\prod_{s=1}^n (1 + v_{I-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}} + \prod_{s=1}^n (1 - v_{I-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}} \right)} \right) \quad (49)$$

is called CCPFEOWG operator.

3. When $\mathfrak{U}(t) = \log\left(\frac{\gamma + (1-\gamma)t^2}{t^2}\right)$, then

$$CCPFAOWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ = \left(\left(\frac{2 \prod_{s=1}^n (u_{R-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}}{\prod_{s=1}^n (1 + (\gamma-1)(1 - u_{R-o(s)}^2))^{\overline{w}_s \nabla_{o(s)}} + (\gamma-1) \prod_{s=1}^n (u_{R-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}} \right)^{\frac{1}{2}} e^{i2\pi \left(\frac{2 \prod_{s=1}^n (u_{I-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}}{\prod_{s=1}^n (1 + (\gamma-1)(1 - u_{I-o(s)}^2))^{\overline{w}_s \nabla_{o(s)}} + (\gamma-1) \prod_{s=1}^n (u_{I-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}} \right)}, \right. \\ \left. \left(\frac{\prod_{s=1}^n (1 + (\gamma-1)v_{R-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}} - \prod_{s=1}^n (1 - v_{R-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}}{\prod_{s=1}^n (1 + (\gamma-1)v_{R-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}} + (\gamma-1) \prod_{s=1}^n (1 - v_{R-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}} \right)^{\frac{1}{2}} e^{i2\pi \left(\frac{\prod_{s=1}^n (1 + (\gamma-1)v_{I-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}} - \prod_{s=1}^n (1 - v_{I-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}}{\prod_{s=1}^n (1 + (\gamma-1)v_{I-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}} + (\gamma-1) \prod_{s=1}^n (1 - v_{I-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}} \right)} \right) \quad (50)$$

is called CCPFHOWG operator. If $\gamma = 1$ in Eq. (50), then we get Eq. (48), if $\gamma = 2$ in Eq. (50), then we get Eq. (49).

4. When $\mathfrak{U}(t) = \log\left(\frac{\gamma-1}{\gamma t^2 - 1}\right)$, then

$$CCPFAOWG((\nabla_1, \beta_1), (\nabla_2, \beta_2), \dots, (\nabla_n, \beta_n)) \\ = \left(\left(\log_{\gamma} \left(1 + \frac{\prod_{s=1}^n (u_{R-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}}{\gamma - 1} \right) \right)^{\frac{1}{2}} e^{i2\pi \left(\log_{\gamma} \left(1 + \frac{\prod_{s=1}^n (u_{I-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}}{\gamma - 1} \right) \right)}, \right. \\ \left. \left(1 - \log_{\gamma} \left(1 + \frac{\prod_{s=1}^n (1 - v_{R-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}}{\gamma - 1} \right) \right)^{\frac{1}{2}} e^{i2\pi \left(1 - \log_{\gamma} \left(1 + \frac{\prod_{s=1}^n (1 - v_{I-o(s)}^2)^{\overline{w}_s \nabla_{o(s)}}}{\gamma - 1} \right) \right)} \right) \quad (51)$$

is called CCPFFOWG operator.

4 MADM Methods

In this section, we develop a MADM method based on the proposed operators and find our best alternative. Some examples are used to do comparative analysis among proposed and existing methods to show the validation of the new approaches. MADM technique plays a valuable role in strategic decision making, and it has been more generally used in certain fields like economics, management sciences, computer science, road signals systems, and software engineering in the last few years. Its spirit is to rank the alternatives under some attributes and select the best one.

Suppose $\beta_i, i = 1, 2, \dots, m$ represent the m-alternatives, and $A_j, j = 1, 2, \dots, n$ represent n-attributes with weight vector $\sum_{j=1}^n \overline{w}_j = 1$. The value of alternative β_i under attribute A_j is represented by CPF numbers $\beta_{ij} = (u_{R-ij} e^{i2\pi(u_{I-ij})}, v_{R-ij} e^{i2\pi(v_{I-ij})}) \quad i = 1, 2, \dots, m, j = 1, 2, \dots, n$ with $0 \leq u_{R-ij}^2 + v_{R-ij}^2 \leq 1, 0 \leq u_{I-ij}^2 + v_{I-ij}^2 \leq 1$. The degree of confidence level ∇_j is also attached with every CPF number with $0 \leq \nabla_j \leq 1, j = 1, 2, \dots, n$. To evaluate the above-cited dilemmas, the decision-making steps are given below:

Stage 1: Because there are two types of the decision information in the decision matrix, i.e., cost type or benefit type, if the information is cost type, then the decision information needs to be normalized, but if the information is benefit type, then no need to be normalized as follows.

$$N = \begin{cases} (u_{R-ij} e^{i2\pi(u_{I-ij})}, v_{R-ij} e^{i2\pi(v_{I-ij})}) & \text{for benefit} \\ (v_{R-ij} e^{i2\pi(v_{I-ij})}, u_{R-ij} e^{i2\pi(u_{I-ij})}) & \text{for cost} \end{cases}$$

For the normalized matrix, we use *Ntoexpressit..*

Stage 2: we get the aggregated values using the CCPFWA, CCPFEWA, CCPFHOWA, CCPFFWA, CCPFWG, CCPFEWG, CCPFHOWG, and CCPFFWG operators.

