Multiplicative Consistency and DEA Cross-Efficiency-Driven Decision-Making Method with Fuzzy Preference Relations

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Abstract Owing to the uncertainty and vagueness of practical decision-making problems, it is more convenient for decision-makers (DMs) to express evaluation information with fuzzy preference relations (FPRs) than precise numerical values. As two important issues in the decision-making process with FPRs, improving consistency and acquiring the priority weights are worth further studying. In this paper, we first provide an approach for constructing the FPR with multiplicative consistency, and then an algorithm of improving consistency is presented to generate an acceptable FPR, in which the initial evaluation information is retained to the largest extent through the local adjustment strategy. Then, an input-oriented CCR DEA model is developed to derive the priority weight vector of alternatives, which is followed that a DEA cross-efficiency model is constructed to discriminate multiple DEA efficient DMUs. Subsequently, the complete rankings of alternatives can be generated by a novel DEA-driven decision-making method. Finally, a numerical example is proposed to verify the feasibility and effectiveness of the developed method. The advantages of the developed method are performed by comparative analysis and sensitivity analysis.

1 Introduction

As an important activity of daily life, decision-making has been widely used in many fields [1]. In the real life, owing to the limitation of the ability of decision makers (DMs) and the vagueness of the decision-making problems, it is more difficult for DMs to express preference information with accurate numerical values [2, 3]. Therefore, more and more DMs utilize fuzzy values to describe evaluation information. The notion of fuzzy sets (FSs) is firstly introduced by Zadeh [4] in 1965, afterwards, intuitionistic fuzzy sets (IFSs) [5], interval-valued fuzzy sets (IVFSs) [6], interval-valued intuitionistic fuzzy sets (IVIFSs) [7], hesitant fuzzy sets (HFSs) [8], interval-valued hesitant fuzzy sets (IVHSs) [9] and other extended forms of FSs were successively proposed.

Preference relation, a powerful and important expression tool, is increasingly attracting the attention of scholars [10], and is employed by many DMs for expressing their evaluation information of one alternative over another. In existing studies, preference relations mainly have two types which are fuzzy preference relations (FPRs) [11] and multiplicative preference relations (MPRs) [12], respectively. In decision-making problems, consistency of preference relations is an important topic [13], lack of consistency may cause unreasonable and inconsistent

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outcomes and then influence the selection of optimal alternative [14].

In terms of measuring consistency of preference relations, numerous approaches are designed to examine whether the preference relations are consistent. Zhang et al. [15] constructed two models to generate the best consistency index (BCI) and the worst consistency index (WCI) of hesitant fuzzy preference relation (HFPR), and discussed the average consistency index (ACI) of HFPR, then those indexes are utilized to reflect the consistency degree of preference relations. Noticing the self-confidence levels, Liu et al. [16] established individual consistency index for evaluating the individual consistency level. Meng et al. [17] studied the incomplete and inconsistent interval fuzzy preference relations (IFPRs), and developed programming models to check whether the IFPRs are of additive and multiplicative consistency. With respect to additive consistency of probabilistic hesitant fuzzy linguistic preference relations (PHFLPRs), Wang et al. [18] defined a method for measuring the additive consistency degree of PHFLPRs.

To achieve acceptable and consistent preference relations, many researchers investigated the methods of consistency improvement. Zhang et al. [19] utilized an integer programming model to enhance the consistency level of fuzzy linguistic preference relations (FLPRs), and then produced acceptable consistent FLPRs. Zhang and Meng [20] established two programming models to repair inconsistent intuitionistic triangular fuzzy preference relations (ITFPRs). For reaching the predefined level of consistency, Li and Wang [21] developed an automatic iterative algorithm to mend the inconsistency of PHFPRs. According to the theory of graph, Wang and Xu [22] designed a selective algorithm and a broken circle algorithm to solve the consistency problem.

In addition, it is also important to derive priority weights of alternatives in the decision-making problems, various methods are designed for acquiring the priority vector of alternatives in existing research. In the group decision-making (GDM) problem with intuitionistic MPRs, Li et al. [23] developed two algorithms to determine DMs' priority weights and overall priority weight vector. Wan et al. [24] adopted a parametric linear program to generate the collective interval priority weights, and then obtain the order of alternatives. Xu [25] derived the priority weight intervals from IMPRs by utilizing the error propagation formula, and then yielded the order of alternatives through sorting the weight intervals. Moreover, data employment analysis (DEA) is also an efficient method in which how to generate the priority weight vector of alternatives and select the optimal alternative. Kao and Liu [26] applied DEA model to derive the order of alternatives and applied it to the case of robot selection. Combined IFPRs with DEA method, Liu et al. [27] proposed a novel DEA model for the priority weight vector derivation.

However, in traditional DEA model, there is a drawback that it may produce multiple efficient decision-making units (DMUs), DEA cross-efficiency can overcome this disadvantage and further discriminate the DEA efficient units [28]. It is the superiority that makes DEA cross-efficiency is widely used in the decision-making process and exploited in different fields. Considering the relationship between different DMUs, Cheng et al. [29] developed a novel cross-efficiency model under the social network environment, and employed it to evaluate the environmental efficiency of Xiang Jiang River Basin. Based on Pareto improvement, Wu et al. [30] provided a method to judge whether the DMUs are Pareto-optimal, and one can ultimately obtain the efficiency of Pareto-optimal through a cross-efficiency model, which can heighten the efficiency of different DMUs. In the problem of multi-attribute decision making, Liu et al. [31] ranked the alternatives by utilizing the stochastic DEA cross-efficiency and illustrated the practicality by an example of evaluating the banks' sustainable development.

According to the above literature reviews, the consistency of preference relations and the derivation of priority weight vector are two vital topics in the decision-making process. Although many researchers developed different methods to solve those issues, shortcomings still exist in some methods. For example, Qian and Feng [32] derived the consistent FPRs by utilizing a convergent iterative algorithm, but all elements of the original FPR matrix are changed in the iterative process, and then the adjusted FPRs with acceptable consistency cannot represent the evaluation information of DMs. Likewise, Liu et al. [27]'s method damaged the initial preference values of DM in the process of consistency-improving, and all elements of the original FPR are revised. Lee [33] proposed a method to construct FPRs that satisfy additive consistency and order consistency, but the method cannot guarantee the constructed FPRs are acceptable for DMs, and the obtained results will decrease the satisfaction degree of DMs. In addition, the final ranking of alternatives is directly induced from the additive FPRs in Lee's [33] method, it may not generate correct and reliable ranking results.

Therefore, to overcome those limitations, our work explores a way to generate acceptable FPRs, and only the most inconsistent elements are revised in the process of constructing the acceptable FPRs. Additionally, DEA cross-efficiency is employed to rank the alternatives based on acceptable multiplicative consistent FPRs. The primary contributions comprise the following five aspects.

(1) A new approach of constructing multiplicative FPRs is provided.

(2) An algorithm for improving consistency is developed, in which a local adjustment strategy is adopted to obtain the FPRs with acceptable consistency, and the original evaluation information of DMs can be retained to the greatest extent.

(3) To rank different alternatives, an input-oriented CCR DEA model is constructed.

(4) To further differentiate multiple DEA efficient DMUs, DEA cross-efficiency is used to determine the complete ranking of alternatives.

(5) The feasibility and practicality of the proposed method are proved by employing a numerical example, comparative analysis and sensitivity analysis.

The remainder of this paper is structured as follows: In Sect. 2, we review some preliminary concepts about FPRs, additive consistency and multiplicative consistency of FPRs. Section 3 shows a new approach for constructing FPRs with multiplicative consistency, and provides a method of measuring consistency level and a consistency-improving algorithm. In Sect. 4, a CCR DEA model which is input-oriented is presented. For discriminating the DMUs with equal weights, DEA cross-efficiency measurement method is also displayed in this section. In Sect. 5, we apply the proposed method to practical decision-making problems. Finally, conclusions and future prospects are highlighted in Sect. 6.

2 Preliminaries

In view of the uncertainty and vagueness of decision-making environment, DMs are increasingly hard to characterize preference information with crisp values. FPR is an important tool that can support DMs to portray evaluation information conveniently. In this section, we briefly describe some prior knowledge about FPRs. Assume that a collection of alternatives is described as $X = \{x_1, x_2, \ldots, x_n\}$, and $n \in N$, $N = \{1, 2, \ldots, n\}$.

Definition 1 [35]. A FPR R on a collection of alternatives $X = \{x_1, x_2, \cdots, x_n\}$ can be denoted by a comparison matrix $R = (r_{ij})_{n \times n}$, and

$$r_{ij} + r_{ji} = 1, r_{ii} = 0.5, \forall i, j \in N, \qquad (1)$$

where $r_{ij} \in [0, 1][0, 1]$, which indicates the fuzzy preference value of alternative x_i to x_j. If $r_{ij} = 0.5$, then implies that alternative x_i is no different from x_j; if $r_{ij} \in (0, 0.5)$, which means that x_j is better than x_i, the greater the r_{ij}, the better alternative x_i than x_j; if $r_{ij} = 1$, then implies that alternative x_i is completely preferred to x_j.

Definition 2 [35]. Let $R = (r_{ij})_{n \times n}$ be a FPR on X, and then $R = (r_{ij})_{n \times n}$ be of additive consistency if it meets the following additive-transitive property:

$$r_{ij} + r_{jk} + r_{ki} = r_{ik} + r_{kj} + r_{ji}, i, j, k \in N, \qquad (2)$$

since $r_{ji} = 1 - r_{ij}, r_{jk} = 1 - r_{kj}, r_{ki} = 1 - r_{ik}$, then we have

$$r_{ij} = r_{ik} + r_{kj} - 0.5, i, j, k \in N. \qquad (3)$$

Definition 3 [36]. Let $R = (r_{ij})_{n \times n}$ be a FPR on X, and then $R = (r_{ij})_{n \times n}$ be of multiplicative consistency if it meets the following multiplicative-transitive property:

$$r_{ij}.r_{jk}.r_{ki} = r_{ik}.r_{kj}.r_{ji}, i, j, k \in N. \qquad (4)$$

Definition 4 [37]. Let $R = (r_{ij})_{n \times n}$ be a FPR on X, and $R = (r_{ij})_{n \times n}$ be of multiplicative consistency, then we have

$$r_{ij} = \frac{w_i}{w_i + w_j}, i, j \in N \qquad (5)$$

where $w = (w_1, w_2, \ldots, w_n)^T$ represents the corresponding normalized priority weight vector, and $w_i \geq 0, \sum_{i=1}^{n} w_i = 1, i \in N$.

