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RESEARCH ARTICLE

Modeling of Passenger Transport Logistics Based on Intelligent Computational Techniques

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Abstract

The paper describes the analysis of the application of classic and optimized triangular norms in the hierarchical structure of weighted fuzzy Petri nets on the example of passenger transport logistics. The research is presented in a scheme with several levels of tasks. Each task includes some knowledge sets which were previously given by the experts and set into tables of "Object–property" type. It leads to the creation of a hierarchical structure of weighted fuzzy Petri net model which includes 125 possible combinations of triples of functions for calculation processing. A classic triple of functions (ZtN, GtN, ZsN) is tested and compared with an optimized triple (ZtN, ZtN, LsN). Additional verification methods are applied to confirm the practical usefulness of these triples. These techniques imply the use of additional triples which can be associated with each other along with practically efficient mathematical calculations.

Keywords Decision-making system · Fuzzy Petri net · Knowledge representation · Modeling of passenger transport logistics · Intelligent computational techniques

1 Introduction

The decision-support systems are becoming increasingly applied in different areas of life with the development of intelligent computational techniques. Various international companies present their own versions of software and algorithms for choosing a specific transport company for the given type of transport. Therefore, it necessitates the advancement of existing technologies and creating new ones to make the idea and systems more competitive and efficient. The authors present the research on the application of different triangular norms in the hierarchical structure of fuzzy Petri nets (FPN) for the analysis of the passenger transport logistics (PTL).

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Earlier studies described in the available literature on the subject area were carried out using various types of classic Petri nets (mainly colored) for many types of transport. It included: aviation, trains, automobile vehicles for various purposes, e.g., traffic control and maintenance, and optimization of these processes [1–4].

One of the applications of hierarchical timed colored Petri net is dedicated to the aviation branch. The experiment showed the easiness of dividing by phases with some number of sub-nets [4]. As a result, a complex scheme is presented with some number of easy-to-interpret nets.

Investigation of tram network performance processes with the use of Petri nets was presented in [5]. Modeling of train branch of transportation was concluded in the paper [6] with the application of parallel-operating objects for passenger trains of different categories. This approach allowed to track dynamic processes of the system. The effectiveness of application of knowledge rules in FPN is presented in [7] which leads to the idea of benefits of FPN application based on expert knowledge. One of the most informative articles on the practical use of weighted fuzzy Petri nets (WFPNs) in various industries is a review [8].

It leads to the idea that the PTL problem can be described as a problem dynamic discrete events, where wFPN serves as a mathematical tool for the representation and the analysis of the problem.

FPNs have met with great interest from scientists and practitioners in the field of artificial intelligence. They are a modification of classic Petri nets to deal with imprecise, unclear or fuzzy information in decision-support systems. In particular, they are used in such systems for modeling fuzzy production rules and automatic rule-based fuzzy inference. The main feature of these nets is that they effectively support the structural organization of information, provide knowledge-based visualization of inference, and facilitate the design of efficient fuzzy inference algorithms. All this makes FPNs a useful methodology for knowledge modeling and inference in decision-support systems [9, 10].

However, to the best of our knowledge, the authors have not found in the scientific literature research papers on the application of FPNs to PTL, in the terms and scope proposed in our papers, discussed briefly below.

The authors' aim of the current research is to test and explore the possibilities of triangular norms in the wFPN for the PTL thematic area, which is represented in a hierarchical conception of the decision-making process for three different types of transport: aviation, railway (train), and automobile vehicles.

A scheme of PTL and the application of FPN as well as generalized FPN (GFPN) for the partial solution of the scheme was presented in paper [11]. Moreover, a practical effectiveness of knowledge tables of "Object–property" type was explained in detail.

The application of weights in FPN (wFPN) which aims to achieve the most concretized numerical result at every output place for the net was presented in the following papers [12–15]. Additionally, an optimized triple of functions (ZtN, ZtN, LsN) for the research with its implementation and a proof of its mathematical effectiveness over a classic one (ZtN, GtN, ZsN) were presented [12, 14, 15]. Since papers [11, 12] gave consistent decisions of the part of PTL scheme, they raised a need to present a hierarchical FPN conception for covering the whole PTL scheme in the future [16]. Along with the development of this theory and the possibility of practical implementation in wFPN, the scheme of PTL levels was presented [11]. This scheme has been described by the structure of interconnected tables with the knowledge provided by experts in the subject area of PTL.

Three-level hierarchical structure of wFPN with additional separate model for PTL was proposed with the application of the three classic triangular norms mentioned above [13].

Another approach which needed to be tested applied an optimized triple of functions (ZtN, ZtN, LsN) on the same scheme for the whole hierarchy [14]. As expected, it resulted in higher numerical values at output places, but the authors were faced with a new question: the change of triple of functions led to the change of decisions. Therefore, the last presented research included the application of triples in the middle (between classic and optimized), which aimed to show the truth probability and effectiveness of application of classic or optimized triples of functions for the given wFPN in the context of PTL development [15].

The following paper is organized as follows. In Sect. 2, the transport logistics problem (TLP) is described in an informal way. The production rules as a knowledge representation for TLP are presented in Sect. 3. Section 4 describes an interpretation of decisions of PTL in the TLP scheme. In Sect. 5, the definition of a weighted fuzzy Petri net (wFPN) and the basic concepts associated with it are recalled. Section 6 presents the transformation of production rules into (w)FPN models. In Sect. 7, the triangular norms and their application in wFPN in the context of PTL are discussed. In Sect. 8, the net models for PTL are presented. A comparative analysis of net models for PTL in the context of advantages and disadvantages is provided in Sect. 9. In Sect. 10, the empirical results and analysis are reported. Concluding remarks are given in Sect. 11. All models are simulated in a special PNeS® software [17].

2 Informally About the Transport Logistics Problem

The general description of the transport logistics problem can be presented graphically as shown in Fig. 1 [11]. The presented scheme of the problem includes 4 levels of tasks, 12 sets of knowledge about objects and their properties, and 18 relations between sets of knowledge.

Each object contains some set of knowledge provided by experts in the field of PTL. Since this scheme can be considered as a relational structure, these knowledge sets can be presented in the tabular form of "Object–property" type [18, 19].

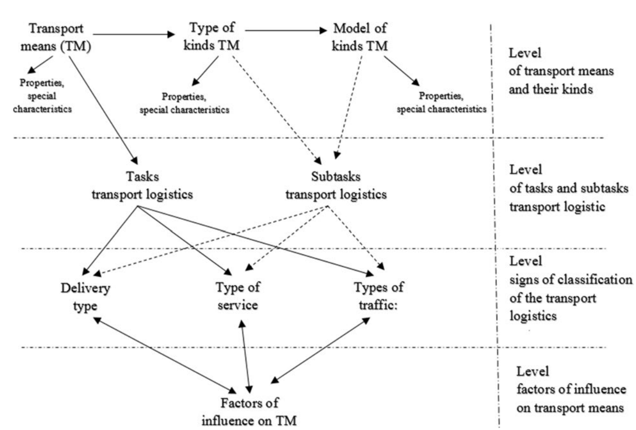

Fig. 1 Scheme description of transport logistic tasks

Table 1 A knowledge table of "Object-property" type

Objects (d_j)/properties (r_i)	r_1	r_2	…	r_q
d_1	w_{11}	w_{21}	…	w_{1q}
d_2	w_{21}	w_{22}	…	w_{2q}
…	…	…	…	…
d_p	w_{p1}	w_{p2}	…	w_{pq}

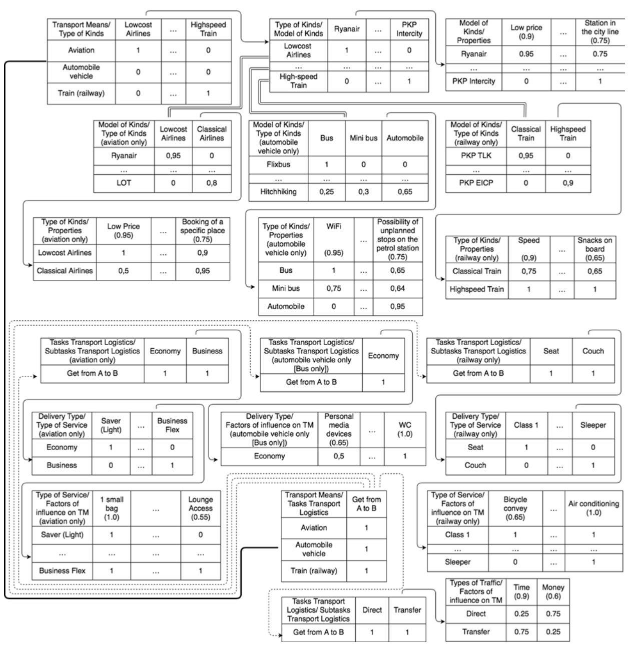

Fig. 2 The completed informative structure of tables of the PTL scheme

Table 1 consists of finite number p of rows (d_j) which are associated with objects and finite number q of columns (r_i) which are associated with properties. Additionally, each cross point between object and property includes some weight w_{ij} in the range [0,1]. If a weight between a property and an object has a value which is greater than zero, then the connection exists and w_{ij} shows its strength. If w_{ij} is equal to "1" then there is the strongest possible connection. If w_{ij} is "0" then there is no connection between the given object and the corresponding property [12–15].