Stage 3: We calculate the Score value of the above-aggregated values.

Stage 4: Ranking these results and find the best one.

Example 3 This analysis gives a demonstrative case to justify the circumstance of the diagnosed technique for resolving mobile medical app (MMA) dilemmas. The

Table 2 The decision matrix contained CPF information

	A_1	A_2	A_3	A_4
β_1	$\left(0.7, 0.6e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.4)} \right)$	$\left(0.71, 0.61e^{i2\pi(0.51)}, 0.51e^{i2\pi(0.41)} \right)$	$\left(0.72, 0.62e^{i2\pi(0.52)}, 0.52e^{i2\pi(0.42)} \right)$	$\left(0.73, 0.63e^{i2\pi(0.53)}, 0.53e^{i2\pi(0.43)} \right)$
β_2	$\left(0.4, 0.5e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.5)} \right)$	$\left(0.41, 0.51e^{i2\pi(0.51)}, 0.51e^{i2\pi(0.51)} \right)$	$\left(0.42, 0.52e^{i2\pi(0.52)}, 0.52e^{i2\pi(0.52)} \right)$	$\left(0.43, 0.53e^{i2\pi(0.53)}, 0.53e^{i2\pi(0.53)} \right)$
β_3	$\left(0.5, 0.8e^{i2\pi(0.7)}, 0.4e^{i2\pi(0.1)} \right)$	$\left(0.51, 0.81e^{i2\pi(0.71)}, 0.41e^{i2\pi(0.11)} \right)$	$\left(0.52, 0.82e^{i2\pi(0.72)}, 0.42e^{i2\pi(0.12)} \right)$	$\left(0.53, 0.83e^{i2\pi(0.73)}, 0.43e^{i2\pi(0.13)} \right)$
β_4	$\left(0.4, 0.5e^{i2\pi(0.4)}, 0.3e^{i2\pi(0.2)} \right)$	$\left(0.41, 0.51e^{i2\pi(0.41)}, 0.31e^{i2\pi(0.21)} \right)$	$\left(0.42, 0.52e^{i2\pi(0.42)}, 0.32e^{i2\pi(0.22)} \right)$	$\left(0.43, 0.53e^{i2\pi(0.43)}, 0.33e^{i2\pi(0.23)} \right)$
β_5	$\left(0.1, 0.5e^{i2\pi(0.6)}, 0.1e^{i2\pi(0.2)} \right)$	$\left(0.11, 0.51e^{i2\pi(0.61)}, 0.11e^{i2\pi(0.21)} \right)$	$\left(0.12, 0.52e^{i2\pi(0.62)}, 0.12e^{i2\pi(0.22)} \right)$	$\left(0.13, 0.53e^{i2\pi(0.63)}, 0.13e^{i2\pi(0.23)} \right)$

Table 3 Aggregated values obtained for weighted averaging operators

	CCPFWA	CCPFEWA	CCPFHWA	CCPFFWA
β_1	$\left(0.5308e^{i2\pi(0.4391)}, 0.62e^{i2\pi(0.531)} \right)$	$\left(0.5204e^{i2\pi(0.4324)}, 0.6419e^{i2\pi(0.5568)} \right)$	$\left(0.5122e^{i2\pi(0.4268)}, 0.5362e^{i2\pi(0.4686)} \right)$	$\left(0.7953e^{i2\pi(0.7613)}, 0.4324e^{i2\pi(0.3687)} \right)$
β_2	$\left(0.3412e^{i2\pi(0.3412)}, 0.7588e^{i2\pi(0.7588)} \right)$	$\left(0.33e^{i2\pi(0.33)}, 0.7932e^{i2\pi(0.7932)} \right)$	$\left(0.3207e^{i2\pi(0.3207)}, 0.6652e^{i2\pi(0.6652)} \right)$	$\left(0.7759e^{i2\pi(0.7759)}, 0.5012e^{i2\pi(0.5012)} \right)$
β_3	$\left(0.6485e^{i2\pi(0.5488)}, 0.6347e^{i2\pi(0.324)} \right)$	$\left(0.6174e^{i2\pi(0.5253)}, 0.6757e^{i2\pi(0.3711)} \right)$	$\left(0.5953e^{i2\pi(0.5078)}, 0.573e^{i2\pi(0.3278)} \right)$	$\left(0.8582e^{i2\pi(0.8236)}, 0.4269e^{i2\pi(0.2213)} \right)$
β_4	$\left(0.3412e^{i2\pi(0.2699)}, 0.6187e^{i2\pi(0.5274)} \right)$	$\left(0.33e^{i2\pi(0.2638)}, 0.6741e^{i2\pi(0.5906)} \right)$	$\left(0.3207e^{i2\pi(0.2585)}, 0.5779e^{i2\pi(0.5139)} \right)$	$\left(0.7759e^{i2\pi(0.7604)}, 0.4105e^{i2\pi(0.3522)} \right)$
β_5	$\left(0.181e^{i2\pi(0.2239)}, 0.7848e^{i2\pi(0.8425)} \right)$	$\left(0.1715e^{i2\pi(0.2077)}, 0.8526e^{i2\pi(0.8915)} \right)$	$\left(0.1638e^{i2\pi(0.1956)}, 0.7226e^{i2\pi(0.747)} \right)$	$\left(0.7895e^{i2\pi(0.7945)}, 0.4967e^{i2\pi(0.5288)} \right)$