Example 1: In some cases, additive transitivity is an unsuitable property to judge whether the FPRs are consistent. Because it may contradict the preference values on the $[0, 1]$ scale [38]. Therefore, we adopt multiplicative-transitive property to model the consistency of FPRs in this paper.

3 Consistency-Improving Algorithm with FPRs

We first introduce a method that is utilized to construct FPRs with multiplicative consistency. Then a criterion for measuring the consistency of FPRs is presented. In the end, a consistency- improving algorithm combined with a local adjustment strategy is designed.

3.1 Multiplicative Consistency Construction Method for FPRs

According to Definition 3, the following theorem can be obtained.

Theorem 1 $R = (r_{ij})_{n \times n}$ *is a FPR on a collection of alternatives* $X = \{x_1, x_2, \ldots, x_n\}$, *then R is of multiplicative consistency if.*

$$r_{ij} = \frac{2n - \sum_{l=1}^{n} r_{il} + 1}{4n - 2\sum_{l=1}^{n}\left(r_{il} + r_{jl}\right) + 2}, \forall i < j \in N. \tag{6}$$

Proof When $i = j$, we have

$$r_{ii} = \frac{2n - \sum_{l=1}^{n} r_{il} + 1}{4n - 2\sum_{l=1}^{n}(r_{il} + r_{il}) + 2} = 0.5, \tag{7}$$

and

$$
\begin{aligned}
r_{ij} + r_{ji} &= \frac{2n - \sum_{l=1}^{n} r_{il} + 1}{4n - 2\sum_{l=1}^{n}\left(r_{il} + r_{jl}\right) + 2} \\
&\quad + \frac{2n - \sum_{l=1}^{n} r_{jl} + 1}{4n - 2\sum_{l=1}^{n}\left(r_{il} + r_{jl}\right) + 2} \\
&= 1.
\end{aligned} \tag{8}
$$

Therefore, $R = (r_{ij})_{n \times n}$ is a FPR.

In what follows, we prove $R = (r_{ij})_{n \times n}$ satisfies the condition of FPR with multiplicative consistency.

Since $r_{ij} = \frac{2n - \sum_{l=1}^{n} r_{il} + 1}{4n - 2\sum_{l=1}^{n}(r_{il} + r_{jl}) + 2}$, then we have

$$
\begin{aligned}
\frac{r_{ij}}{r_{ji}} &= \frac{2n - \sum_{l=1}^{n} r_{il} + 1}{4n - \sum_{l=1}^{n}\left(r_{il} + r_{jl}\right) + 2} \cdot \frac{4n - \sum_{l=1}^{n}\left(r_{il} + r_{jl}\right) + 2}{2n - \sum_{l=1}^{n} r_{jl} + 1} \\
&= \frac{2n - \sum_{l=1}^{n} r_{il} + 1}{2n - \sum_{l=1}^{n} r_{jl} + 1},
\end{aligned} \tag{9}
$$

and

$$
\frac{r_{il} \cdot r_{ji}}{r_{jl} \cdot r_{li}} = \frac{\frac{2n - \sum_{l=1}^{n} r_{il} + 1}{4n - \sum_{l=1}^{n}(r_{il} + r_{ll}) + 2} \cdot \frac{2n - \sum_{l=1}^{n} r_{ll} + 1}{4n - \sum_{l=1}^{n}(r_{jl} + r_{ll}) + 2}}{\frac{2n - \sum_{l=1}^{n} r_{jl} + 1}{4n - \sum_{l=1}^{n}(r_{jl} + r_{ll}) + 2} \cdot \frac{2n - \sum_{l=1}^{n} r_{ll} + 1}{4n - \sum_{l=1}^{n}(r_{il} + r_{ll}) + 2}} = \frac{2n - \sum_{l=1}^{n} r_{il} + 1}{2n - \sum_{l=1}^{n} r_{jl} + 1}. \tag{10}
$$

According to Eqs. (9) and (10), it can be shown that

$$\frac{r_{ij}}{r_{ji}} = \frac{r_{il} \cdot r_{lj}}{r_{jl} \cdot r_{li}}. \tag{11}$$

Thus,

$$r_{ij} \cdot r_{jl} \cdot r_{li} = r_{il} \cdot r_{lj} \cdot r_{ji}. \tag{12}$$

The proof of Theorem 1 is completed. ∎

Theorem 1 suggests the necessary conditions of multiplicative consistency for FPRs, on the basis of Theorem 1, the following method for constructing multiplicative FPRs can be deduced.

Theorem 2 *Let $R = (r_{ij})_{n \times n}$ be an original FPR provided by a DM, we have*

$$\overline{r_{ij}} = \begin{cases} \dfrac{2n - \sum_{l=1}^{n} r_{il} + 1}{4n - 2\sum_{l=1}^{n}\left(r_{il} + r_{jl}\right) + 2}, & i < j, \\ 0.5, & i = j, \\ 1 - \overline{r_{ji}}, & i > j, \end{cases} \tag{13}$$

then $\overline{R} = (\overline{r_{ij}})_{n \times n}$ is a FPR with multiplicative consistency.

Based on the above theorems, a multiplicative consistent FPR $\overline{R} = (\overline{r_{ij}})_{n \times n}$ can be generated from an original FPR $R = (r_{ij})_{n \times n}$.

Example 1 Suppose that a DM provides its appraisal information over four alternatives x_1, x_2, x_3, x_4, and then we can derive an original FPR $R = (r_{ij})_{n \times n}$ as follows:

$$R = \begin{pmatrix} 0.5 & 0.4 & 0.6 & 0.7 \\ 0.6 & 0.5 & 0.8 & 0.4 \\ 0.4 & 0.2 & 0.5 & 0.9 \\ 0.3 & 0.6 & 0.1 & 0.5 \end{pmatrix}.$$

Based on Theorem 2, we can generate a multiplicative consistent FPR as follows:

$$\overline{R} = \begin{pmatrix} 0.5000 & 0.7556 & 0.7083 & 0.6415 \\ 0.2444 & 0.5000 & 0.7128 & 0.6442 \\ 0.2917 & 0.2872 & 0.5000 & 0.6364 \\ 0.3585 & 0.3558 & 0.3636 & 0.5000 \end{pmatrix}.$$

3.2 Consistency Measure for FPRs

For most DMs, providing an absolutely consistent FPR is difficult in practical decision-making problems. However, lacking consistency may cause inconsistent evaluation outcomes, it is necessary to design an algorithm to improve consistency level of FPRs. To retain DM's original evaluation information to the greatest extent, we utilize the local adjustment strategy to heighten the consistency degree of FPRs.

Definition 6 [39]. Let $R = (r_{ij})_{n \times n}$ is a FPR provided by a DM, and $\overline{R} = (\overline{r_{ij}})_{n \times n}$ is a corresponding multiplicative consistent FPR, then the distance between R and $\overline{R}$ is expressed by the following equation:

$$d(R, \overline{R}) = \sqrt{\frac{2}{n(n-1)} \sum_{i<j}^{n} (r_{ij} - \overline{r_{ij}})^2}, \tag{14}$$

where $i \neq j, i, j \in N$.

Then, the consistency index of R is presented below:

$$CI(R) = d(R, \overline{R}), \tag{15}$$

where $CI(R) \in [0, 1]$, the closer the value of $CI(R)$ is to 0, the higher the consistency level of R is. If $CI(R) = 0$, it means that R is of complete consistency.

Definition 7 Let $R = (r_{ij})_{n \times n}$ be a FPR, $\overline{CI}$ be a predefined threshold value, if

$$CI(R) \leq \overline{CI}, \tag{16}$$

then R is called an acceptable multiplicative consistent FPR.

3.3 An Algorithm of Improving Consistency for FPRs

Improving consistency is an important problem in the decision-making process. Assume that a collection of alternatives denoted as $X = \{x_1, x_2, \ldots, x_n\}$, $R = (r_{ij})_{n \times n}$ is an original FPR provided by a DM. By comparing the consistency index of FPR with the predefined threshold, whether the FPR is of acceptable consistency can be determined. If the FPR is unacceptable consistent, then it should be adjusted with the following algorithm.

In the process of adjustment, to maintain the DM's initial evaluation information to the greatest extent, a local consistency adjustment method is adopted to enhance the consistency degree of FPRs, in which only the most inconsistent element will be adjusted. Let the most inconsistent element be $r_{i^*j^*}$, where $|r_{i^*j^*} - \overline{r_{i^*j^*}}| = \max_{i<j} |r_{ij} - \overline{r_{ij}}|$, and it should be updated in each iteration. The procedures of consistency-improving are displayed in the following algorithm.

Algorithm 1

Input An original FPR $R = (r_{ij})_{n \times n}$, a predefined consistency threshold $\overline{CI}$, and the adjusted parameter δ, where $\delta \in (0, 1)$.

Output A FPR $\widetilde{R} = (\widetilde{r_{ij}})_{n \times n}$ with acceptable multiplicative consistency

Step 1 Let iteration $t = 0$ and
$$\text{R}^{(t)} = \left(r_{ij}^{(t)}\right)_{n \times n} = \text{R}^{(0)} = (r_{ij})_{n \times n}.$$

Step 2 According to Theorem 2, a FPR $\overline{R} = (\overline{r_{ij}})_{n \times n}$ with multiplicative consistency is derived.

Step 3 Utilize Eq. (15) to obtain the consistency index $CI(R^{(t)})$. If $CI(R^{(t)}) < \overline{CI}$, which means that R is multiplicative consistent, then output $\widetilde{R} = (\widetilde{r_{ij}})_{n \times n}$;
if $CI(R^{(t)}) > \overline{CI}$, go to the next step.

Step 4 Find and update the most inconsistent element $r_{i^*j^*}$, then the elements of the adjusted FPR $R^{(t+1)} = (r_{ij}^{(t+1)})_{n \times n}$ can be constructed as follows:

$$r_{ij}^{(t+1)} = \begin{cases} (1 - \delta)r_{i^*j^*}^{(t)} + \delta\overline{r_{i^*j^*}^{(t)}}, & i = i^*, j = j^*, \\ r_{ij}^{(t)}, & \text{otherwise}, \\ 1 - r_{ji}^{(t+1)}, & i = j^*, j = i^*, \end{cases} \tag{17}$$

afterward, let $t = t + 1$, return the step 2

Step 5 Let $\widetilde{R} = R^{(t)}$. Output a FPR $\widetilde{R} = (\widetilde{r_{ij}})_{n \times n}$ is of acceptable multiplicative consistency.

Step 6 End.