In case of applying a simplified (determined) version of the knowledge table, the weights are missing and cross points are filled with binary values "0" or "1". They indicate the existence of connection without showing its degree of truth and its strength.

Figure 2 presents 20 interconnected knowledge tables of "Object–property" type. Each arrow in the structure represents a connection between tables of "Object–property" type in accordance to the scheme in Fig. 1.

The bold line reveals a connection of objects "Aviation", "Automobile vehicle", "Train/Railway" from level 1 of the PTL scheme to the transfer table which represents level 2 of the scheme (Fig. 1) allowing to connect the structure with levels 3 and 4, respectively. The dotted lines represent the connection between this transfer table and the four additional transfer tables.

The three additional transfer tables represent the three types of transport separately at levels 3 and 4 in Fig. 1. Moreover, the last additional table represents a separate task "Types of Traffic" in Fig. 1. Each transfer table linked with dotted line in Fig. 2 represents level 2 of the scheme in Fig. 1.

The double lines between levels 1 and 2 of the structure in Fig. 2 describe in detail three types of transport at level 1 in Fig. 1. This division establishes a hierarchy that will be used to simplify the problem-solving path.

It should be noted that the development of the automobile vehicle branch on level 3 in Fig. 1 includes only bus companies, since their objects and properties are stable and can be predicted in advance. Other automobile vehicle alternatives which imply an automobile or mini bus using an application (e.g., "Blablacar") or even hitchhiking for getting from point A to B cannot be predicted in advance. Therefore, these options are not included in the tables on the levels 3 and 4 of knowledge representation in Fig. 1. Also, it is worth noting that "Delivery Type" and "Type of Service" tasks on level 3 in Fig. 1 were combined into one meaning "Delivery Type" for the bus companies because these knowledge sets are approximated in their meanings.

3 Production Rules as a Representation of Knowledge for TLP

To make a formal description of connections between object and property presented in the knowledge tables, there is a need to create production rules with a connection of "One-to-One" or "Many-to-One" type.

An example of production rule with a "One-to-One" connection type is as follows:

$$\text{IF } r_i \text{ THEN } d_j, \tag{1}$$

where r_i is the property and d_j is the object.

Logical operators AND and OR are implied to connect some number of properties to one object forming the connection of "Many-to-One" type. The extended version of production rule is as follows:

$$\text{IF } r_{i1} \text{ AND(OR)} \ldots \text{AND(OR)} r_{in} \text{ THEN } d_j. \tag{2}$$

Formulas (1) and (2) are applied in classic FPN, but the authors suggest the use of weighted FPN for achieving more concrete and precise results. Consequently, formulas (1) and (2) should be modified according to Table 1, which includes weights at intersection points, to the form shown in formulas (3) and (4), respectively.

$$\text{IF } r_{i1} \cdot w_{i1} \text{ THEN } d_j. \tag{3}$$

$$\text{IF } r_{i1} \cdot w_{i1} \text{ AND(OR)} \ldots \text{AND(OR)} r_{in} \cdot w_{in} \text{THEN } d_j. \tag{4}$$

In this manner, a connection between objects and properties was established at the internal level of the knowledge table. It can be observed in Fig. 1 that some objects may have a connection with both properties as well as with other objects. Therefore, “Object–property” tables are convenient in the application since they also allow matching the connections at the external level—between the tables.

The same structure of production rules as presented in formulas (1–4) is applied for interconnection at the outer level. The difference lies in the interpretation of properties and objects. Interconnected tables of the “Object–property” type have a feature to make a transition of object from the previous table into a property in the following table. This strategy can only be applied between two tables that are interconnected between each other. Thus, according to formulas (1–4), property r_i is considered as a resulting object from the previous table and the object d_j becomes a property in the following table at the outer level.

The complete list of the obtained production rules after processing the knowledge tables in Fig. 2 contains 15 rules.

The list of rules implies the connection of “Many-to-Many” type. In addition, the output meanings are linked by the logical operator OR, which allows dividing the production rule into rules with the "Many-to-One" connection type. This makes it possible to separate objects on the same level, making their results completely independent.

It leads to the simplification in calculations and interpretation of results (decisions). On the other hand, this method increases the number of production rules, because their number will be increased to the number of objects which should be separated. If there are N resulting objects connected with logical OR, then N separated production rules with unique resulting object will be created.

After the division in accordance to these principles, the number of production rules was increased to 68 production rules. Here is just a sample of these rules:

1. IF “Ryanair” AND “Wizzair”, THEN “Lowcost Airlines”.
2. IF “Air France–KLM” AND “LOT” AND “Lufthansa”, THEN “Classic Airlines”.
3. IF “Low Price” AND “Business class” AND “Loyalty Programs” AND “Airport in the city line” AND “Only Hand Luggage” AND “Different types of comfort” AND “Seat comfort” AND “Classification by classes of comfort” AND “Booking of a specific place”, THEN “Low-cost Airlines”.

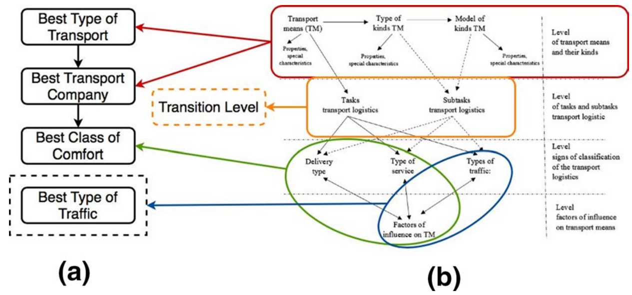

Fig. 3 **a** Target decisions of PTL. **b** Interpretation of decisions of PTL in the TLP scheme

4 TLP and PTL

When the knowledge is provided by the experts, the next step is to clarify the target decisions in accordance to the scheme presented in Fig. 1. The target decisions and their logical sequence are presented in Fig. 3a.

As it can be observed in this figure, the decision-making process of TLP (Fig. 1) in the context of passenger transport consists of four partial decisions: best type of transport mean, best transport company, best class of comfort and best type of traffic. The first three decisions should be obtained in the corresponding sequence as presented in Fig. 3a. They establish a hierarchical structure for obtaining necessary decisions, where future decisions derive from the previous ones. The last component of Fig. 3a is indicated by a black stroke line—best type of traffic. This decision is separated from all others and is independent of them. It should be obtained without influence of decisions which were previously achieved.

A broad explanation is graphically presented in Fig. 3b.

Level 1 (red color) of the scheme is the most detailed in the description and complex in its interpretation. Since objects presented on this level are strongly interconnected between each other, they cannot be considered separately in the scheme.

Yet, with a broad description of level 1, the first two decisions can be extracted: best type of transport and best transport company.

Note: the very first decisions (best type of transport mean) is the most crucial because: (a) it is the starting point of the hierarchy; (b) therefore, it has the hugest influence on the following decisions (Fig. 3b).

Level 2 (orange color) is a transfer level which does not give any concrete influential decisions. In the context of PTL, it can be considered as the goal: “Get a passenger from point A to point B" and is represented with transfer tables in Fig. 2.

Level 3 and 4 (green and blue colors, respectively) have a notable interconnection, since the input knowledge for level 3 is presented at level 4 (i.e., objects are presented on level 3 and properties on the 4th level). These decisions totally depend on the factors (properties) presented on level 4. In addition, the meanings of "Delivery Type" and "Type of Service" are combined into the meaning of "Class of Comfort". Moreover, they have a similar net structure for each type of transport. The decision on the best class of comfort, however, depends on previous decisions from the first two levels in Fig. 3a and level 4 in Fig. 1. The decision on the best type of traffic depends only on TLP level 4 (Fig. 1) and it is separated and independent in the hierarchy (i.e., previous decision in Fig. 3 does not influence the decision on the best type of traffic).

5 Weighted Fuzzy Petri Nets as a Formal PTL Model

Although Petri nets have wide application areas, they have not been able to present fuzzy data used in systems with uncertainty. To overcome this disadvantage, fuzzy Petri nets were developed [20]. In this section, definition of weighted fuzzy Petri nets (abbreviated as wFPN) [21] and the basic concepts related to it will be recalled. When the knowledge is represented in the knowledge tables and production rules are created, the next step is to create wFPN.

A wFPN is a tuple $N=(P, T, I, O, W, S, \alpha, \beta, \gamma, M_0)$, where:

1. P, T, S are finite sets, disjoint pairs, and P, S with the same cardinality;
2. I, O are functions in the domain T and the codomain 2^P (the power set of the set P) and are called the *input function* and the *output function*, respectively;
3. W is a function in the domain $(P\times T)\cup(T\times P)$ and the codomain [0,1] (the real number interval from 0 to 1) and is called the *weight function*;
4. α is a function in the domain P and the codomain S and is called the *statement binding function*;
5. β, γ are functions in the domain T and the codomain [0,1] and are called the *truth degree function* and the *threshold function*, respectively;
6. M_0 is a function in the domain P and the codomain [0,1] and is called the *initial marking*.

The elements of P are called *places* and the elements of T are called *transitions*. The elements of $I(t)$ and $O(t)$ for a transition t are called *input places* and *output places* of the transition t, respectively. The function W is the *weight* of the directed arcs from places to transitions and vice versa. The elements of S are called *statements*. They are attached to places with the function α. The elements of M_0 are called *tokens*.