MMA company has constructed quickly and has gotten a various scale thanks to the broad commercialization of mobile internet and smartphones in experts' everyday life, the simultaneous modifying the quality of various modified techniques, as well as the decline of the construction cost and threshold of mobile qualities and features. Based on the survey, many medical apps are working in the market currently. But based on their usage, these apps can be divided into platforms and function as mobile medical devices. Platform MMAs, such as a doctor, good doctor, personal doctor, and thumb doctor, divided the following features: online consolation, appointment registrations, disease analysis, and health consultation where the function MMAs, such as meeting you, Baobao ahi Dao, well tang, are the features, such as female period, diabetes mellitus, which is very beneficial for some peoples. Albeit various portable clinical applications have arisen in the market, which has carried comfort to individuals' day-to-day existence, there is still a lack in these applications, like inadequate client security insurance, troublesome confirmation of data realness, and high gamble of telemedicine finding and misleading clinical notices. To better understand the versatile clinical application market, five well-known Chinese portable clinical applications are

considered for assessment, thus secretly denoted by: $\beta_i, i = 1, 2, 3, 4, 5$, represented the alternatives and they are evaluated by the features or functions in MMAs which represented the criteria, called A_1 : Safety, A_2 : interface, A_3 : reliability and A_4 : functionality. To evaluate the above-cited dilemmas, the decision-making matrix is given in Table 2:

- Stage 1: Because the information in Table 2 is the same as benefit type, it does not need to be normalized.
- Stage 2: Get the aggregated values using the CCPFWA, CCPFEWA, CCPFHW, CCPFFWA, CCPFWG, CCPFEWG, CCPFHWG, and CCPFFWG operators, see Tables 3 and 4.
- Stage 3: Obtain the Score value of the above-aggregated values, see Tables 5 and 6. The geometrical interpretation of the information in Table 5 is described in t Fig. 4. The geometrical interpretation of the information in Table 6 is described in Fig. 5.
- Stage 4: Get the ranking results and find the best one, see Table 7.

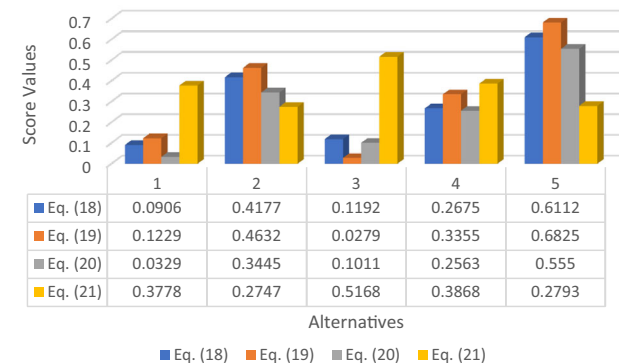
Form Table 7, we can get that the CCPFFWA aggregation operator and CCPFFWG aggregation operator