Remark 2 The adjusted parameter δ reflects the degree to which the initial evaluation information is preserved, the larger the value of δ is, the less the original evaluation information is retained. Particularly, when $\delta = 0$, the initial preference information of DMs is absolutely maintained. In the process of consistency improvement, as the value of δ increases, the iteration times of reaching desirable consistency level decrease.

Based on Algorithm 1, the following theorem can be deduced.

Theorem 3 *Let R be an original FPR, δ be the adjusted parameter, $\{R^{(t)}\}$ be a sequence of FPR in Algorithm 1, and the consistency index of $R^{(t)}$ is represented by $CI(R^{(t)})$, then the following formula can be generated at each iteration.*

$$CI(R^{(t+1)}) < CI(R^{(t)}). \tag{18}$$

Proof According to Theorem 2, for $\forall i < j$, we have

$$\overline{r_{ij}^{(t)}} = \frac{2n - \sum\limits_{l=1}^{n} r_{il}^{(t)} + 1}{4n - 2\sum\limits_{l=1}^{n}\left(r_{il}^{(t)} + r_{jl}^{(t)}\right) + 2}, \overline{r_{ij}^{(t+1)}}$$

$$= \frac{2n - \sum\limits_{l=1}^{n} r_{il}^{(t+1)} + 1}{4n - 2\sum\limits_{l=1}^{n}\left(r_{il}^{(t+1)} + r_{jl}^{(t+1)}\right) + 2} \qquad (19)$$

From Algorithm 1, we have $r_{ij}^{(t)} = r_{ij}^{(t+1)}(i<j, i\neq i^*, j\neq j^*)$, therefore, we can obtain $\overline{r_{ij}^{(t)}} = \overline{r_{ij}^{(t+1)}}$. Then, for each iteration t, we have

$$\begin{aligned}
CI(R^{(t+1)}) &= \sqrt{\frac{2}{n(n-1)}\sum_{i<j}^{n}(r_{ij}^{(t+1)} - \overline{r_{ij}^{(t+1)}})^2}\\
&= \sqrt{\frac{2}{n(n-1)}\left((r_{i^*j^*}^{(t+1)} - \overline{r_{i^*j^*}^{(t+1)}})^2 + \sum_{i<j,(i,j)\neq(i^*,j^*)}^{n}(r_{ij}^{(t+1)} - \overline{r_{ij}^{(t+1)}})^2\right)}\\
&= \sqrt{\frac{2}{n(n-1)}\left(((1-\delta)r_{i^*j^*}^{(t)} + \delta\overline{r_{i^*j^*}^{(t)}} - \overline{r_{i^*j^*}^{(t)}})^2 + \sum_{(i,j)\neq(i^*,j^*),i<j}^{n}(r_{ij}^{(t)} - \overline{r_{ij}^{(t)}})^2\right)}\\
&= \sqrt{\frac{2}{n(n-1)}\left(((1-\delta)(r_{i^*j^*}^{(t)} - \overline{r_{i^*j^*}^{(t)}}))^2 + \sum_{(i,j)\neq(i^*,j^*),i<j}^{n}(r_{ij}^{(t)} - \overline{r_{ij}^{(t)}})^2\right)}\\
&< \sqrt{\frac{2}{n(n-1)}\left((r_{i^*j^*}^{(t)} - \overline{r_{i^*j^*}^{(t)}})^2 + \sum_{(i,j)\neq(i^*,j^*),i<j}^{n}(r_{ij}^{(t)} - \overline{r_{ij}^{(t)}})^2\right)}\\
&= \sqrt{\frac{2}{n(n-1)}\sum_{i<j}^{n}(r_{ij}^{(t)} - \overline{r_{ij}^{(t)}})^2}\\
&= CI(R^{(t)}).
\end{aligned}$$

$$\qquad (20)$$

The proof of Theorem 3 is completed. ∎

The following inference can be obtained from Theorem 3.

Corollary 1 *For each iteration t, we have $CI(R^{(t)}) > CI(R^{(t+1)})$, and $CI(R^{(t)}) > 0$. Consequently, the sequence $\{CI(R^{(t)})\}$ is monotonically decreasing and has lower bounds.*

Example 2 Following Example 1, we know that $\overline{R}$ is a FPR with multiplicative consistency. Let consistency threshold $\overline{CI} = 0.21$, adjusted parameter $\delta = 0.2$, then the following acceptable multiplicative consistent FPR $\widetilde{R}$ can be yielded from Algorithm 1:

Input: An original FPR $R = (r_{ij})_{n \times n}$.

Output   The final ranking results and the best alternative $\tilde{x}$.

Stage I   Consistency-improving process

  Step 1   Based on the given original FPR $R = (r_{ij})_{n \times n}$, we utilize Algorithm 1 to improve the consistency degree of original FPR, and then we can derive an acceptable multiplicative consistent FPR $\widetilde{R} = (\widetilde{r}_{ij})_{n \times n}$.

Stage II   Priority weights and efficiency values determining process

  Step 2   Based on the model (21) and Theorem 4, the priority weight vector $w = (w_1, w_2, \cdots, w_n)^T$ of alternatives can be generated. If the weights of different alternatives are equal, then go to step 3; otherwise, skip to step 4.

  Step 3   Based on models (29) and (30), Eqs. (31) and (32), calculating the DEA cross-efficiency values of different alternatives.

Stage III   The best alternative selecting process

  Step 4   Generate the final ranking results and select the best alternative. If there are some alternatives with equal weights, the alternative with maximum cross-efficiency is picked; otherwise, we select the alternative with maximum weight.

$$\widetilde{R} = \begin{pmatrix} 0.5 & 0.4711 & 0.6 & 0.7 \\ 0.5289 & 0.5 & 0.8 & 0.4 \\ 0.4 & 0.2 & 0.5 & 0.9 \\ 0.3 & 0.6 & 0.1 & 0.5 \end{pmatrix}.$$

4 DEA-Driven Decision-Making Method with FPRs

This section explores how DEA can be employed to generate ranking results of alternatives and establish a decision-making method. First, an input-oriented DEA model is

constructed for deriving the priority weights of alternatives. Then, when the weights of different alternatives are equal, a DEA cross-efficiency measurement method is introduced to differentiate DMUs. Finally, based on the input-oriented DEA model and DEA cross-efficiency, a DEA-driven decision-making method with FPRs is designed.

4.1 Generating Priority Weights Based on DEA

In this subsection, we provide a DEA model which is input-oriented to obtain the priority weight vector.

We denote $X = \{x_1, x_2, \ldots, x_n\}$ as a collection of alternatives, $\widetilde{R} = (\widetilde{r}_{ij})_{n \times n}$ is an acceptable multiplicative consistent FPR on X. In the alternative selecting process, each alternative x_i represents a DMU, and each column of $\widetilde{R} = (\widetilde{r}_{ij})_{n \times n}$ represents an input. Then we evaluate each alternative x_i by constructing an input-oriented CCR DEA model. In the following, the virtual outputs take the value of 0.5 for all alternatives [40].

$$\min \theta_i$$
$$s.t. \begin{cases} \sum_{p=1}^{n} u_p \widetilde{r}_{pk} \leq \theta_i \widetilde{r}_{ik}, \ k \in N, \\ \sum_{p=1}^{n} 0.5 u_p \geq 0.5, \\ u_p \geq 0, p \in N, \\ \theta_i \text{ is free.} \end{cases} \tag{21}$$

where u_p indicates the input weight of the corresponding DMU_p, the efficiency score of x_p can be obtained by minimizing inputs with limited outputs. θ_i^* is the optimal solution of model (21), which represents the efficiency score of x_i, and if $\theta_i^* < 1$, which means that x_i is inefficient.

According to model (21), the following theorem can be deduced to generate the priority weights.

Theorem 4 *Let $R = (r_{ij})_{n \times n}$ be a multiplicative consistent FPR, the optimal solution of model (21) is θ_i^*, $\sigma : N \to N$ is a permutation, and we have $\theta_{\sigma(1)}^* \geq \theta_{\sigma(2)}^* \geq \cdots \geq \theta_{\sigma(n)}^*$, then the generated priority weights meet that $w_{\sigma(1)} \leq w_{\sigma(2)} \leq \cdots \leq w_{\sigma(n)}$, and*

(1) $\quad w_{\sigma(n)} = 1 \Big/ \sum_{i=1}^{n} \frac{\theta_{\sigma(n)}^*}{2\theta_{\sigma(i)}^* - \theta_{\sigma(n)}^*}$

(2) $\quad w_{\sigma(i)} = \frac{\theta_{\sigma(n)}^* \cdot w_{\sigma(n)}}{2\theta_{\sigma(i)}^* - \theta_{\sigma(n)}^*} i \in N$

Proof Due to $R = (r_{ij})_{n \times n}$ is a FPR with multiplicative consistency, from Eq. (5), we know that there is a priority weight vector $w = (w_1, w_2, \cdots, w_n)^T$, and it satisfies $r_{ij} = \frac{w_i}{w_i + w_j}$, where $w_i > 0, \sum_{i=1}^{n} w_i = 1$. Thus, model (21) can be transformed into the following model:

$$\min \theta_i$$
$$s.t. \begin{cases} \sum_{p=1}^{n} u_p \frac{w_p}{w_p + w_k} \leq \theta_i \frac{w_i}{w_i + w_k}, k \in N, \\ \sum_{p=1}^{n} u_p \geq 1, \\ u_p \geq 0, p \in N, \\ \theta_i \text{ is free.} \end{cases} \tag{22}$$

Generally speaking, we assume that $w_1 \leq w_2 \leq \cdots \leq w_n$, then for $\forall i \in N$, $\frac{w_i}{w_i + w_k} \geq 0$. Thus, the objective function θ_i is minimal only if the constraint $\sum_{p=1}^{n} u_p \geq 1$ turns into $\sum_{p=1}^{n} u_p = 1$.