Graphically, each wFPN can be represented with some number of places (circles) and transitions (rectangles) connected with arcs. Each place contains one token from [0,1] which is placed in that place. If the token is 0, the place is empty. Moreover, there are the following assumptions: (1) if the weight of the directed arc is 1, this number is not shown in the net graph; (2) if the weight of an arc is 0, then that arc is not shown in the net graph. For each transition t, there are assigned two real numbers and a triple consisting of three operators: the value of the truth degree function $\beta(t)$, the value of the threshold function $\gamma(t)$ and three operators IN, OUT_1, OUT_2, where IN corresponds to the input function $I(t)$ and OUT_1, OUT_2 correspond to the output function $O(t)$. The values for the truth degree and threshold function are set by experts.

The initial marking of a wFPN can be generally changed into a successor marking to certain rules and this can itself transform in turn into successor markings. The rules describing the possible changes from one marking to the next one are called *firing rules*, and the change occurring itself is called a *firing*. Throughout such firings, the distribution of tokens over the places of a wFPN can change and thereby the whole view of the net changes. To fire a transition, the following condition should be satisfied:

$$IN(w_{i1}\cdot M(p_{i1}), w_{i2}\cdot M(p_{i2}), \ldots, w_{ik}\cdot M(p_{ik})) \geq \gamma(t) > 0, \tag{5}$$

where IN is an input operator, w_i is the weight which describes the strength of connection between input place and transition, $M(p_i)$ is the token of the place. If the prerequisite is satisfied, then operator IN takes as input tokens from input places multiplied with corresponding weights and the value of the threshold function $\gamma(t)$. The resulting value of the IN operator becomes the first input for the second operator OUT_1, while the second input value is the truth degree function $\beta(t)$. The resulting value of the second operator becomes the first input for the third operator OUT_2, while the second value are tokens from the output places.

Formally, if M is a marking of N enabling transition t and M' the marking derived from M by the firing transition t, then for each $p\in P$:

$$M'(p) = \begin{cases} OUT_2(OUT_1(IN(w_{i1}\cdot M(p_{i1}), \ldots, w_{ik}\cdot M(p_{ik})), \beta(t)), M(p) & \text{if } p \in 0(t) \\ M(p) & \text{otherwise} \end{cases}$$

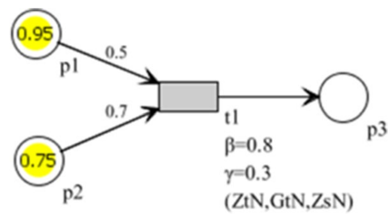

Fig. 4 An example of wFPN model

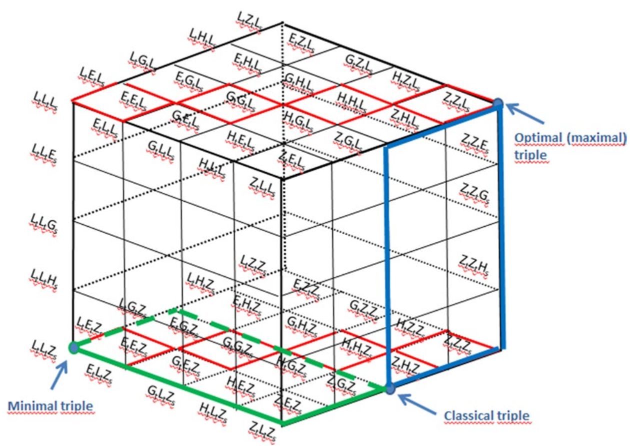

Fig. 5 A cube with all possible combinations of triples of t-norms and s norms that are considered in this paper

Each operator from the triple (IN, OUT_1, OUT_2) will be instantiated by the corresponding t-norm/s-norm, respectively [22].

In this paper, we use the following t/s-norms (see Sect. 10):

1. $LtN(a,\ b) = max(0,\ a+b–1)$, $LtN(a,\ b) = min(1,\ a+b)$ (*Łukasiewicz t/s – norm);*
2. $EtN(a,\ b) = \frac{ab}{2-(a+b-ab)}$, $EsN(a,\ b) = \frac{a+b}{1+ab}$ (*Einstein t/s – norm);*
3. $GtN(a,\ b) = ab$, $GsN(a,\ b) = a+b-ab$ (*Goguen t/s – norm);*
4. $HtN(a,\ b) = \begin{cases} 0\ for\ a=b=0 \\ \frac{ab}{a+b-ab} otherwise \end{cases}$;

   $HsN(a,\ b) = \begin{cases} 1\ for\ a=b=1 \\ \frac{a+b-2ab}{1-ab} otherwise \end{cases}$

   (*Hamacher t/s – norm);* $\begin{cases} 1\ for\ a=b=1 \\ \frac{a+b-2ab}{1-ab} otherwise \end{cases}$
5. $ZtN(a,\ b) = min(a,\ b)$, $ZsN(a,\ b) = max(a,\ b)$ (*Zadeh t/s – norm).*

All interesting combinations of the triples of t-norms and s-norms that were considered in [15, 23] can be graphically represented as a cube (Fig. 5).

Figure 4 presents a wFPN model with the initial marking before firing of the transition t_1. The net consists of two input places with markings being equal to $M(p_1)=0.95$, $M(p_2)=0.75$. On the right, an empty output place can be found where $M(p_3)=0$. Each input place is connected with a directed arc with the transition t_1 and the transition is in turn connected to the output place with the directed arc. Each input arc includes some fuzzy value of weight: $w_1=0.5$, $w_2=0.7$. Three parameters: beta, gamma and triple of functions are attached to the transition t_1. In this example, a classic triple of functions is presented (ZtN, GtN, ZsN) with the truth degree function $\beta(t_1)=0.8$ and the threshold function $\gamma(t_1)=0.3$. Initially, the condition (5) is checked whether the transition is ready to fire. From the formula, it can be spotted that it takes the first function of the triple with the input values multiplied with weights. Thus, it has the following implementation:

$$\mathrm{ZtN}(0.5 \cdot 0.95, 0.7 \cdot 0.75) \geq 0.3 > 0$$

$$0.475 \geq 0.3 > 0.$$

It means the condition for firing a transition is satisfied and the next step is to apply the second element of the triple:

$$\mathrm{GtN}(0.475, \beta(t_1)) = 0.475 \cdot 0.8 = 0.38$$

The last step is to apply the third element of the triple:

$$\mathrm{ZsN}(0.38, M(p_3)) = 0.38$$

where the result is set on the output place. This value in the net drawing is ignored.

6 Transformation of Production Rules into (w)FPN

Each production rule which was created on the basis of the knowledge table is interpreted into the wFPN model. The structure of the production rule and wFPN is similar and consists of some number of inputs, connection operator and some number of outputs. In case of considering production rules: inputs—properties, connection operator—THEN, outputs—objects. In case of considering wFPN: inputs—input places (incl. marking of places), connection operator—transition, outputs—output places (incl. marking of places). Thus, there is a direct relationship of meanings between the production rules and wFPN: properties are assigned to the input places, operator IF…THEN is interpreted as the transition, and objects are assigned to the output places. Another important realization of transformation lies in the numerical value equivalence that is used during the transformation. As mentioned above, each property in the knowledge table is assigned with the fuzzy value in the range [0,1]. The fuzzy value of the property in the knowledge table is directly assigned to the input place as a marking of a place in the wFPN. It initiates the mathematical interpretation of the knowledge allowing future calculations in the decision-support system which applies intelligent computational techniques. In the same manner, the achieved numerical marking of the output place is interpreted in the knowledge table as truth degree of the object. In case an array of objects exists, the preferred decision is the object with the highest truth degree, as it is suggested to be the most effective. Additional important observation that should be taken into account when analyzing the results is the difference in the degree of truthfulness between objects: as the difference increases, the advantage of an object with a higher numerical value increases consistently.

This paper suggests applying weighted FPN, since they are more precise and accurate. Therefore, weights should have a broader explanation. As mentioned in the description of Table 1, weights describe the strength of connection between property and object with some fuzzy (truth-degree) value. Since each output object is unique in the FPN model, weights describe the strength of connection between an input place (which is a property) and a transition (which is a connection operator). Therefore, weight is assigned strictly to the arc which connects input place to the transition. It cannot be assigned vice versa. The transition receives the input value for the calculations in a form of multiplication of markings of input place with the weight assigned to the arc.

Additionally, each production rule applies logical operator(-s) AND/OR as a binding operator for properties (in case their number is greater than one) and for objects at the output. Note: as it was stated in Sect. 3, it is not suggested to connect output objects with logical operators, since it complicates the structure of the net leading to the confusion in its understandability and interpretation. Every object which is connected with logical OR at the output is suggested to be extracted into a separate production rule. It simplifies and improves the accuracy of the transformation into the (w)FPN model. This approach allows achieving equivalence of the correspondence of the number of production rules to the number of transitions in the (w)FPN model (i.e., every production rule includes operator THEN, which is represented in the (w)FPN model as the transition t).

The choice of triangular norm for the triple which is placed under the transition directly corresponds to the type of connection (AND/OR) which was previously applied in the production rule. One of the t-norms' function will be chosen in case of applying logical AND. Otherwise (in case of applying logical OR), a function from a list of s-norms should be chosen.