Table 4 Aggregated values obtained for weighted geometric operators

	CCPFWG	CCPFEWG	CCPFHWG	CCPFFWG
β_1	$\left(\begin{matrix} 0.7041e^{i2\pi(0.62)} \\ 0.4391e^{i2\pi(0.3503)} \end{matrix} \right)$	$\left(\begin{matrix} 0.7207e^{i2\pi(0.6419)} \\ 0.4324e^{i2\pi(0.3465)} \end{matrix} \right)$	$\left(\begin{matrix} 0.598e^{i2\pi(0.5362)} \\ 0.4268e^{i2\pi(0.3432)} \end{matrix} \right)$	$\left(\begin{matrix} 0.4943e^{i2\pi(0.4324)} \\ 0.7613e^{i2\pi(0.7322)} \end{matrix} \right)$
β_2	$\left(\begin{matrix} 0.7588e^{i2\pi(0.7588)} \\ 0.3412e^{i2\pi(0.3412)} \end{matrix} \right)$	$\left(\begin{matrix} 0.7932e^{i2\pi(0.7932)} \\ 0.33e^{i2\pi(0.33)} \end{matrix} \right)$	$\left(\begin{matrix} 0.6652e^{i2\pi(0.6652)} \\ 0.3207e^{i2\pi(0.3207)} \end{matrix} \right)$	$\left(\begin{matrix} 0.5012e^{i2\pi(0.5012)} \\ 0.7759e^{i2\pi(0.7759)} \end{matrix} \right)$
β_3	$\left(\begin{matrix} 0.8982e^{i2\pi(0.8398)} \\ 0.2996e^{i2\pi(0.0792)} \end{matrix} \right)$	$\left(\begin{matrix} 0.9052e^{i2\pi(0.8542)} \\ 0.2941e^{i2\pi(0.079)} \end{matrix} \right)$	$\left(\begin{matrix} 0.7435e^{i2\pi(0.7058)} \\ 0.2892e^{i2\pi(0.0789)} \end{matrix} \right)$	$\left(\begin{matrix} 0.6146e^{i2\pi(0.57)} \\ 0.7541e^{i2\pi(0.7173)} \end{matrix} \right)$
β_4	$\left(\begin{matrix} 0.7588e^{i2\pi(0.6939)} \\ 0.2017e^{i2\pi(0.1357)} \end{matrix} \right)$	$\left(\begin{matrix} 0.7932e^{i2\pi(0.739)} \\ 0.199e^{i2\pi(0.1348)} \end{matrix} \right)$	$\left(\begin{matrix} 0.6652e^{i2\pi(0.626)} \\ 0.1965e^{i2\pi(0.134)} \end{matrix} \right)$	$\left(\begin{matrix} 0.5012e^{i2\pi(0.4587)} \\ 0.7484e^{i2\pi(0.7397)} \end{matrix} \right)$
β_5	$\left(\begin{matrix} 0.9288e^{i2\pi(0.9472)} \\ 0.037e^{i2\pi(0.0708)} \end{matrix} \right)$	$\left(\begin{matrix} 0.9466e^{i2\pi(0.9587)} \\ 0.0369e^{i2\pi(0.0701)} \end{matrix} \right)$	$\left(\begin{matrix} 0.7808e^{i2\pi(0.7883)} \\ 0.0368e^{i2\pi(0.0694)} \end{matrix} \right)$	$\left(\begin{matrix} 0.5777e^{i2\pi(0.589)} \\ 0.7797e^{i2\pi(0.7809)} \end{matrix} \right)$

Table 5 Score values obtained from the information given in Table 3

	CCPFWA	CCPFEWA	CCPFHWA	CCPFFWA
β_1	0.0906	0.1229	0.0329	0.3778
β_2	0.4177	0.4632	0.3445	0.2747
β_3	0.1192	0.0279	0.1011	0.5168
β_4	0.2675	0.3355	0.2563	0.3868
β_5	0.6112	0.6825	0.555	0.2793

**Fig. 4** Geometrical interpretation of the information in Table 5

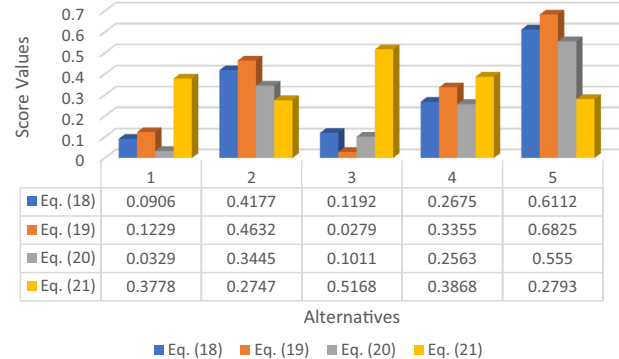
produce the different results like β_3 and β_1 . Further, the remaining ranking results available in Table 6 are the same, which give the best optimal as a β_5 .

5 Comparative Analysis

It is necessary to compare the proposed approaches with some existing methods to illustrate the validation and effectiveness of the developed approaches. For this, we used the following operators, which are based on different ideas, called aggregation operators (AOs) for IFS proposed by Xu [36], Einstein AOs (EAOs) for IFS developed by

Table 6 Score values obtained from the information given in Table 4

	CCPFWG	CCPFEWG	CCPFHWG	CCPFFWG
β_1	0.2674	0.2918	0.1821	0.2834
β_2	0.4177	0.4632	0.3445	0.2747
β_3	0.6796	0.6931	0.5406	0.1421
β_4	0.5577	0.5992	0.4804	0.2642
β_5	0.8841	0.8992	0.7314	0.1969

**Fig. 5** Geometrical interpretation of the information in Table 6

Zhao and Wei [37], Hamacher AOs (HAOs) for IFS founded by Huang [40], HAOs for PFS developed by Wu and Wei [43], AOs for PFS proposed by Feng et al. [44], AOs for CIFS proposed by Garg and Rani [26], HAOs for CIFS proposed by Akram et al. [29], AOs for CPFSSs diagnosed by Mahmood et al. [46]. Using the information in Table 2, the comparative analysis is shown in Table 8.

From Table 8, the methods in Ref. [26, 29, 36, 37, 40, 43, 44, 46] are not evaluated the information given in Table 2 because of certain restrictions. All reasons behind these issues are explained below.