Moreover, since $w_1 \leq w_2 \leq \cdots \leq w_n$, we have $0 \leq \frac{w_1}{w_1 + w_k} \leq \frac{w_2}{w_2 + w_k} \leq \cdots \leq \frac{w_n}{w_n + w_k}$ for $\forall k \in N$. Thus, the optimal solutions are $\{u_1^*, u_2^*, \ldots, u_n^*\} = \{1, 0, \ldots, 0\}$, then the first constraint of the model (22) is transformed into $\frac{w_1}{w_1 + w_k} \leq \theta_i \frac{w_i}{w_i + w_k}, k \in N$, which means that $\theta_i \geq \frac{w_1(w_i + w_k)}{w_i(w_1 + w_k)}$. Hence, the optimal solution of model (22) can be determined as follows:

$$\theta_i^* = \max_{k \in N} \left\{ \frac{w_1(w_i + w_k)}{w_i(w_1 + w_k)} \right\} = \frac{w_1(w_i + w_n)}{w_i(w_1 + w_n)}$$
$$= \frac{w_1}{w_1 + w_n} (1 + \frac{w_n}{w_i}). \tag{23}$$

From Eq. (23) and $w_1 \leq w_2 \leq \cdots \leq w_n$, we can obtain $\theta_1^* \geq \theta_2^* \geq \cdots \geq \theta_n^*$, therefore

$$\max_{i \in N} \{\theta_i^*\} = \theta_1^* = \frac{w_1(w_1 + w_n)}{w_1(w_1 + w_n)} = 1,$$
$$\min_{i \in N} \{\theta_i^*\} = \theta_n^* = \frac{w_1(w_n + w_n)}{w_n(w_1 + w_n)} = \frac{2w_1}{w_1 + w_n}. \tag{24}$$

According to Eqs. (23) and (24), we have

$$2\theta_i^* = \frac{2w_1(w_i + w_n)}{w_i(w_1 + w_n)} = \frac{2w_1}{w_1 + w_n} \cdot \frac{w_i + w_n}{w_i}$$
$$= \min_{i \in N} \{\theta_i^*\} \cdot \frac{w_i + w_n}{w_i}. \tag{25}$$

Then, we have.

$$w_i = \frac{w_n \cdot \min_{i \in N} \{\theta_i^*\}}{2\theta_i^* - \min_{i \in N} \{\theta_i^*\}} \tag{26}$$

Therefore,

$$1 = \sum_{i=1}^{n} w_i = \sum_{i=1}^{n} \frac{w_n \cdot \min_{i \in N} \{\theta_i^*\}}{2\theta_i^* - \min_{i \in N} \{\theta_i^*\}} = w_n \cdot \sum_{i=1}^{n} \frac{\min_{i \in N} \{\theta_i^*\}}{2\theta_i^* - \min_{i \in N} \{\theta_i^*\}}, \text{ then we}$$

obtain that

$$w_n = 1 \Big/ \sum_{i=1}^{n} \frac{\min_{i \in N} \{\theta_i^*\}}{2\theta_i^* - \min_{i \in N} \{\theta_i^*\}} \tag{27}$$

The proof of Theorem 4 is completed. ∎

Remark 3 We employ the input-oriented CCR DEA model to generate the weights of alternatives for the following reasons: firstly, DEA model is a great method for selecting most efficient DMU, and it is widely used in many fields. Secondly, the input-oriented CCR DEA model is suitable for FPR. According to the features of FPR, we can regard each alternative as a DMU, and view each column of FPR as an input. Consequently, the input-oriented CCR DEA model can be utilized to derive the priority weights of alternatives. Lastly, the difference between our CCR DEA model and the existing literature is that this DEA model is input-oriented, while most of the existing literature is output-oriented, such as [27, 34, 41]. Deriving the weights of alternatives by input-oriented CCR DEA model not only enriches the existing research, but also provides a simple and convenient way to generate the rankings of alternatives.

Example 3 Following Example 2, we can obtain the priority weights by utilizing model (21) from the FPR $\widetilde{R}$. And then the efficiency score of the alternative x_1 can be derived from the following model (28):

$$\min \theta_1$$

$$s.t. \begin{cases} 0.5u_1 + 0.5289u_2 + 0.4u_3 + 0.3u_4 \leq 0.5\theta_1, \\ 0.4711u_1 + 0.5u_2 + 0.2u_3 + 0.6u_4 \leq 0.4711\theta_1, \\ 0.6u_1 + 0.8u_2 + 0.5u_3 + 0.1u_4 \leq 0.6\theta_1, \\ 0.7u_1 + 0.4u_2 + 0.9u_3 + 0.5u_4 \leq 0.7\theta_1, \\ u_1 + u_2 + u_3 + u_4 \geq 1, \\ u_i \geq 0, \\ \theta_1 \text{ is free.} \end{cases}$$

$$(28)$$

The optimal solution of model (28) is $\theta_1^* = 0.8725$. Similarly, alternatives x_2, x_3 and x_4 can also be evaluated, and $\theta_2^* = 1, \theta_3^* = 1, \theta_4^* = 1$, then we have $\theta_1^* < \theta_2^* = \theta_3^* = \theta_4^*$. According to Theorem 4, we have $w_1 > w_2 = w_3 = w_4$, and the priority weights of the four alternatives can be calculated as follows: $w = \{w_1, w_2, w_3, w_4\}^T = \{0.2764, 0.2139, 0.2139, 0.2139\}^T$.

4.2 Deriving Complete Ranking Based on DEA Cross-Efficiency

From Example 3, it is observed that the weights of different DMUs may same, which means that there exist multiple DEA efficient DMUs [41], and we do not derive the complete ranking of alternatives. Therefore, to discriminate multiple DEA efficient DMUs, DEA cross-efficiency is used to further measure the efficiencies of DMUs.

Firstly, we establish a CCR-DEA model to generate the self-evaluation efficiency of $DMU_P (p \in N)$ based on the derived acceptable multiplicative consistent FPRs, and each alternative represents a DMU, each column of $\widetilde{R} = (\widetilde{r_{ij}})_{n \times n}$ represents an output. At the same time, the virtual input value of all DMUs is taken as 0.5. The CCR-DEA model is established as follows:

$$\max E_{ii} = \sum_{p=1}^{n} u_{pi} \widetilde{r_{ip}}$$

$$s.t. \begin{cases} \sum_{p=1}^{n} u_{pi} \widetilde{r_{jp}} \leq 0.5 v_i, j \in N, \\ 0.5 v_i = 1, \\ u_{pi} \geq 0, p \in N. \end{cases}$$

$$(29)$$

where v_i represents the weight of input, $u_{pi} (p \in N)$ denotes the weight of output, and E_{ii} indicates the self-evaluated efficiency score of DMU_i.

Then, based on the optimal solution of model (29) $E_{ii}^* = \{E_{11}^*, E_{22}^*, \cdots, E_{nn}^*\}$, the following DEA cross-efficiency model is established:

$$\min \sum_{j=1}^{n} d_{ji}$$

$$s.t. \begin{cases} \sum_{p=1}^{n} u_{pi} (\sum_{j=1}^{n} \widetilde{r_{jp}}) + v_i x = n, i \in N, \\ \sum_{p=1}^{n} u_{pi} \widetilde{r_{ip}} - E_{ii}^* v_i x = 0, i \in N, \\ \sum_{p=1}^{n} u_{pi} \widetilde{r_{jp}} - E_{jj}^* v_i x + d_{ji} = 0, i, j \in N, \\ u_{pi}, v_i, d_{ji} \geq 0, i, j, p \in N. \end{cases}$$

$$(30)$$

where x represents the virtual input variable of each DMU, i.e., $x = 0.5$, the weights of associated inputs and outputs are expressed as v_i, u_{pi}, the deviation variable is denoted as d_{ji}. With the optimal weights $\{u_{1i}^*, u_{2i}^*, \cdots, u_{ni}^*, v_i^*\}$ of DMU_i, the self-evaluation efficiency and peer-evaluation efficiency of DMU_i can be obtained as follows:

$$E_{ij} = \frac{\sum_{p=1}^{n} u_{pi}^* \widetilde{r_{jp}}}{v_i^* x}, i \neq j, i, j \in N \tag{31}$$

According to the above analysis, we can obtain the following cross-efficiency matrix $E = (E_{ij})_{n \times n}$, which is composed of self-evaluation efficiency values and peer-evaluation efficiency values.

$$E = \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1n} \\ E_{21} & E_{22} & \cdots & E_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ E_{n1} & E_{n2} & \cdots & E_{nn} \end{bmatrix}.$$

Eventually, we can derive the final efficiency of DMU_j by a weighted arithmetic average operator,

$$E_j = \frac{1}{n} \sum_{i=1}^{n} E_{ij}, j \in N \qquad (32)$$

The final efficiency of the alternative x_j is arithmetic average of the j column of matrix E, then we can acquire the complete rankings by sorting the efficiency values of all alternatives in descending order.

Example 4 Following example 3, we can utilize model (29) to obtain the optimal self-evaluated efficiency $E_{ii}^* = (E_{11}^*, E_{22}^*, E_{33}^*, E_{44}^*) = (1, 1, 1, 1)$, then the cross-efficiency matrix is generated by using the model (30) and Eq. (31),

$$E = \begin{bmatrix} 1.0000 & 0.8217 & 0.8458 & 1.0000 \\ 1.0000 & 1.0000 & 0.6597 & 1.0000 \\ 1.0000 & 0.7340 & 1.0000 & 0.8998 \\ 1.0000 & 0.8217 & 0.8458 & 1.0000 \end{bmatrix}.$$

Then, we use Eqs. (31) and (32) to calculate self-evaluation efficiency and peer-evaluation efficiency of four alternatives, the final efficiency scores are $E = (E_1, E_2, E_3, E_4) = (1, 0.8443, 0.8378, 0.9749)$, then the ranking result is $x_1 \succ x_4 \succ x_2 \succ x_3$. Consequently, the best alternative is x_1, which has the maximum cross-efficiency.

4.3 DEA-Driven Decision-Making Method with FPRs

In the light of the above analysis, we develop the following DEA-driven decision-making method (Algorithm 2), which can improve the consistency level of FPRs and determine the complete rankings of alternatives.