Also, each transition includes two more elements that should be explained: the truth degree function $\beta(t)$ and the threshold function $\gamma(t)$. The following formula was proposed to synchronize the truth degree value for the PTL [11]:

$$\beta = k/(k+1), \tag{6}$$

Meanwhile, the rule for establishing a value for the threshold function in the PTL remains the same as for the general case: it is being set by the experts. It can be explained in the following way: experts take control of the net effectiveness and resulting decisions, because $\gamma(t)$ plays a vital role in firing a transition and therefore the development of the mathematical processing in the net.

7 Triangular Norms and Their Application in FPN in the Context of PTL

Figure 5 includes the cubic representation form of all 125 possible combinations of triples of functions. Three triples of functions should be highlighted in this cube: (LtN, LtN, ZsN), (ZtN, GtN, ZsN), and (ZtN, ZtN, LsN). Triple (LtN, LtN, ZsN) is called the minimal triple since this combination of functions leads to the lowest possible numerical values at the output places, while triple (ZtN, ZtN, LsN) is called an optimal (maximal) triple since this combination of functions leads to the maximal possible numerical values at the output places. A triple of functions (ZtN, GtN, ZsN) is called classic and is the most often-in-use triple. It is also used as a starting point for analysis of different combinations of triples of functions.

One of the goals of wFPN is to provide relatively large numerical values at the output places, which in turn represent the degree of truthfulness of the properties of the objects to which these locations are assigned. A similar goal is to compare the difference between the numerical values at the output places. The last point is relevant because it offers a great possibility for the analysis of achieved values and corresponding decisions as well as for analysis of the preference in selecting of some specific object.

Papers [13–15] challenged the authors with the problem of the change of decisions with the change of triples. For this reason, the authors applied two strategies: (1) to apply all other combinations of triples which are in between previously chosen couple of triples for the analysis of truth probability of their resulting outputs (i.e., the preferred triple of function from the couple is the one that has the same decision as the decision achieved by the majority of triple of functions in between); (2) to find the average mathematical meaning of every resulting output place which is associated with the corresponding object after applying all the chosen triples of functions [15].

The formula for this strategy is as follows:

$$Res(Obji) = \frac{Res(ZtN, GtN, ZsN)Obj_i + \cdots + Res(ZtN, ZtN, LsN)Obj_i}{Num\_of\_triples} \tag{7}$$

Additional remark on application of triples of functions refers to the specificity of PTL. The wFPN model is presented in a step-by-step mode, which means that the first level of input places is filled with fuzzy values only at the very beginning (commencement) of the simulation, while all other levels of places are empty. The firing of transition is also done in the step-by-step mode, which means the consequent firing of levels of places: the first level of transitions is fired, then the second level, etc. This leads to the conclusion that the third operator OUT_2 of the triple (IN, OUT_1, OUT_2) never affects the resulting calculations, because the output place is always empty before firing the transition in the model under consideration. OUT_2 operator takes zero value from the output place as the second input function for itself, which makes no influence on the result in accordance to the description of every possible s-norm that can be chosen for the second output operator.

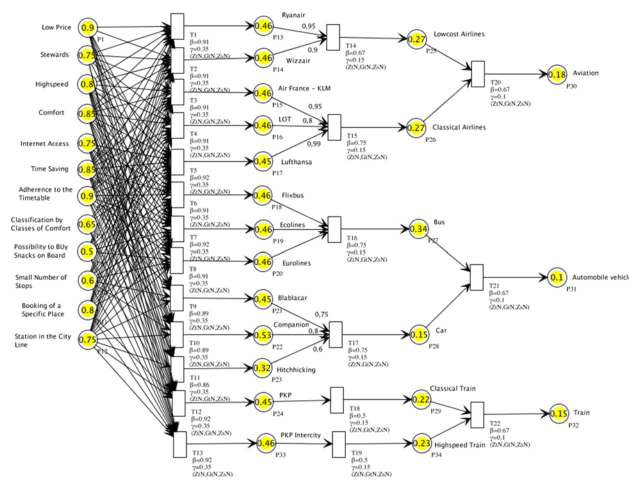

Fig. 6 wFPN model for the best type of transport

8 Net Models for PTL

This section covers the application of wFPN in the context of a PTL development with two opposite approaches: non-hierarchical (Sect. 8.1) and hierarchical one (Sect. 8.2) including the further analysis of the pros and cons of each strategy presented in Sect. 9.

8.1 Non-hierarchical Structure

This part describes a non-hierarchical approach of applying wFPN in the context of solving PTL. Figure 3a gives a vision that four decisions are expected. Therefore, at least four wFPN models should be considered with the development of each type of transport. This part gives a detailed description of the wFPN models which apply classic triple of functions.

Note: if the figure contains many arcs between the input places and transitions, the weights are not shown in the figure, but are included in the calculations (Figs. 6, 8, 12, 15, 16). Also, if the arc does not have a weight assigned to it in the figure, the connection of the weight is equal to 1.

Figure 6 consists of four levels of places and three levels of transitions. The net starts from common input features which describe three types of transport located at the output: aviation, automobile vehicle, and train.

Two levels of places in the middle aims to generalize a wide range of options to achieve three objects at the end. It should be noted that these decisions have a huge difference in their interpretation.

In accordance to Fig. 3a, the following decision should be the best transport company. It is not obligatory to search for the best transport company for all three types of transport, because the decision on the best transport mean is already received (Fig. 6).

Obviously, the best transport company should be chosen in accordance to this transport mean. In the same manner, the best class of comfort (third decision in Fig. 3a) should be achieved resulting from the best transport company which has been previously chosen for the defined transport mean (Fig. 6).

The difference is that the decision about the best class of comfort can be generalized for the list of transport companies of the appropriate transport mean (making a direct link between a second and a third decision in Fig. 3a). This generalization cannot be applied to the relationship between the first and second decisions (Fig. 3a) as its commonality has already been used in the wFPN model (Fig. 6) to establish a hierarchy with a branching to the types of transport (Figs. 2, 3a, 14, 15, 16). Therefore, the decision made in the net in Fig. 6 establishes a hierarchical structure. To clearly describe the research, all possible developments will be considered for each type of transport. The last decision ("Type of traffic") in Fig. 3a is always considered separately. Thus, the wFPN model for this decision (Fig. 13) is also always independent.

The development of the aviation branch with levels 2 and 3 of decisions (best "Transport Company" and "Class of Comfort") is presented in Figs. 7 and 8 correspondingly.

Figure 7 consists of three levels of places and two levels of transitions. The net starts from the detailed description of features that are common for airline companies which are located at the output. The level of places in the middle works similarly to Fig. 6, generalizing and classifying by type of aviation industry. Finally, there is the last level of places which represent the decisions (airline companies) for this wFPN model coming from the defined types of kinds of aviation on the second level of the places.

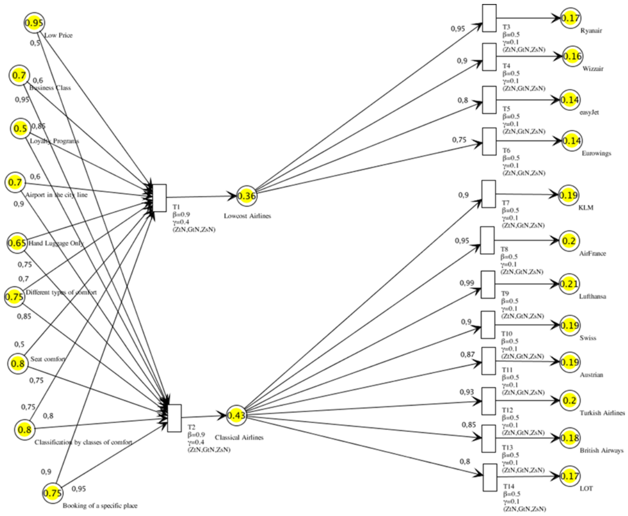

Fig. 7 wFPN model with the decision on the best transport company (aviation branch)

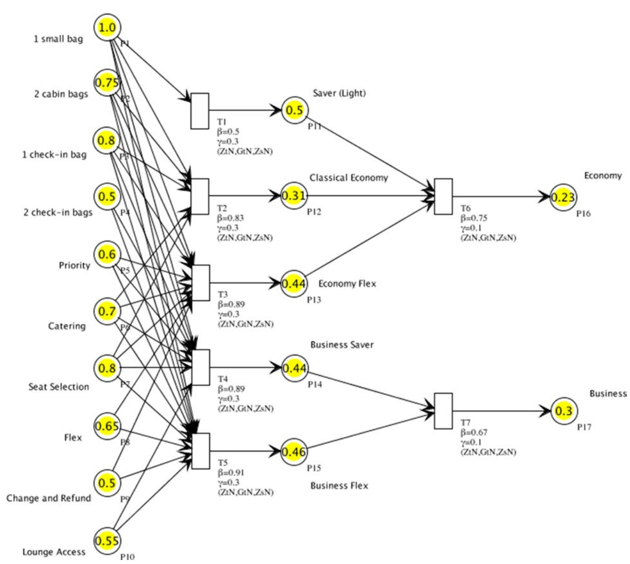

Fig. 8 wFPN model with the decision on the best class of comfort (aviation branch)

Figure 8 has a similar structure of the net to that in Fig. 7 with the same number of levels of places and transitions. The difference is that this model statically decreases in the number of output places after each level of transitions and does not have such a branching structure which is defined on the second level of places in Fig. 7. This net starts with the input characteristics for the type of delivery, and finally comes the middle definition of the type of service in the context of aviation passenger transport. The development of the automobile vehicle branch with levels 2 and 3 of decisions (best transport company and class of comfort) is presented in Figs. 9 and 10 correspondingly.