1. Xu [36] developed simple AOs for IFS, Zhao and Wei [37] evaluated EAOs for IFS, and Huang [40], and the IFS

Table 7 Ranking values obtained from Tables 5 and 6

Methods	Ranking values	Best decision
CCPFWA operator	$\beta_5 > \beta_2 > \beta_4 > \beta_3 > \beta_1$	β_5
CCPFEWA operator	$\beta_5 > \beta_2 > \beta_4 > \beta_1 > \beta_3$	β_5
CCPFHWA operator	$\beta_5 > \beta_2 > \beta_4 > \beta_3 > \beta_1$	β_5
CCPFFWA operator	$\beta_3 > \beta_4 > \beta_1 > \beta_5 > \beta_2$	β_3
CCPFWG operator	$\beta_5 > \beta_3 > \beta_4 > \beta_2 > \beta_1$	β_5
CCPFEWG operator	$\beta_5 > \beta_3 > \beta_4 > \beta_2 > \beta_1$	β_5
CCPFHWG operator	$\beta_5 > \beta_3 > \beta_4 > \beta_2 > \beta_1$	β_5
CCPFFWG operator	$\beta_1 > \beta_2 > \beta_4 > \beta_5 > \beta_3$	β_1

Table 8 Representation of the comparative analysis

Methods	Score values	Ranking results
Xu [36]	Not evaluated information	Not evaluated information
Zhao and Wei [37]	Not evaluated information	Not evaluated information
Huang [40]	Not evaluated information	Not evaluated information
Wu and Wei [43]	Not evaluated information	Not evaluated information
Feng et al. [44]	Not evaluated information	Not evaluated information
Garg and Rani [26]	Not evaluated information	Not evaluated information
Akram et al. [29]	Not evaluated information	Not evaluated information
Mahmood et al. [46]	0.0815,0.4266,0.1281,0.2764,0.6201	$\beta_5 > \beta_2 > \beta_4 > \beta_3 > \beta_1$
CCPFWA operator	0.0906,0.4177,0.1192,0.2675,0.6112	$\beta_5 > \beta_2 > \beta_4 > \beta_3 > \beta_1$
CCPFEWA operator	0.1229,0.4632,0.0279,0.3355,0.6825	$\beta_5 > \beta_2 > \beta_4 > \beta_1 > \beta_3$
CCPFHWA operator	0.0329,0.3445,0.1011,0.2563,0.555	$\beta_5 > \beta_2 > \beta_4 > \beta_3 > \beta_1$
CCPFFWA operator	0.3778,0.2747,0.5168,0.3868,0.2793	$\beta_3 > \beta_4 > \beta_1 > \beta_5 > \beta_2$
CCPFWG operator	0.2674,0.4177,0.6796,0.5577,0.8841	$\beta_5 > \beta_3 > \beta_4 > \beta_2 > \beta_1$
CCPFEWG operator	0.2918,0.4632,0.6931,0.5992,0.8992	$\beta_5 > \beta_3 > \beta_4 > \beta_2 > \beta_1$
CCPFHWG operator	0.1821,0.3445,0.5406,0.4804,0.7314	$\beta_5 > \beta_3 > \beta_4 > \beta_2 > \beta_1$
CCPFFWG operator	0.2834,0.2747,0.1421,0.2642,0.1969	$\beta_1 > \beta_2 > \beta_4 > \beta_5 > \beta_3$

is the specific case of the CPFS proposed in this paper, and one more weakness in Ref. [36, 37, 40] is that they cannot contain confidence levels, so due to these issues, Refs. [36, 37, 40] do not evaluated the information given in Table 2. If we remove the confidence degree from Table 2, then the operators in Ref. [36, 37, 40] still have failed because of their mathematical structure.

- Wu and Wei [43] developed simple HAOs for PFS and Feng et al. [44] developed AOs for PFS, and the PFS is the specific case of the CPFS proposed in this paper, and one more weakness in Ref. [43, 44] is that they cannot contain confidence levels, so due to these issues, Refs. [43, 44] do not evaluate the information given in Table 2. If we remove the confidence degree from Table 2, then the operators in Ref. [43, 44] still have failed because of their mathematical structure.
- Garg and Rani [26] proposed simple AOs for CIFS and Akram et al. [29] developed HAOs for CIFS, and the CIFS is the specific case of the CPFS proposed in this

paper, and one more weakness in Ref. [26, 29] is that they cannot contain confidence levels, so due to these issues, Refs. [26, 29] do not evaluate the information given in Table 2. If we remove the confidence degree from Table 2, then the operators in Ref. [26, 29] still have failed because of their mathematical structure.

- Mahmood et al. [26] proposed simple AOs for CPFS using confidence level. The operator in [26] is the special case of the proposed work, and in this paper, we have given the general form of the aggregation operators, and then given different operators based on different t-norm and t-conorms. They are available in Table 8 which gave the ranking results: $\beta_5 > \beta_2 > \beta_4 > \beta_3 > \beta_1$. Here, we obtained the best optimal β_5 .