According to consistency-improving and DEA evaluation method, the above decision-making process is presented in Fig. 1.
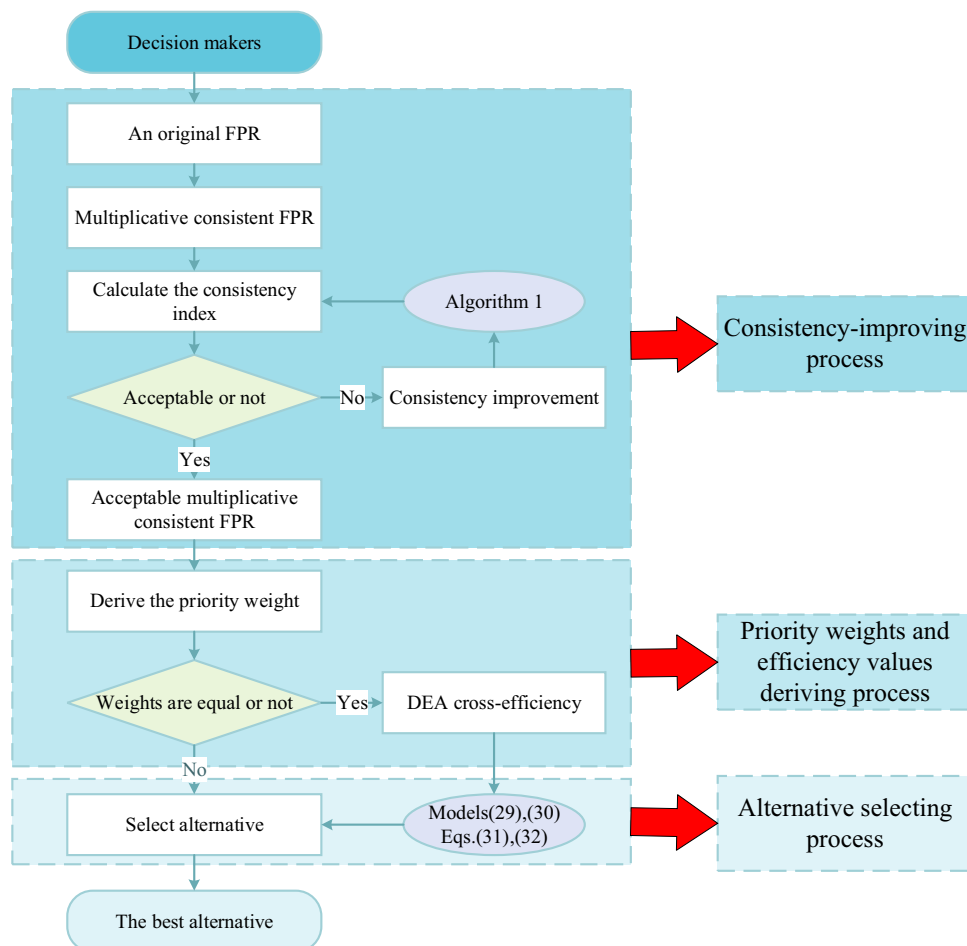

Fig. 1 DEA-driven decision-making method with FPRs

Algorithm 2

$$R = \begin{pmatrix} 0.5 & 0.75 & 0.65 & 0.6 & 0.8 \\ 0.25 & 0.5 & 0.6 & 0.7 & 0.55 \\ 0.35 & 0.4 & 0.5 & 0.9 & 0.68 \\ 0.4 & 0.3 & 0.1 & 0.5 & 0.72 \\ 0.2 & 0.45 & 0.32 & 0.28 & 0.5 \end{pmatrix}.$$

Next, Algorithm 2 is adopted for generating the rankings of alternatives. Let consistency threshold $\overline{CI} = 0.1$, adjusted parameter $\delta = 0.2$, iteration $t = 0$. From Theorem 2, a multiplicative consistent FPR is yielded which is presented below:

$$\overline{R} = \begin{pmatrix} 0.5000 & 0.7549 & 0.7906 & 0.6778 & 0.6471 \\ 0.2451 & 0.5000 & 0.7540 & 0.6583 & 0.6316 \\ 0.2094 & 0.2460 & 0.5000 & 0.6642 & 0.6363 \\ 0.3222 & 0.3417 & 0.3358 & 0.5000 & 0.6210 \\ 0.3529 & 0.3684 & 0.3637 & 0.3790 & 0.5000 \end{pmatrix}.$$

Then, Eqs. (14) and (15) are applied to acquire the consistency index, where $CI(R^{(0)}) = 0.1219$. Due to $CI(R^{(0)}) > \overline{CI} = 0.1$, which signifies the current consistency level is unacceptable, the consistency improvement process is executed. Since $\max_{i<j} \left| r_{ij}^{(0)} - \overline{r}_{ij}^{(0)} \right| = \left| r_{34}^{(0)} - \overline{r}_{34}^{(0)} \right| = 0.2358$, the most inconsistent element r_{35} should be revised by Eq. (17), and the following updated FPR can be obtained.

$$R^{(1)} = \begin{pmatrix} 0.5 & 0.75 & 0.65 & 0.6 & 0.8 \\ 0.25 & 0.5 & 0.6 & 0.7 & 0.55 \\ 0.35 & 0.4 & 0.5 & 0.8528 & 0.68 \\ 0.4 & 0.3 & 0.1472 & 0.5 & 0.72 \\ 0.2 & 0.45 & 0.32 & 0.28 & 0.5 \end{pmatrix}.$$

By Eqs. (14) and (15), the adjusted consistency index $CI(R^{(1)}) = 0.1134 > \overline{CI}$, the consistency level remains unacceptable. Therefore, the FPR $R^{(1)}$ should be further modified. In the same way, the acceptable multiplicative consistent FPR $R^{(4)}$ can be generated when $t = 4$, and the consistency index $CI(R^{(4)}) = 0.0996 < 0.1$.

$$R^{(4)} = \begin{pmatrix} 0.5 & 0.75 & 0.65 & 0.6 & 0.7694 \\ 0.25 & 0.5 & 0.6 & 0.7 & 0.55 \\ 0.35 & 0.4 & 0.5 & 0.7849 & 0.68 \\ 0.4 & 0.3 & 0.2151 & 0.5 & 0.72 \\ 0.2306 & 0.45 & 0.32 & 0.28 & 0.5 \end{pmatrix}.$$

Subsequently, the efficiency values of alternatives can be derived with model (21), i.e., $\theta_i^* = (0.6499, 0.9224, 0.9215, 1, 1)$. Then, according to Theorem 4, the priority weights of alternatives are $w = (0.3277, 0.1782, 0.1785, 0.1578, 0.1578)^T$. Due to the weights of alternatives x_4 and x_5 are identical, models

(29) and (30) are employed to discriminate alternatives and generate the final rankings.

By running models (29) and (30), the optimal weights of inputs and outputs are acquired, and the following cross-efficiency matrix is constructed.

$$E = \begin{pmatrix} 1.0000 & 0.8386 & 1.0000 & 0.9077 & 0.5997 \\ 1.0000 & 1.0000 & 1.0000 & 0.5985 & 0.4985 \\ 1.0000 & 0.8386 & 1.0000 & 0.9077 & 0.5997 \\ 1.0000 & 0.7148 & 0.8838 & 0.9358 & 0.6499 \\ 1.0000 & 0.7148 & 0.8838 & 0.9358 & 0.6499 \end{pmatrix}.$$

Thus, the final efficiency scores of alternatives are $E = (E_1, E_2, E_3, E_4, E_5) = (1, 0.8214, -$, and the ranking result is $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$.

5 Numerical Example and Model Discussion

This section offers a practical example for illustrating the feasibility and practicability of the decision-making method, and then comparative analysis is conducted to highlight the merits of the developed method. Finally, sensitivity analysis is utilized to discuss the influence of essential parameters on the rankings of alternatives.

5.1 Numerical Example

Owning to complexity, suddenness and destructiveness, emergency events not only threaten the life safety of public, but also lead to huge economic and property losses [42]. In recent years, as the frequent occurrence of emergency events, such as COVID-19, waterlogging, earthquakes, and others, many researchers are absorbed in emergency decision-making [43, 44]. In the process of emergency management, sufficient material supply plays a pivotal and important role in guarantying the life health of public and maintaining social order. Due to the lack of emergency events information and the urgency of decision-making time, how to quickly and accurately deliver goods to victims is significant when emergencies happened. To safeguard public safety and reduce the damages of emergencies to society, it is necessary to set up temporary warehouse of emergency materials, which can cut down the time of emergency goods dispatched to demand points and improve the efficiency of emergency decision-making. Therefore, selecting an optimal location of temporary warehouse for emergency materials deserves further study.

To improve the efficiency of preventing and coping with emergency events, we need to choose an optimal storage position for emergency materials. After investigation and consideration of various factors, we assume that there are four alternatives which are denoted as $X = \{x_1, x_2, x_3, x_4\}$.

The evaluation information of four temporary warehouse locations is given by a DM and expressed as a FPR $R = (r_{ij})_{4 \times 4}$, the detailed information of the FPR R is as follows:

$$R = \begin{pmatrix} 0.5 & 0.3 & 0.6 & 0.7 \\ 0.7 & 0.5 & 0.8 & 0.4 \\ 0.4 & 0.2 & 0.5 & 0.6 \\ 0.3 & 0.6 & 0.4 & 0.5 \end{pmatrix}.$$

We set the consistency threshold $\overline{CI} = 0.135$, and the adjusted parameter $\delta = 0.2$ [34]. In the following, we utilize the established decision-making method to derive the ranking results and select an optimal temporary position from four alternatives.

Stage I Consistency-Improving Process

Step 1 Let iteration $t = 0$ and $R^{(0)} = R = (r_{ij})_{4 \times 4}$, $\overline{CI} = 0.135$ and $\delta = 0.2$, based on the original FPR $R^{(0)} = (r_{ij}^{(0)})_{4 \times 4}$, we can utilize Theorem 2 to obtain multiplicative consistent FPR $\overline{R} = (\overline{r_{ij}})_{4 \times 4}$ as follows:

$$\overline{R} = \begin{pmatrix} 0.5000 & 0.7667 & 0.6635 & 0.6765 \\ 0.2333 & 0.5000 & 0.6735 & 0.6875 \\ 0.3365 & 0.3265 & 0.5000 & 0.6636 \\ 0.3235 & 0.3125 & 0.3364 & 0.5000 \end{pmatrix}.$$

Then, we calculate the consistency index of the original FPR $R^{(0)}$ by using the Eqs. (14) and (15), we have $CI(R^{(0)}) = d(R^{(0)}, \overline{R}) = 0.2328 > \overline{CI} = 0.135$, which means that $R^{(0)}$ is a FPR with unacceptable multiplicative consistency. Therefore, we should improve the consistency level of $R^{(0)}$. Because $\left| r_{12}^{(0)} - \overline{r_{12}^{(0)}} \right| = 0.4667 = \max_{i<j} \left| r_{ij}^{(0)} - \overline{r_{ij}^{(0)}} \right|$, $r_{12}^{(0)}$ is the most inconsistent element of $R^{(0)}$. After that, Eq. (17) is adopted to adjust $r_{12}^{(0)}$, we can obtain a new element $r_{12}^{(1)} = (1 - 0.2)r_{12}^{(0)} + 0.2\overline{r_{12}^{(0)}} = 0.3933 r_{12}^{(1)}$. Thus, the following adjusted FPR is generated:

$$R^{(1)} = \begin{pmatrix} 0.5 & 0.3933 & 0.6 & 0.7 \\ 0.6067 & 0.5 & 0.8 & 0.4 \\ 0.4 & 0.2 & 0.5 & 0.6 \\ 0.3 & 0.6 & 0.4 & 0.5 \end{pmatrix}.$$