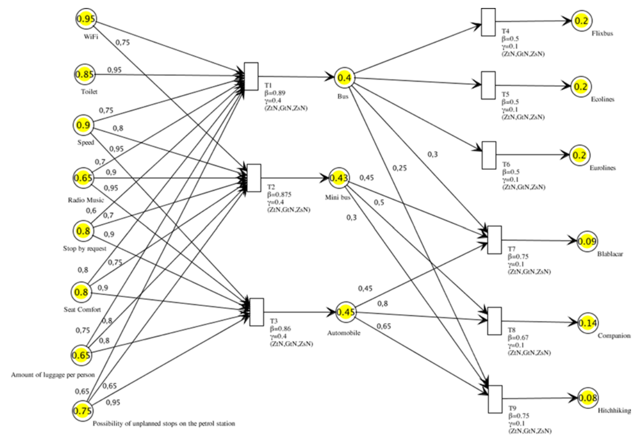

Fig. 9 wFPN model with the decision on the best transport company (automobile vehicle branch)

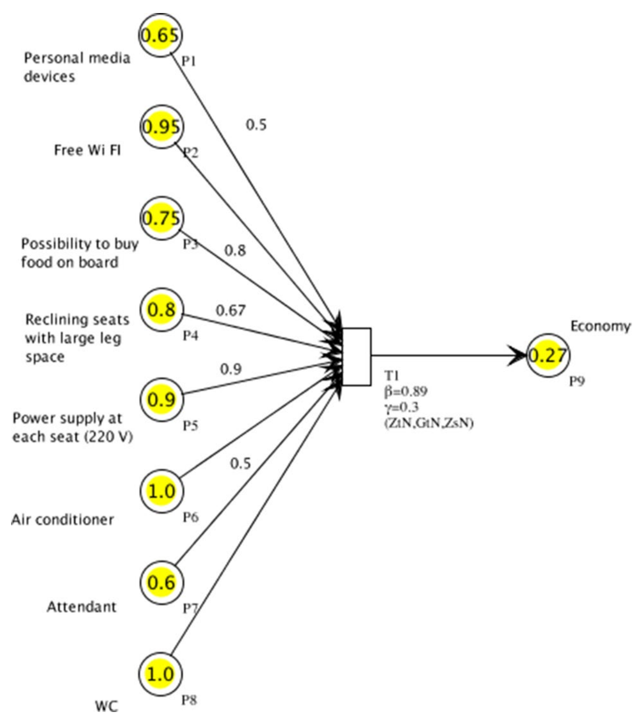

Fig. 10 wFPN model with the decision on the best class of comfort (automobile vehicle branch)

Figure 9 is similar in its structure to Fig. 7, since they both represent the same level of knowledge processing for different types of transport. This wFPN model also consists of three levels of places as well as two levels of transitions with a branching on the second level of places and ends up with the decision on the best transport company for the automobile vehicle type of transportation.

As stated in Sect. 2, the automobile vehicle branch has its own specification in the structure description of PTL. In

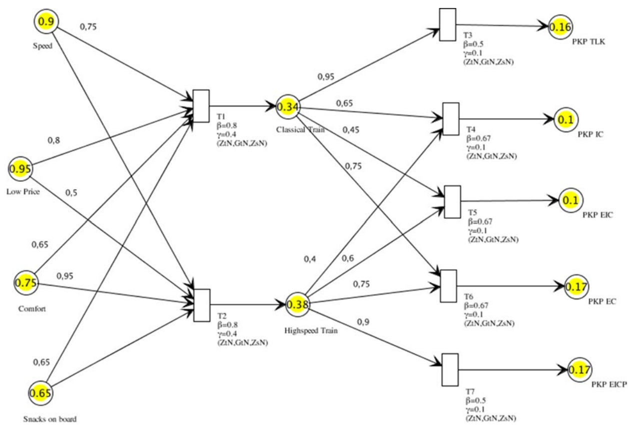

Fig. 11 wFPN model with the decision on the best type of company (railway/train branch)

case the decision is different from that of the bus company, then the resulting value for this branch is taken from Fig. 9, because this decision cannot have a further description (development) as it cannot be defined in advance. Moreover, the meanings of "Type of service" and "Delivery type" are related to the bus companies only and are conceptually similar. Therefore, they were united into one meaning—"Delivery type". The resulting net for the best type of comport is presented in Fig. 10.

Figure 10 is a good example of knowledge processing in a single table of "Object–property" type. It has a "Many-to-one" connection, where some number of properties (input places) are connected to the single object (output place) which is a unique decision in this case. This structure includes only a single transition, which means that a single production rule was created.

In fact, such models are not highly effective since they have no alternatives at the output, achieving only one possible decision. Yet, it is worth checking if it could be achieved (whether the transition is fired). Additionally, it gives a broader description of the study.

Figures 11 and 12 provide a description of the railway branch if the decision was "Train" in the model presented in Fig. 6.

Figure 11 is created in the same way as Figs. 7 and 9, because they are on the same level of knowledge processing. Starting from the input properties, it is generalized for a type of kinds and then delivered to the respective output places which describe railway passenger transport companies.

Remark The Polish railway has been chosen as a representative system for the railway structure. It is a part of the EU network operation and can be easily adapted to the system of foreign country by necessity.

Figure 12 is created in the same manner as Fig. 8. It starts from properties referring to the railway branch (i.e., features of passenger trains) with a generalization by types of service in the middle level of places, leading to more generalized options at the output.

At this moment, a decision on the best type of transport and two decisions for each type of transport covering the first three tasks are presented in Fig. 3a. Additionally, this figure requires one more decision: best type of traffic which is independent of nets presented in Figs. 6, 7, 8, 9, 10, 11 and 12 (i.e., at least one situation should be developed for the activation of this net).

Figure 13 presents the simple wFPN model which gives a decision on the best type of traffic. The structure of the net represents the strategy: "Time is money". Therefore, the decision completely depends on the user's preferences.

The net operates as follows: if "Time" is a priority with a higher fuzzy value at the input, then the decision will be a "Direct" (trip), otherwise a "Transfer" (trip).

8.2 Hierarchical Structure

This chapter discusses the application of the hierarchical structure of wFPN models in the context of PTL development.

Figure 14 illustrates the interpretation of hierarchical structure in Fig. 2 for solving PTL. It consists of the same three levels: best type of transport, best transport company, and best class of comfort.

Additionally, the structure is divided into three branches: aviation, automobile vehicle, and railway. It simplifies

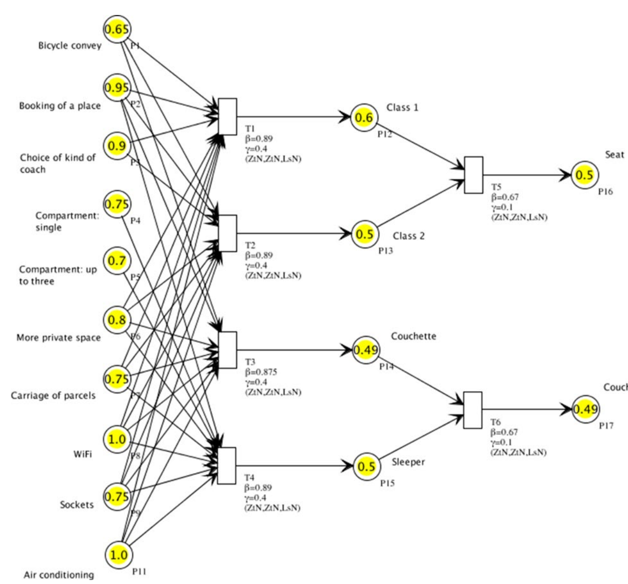

Fig. 12 wFPN model with the decision on the best type of company (railway/train branch)

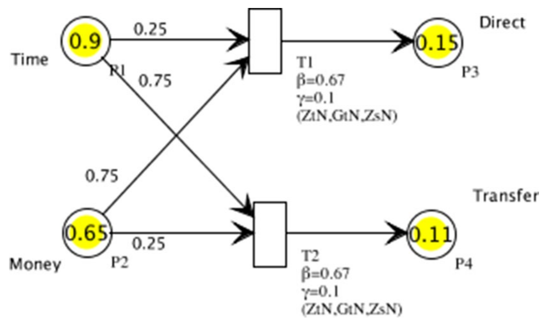

Fig. 13 wFPN model with classic triple of functions (ZtN, GtN, ZsN) for the best type of traffic