Noticed that the diagnosed operators are massively valuable and dominant compared to prevailing operators because by using different values of t-norm and t-conorm, we can easily obtain the prevailing aggregation operators.

Therefore, the diagnosed theory is much more accurate and effective than exiting work [26, 29, 36, 37, 40, 43, 44, 46].

6 Conclusion

The main conclusions of this manuscript are described below:

1. We developed the fundamental Archimedean operational laws.
2. We proposed the CCPFSAWA, CCPFSAOWA, CCPFSAWG, and CCPFSAOWG operators and explored their valuable results and properties.
3. We developed the CCPFWA, CCPFOWA, CCPFEWA, CCPFEOWA, CCPFHOWA, CCPFFWA, CCPFFOWA, CCPFFWG, CCPFFOWG, CCPFEWG, CCPFEOWG, CCPFHWG, CCPFHOWG, CCPFFWG, and CCPFFOWG operators.
4. We proposed a MADM method based on the evaluated operators.
5. We used many examples to show the validation of the new approaches.

6.1 Limitations

The Archimedean aggregation operators based on CPF information are massively powerful, but in many situations, they have no working feasibility, for instance, if someone provided information that cannot satisfy the condition of CPF information or if someone provided information in the shape of three dimension, then the operators based on CPF information have been neglected, for this, we need to develop the Archimedean aggregation operators based on complex q-rung orthopair, complex picture, complex spherical and complex T-spherical fuzzy information.

6.2 Future Works

In the future, it is necessary to extend the proposed operators to the Maclaurin operators [47], Aczel-Alsina operators [48], Bonferroni operators [49], TOPSIS technique [50], fuzzy N-soft sets [51], Hesitant fuzzy N-soft sets [52], complex Pythagorean fuzzy N-soft sets [53], Pythagorean m-polar fuzzy sets [54], q-rung orthopair m-polar fuzzy sets [55], ELECTRE-I technique [56], and m-polar fuzzy graphs [57] in the environment CPF information.

References

1. Atanassov, K.: Intuitionistic fuzzy sets. In VII ITRK's Session; Deposited in Central Sci.—Techn. Library of Bulg. Acad. of Sci., 1697/84; Sofia, Bulgaria, June 1983. (in Bulgarian)
2. Atanassov, K.: Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **20**(1), 87–96 (1986)
3. Zadeh, L.A.: Fuzzy sets and systems. *Inf. Control* **8**(3), 338–353 (1965)
4. Mahmood, T.: A novel approach toward bipolar soft sets and their applications. *J. Math.* **2020**, 4690808 (2020)
5. Liu, S., Yu, W., Chan, F.T., Niu, B.: A variable weight-based hybrid approach for multi-attribute group decision making under interval-valued intuitionistic fuzzy sets. *Int. J. Intell. Syst.* **36**(2), 1015–1052 (2021)
6. Thao, N.X.: Some new entropies and divergence measures of intuitionistic fuzzy sets based on Archimedean t-conorm and application in supplier selection. *Soft. Comput.* **25**(7), 5791–5805 (2021)
7. Gohain, B., Dutta, P., Gogoi, S., Chutia, R.: Construction and generation of distance and similarity measures for intuitionistic fuzzy sets and various applications. *Int. J. Intell. Syst.* **36**(12), 7805–7838 (2021)
8. Xue, Y., Deng, Y.: Decision-making under measure-based granular uncertainty with intuitionistic fuzzy sets. *Appl. Intell.* **51**(8), 6224–6233 (2021)
9. Ecer, F., Pamucar, D.: MARCOS technique under intuitionistic fuzzy environment for determining the COVID-19 pandemic performance of insurance companies in terms of healthcare services. *Appl. Soft Comput.* **104**, 107199 (2021)
10. Augustine, E.P.: The novel correlation coefficient for intuitionistic fuzzy sets and its application to multi-criteria decision-making problems. *Int. J. Fuzzy Syst. Appl. (IJFSA)* **10**(2), 39–58 (2021)
11. Yager, R.R.: Pythagorean membership grades in multicriteria decision making. *IEEE Trans. Fuzzy Syst.* **22**(4), 958–965 (2013)
12. Rani, P., Mishra, A.R., Saha, A., Pamucar, D.: Pythagorean fuzzy weighted discrimination-based approximation approach to the assessment of sustainable bioenergy technologies for agricultural residues. *Int. J. Intell. Syst.* **36**(6), 2964–2990 (2021)
13. Ejegwa, P.A., Adah, V., Onyeke, I.C.: Some modified Pythagorean fuzzy correlation measures with application in determining some selected decision-making problems. *Granul. Comput.* **7**(2), 381–391 (2022)
14. Sarkar, B., Biswas, A.: Pythagorean fuzzy AHP-TOPSIS integrated approach for transportation management through a new distance measure. *Soft Comput.* **25**(5), 4073–4089 (2021)
15. Riaz, M., Naeem, K., Afzal, D.: Pythagorean m-polar fuzzy soft sets with TOPSIS method for MCGDM. *Punjab Univ. J. Math.* **52**(3), 21–46 (2020)
16. Akram, M., Luqman, A., Kahraman, C.: Hesitant Pythagorean fuzzy ELECTRE-II method for multi-criteria decision-making problems. *Appl. Soft Comput.* **108**, 107479 (2021)
17. Shahzadi, G., Akram, M., Al-Kenani, A.N.: Decision-making approach under Pythagorean fuzzy Yager weighted operators. *Mathematics* **8**(1), 70 (2020)
18. Akram, M., Luqman, A., Alcantud, J.C.R.: An integrated ELECTRE-I approach for risk evaluation with hesitant Pythagorean fuzzy information. *Expert Syst. Appl.* **200**, 116945 (2022)
19. Luqman, A., Akram, M., Alcantud, J.C.R.: Digraph and matrix approach for risk evaluations under Pythagorean fuzzy information. *Expert Syst. Appl.* **170**, 114518 (2021)
20. Ramot, D., Milo, R., Friedman, M., Kandel, A.: Complex fuzzy sets. *IEEE Trans. Fuzzy Syst.* **10**(2), 171–186 (2002)