We utilize Eqs. (14) and (15) to calculate the consistency index of adjusted FPR, and then we can obtain $CI(R^{(1)}) = d(R^{(1)}, \overline{R}) = 0.2028 > \overline{CI} = 0.135$, $R^{(1)} = (r_{ij}^{(1)})_{4 \times 4}$ is unacceptable multiplicative FPR. As $\left| r_{12}^{(1)} - \overline{r_{12}^{(1)}} \right| = 0.3734 = \max_{i<j} \left| r_{ij}^{(1)} - \overline{r_{ij}^{(1)}} \right|$, $r_{12}^{(1)}$ is the most inconsistent element of $R^{(1)}$ and it should be revised. Then, we can obtain the new element $r_{12}^{(2)} = (1 - 0.2)r_{12}^{(1)} + 0.2\overline{r_{12}^{(1)}} = 0.4680$, the adjusted FPR $R^{(2)} = (r_{ij}^{(2)})_{4 \times 4}$ as follows:

$$R^{(2)} = \begin{pmatrix} 0.5 & 0.4680 & 0.6 & 0.7 \\ 0.5320 & 0.5 & 0.8 & 0.4 \\ 0.4 & 0.2 & 0.5 & 0.6 \\ 0.3 & 0.6 & 0.4 & 0.5 \end{pmatrix}.$$

We utilize Eqs. (14) and (15) to calculate the consistency index of adjusted FPR, and then we can obtain $CI(R^{(2)}) = d(R^{(2)}, \overline{R}) = 0.1810 > \overline{CI} = 0.135$, which indicates $R^{(2)}$ is of unacceptable consistency. Due to $\left| r_{12}^{(2)} - \overline{r_{12}^{(2)}} \right| = 0.2987 = \max_{i<j} \left| r_{ij}^{(2)} - \overline{r_{ij}^{(2)}} \right|$, $r_{12}^{(2)}$ is the most inconsistent element of $R^{(2)}$, and then the new element is derived by using Eq. (17), we have $r_{12}^{(3)} = (1 - 0.2)r_{12}^{(2)} + 0.2\overline{r_{12}^{(2)}} = 0.5277$, the adjusted FPR $R^{(3)} = (r_{ij}^{(3)})$ as follows:

$$R^{(3)} = \begin{pmatrix} 0.5 & 0.5277 & 0.6 & 0.7 \\ 0.4723 & 0.5 & 0.8 & 0.4 \\ 0.4 & 0.2 & 0.5 & 0.6 \\ 0.3 & 0.6 & 0.4 & 0.5 \end{pmatrix}.$$

Since $CI(R^{(3)}) = d(R^{(3)}, \overline{R}) = 0.1665 > \overline{CI} = 0.135$, $R^{(3)}$ is of unacceptable consistency. As $\left| r_{24}^{(3)} - \overline{r_{24}^{(3)}} \right| = 0.2875 = \max_{i<j} \left| r_{ij}^{(3)} - \overline{r_{ij}^{(3)}} \right|$, $r_{24}^{(3)}$ is the most inconsistent element of $R^{(3)}$, and then we can acquire the new element by using Eq. (17), we have $r_{24}^{(4)} = (1 - 0.2)r_{24}^{(3)} + 0.2\overline{r_{24}^{(3)}} = 0.4575$, the adjusted FPR $R^{(4)} = (r_{ij}^{(4)})_{4 \times 4}$ as follows:

$$R^{(4)} = \begin{pmatrix} 0.5 & 0.5277 & 0.6 & 0.7 \\ 0.4723 & 0.5 & 0.8 & 0.4775 \\ 0.4 & 0.2 & 0.5 & 0.6 \\ 0.3 & 0.5225 & 0.4 & 0.5 \end{pmatrix}.$$

As $CI(R^{(4)}) = d(R^{(4)}, \overline{R}) = 0.1498 > \overline{CI} = 0.135$, $R^{(4)}$ is of unacceptable consistency. Owing to $\left| r_{12}^{(4)} - \overline{r_{12}^{(4)}} \right| = 0.2390 = \max_{i<j} \left| r_{ij}^{(4)} - \overline{r_{ij}^{(4)}} \right|$, $r_{12}^{(4)}$ is the most

inconsistent element of $R^{(4)}$, and then the new element is obtained by using Eq. (17), we have $r_{12}^{(5)} = (1 - 0.2)r_{12}^{(4)} + 0.2\overline{r_{12}^{(4)}} = 0.5755$, the adjusted FPR $R^{(5)} = (r_{ij}^{(5)})_{4\times4}$ as follows:

$$R^{(5)} = \begin{pmatrix} 0.5 & 0.5755 & 0.6 & 0.7 \\ 0.4245 & 0.5 & 0.8 & 0.4775 \\ 0.4 & 0.2 & 0.5 & 0.6 \\ 0.3 & 0.5225 & 0.4 & 0.5 \end{pmatrix}.$$

Since $CI(R^{(5)}) = d(R^{(5)}, \overline{R}) = 0.1325 < 0.135$, $R^{(5)} = (r_{ij}^{(5)})_{4\times4}$ is of acceptable consistency, the iteration process terminates at this step. Thus, the acceptable multiplicative consistent FPR $\widetilde{R} = (\widetilde{r}_{ij})_{4\times4} = R^{(5)}$ is as follows:

$$\widetilde{R} = \begin{pmatrix} 0.5 & 0.5755 & 0.6 & 0.7 \\ 0.4245 & 0.5 & 0.8 & 0.4775 \\ 0.4 & 0.2 & 0.5 & 0.6 \\ 0.3 & 0.5225 & 0.4 & 0.5 \end{pmatrix}.$$

For convenience, the consistency index $CI(R^{(t)})$, the most inconsistent element $r_{i^*j^*}$ and the adjusted FPR for each iteration are presented in Table 1.

Stage II Priority weight vector deriving process

Step 2 Based on the acceptable multiplicative consistent FPR $\widetilde{R} = (\widetilde{r}_{ij})_{4\times4}$ derived from stage I, we utilize model (21) to obtain the efficiency scores of four alternatives, respectively, the efficiency scores are $\theta^* = (\theta_1, \theta_2, \theta_3, \theta_4) = (0.7516, 1, 1, 1)$. Then, we utilize Theorem 4 to obtain the priority weights as follows:
$w = (0.3562, 0.2146, 0.2146, 0.2146)^T$.
Since there exist multiple efficient DMUs from the above result, we cannot obtain a complete ranking among those alternatives whose weights are same. Therefore, we go to step 3 and employ DEA cross-efficiency to further discriminate DMUs and generate the final rankings.

Step 3 On the basis of the acceptable FPR with multiplicative consistency $\widetilde{R} = (\widetilde{r}_{ij})_{4\times4}$, we calculate the self-evaluated efficiency scores by using model (29), and then we have $E_{ii}^* = (E_{11}^*, E_{22}^*, E_{33}^*, E_{44}^*) = (1, 1, 0.8571, 0.9048)$. Next, we use model (30) and Eq. (31) to obtain the cross-efficiency matrix as follows:

Table 1 The specific iteration processes

Iteration (t)	CI(R$^{(t)}$)	$r_{i^*j^*}$	$R^{(t)}$
0	0.2328	r_{12}	$R^{(0)} = \begin{pmatrix} 0.5 & 0.3 & 0.6 & 0.7 \\ 0.7 & 0.5 & 0.8 & 0.4 \\ 0.4 & 0.2 & 0.5 & 0.6 \\ 0.3 & 0.6 & 0.4 & 0.5 \end{pmatrix}$
1	0.2018	r_{12}	$R^{(1)} = \begin{pmatrix} 0.5 & 0.3933 & 0.6 & 0.7 \\ 0.6067 & 0.5 & 0.8 & 0.4 \\ 0.4 & 0.2 & 0.5 & 0.6 \\ 0.3 & 0.6 & 0.4 & 0.5 \end{pmatrix}$
2	0.1810	r_{12}	$R^{(2)} = \begin{pmatrix} 0.5 & 0.4680 & 0.6 & 0.7 \\ 0.5320 & 0.5 & 0.8 & 0.4 \\ 0.4 & 0.2 & 0.5 & 0.6 \\ 0.3 & 0.6 & 0.4 & 0.5 \end{pmatrix}$
3	0.1665	r_{24}	$R^{(3)} = \begin{pmatrix} 0.5 & 0.5277 & 0.6 & 0.7 \\ 0.4723 & 0.5 & 0.8 & 0.4 \\ 0.4 & 0.2 & 0.5 & 0.6 \\ 0.3 & 0.6 & 0.4 & 0.5 \end{pmatrix}$
4	0.1498	r_{12}	$R^{(4)} = \begin{pmatrix} 0.5 & 0.5277 & 0.6 & 0.7 \\ 0.4723 & 0.5 & 0.8 & 0.4775 \\ 0.4 & 0.2 & 0.5 & 0.6 \\ 0.3 & 0.5225 & 0.4 & 0.5 \end{pmatrix}$
5	0.1325	–	$R^{(5)} = \begin{pmatrix} 0.5 & 0.5755 & 0.6 & 0.7 \\ 0.4245 & 0.5 & 0.8 & 0.4775 \\ 0.4 & 0.2 & 0.5 & 0.6 \\ 0.3 & 0.5225 & 0.4 & 0.5 \end{pmatrix}$

$$E = \begin{pmatrix} 1.0000 & 1.0000 & 0.8456 & 0.6911 \\ 1.0000 & 1.0000 & 0.8456 & 0.6911 \\ 1.0000 & 0.6833 & 0.8571 & 0.7142 \\ 1.0000 & 0.8748 & 0.3538 & 0.9048 \end{pmatrix}.$$

Then, we can obtain the final efficiency scores of four temporary warehouse locations by utilizing Eq. (32), which are $E = (E_1, E_2, E_3, E_4) = (1, 0.8895, 0.7255, 0.7503)$. Stage III The best alternative selecting process

Step 4   From Stage II, we know that $E_1 < E_2 < E_4 < E_3$, then the ranking results of four alternatives are $x_1 \succ x_2 \succ x_4 \succ x_3$. Therefore, the optimal location of emergency materials is x_1.

5.2 Comparative Analysis

In this subsection, the comparison between the existing method and our developed method is displayed to verify the feasibility and effectiveness of our method.

(1) Now, we utilize the Algorithm and the priority generation method proposed by Qian and Feng [32] to solve the problem, the details are as follows.

First, we use the Algorithm in [32] to obtain the acceptable FPR, the steps are as follows:

Step 1   Let $B^{(0)} = (b_{ij}^{(0)})_{4 \times 4} = R$, iteration $k = 0$, adjusted parameter $\alpha = 0.8$, and threshold $\delta = 0.135$.