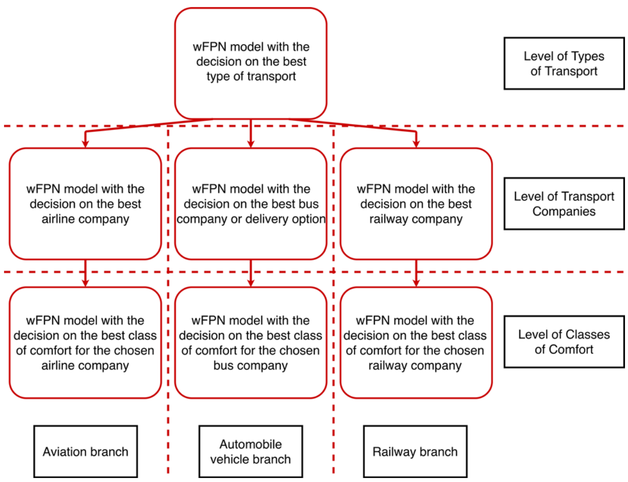

Fig. 14 Conception of hierarchical structure of wFPN models for the subject area of PTL

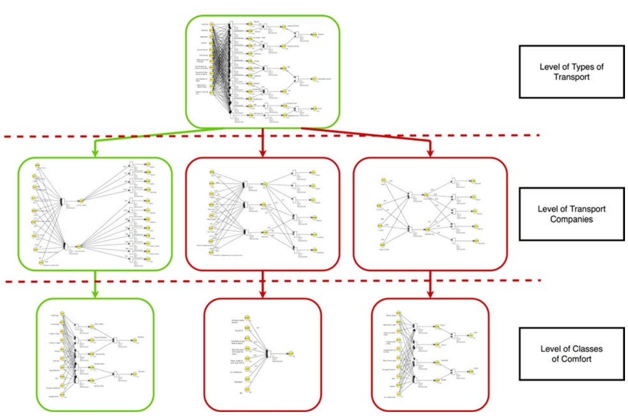

Fig. 15 The hierarchical application of wFPN with the classic triple of functions (ZtN, GtN, ZsN) for the subject area of PTL

the path to the solution because only one branch will be activated.

The decision on the activation of the specific branch lies in the wFPN model on level 1 (Fig. 14). Thus, the first wFPN model located on level 1 is the key component of this hierarchy. The root wFPN model on level 1 is a starting point and is always activated. Whereas only one branch is activated on the basis of the decision of the root wFPN model, the total number of applied wFPN models will be equal to three (the one main model and two additional models from the concrete branch). Still, this hierarchical structure allows considering all possible developments including in it seven wFPN models in total [13, 14].

Additionally, this conception implies the strategy, where the following decisions are totally dependent on the achieved decisions on the previous levels. Thus, level 3 is dependent on level 2 and level 2—on level 1. In this way, there will be one branch activated in accordance with the decision made on (root) level 1. This approach simplifies the structure and the search for the solution as well as reduces the calculation time. On the other hand, all other developments of the situation are practically excluded and, therefore, some good alternative(-s) can be omitted.

Moreover, wFPN model on the best type of traffic is excluded from the hierarchical structure in Fig. 14 because it is considered separately in the sequence presented in Fig. 3a. What is more, it is independent. This is the only net which does not depend on the decision on level 1 in Fig. 6 as well as on the previous decisions in the hierarchy. Thus, it is activated in any development of the situation.

The application of the hierarchical structure (Fig. 14) for the nets in Figs. 6, 7, 8, 9, 10, 11 and 12 is shown in Fig. 15.

Figure 15 shows a net PTL model using a hierarchical approach. This structure includes 7 wFPN models with classic triple of functions (ZtN, GtN, ZsN). The (root) level 1

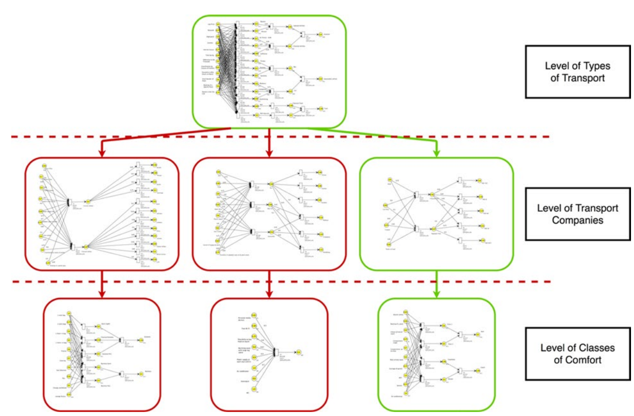

Fig. 16 The hierarchical application of wFPN with the optimized triple of functions (ZtN, ZtN, LsN) for the subject area of PTL

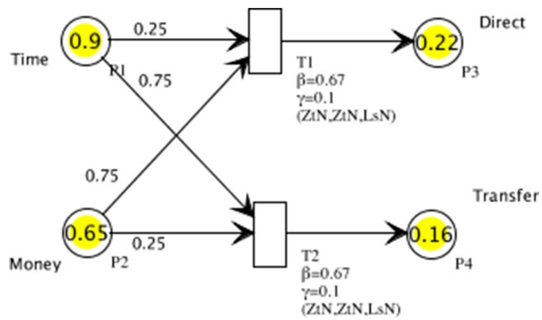

Fig. 17 wFPN model with the optimized triple of functions (ZtN, ZtN, LsN) for the best type of traffic

with the main wFPN gives an "Aviation" decision as the best type of transport with the highest resulting value at the output place equal to 0.18. It leads to the activation of the aviation branch, which is highlighted in green in Fig. 15. The decisions are as follows: "Lufthansa" (highest resulting value: 0.21) as the best transport company of the aviation branch and "Business" (highest resulting value: 0.5) as the best class of comfort for the previously chosen airline company.

Another approach that is supposed to be tested is the hierarchical structure of wFPN models with application of an optimized triple of functions (ZtN, ZtN, LsN). The result is presented in Fig. 16.

Figure 16 shows the decision-making sequence for PTL using the optimized triple of triangular norms (ZtN, ZtN, LsN). The (root) level 1 with the main wFPN gives a "Train" decision as the best type of transport with the highest resulting value at the output place equal to 0.49. It leads to activation of railway branch with the following decisions: "PKP EICP" (resulting value: 0.43) as the best railway passenger company and "Coach" place (resulting value: 0.5) as the best option for the chosen train.

Additional wFPN model for the best type of traffic is always excluded from the hierarchical structure (Fig. 3a). The use of the optimized triple for the best type of traffic results in the wFPN model presented in Fig. 17. Here, the same decision was achieved as in Fig. 15.

9 Comparative Analysis of Net Models for PTL: Advantages and Disadvantages

This section covers two alternative approaches for solving PTL with classic triple of functions (ZtN, GtN, ZsN).

A non-hierarchical approach is presented in Figs. 6, 7, 8, 9, 10, 11, and 12. Moreover, an independent wFPN model in Fig. 13 was also included in the list of decisions to complete the list of target decisions presented in Fig. 3a.

The advantages of the non-hierarchical approach:

- all situations are considered and developed;
- all alternatives can be treated and analyzed equally;
- the results of the same level of the structure can be analyzed;
- there is a possibility to choose a different (more preferred) decision for undefined reasons.

The disadvantages of the non-hierarchical approach:

- the number of nets can be enormous (coming from the first argument of advantages);
- the increase of the calculation time is expected;
- it demands more computer resources for calculations hypothetically;
- it includes useless decisions which can be unnecessary (requirement of time and computer resources for these decisions);
- the increased number of nets complicates the analysis of decisions (outcomes).

The hierarchical approach with the same wFPN models and the same triple of functions is presented in Fig. 15. Note, Fig. 13 is also considered in this structure, but it is mentioned independently to complete the solution search and its strategy for the PTL. Therefore, the full study is presented in Figs. 13 and 15.

The advantages of the non-hierarchical approach:

- allows considering all possible developments, but activates only the branch with the highest probability to become true;
- simplifies the calculations;
- reduces time for the solution search;
- saves computer resources.

The disadvantages of the non-hierarchical approach:

- only one development is considered (in case equal highest numerical result is received at two or more output places, then two or more corresponding branches associated with these output places will be considered);
- decisions can be compared only within the same net;
- there is only one net at each level for analysis;
- there is limited possibility to include every wFPN model into the hierarchy (Fig. 13 is not included since it is independent from other decisions).

Thus, each approach has its specification and properties. Yet, the hierarchical structure gives more valuable benefits, which result in the simplification of the solution search. Therefore, a hierarchical approach to research related to PTL is preferable.

Table 2 The results of wFPN model on level 1

	Aviation	Automobile vehicle	Train
(ZtN, GtN, ZsN)	**0.18**	0.1	0.15
(ZtN, ZtN, LsN)	0.4	0.22	**0.49**

10 Simulation Results and Discussion

This section analyzes the achieved decisions by the classic triple (ZtN, GtN, ZsN) and optimized one (ZtN, ZtN, LsN) in the hierarchical structures presented in Figs. 15 and 16 with additional models depicted in Figs. 13 and 17, respectively.

As mentioned in Sect. 7, the optimized triple of functions assumes achieving the highest possible numerical values at the output of any wFPN model. The models shown in Figs. 16 and 17 confirm this suggestion in comparison to the models that used the classic triple in Figs. 13 and 15.

The results from models on level 1 of the hierarchy are presented in Table 2, where the highest value (decision) is marked in bold. Later, in Tables 3, 4 and 5, decisions are marked in the same way.