21. Liu, P., Ali, Z., Mahmood, T.: The distance measures and cross-entropy based on complex fuzzy sets and their application in decision making. *J. Intell. Fuzzy Syst.* **39**(3), 3351–3374 (2020)
22. Mahmood, T., Ali, Z., Gumaei, A.: Interdependency of complex fuzzy neighborhood operators and derived complex fuzzy coverings. *IEEE Access* **9**, 73506–73521 (2021)
23. Hu, B., Bi, L., Dai, S.: The orthogonality between complex fuzzy sets and its application to signal detection. *Symmetry* **9**(9), 175 (2017)
24. Thirunavukarasu, P., Suresh, R., Ashokkumar, V.: Theory of complex fuzzy soft set and its applications. *Int. J. Innov. Res. Sci. Technol.* **3**(10), 13–18 (2017)
25. Alkouri, A.M.D.J.S., Salleh, A.R.: Complex intuitionistic fuzzy sets. *AIP Conf. Proc. Am. Inst. Phys.* **1482**(1), 464–470 (2012)
26. Garg, H., Rani, D.: Novel aggregation operators and ranking method for complex intuitionistic fuzzy sets and their applications to decision-making process. *Artif. Intell. Rev.* **53**(5), 3595–3620 (2020)
27. Garg, H., Rani, D.: Robust averaging–geometric aggregation operators for complex intuitionistic fuzzy sets and their applications to MCDM process. *Arab. J. Sci. Eng.* **45**(3), 2017–2033 (2020)
28. Garg, H., Rani, D.: Generalized geometric aggregation operators based on t-norm operations for complex intuitionistic fuzzy sets and their application to decision-making. *Cogn. Comput.* **12**(3), 679–698 (2020)
29. Akram, M., Peng, X., Sattar, A.: A new decision-making model using complex intuitionistic fuzzy Hamacher aggregation operators. *Soft Comput.* **25**(10), 7059–7086 (2021)
30. Ali, Z., Mahmood, T., Aslam, M., Chinram, R.: Another view of complex intuitionistic fuzzy soft sets based on prioritized aggregation operators and their applications to multiattribute decision making. *Mathematics* **9**(16), 1922 (2021)
31. Ullah, K., Mahmood, T., Ali, Z., Jan, N.: On some distance measures of complex Pythagorean fuzzy sets and their applications in pattern recognition. *Complex Intell. Syst.* **6**(1), 15–27 (2020)
32. Ali, Z., Mahmood, T., Ullah, K., Khan, Q.: Einstein geometric aggregation operators using a novel complex interval-valued pythagorean fuzzy setting with application in green supplier chain management. *Rep. Mech. Eng.* **2**(1), 105–134 (2021)
33. Akram, M., Khan, A., Borumand Saeid, A.: Complex Pythagorean Dombi fuzzy operators using aggregation operators and their decision-making. *Expert. Syst.* **38**(2), e12626 (2021)
34. Janani, K., Veerakumari, K.P., Vasanth, K., Rakkiyappan, R.: Complex Pythagorean fuzzy einstein aggregation operators in selecting the best breed of Horsegram. *Expert Syst. Appl.* **187**, 115990 (2022)
35. Akram, M., Peng, X., Al-Kenani, A.N., Sattar, A.: Prioritized weighted aggregation operators under complex Pythagorean fuzzy information. *J. Intell. Fuzzy Syst.* **39**(3), 4763–4783 (2020)
36. Xu, Z.: Intuitionistic fuzzy aggregation operators. *IEEE Trans. Fuzzy Syst.* **15**(6), 1179–1187 (2007)
37. Zhao, H., Xu, Z., Ni, M., Liu, S.: Generalized aggregation operators for intuitionistic fuzzy sets. *Int. J. Intell. Syst.* **25**(1), 1–30 (2010)
38. Zhao, X., Wei, G.: Some intuitionistic fuzzy Einstein hybrid aggregation operators and their application to multiple attribute decision making. *Knowl.-Based Syst.* **37**, 472–479 (2013)
39. Xu, Z., Yager, R.R.: Some geometric aggregation operators based on intuitionistic fuzzy sets. *Int. J. Gen Syst* **35**(4), 417–433 (2006)