Step 2   We can obtain the $FGBI(B^{(0)}) = 0.167$ by employing the following Quadratic programming (QP) model in [32], and the weight vector $v = (v_1, v_2, v_3, v_4)^T = (0.3, 0.45, 0.1, 0.15)^T$ can also be obtained.

$$\min FGCI = \frac{2}{(4-1)(4-2)} \sum_{i<j} (2b_{ij} - v_i - v_j)^2$$
$$s.t. \begin{cases} v_1 + v_2 + v_3 + v_4 = 1 \\ v_1, v_2, v_3, v_4 \geq 0 \end{cases}$$

Step 3   Since $FGBI(B^{(0)}) = 0.167 > \delta = 0.135$, which means that $B^{(0)}$ is unacceptable FPR.

Step 4   We need to adjust all elements of FPR $B^{(0)}$ by using Eq. (10) in [32]. Therefore, we can obtain the adjusted FPR $B^{(1)} = (b_{ij}^{(1)})_{4 \times 4}$ as follows:

$$B^{(1)} = \begin{pmatrix} 0.5 & 0.325 & 0.6 & 0.675 \\ 0.675 & 0.5 & 0.775 & 0.45 \\ 0.4 & 0.225 & 0.5 & 0.575 \\ 0.325 & 0.55 & 0.425 & 0.5 \end{pmatrix}$$

Step 5:   Let $k = 1$, we can calculate the $FGBI(B^{(1)}) = 0.1067$ by using the QP model in [32], and the weight vector is $v = (v_1, v_2, v_3, v_4)^T = (0.3, 0.45, 0.1, 0.15)^T$. As $FGBI(B^{(1)}) = 0.1067 < 0.135$, it means that $B^{(1)}$ is satisfactory consistency and $B^{(1)}$ is of acceptable consistency.

Step 6   Output the following modified acceptable consistent FPR:

$$B^{(1)} = \begin{pmatrix} 0.5 & 0.325 & 0.6 & 0.675 \\ 0.675 & 0.5 & 0.775 & 0.45 \\ 0.4 & 0.225 & 0.5 & 0.575 \\ 0.325 & 0.55 & 0.425 & 0.5 \end{pmatrix}.$$

To check the effectiveness of modification, the criteria of modification effectiveness should be calculated. We have $d_1^{(1)} = 0.025 < 0.2$, $d_2^{(1)} = 0.025 < 0.1$. Therefore, the modification is regarded as acceptable.

Step 7   We utilize the priority generation method of [32] to derive the interval weights based on the FPR $B^{(1)}$ and threshold δ. We can generate the interval weights of four alternatives by using the nonlinear programming model in [32]. Then the following interval weights of four alternatives can be derived:

$[v_1^L, v_1^U] = [0.1738, 0.4262], [v_2^L, v_2^U] = [0.3238, 0.5762],$
$[v_3^L, v_3^U] = [0.0000, 0.2262], [v_4^L, v_4^U] = [0.0238, 0.2762].$

Thus, the ranking result of four alternatives is $x_2 \succ x_1 \succ x_4 \succ x_3$, and the optimal alternative is x_2.

(2) Now, we utilize Lee [33]'s method to solve the problem.

Step 1 Let $P^* = (p_{ij})_{4 \times 4} = R$, based on the complete FPR P^*, we can construct the additive consistent matrix $\overline{P}$ by using Eq. (42) in [33], which is shown below:

$$\overline{P} = \begin{pmatrix} 0.5 & 0.5 & 0.6 & 0.575 \\ 0.5 & 0.5 & 0.675 & 0.475 \\ 0.4 & 0.325 & 0.5 & 0.525 \\ 0.425 & 0.525 & 0.475 & 0.5 \end{pmatrix},$$

Step 2 Utilizing Eq. (43) in [33], the ranking value $RV(x_i)$ of alternative $x_i (i = 1, 2, 3, 4)$ can be calculated as follows:

$$RV(x_1) = \frac{2}{4^2} \sum_{j=1}^{4} p_{1j} = \frac{2}{4^2}(0.5 + 0.5 + 0.6 + 0.575) = 0.2719,$$

$$RV(x_2) = \frac{2}{4^2} \sum_{j=1}^{4} p_{2j} = \frac{2}{4^2}(0.5 + 0.5 + 0.675 + 0.475) = 0.2688,$$

$$RV(x_3) = \frac{2}{4^2} \sum_{j=1}^{4} p_{3j} = \frac{2}{4^2}(0.4 + 0.325 + 0.5 + 0.525) = 0.2188$$

$$RV(x_4) = \frac{2}{4^2} \sum_{j=1}^{4} p_{4j} = \frac{2}{4^2}(0.425 + 0.525 + 0.475 + 0.5) = 0.2406.$$

Step 3 Due to $RV(x_1) > RV(x_2) > RV(x_4) > RV(x_3)$, the rankings of the four alternatives are $x_1 \succ x_2 \succ x_4 \succ x_3$, where the best alternative is x_1.

(3) Now, Liu et al. [27]'s method is employed to generate the orders of alternatives.

Step 1 Let initial FPR $R^{(0)} = (r_{ij}^{(0)})_{4 \times 4} = R$, iteration $\gamma = 0$, consistency threshold $\overline{CI} = 0.135$, and parameter $\theta = 0.8$.

Step 2 Calculating the consistency index $CI(R^{(0)})$ by Eq. (13).

Step 3 As $CI(R^{(0)}) = 0.2500 > 0.135$, which means that the current consistency degree is unacceptable.

Step 4 According to Theorem 2 in [27], the following additive consistent fuzzy preference relation is derived

$$R^{(0)} = \begin{pmatrix} 0.5000 & 0.4797 & 0.5281 & 0.5171 \\ 0.5203 & 0.5000 & 0.5484 & 0.5374 \\ 0.4719 & 0.4516 & 0.5000 & 0.4890 \\ 0.4829 & 0.4626 & 0.5110 & 0.5000 \end{pmatrix}$$

Step 5 Employing Eq. (15) of [27] to improve the consistency level of R. After 3 iterations, the adjusted acceptable FPR is generated as follows:

$$R^{(3)} = \begin{pmatrix} 0.5000 & 0.3877 & 0.5649 & 0.6107 \\ 0.5041 & 0.5000 & 0.6772 & 0.4671 \\ 0.4351 & 0.3228 & 0.5000 & 0.5458 \\ 0.3893 & 0.5329 & 0.4542 & 0.5000 \end{pmatrix}.$$

Then, the modified consistency index $CI(R^{(3)}) = 0.1246 < 0.135$, FPR $R^{(3)}$ meets the acceptable additive consistency. **Step 6** Based on $R^{(3)}$, the preference values of four alternatives are derived and displayed below:

$p_1 = 0.5158, p_2 = 0.5371, p_3 = 0.4509, p_4 = 0.4671.$

Consequently, the order of alternatives is $x_2 \succ x_1 \succ x_4 \succ x_3$.

(4) Wu [45]'s method is utilized to pick the optimal alternative

Step 1 Let FPR $R = (r_{ij})_{4 \times 4}$, then Model (1) in [45] is employed to derive the optimal weights, and the CCR efficiency of each alternative E_{dd} and parameter α_d can be calculated as follows:

$E_{dd} = (1, 1, 0.8571, 1)$

$\alpha_d = (0.9375, 0.7394, 0.7885, 0.7321)$

Step 2 Solving model (2) in [45], the cross-efficiency values E_{dj} can be calculated by using Eq. (3).

$$E_{dj} = \begin{pmatrix} 1.0000 & 0.5714 & 0.8571 & 0.7143 \\ 0.7500 & 1.0000 & 0.6250 & 0.5000 \\ 1.0000 & 0.5714 & 0.8571 & 0.7143 \\ 0.6296 & 1.0000 & 0.4444 & 1.0000 \end{pmatrix}$$

Step 3 According to Eqs. (6) and (7) in [45], the following consistency FPR $B = (b_{dj})_{4 \times 4}$ can be obtained:

$$B = \begin{pmatrix} 0.5000 & 0.4989 & 0.5290 & 0.5072 \\ 0.5011 & 0.5000 & 0.5301 & 0.5083 \\ 0.4710 & 0.4699 & 0.5000 & 0.4781 \\ 0.4928 & 0.4917 & 0.5219 & 0.5000 \end{pmatrix}$$

Step 4: Based on Eq. (8), the ranking weights of four alternatives are generated, we have $w = (0.2529, 0.2533, 0.2432, 0.2505)$. Therefore, the final ranking is $x_2 \succ x_1 \succ x_4 \succ x_3$.

Table 2 The derived ranking results from different methods

Methods	The ranking results	The best alternative
Qian and Feng [32]'s method	$x_2 \succ x_1 \succ x_4 \succ x_3$	x_2
Lee [33]'s method	$x_1 \succ x_2 \succ x_4 \succ x_3$	x_1
Liu et al. [27]'s method	$x_2 \succ x_1 \succ x_4 \succ x_3$	x_2
Wu [45]'s method	$x_2 \succ x_1 \succ x_4 \succ x_3$	x_2
Our method	$x_1 \succ x_2 \succ x_4 \succ x_3$	x_1
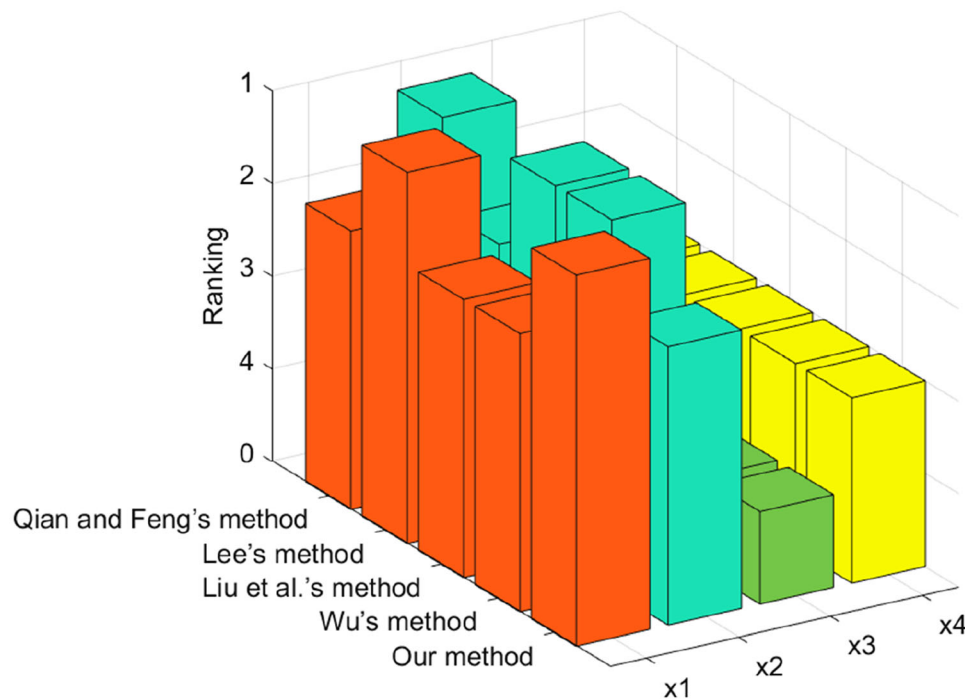
Fig. 2 The rankings of alternatives with different methods

Based on the above comparative analysis, Table 2 and Fig. 2 reveal the derived ranking results from different methods.