Table 2 gives a consistent vision of results of wFPN models in the root of the hierarchy. Each object representing each type of transport achieved higher numerical value at the output with the application of optimized triple of functions:

- Aviation: 0.4 > 0.18.
- Automobile vehicle: 0.22 > 0.1.
- Train: 0.49 > 0.15.

In the same manner, every model in the hierarchical structures can be compared with each other in Figs. 15 and 16 as well as in Figs. 13 and 17. In every single case, the optimized triple of functions justified expectations by achieving higher numerical values at the output.

The decisions on the best type of transport (i.e., highest numerical values in the array) for each triple of functions are highlighted in bold in Table 2. Different triples of functions achieved different decisions: classic triple of functions made a decision on “Aviation” with activation of the corresponding branch in the hierarchy, while optimized triple of functions made a decision “Train” and also activated the corresponding branch. In both cases, these branches were highlighted in green in Figs. 15 and 16.

The observations revealed that the different triples of functions lead to the appreciable different decisions. Therefore, the question now arises about the correctness of the choice from the human point of view. To answer this question, there is a need to have a closer look on a blue rectangle of the cube presented in Fig. 5.

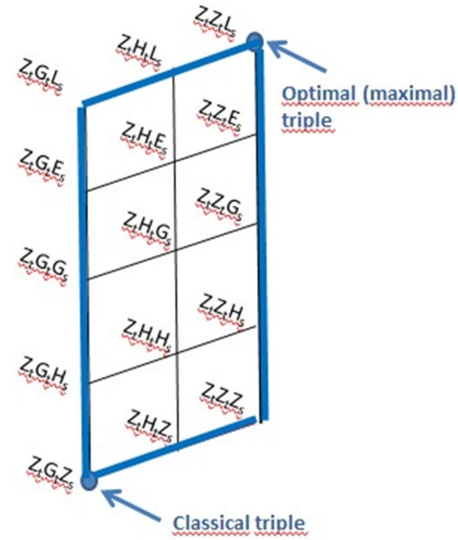

Fig. 18 The rectangle with all possible combinations of triples in-between classic and optimized one

Table 3 The results of wFPN model on level 1

	Aviation	Automobile vehicle	Train
(ZtN, GtN, ZsN)	**0.18**	0.1	0.15
(ZtN, HtN, ZsN)	**0.29**	0.18	0.28
(ZtN, ZtN, LsN)	0.4	0.22	**0.49**

Table 4 The results of wFPN model on level 1

	Aviation	Automobile vehicle	Train
(ZtN, GtN, ZsN)	**0.18**	0.1	0.15
(ZtN, HtN, ZsN)	**0.29**	0.18	0.28
(ZtN, ZtN, ZsN)	0.4	0.22	**0.49**

Figure 18 describes all 13 possible combinations of triples that lie between the classic (ZtN, GtN, ZsN) and the optimized triple (ZtN, ZtN, LsN).

From Fig. 18, the location of each function in the triple can be explained in the following way:

- the first (*IN*) function is always ZtN, because the rectangle (Fig. 18) is on the plane of the cube (Fig. 5) and ZtN is fully associated with this plane;
- the second (OUT_1) function can be chosen from the vertical lines of the rectangle (3 options);
- the third (OUT_2) function can be chosen from the horizontal lines of the rectangle (5 options).

In total, 15 combinations of triples can be formed which are represented in Fig. 18. The goal is to test 13 triples that

Table 5 The results of wFPN model on level 1

	Aviation	Automobile vehicle	Train
(ZtN, GtN, ZsN)	**0.18**	0.1	0.15
(ZtN, GtN, HsN)	**0.18**	0.1	0.15
(ZtN, GtN, GsN)	**0.18**	0.1	0.15
(ZtN, GtN, EsN)	**0.18**	0.1	0.15
(ZtN, GtN, LsN)	**0.18**	0.1	0.15
(ZtN, HtN, ZsN)	**0.29**	0.18	0.28
(ZtN, HtN, HsN)	**0.29**	0.18	0.28
(ZtN, HtN, GsN)	**0.29**	0.18	0.28
(ZtN, HtN, EsN)	**0.29**	0.18	0.28
(ZtN, HtN, LsN)	**0.29**	0.18	0.28
(ZtN, ZtN, ZsN)	0.4	0.22	**0.49**
(ZtN, ZtN, HsN)	0.4	0.22	**0.49**
(ZtN, ZtN, GsN)	0.4	0.22	**0.49**
(ZtN, ZtN, EsN)	0.4	0.22	**0.49**
(ZtN, ZtN, LsN)	0.4	0.22	**0.49**

are between classic and optimized to prove the truth of the results obtained by the above-mentioned triples (i.e., classic or optimized triple).

Additionally, the specification and the structure of the task should be considered properly. As mentioned in Sect. 7: the third function (OUT_2) does not influence the structure of the net, since it is supposed to be activated by the step-by-step strategy. It means that the output place is always empty and its numerical value is equal to 0, making the second input value for this function also equal to 0, which leads to the invariance of the result after the second function OUT_1. Therefore, all five possible functions located on the horizontal lines in Fig. 18 are not influential in the triples which apply this strategy.

Thus, the first function is always the same—ZtN, the only second function which has not been used yet is HtN (GtN and ZtN as the second functions were already applied), and the choice of the third function does not influence the net. Therefore, the following triple is obtained (ZtN HtN, ZsN), which is the only one to be tested.

Table 3 presents the extended version of Table 2 including the new triple in the middle (ZtN, HtN, ZsN).

The observation of the results should be divided into two parts: a) according to Fig. 18, the calculation of the triple in the middle gave the values that are in the middle between results achieved by the classical and optimized triples; (b) the suggestion of effectiveness of classic triple for the given net.

After application of triple (ZtN, HtN, ZsN), every obtained numerical value for every type of transport is located in the middle between results achieved by the classic and optimized triple of functions. Thus, the following relationship (ZtN, GtN, ZsN) ≤ (ZtN, HtN, ZsN) ≤ (ZtN, ZtN, LsN) is true:

- Aviation: $0.18 \leq 0.29 \leq 0.4$.
- Automobile vehicle: $0.1 \leq 0.18 \leq 0.22$.
- Train: $0.15 \leq 0.28 \leq 0.49$.

Additional critical observation concerns the achieved decisions of the triple (ZtN, HtN, ZsN). The highest resulting value is equal to 0.29 for the object which is associated with the meaning "Aviation". This is the same decision that was made by the classic three functions (ZtN, GtN, ZsN). In this way, the classic triple of functions proves its effectiveness for this type of net (that is in a step-by-step format).

For simplicity and purity of analysis, the following suggestion is true: the results of the optimized triple (ZtN, ZtN, LsN) are equal to the results achieved by (ZtN, ZtN, ZsN) for the structure of the net presented in Fig. 6 forming Table 4.

Table 4 gives a clear understanding that the second function is a defining function of the net which applies triples in-between classic and optimized ones for the case of the step-by-step transition firing sequence. Additionally, Table 4 can be extended to include all 15 function triples presented in Fig. 18.

Table 5 covers the solution of all possible triples of functions presented in Fig. 18 for the net structure presented in Fig. 6. Additionally, Table 5 reveals observations that the first function is always the same as well as the third function does not influence the result. Only combinations of the first two functions have had an impact on the calculations. In fact, the first function ZtN was stable in this case, thereby the defining function was always the second one. There are three options for the second function: GtN, HtN, ZtN. The highest numerical results for both functions GtN and HtN are assigned to the same object "Aviation".

From Table 3 (with modified and extended versions in Tables 4 and 5), the authors concluded that the triple of functions (ZtN, HtN, ZsN) achieved the decision "Aviation" as the best one, because of the highest achieved numerical result at the output being equal to 0.29. Yet, the analysis of other results for other objects of the same network showed that the object "Train" obtained smaller outcome, being equal to 0.28. It means that the difference between objects "Aviation" and "Train" is 0.01, which is certainly subtle and not decisive.

It necessitates to analyze the results achieved by the classic triple (ZtN, GtN, ZsN) and the optimized triple (ZtN, ZtN, LsN), respectively. The classic triple also suggests the object "Aviation", as it has the highest numerical value at the output being equal to 0.18. Yet, the difference between objects "Aviation" and "Train" after applying a classic triple is equal to 0.03, which is also not representative (border limit). The difference between the same objects after application of the optimized triple is 0.09. It approximates to 0.1, which is more influential compared to the previously achieved differences: 0.03 and 0.09 correspondingly. Moreover, the optimized triple suggested “Train” as an alternative decision.

For this reason, an additional approach is proposed to justify the truth–probability of the effectiveness of applying classic or optimized triple of functions. This approach applies formula 7 to calculate the average value of each resulting output place, which is associated with the object after applying all selected triples of functions.