40. Huang, J.Y.: Intuitionistic fuzzy Hamacher aggregation operators and their application to multiple attribute decision making. *J. Intell. Fuzzy Syst.* **27**(1), 505–513 (2014)
41. Khan, A.A., Ashraf, S., Abdullah, S., Qiyas, M., Luo, J., Khan, S.U.: Pythagorean fuzzy Dombi aggregation operators and their application in decision support system. *Symmetry* **11**(3), 383 (2019)
42. Peng, X., Yuan, H.: Fundamental properties of Pythagorean fuzzy aggregation operators. *Fund. Inform.* **147**(4), 415–446 (2016)
43. Wu, S.J., Wei, G.W.: Pythagorean fuzzy Hamacher aggregation operators and their application to multiple attribute decision making. *Int. J. Knowl.-Based Intell. Eng. Syst.* **21**(3), 189–201 (2017)
44. Feng, J., Zhang, Q., Hu, J.: Group generalized Pythagorean fuzzy aggregation operators and their application in decision making. *IEEE Access* **8**, 138004–138020 (2020)
45. Sarkar, A., Biswas, A.: Multicriteria decision-making using Archimedean aggregation operators in Pythagorean hesitant fuzzy environment. *Int. J. Intell. Syst.* **34**(7), 1361–1386 (2019)
46. Mahmood, T., Ali, Z., Ullah, K., Khan, Q., AlSalman, H., Gumaei, A., Rahman, S.M.M.: Complex pythagorean fuzzy aggregation operators based on confidence levels and their applications. *Math. Biosci. Eng.* **19**(1), 1078–1107 (2022)
47. Ullah, K.: Picture fuzzy maclaurin symmetric mean operators and their applications in solving multiattribute decision-making problems. *Math. Probl. Eng.* (2021). <https://doi.org/10.1155/2021/1098631>
48. Hussain, A., Ullah, K., Yang, M.S., Pamucar, D.: Aczel-Alsina aggregation operators on T-spherical fuzzy (TSF) information with application to TSF multi-attribute decision making. *IEEE Access* **10**, 26011–26023 (2022)
49. Akram, M., Ullah, K., Pamucar, D.: Performance evaluation of solar energy cells using the interval-valued T-spherical fuzzy bonferroni mean operators. *Energies* **15**(1), 292 (2022)
50. Ali, Z., Mahmood, T., Yang, M.S.: TOPSIS method based on complex spherical fuzzy sets with Bonferroni mean operators. *Mathematics* **8**(10), 1739 (2020)
51. Akram, M., Adeel, A., Alcantud, J.C.R.: Fuzzy N-soft sets: a novel model with applications. *J. Intell. Fuzzy Syst.* **35**(4), 4757–4771 (2018)
52. Akram, M., Adeel, A., Alcantud, J.C.R.: Hesitant fuzzy N-soft sets: a new model with applications in decision-making. *J. Intell. Fuzzy Syst.* **36**(6), 6113–6127 (2019)
53. Akram, M., Wasim, F., Al-Kenani, A.N.: A hybrid decision-making approach under complex pythagorean fuzzy N-soft sets. *Int. J. Comput. Intell. Syst.* **14**(1), 1263–1291 (2021)
54. Riaz, M., Hashmi, M.R.: Soft rough Pythagorean m-polar fuzzy sets and Pythagorean m-polar fuzzy soft rough sets with application to decision-making. *Comput. Appl. Math.* **39**(1), 1–36 (2020)
55. Riaz, M., Hamid, M.T., Afzal, D., Pamucar, D., Chu, Y.M.: Multi-criteria decision making in robotic agri-farming with q-rung orthopair m-polar fuzzy sets. *PLoS ONE* **16**(2), e0246485 (2021)
56. Akram, M., Waseem, N., Liu, P.: Novel approach in decision making with m-polar fuzzy ELECTRE-I. *Int. J. Fuzzy Syst.* **21**(4), 1117–1129 (2019)

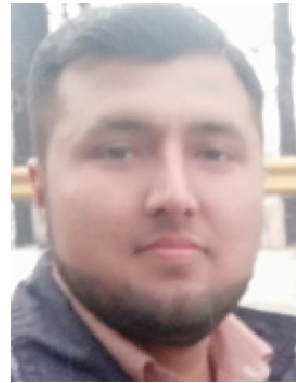
57. Akram, M., Younas, H.R.: Certain types of irregular m-polar fuzzy graphs. *J. Appl. Math. Comput.* **53**(1), 365–382 (2017)

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