From Table 2 and Fig. 2, we find that the generated final ranking results with our method are different from Qian and Feng [32]'s method, Liu et al. [27]'s method and Wu [45]'s method. Qian and Feng [32]'s method proposed an iterative algorithm to obtain an acceptable FPR with satisfactory consistency. But all original evaluation information is changed in the process of iterative, and then the elements of the acceptable consistent FPR cannot represent the DM's initial evaluation information. Similarly, Liu et al. [27]'s method modified all elements of initial FPR in the process of improving consistency degree, and the original preference of DM is destroyed. Based on the DEA evaluation method, Wu [45] utilized cross-efficiency values to construct consistency FPR, but the method neglected the consistency level of FPR, and there is no guarantee that the established consistency FPR is accepted by DMs.

Additionally, from the multiplicative consistent FPR $\overline{R} = (\overline{r_{ij}})_{4 \times 4}$, the preference value of alternative x_1 is higher than x_2. Therefore, the derived ranking results with those methods are unreliable and unreasonable.

According to Table 2 and Fig. 2, it is observed that our method and Lee [33]'s method generate unanimous ranking results. Lee [33] constructed an additive consistent FPR based on a complete FPR, but Lee [33]'s method ignored the consistency degree of FPR, which not ensure the FPR with additive consistency is acceptable, and then may lead to unreliable decision-making results. And Lee [33]'s method directly utilized the preference information of each row of FPR to derive the order of alternatives by a weighted arithmetic average operator, which may be unreliable and inaccurate.

However, our decision-making model not only guarantees the original FPR is consistent, but also applies a consistency-improving algorithm to ensure that the multiplicative consistent FPR is acceptable. Additionally, we adopt local consistency adjust strategy to improve the

consistency level of FPR, which retains the DM's original preference information to the largest extent, and the adjusted FPR meets the expected consistency threshold. Moreover, the DEA cross-efficiency is applied to derive the rankings of alternatives and then the alternative with maximum efficiency scores is selected, the obtained decision-making results from our method are more convincing and reliable.

5.3 Sensitivity Analysis

In this subsection, we carry out a sensitivity analysis to investigate the effects of the essential parameter δ on the consistency level of FPR and the final ranking results of alternatives.

5.3.1 Effects of Adjusted Parameter δ on the Consistency Level of FPR

The adjusted parameter δ expresses the extent to which the initial preference information is retained, the smaller the value of δ is, the more the original preference of DM is held.

From Table 3 and Fig. 3, it can be seen that as the adjusted parameter increases, the number of iterations will converge to a certain numerical value. Specifically, when $0.1 \le \delta < 0.5$, the number of iterations will gradually decline, and the declining rate of the number of iterations is decreasing. When $0.5 \le \delta \le 0.9$, the number of iterations remains unchanged. Additionally, with the increase of adjusted parameter δ, the speed of reaching an acceptable consistency level is becoming faster. Since for each iteration, the distance between the adjusted FPR and the FPR with multiplicative consistency is getting smaller. And the larger the value of δ is, the closer the adjusted element is to the desired value. When δ is lager enough, i.e., $\delta \ge 0.5$, the number of iterations keeps unchanging attribute to the consistency level meets the expected threshold.

Moreover, as the value of adjusted parameter δ increases, the consistency index of the acceptable FPR $\widetilde{R}$ will continue to decline except for $\delta = 0.5$, which means that the larger the value of δ is, the higher the consistency degree of FPR $\widetilde{R}$ is. Because in the iteration process, the distance between the original FPR and the adjusted FPR gradually decreases. And with the increase of δ, the inconsistent element $r_{i^*j^*}$ of initial FPR is modified to a greater extent, then the consistency index gradually
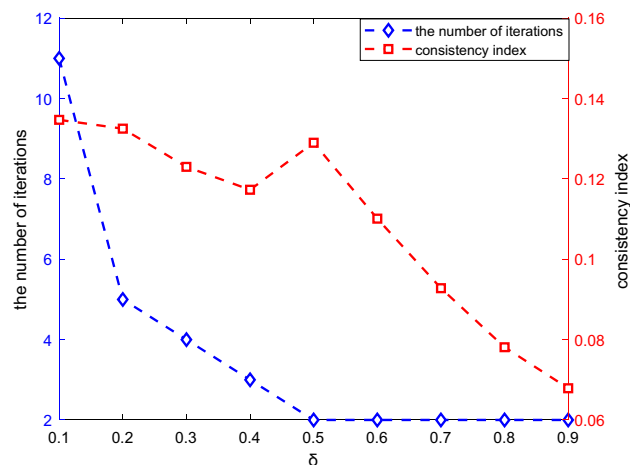

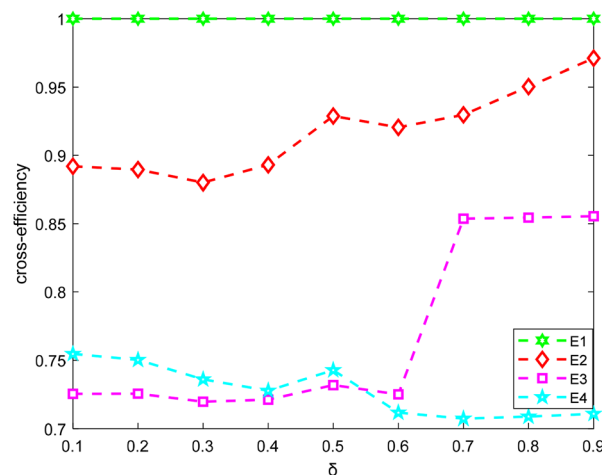**Fig. 3** The influence on the consistency index and the number of iterations

Fig. 4 The influence on the cross-efficiency of alternatives

reduces, while the consistency degree of FPR progressively heightens.

5.3.2 Effects of Adjusted Parameter δ on the Final Ranking Results

As can be seen in Fig. 4 and Table 4, with the increase of the adjusted parameter δ, the ranking results of alternatives keep unchanging until it exceeds a certain value, i.e., $\delta = 0.5$. Concretely, for different δ, alternative x_1 still possesses the maximum cross-efficiency values. In other words, alternative x_1 is always the optimal selection. Since

δ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$CI(R)$	0.1347	0.1325	0.123	0.1173	0.129	0.1101	0.0928	0.0781	0.0679
Number of iterations	11	5	4	3	2	2	2	2	2

Table 3 The influence on the consistency index and the number of iterations

Table 4 Cross-efficiency values and final rankings of alternatives

δ	Cross-efficiency values				Ranking results	Optimal alternative
	x_1	x_2	x_3	x_4		
0.1	1.0000	0.8919	0.7254	0.7547	$x_1 \succ x_2 \succ x_4 \succ x_3$	x_1
0.2	1.0000	0.8895	0.7255	0.7503	$x_1 \succ x_2 \succ x_4 \succ x_3$	x_1
0.3	1.0000	0.8800	0.7196	0.7360	$x_1 \succ x_2 \succ x_4 \succ x_3$	x_1
0.4	1.0000	0.8930	0.7211	0.7277	$x_1 \succ x_2 \succ x_4 \succ x_3$	x_1
0.5	1.0000	0.9288	0.7318	0.7428	$x_1 \succ x_2 \succ x_4 \succ x_3$	x_1
0.6	1.0000	0.9205	0.7250	0.7116	$x_1 \succ x_2 \succ x_3 \succ x_4$	x_1
0.7	1.0000	0.9297	0.8357	0.7072	$x_1 \succ x_2 \succ x_3 \succ x_4$	x_1
0.8	1.0000	0.9504	0.8545	0.7087	$x_1 \succ x_2 \succ x_3 \succ x_4$	x_1
0.9	1.0000	0.9710	0.8555	0.7107	$x_1 \succ x_2 \succ x_3 \succ x_4$	x_1

parameter δ has little influence on the cross-efficiency value of alternative x_1, which still equal to 1. In addition, the cross-efficiency of alternative x_4 is larger than x_3 when $0.1 \leq \delta \leq 0.5$, and the cross-efficiency of alternative x_3 will over alternative x_4 when $0.6 \leq \delta \leq 0.9$. The reason is that as δ increases, the cross-efficiency score of alternative x_3 gradually rises, while the cross-efficiency value of alternative x_4 gradually declines, and then the efficiency of x_4 is exceeded by x_3 when $\delta > 0.5$.

Based on the above analysis, it is clear that the selection of optimal alternative is not sensitive to the changes of value of δ, which illustrates the robustness of our developed decision-making method.

6 Conclusion

In this study, a DEA-driven decision-making method is developed under fuzzy environment, in which FPR is employed to express evaluation information of DM. Firstly, a novel approach is designed to establish a multiplicative consistent FPR. Then, to guarantee FPR is acceptable for DM, we develop a consistency index to check the consistency level of multiplicative consistent FPR. If the initial consistency level does not meet the threshold, a consistency-improving algorithm is constructed to enhance the consistency degree of FPR, in which only the most inconsistent element is revised, and the initial evaluation information of DM is maintained to the greatest extent. Subsequently, for assessing the performance of alternatives, an input-oriented CCR DEA model is provided, and the DEA cross-efficiency measurement method is utilized to further distinguish DMUs with the same weight. After that, a DEA-driven decision-making method with FPR is developed to obtain complete ranking results of alternatives. In the end, a numerical example is conducted to verify the feasibility of the proposed method, comparative analysis and sensitivity analysis are given for highlighting the merits and robustness of the design decision-making method.

However, how to determine the appropriate consistency threshold is not studied in our work. Moreover, we assume the FPR provided by DM is complete, and the situation that FPRs are incomplete is neglected. Therefore, a possible future research direction is how to determine the appropriate consistency threshold for different decision-making problems. In addition, it is interesting for incomplete FPRs to discuss decision-making models.

Declarations

Conflict of interest None.

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