The results and decision achieved in Tables 3 and 5 are supposed to be equal. Table 3 includes the unique triple which is in the middle of the triples where the defining function is the second one, HtN. Meanwhile, Table 5 includes all triples from Fig. 18. Yet, the same principle is applied to Table 5: only the second function influences the calculations. Therefore, the influence of the second function is suggested to be also verified by formula (7). The achieved calculations are as follows:

- Aviation: $\frac{0.18+0.29+0.4}{3} = \frac{5\cdot0.18+5\cdot0.29+5\cdot0.4}{15} = 0.29$
- Automobile vehicle: $\frac{5\cdot0.1+5\cdot0.18+5\cdot0.22}{15} = 0.16.$
- Train: $\frac{0.15+0.28+0.49}{3} = \frac{5\cdot0.15+5\cdot0.28+5\cdot0.49}{15} = 0.30.$

From the obtained calculations, the object “Train” has the highest truth–probability compared to other objects. The difference between two objects with the highest values (“Train” and “Aviation”) approximates to 0.01(6). Additionally, the triple (ZtN, HtN, ZsN) in Table 3 made the difference between two objects "Train" and “Aviation” equal to 0.01 The problem is that two different approaches gave alternative decisions with almost equal numerical difference between these two objects. In other words, their truth probabilities approximate each other. Therefore, the prerequisites should be carefully analyzed. Additionally, it is worth mentioning that the first approach with the application of all possible triples of functions, which are in between optimized and classic triples, suggested the same decision (“Aviation”) as that achieved by the classic triple. On the other hand, the approach with the calculation of the average mathematical meaning suggested an alternative decision, which is the same decision as that achieved by the optimized triple. The decision achieved by the last approach suggested “Train” as the best type of transport.

The difference in decisions can be explained by the influence of the difference of the resulting values achieved by every triple of functions for two objects with the highest results which is clearly represented in Table 3. Even the quantitative advantage, where two similar decisions were assigned to the same object opposing one alternative, did

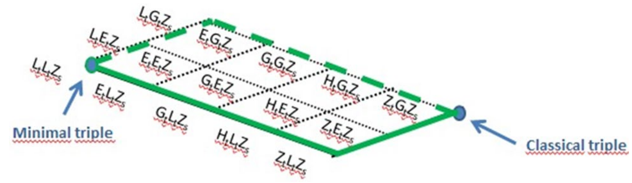

Fig. 19 The rectangle with all possible combinations of triples in-between minimal and classic one

not overcome the difference represented by the numerical values for the same objects.

The total difference of the results obtained by triples [(ZtN, GtN, ZsN) and (ZtN, HtN, ZsN)] for the objects "Aviation" and "Train" is calculated as follows: (0.18 − 0.15) + (0.29 − 0.28) = 0.04. At the same time, the numerical difference for the same objects after applying the optimized triple is calculated as follows: 0.49 − 0.4 = 0.09. The sum of the differences, which is 0.04 for the two different triples that led to the same decision ("Aviation"), did not overcome the difference of 0.09 for the alternate decision ("Train") obtained by the optimized triplet.

Thus, the results can be interpreted with two different approaches:

Decision is considered as a fixed fact. Then, the final decision relies on the majority of similar decisions achieved by triples in the middle between classic and optimized. In the current study, such an approach results in the benefit of using three classic triangular norms (ZtN, GtN, ZsN) and its practical implementation is presented in Figs. 13, 15.

Decision is considered as a mathematical value. Then, the final decision relies on the comparison of achieved numerical values by all 15 triples of functions presented in the blue rectangle (Fig. 18). The average mathematical value presented in formula 7 can be a good approach to justify and analyze the achieved decisions. This approach is based on analyzing the difference between the resulting values at the output places.

The analysis of the research outcomes brings the authors to the idea for further researches, applying additional triples of functions for verification of decisions achieved by classic and optimized triples. It is proposed to test triples of functions which are located in between minimal triple (LtN, LtN, ZsN) and a classic one (ZtN, GtN, ZsN).

Figure 19 also consists of 15 triples of functions, 13 of which are located between the minimal (LtN, LtN, ZsN) and a familiar classic triple (ZtN, GtN, ZsN). The benefit for the research of the current rectangle lies in a larger number of possible combinations to be tested. This can be explained by the different positions of the rectangle in the plane of the cube (Fig. 5). It leads to different interpretation of location of functions in the triple. The selection of the first function (*IN*) is on the horizontal lines. It means that there are five possible functions to choose from as the first function of the triple. The selection of the second function (OUT_1) is on the vertical lines. It allows testing 3 different functions as the second part of the triple. The last third (OUT_2) function is stable in this rectangle and is Zadeh s-norm (ZsN). It can be explained in the following way: this rectangle has a horizontal location on another plane of the cube with all possible triples of functions.

Therefore, this rectangle also achieves 15 combinations that can be tested.

This rectangle enables the better disclosure of the wFPN structure for solving PTL. The structure of the net was always created for a firing in a step-by-step mode, where the following levels of places were always empty at the beginning disabling the influence of the third function of the triple. In this case, the third function is stable and it is always ZsN. Therefore, the net has a better disclosure of all 15 possible combinations of triples. It leads to the proportional equality of the results achieved by every combination of the triple in the green rectangle.

Comparing the rectangles shown in Figs. 18 and 19, it can be concluded that the last one gives more possibilities for analysis. In the blue rectangle, five different functions to be set as a third function were neglected because of the specificity of the structure. In the green rectangle, only one function is neglected which is stable at the same time. Thus, the blue rectangle has actually lost five functions compared to one function in the green rectangle because of the net structure. An additional benefit of the green rectangle is that this stable function is located as the third function of the triple which is neglected, while a blue rectangle has a stable function (Zadeh t-norm) on the first place of the triple. It leads to the reduction of the number of possible combinations applied for the already mentioned specific structure of the wFPN.

Thus, the determinative functions for the result in a blue (vertical) rectangle are: (1) the unique stable input functions ZtN, located on the plane of the cube (Fig. 5); (2) three functions located on the vertical lines. It leads to three practically different combinations of triples which were tested and their results are presented in Table 3. Table 5 proved this suggestion by applying all 15 combinations which led to the same three different combinations of results.

On the other side, the determinative functions for the outcomes in a green (horizontal) rectangle are five different input functions, which are located on the horizontal lines of the rectangle, and three functions, which are located on the vertical lines. This leads to proportionally equal combinations with a neglected third stable function ZsN.

Such an approach is suggested to achieve a bigger number of results and decisions for the analysis to confirm the effectiveness of either classic or optimized triple of functions. These combinations will be applied in the wFPN models referring to the subject area of PTL. Moreover, the new study may lead to an additional proposal to use another triple that will be more effective in practice.

11 Conclusion

This paper presented the extended practical implementation of weighted fuzzy Petri nets with different triple of functions in the context of PTL research based on the combination of knowledge representation in the interconnected tables of type “Object–property”, production rules and intelligent computation techniques (wFPN). Such techniques were implemented to give a practical realization of the scheme presented in Fig. 1: knowledge representation in tables of type “Object–property”, production rules, fuzzy Petri nets (incl. its modification with weights), hierarchical approach for creating wFPN, mathematical tools on which these nets are based, formed triples of triangular norms (t/s—norms) as well as comparison of the classic (ZtN, GtN, ZsN) with the optimized triple (ZtN, ZtN, LsN) proposing different approaches for verification of their results.

The scheme of TLP was interpreted in an easy-to-understand concept of target decisions as presented in Fig. 3. The application techniques of “Object–property” tables to process the knowledge provided by the experts were introduced and to make its representation in the FPN (incl. wFPN). Moreover, the mathematical tool of FPN implying t-/s-norm functions was graphically described in a form of cube (Fig. 5) suggesting choosing from 125 possible combinations.

To sum up, this paper introduced a flowchart with a step-by-step processing of the knowledge with the application of intelligent computational techniques which results in the application in the decision-support system PNeS® [17]. To finalize the scheme of PTL, this paper included a conception of the hierarchical structure implying (w)FPN for solving PTL. The benefit of such strategy theoretically lies in the reduction of calculations as well as in the speed increase, since three wFPN models out of seven are constantly activated. At the same time, all seven nets describe all possible developments of the situation in accordance to the achieved decision on the previous levels. There were tested classic (ZtN, GtN, ZsN) and optimized (ZtN, ZtN, LsN) combinations of triangular norms which resulted with different developments (Figs. 15, 16). The decision on the best type of traffic is a separated decision presented in both Figs. 3a and 14. It is an independent wFPN (Figs. 13, 17) which is not influenced by other wFPN models. As far as there were different developments of the decisions, different strategies were applied for their verification. Each of these approaches has its pros and cons depending on the requirements.

The practical implementation of this theoretical knowledge requires further researches in the development of wFPN for solving PTL. The objective is to find the proper triple of functions presented in Fig. 5 which will benefit in the application of the intelligent computation techniques for solving complex-in-description TLP (incl. PTL) [11]. Therefore, the approach proposed in [21] will be analyzed in the next paper.

Moreover, for further researches, other classes of wFPN can be considered: parameterized wFPN (wPFPN) (cf. [24]), FGFPN [25], T2GFPN [26]. wPFPN includes parameter v which is associated with triangular norms. Thus, the change of this parameter directly influences the calculations of triangular norms, which can lead to a confirmation of already achieved decisions or propose some additional alternatives. T2GFPN allows representing values in some range. It leads to the extension of possibilities to the knowledge representation. Yet, it should be analyzed if this approach results in accurate outputs due to the fuzziness in the range representation. FGFPN is the most advanced type of FPN, since it includes the widest range of triangular norm for the analysis as well as the possibility to choose the optimal one for the specific purpose.

Declaration

Conflicts of interest The authors declare that they have no conflict of interest.

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