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Injective Choice Functions

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Preface

A marriage of a family F of sets is an injective choice function for F. The marriage problem consists in establishing necessary and sufficient criteria which decide if a family has an injective choice function. First P. Hall formulated his well-known criterion for finite families in 1935. This criterion was generalized by M. Hall to infinite families which have finite members only. A detailed discussion of the results up to 1970 and many applications can be found in Mirsky's book [Mi]. In the seventies the research on the marriage problem took a rapid development. Several necessary and sufficient conditions for countable families were found ; on the one hand transfinite versions of Hall's Theorem, as for example in [N2], on the other hand extensions of the Compactness Theorem as in [HPS 1]. In Chapter III we are going to present these criteria and show that they are all equivalent.

But only three years ago, R. Aharoni, C.St.J.A. Nash-Williams, and S. Shelah published the first necessary and sufficient criterion for arbitrary families. Its form follows the one of P. Hall's Theorem: a family has a marriage if and only if it does not contain one of a set of "forbidden" substructures. Similar criteria can be found in the second chapter of this book.

The Aharoni - Nash-Williams - Shelah - criterion, which we obtain as a consequence of a criterion of K.P. Podewski in this book, has been successfully applied by Aharoni to solve some famous problems in graph theory. His main result is the proof of a strong form of König's Duality Theorem, suggested by P. Erdös. As a consequence he could prove a strong version of Menger's Theorem for graphs which contain no infinite path. One aim of this book is a self-contained representation of these intricate theorems. For this reason we have inserted a separate chapter on set theory for those readers who are not so familiar with transfinite methods. We suggest reading the introduction after the study of Chapter I.

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