Signal Processing 33 (1993) 235–236
Elsevier

Short communication

Comments on "Fixed-point error analysis of fast Hartley transform"

Guey-Shya Chen, Wei-Jou Duh and Ja-Ling Wu
Department of Computer Science and Information Engineering, National Taiwan University, Taipei, Taiwan, 10764, ROC

Received 26 November 1990

Abstract. This note contains a complete work on and a correction to a recently published *Signal Processing* Paper.

Zusammenfassung. Dieser Kurzbeitrag enthält eine vollständige Arbeit und eine Korrektur zu einem kürzlich in *Signal Processing* erschienenen Aufsatz.

Résumé. Cette contribution brève contient un travail complet et une correction à une article recente publiée dans *Signal Processing*.

Keywords. Hartley transform; error analysis.

In [1], Prabhu and Narayanan have proposed the scaling scheme for fast Hartley transform (FHT) with transform length $N=2^{2k}$, which is a power of 4. In this note, we derive $N=2^{2k+1}$ for both decimation-in-frequency (DIF) and decimation-in-time (DIT) cases to complete the work of fixed-point error analysis of FHT.

Such a scaling is the same as that of in the $N=2^{2k}$ cases, but it is required one additional scaling factor $\frac{1}{2}$ in the stage $k+1$ for DIF and in the stage $k+2$ for DIT. The average truncation error variance $[\sigma_{\mathrm{T}}^2]_{\mathrm{av}}$, round-off error variance $[\sigma_{\mathrm{R}}^2]_{\mathrm{av}}$, and signal-to-noise ratio $[\sigma_{\mathrm{H}}^2/\sigma_{\mathrm{AVN}}^2]_{\mathrm{av}}$ at the output are, respectively, given by

(1) DIF algorithm

$$[\sigma_{\mathrm{T}}^2]_{\mathrm{av}}=\sigma_{t1}^2\left[\frac{2}{\sqrt{2}}N^{1/2}\right] + \sigma_{t2}^2\left[\frac{8}{7\sqrt{2}}N^{1/2}-\frac{16}{7}N^{-1}\right],$$

$$[\sigma_{\mathrm{R}}^2]_{\mathrm{av}}=\sigma_r^2\left[\frac{18}{7\sqrt{2}}N^{1/2}-2k-\frac{38}{5}-\frac{8}{7}N^{-1}+\frac{32}{15}N^{-2}\right], \tag{1}$$

$$\left[\frac{\sigma_{\mathrm{H}}^2}{\sigma_{AVN}^2}\right]_{\mathrm{av}}=2^{2t}\Big/N\left[\frac{15}{7\sqrt{2}}N^{1/2}-\frac{1}{2}k-\frac{19}{30}-\frac{25}{14}N^{-1}+\frac{8}{16}N^{-2}\right].$$

(2) DIT algorithm

$$[\sigma_{\mathrm{T}}^2]_{\mathrm{av}}=\sigma_{t1}^2\left[\frac{1}{\sqrt{2}}N^{1/2}+12N^{-1}\right]$$

Correspondence to: Prof. J.-L. Wu, Department of Computer Science and Information Engineering, National Taiwan University, Taipei, Taiwan, 10764, ROC.

$$+\sigma_{r2}^2\left[\frac{4}{7\sqrt{2}}N^{1/2}-\frac{64}{7}N^{-1}\right],$$

$$[\sigma_{\mathrm{R}}^2]_{\mathrm{av}}=\sigma_r^2\left[\frac{18}{7\sqrt{2}}N^{1/2}-\frac{14}{3}-\frac{88}{21}N^{-1}\right], \tag{2}$$

$$\left[\frac{\sigma_{\mathrm{H}}^2}{\sigma_{\mathrm{AVN}}^2}\right]_{\mathrm{av}}=\frac{2^{2t}}{N\left[\frac{39}{28\sqrt{2}}N^{1/2}-\frac{7}{6}-\frac{19}{42}N^{-1}\right]}.$$

There are some minor errors in the DIF error expressions of the original paper in eqs. (11), (13) and (14). By using the same notations defined earlier, the correct expressions should be

$$[\sigma_{\mathrm{R}}^2]_{\mathrm{av}}=\sigma_r^2\left[\frac{15}{7}N^{1/2}-m-\frac{32}{15}-\frac{8}{7}N^{-1}+\frac{32}{15}N^{-2}\right], \tag{3}$$

$$\sigma_{\mathrm{AVN}}^2=[\sigma_{\mathrm{T}}^2]_{\mathrm{av}}+[\sigma_{\mathrm{R}}^2]_{\mathrm{av}}$$

$$=\frac{2^{-2t}}{12}\left[\frac{57}{7}N^{1/2}-m-\frac{32}{15}-\frac{50}{7}N^{-1}+\frac{32}{15}\mathrm{N}^{-2}\right]$$

$$\simeq\frac{2^{-2t}}{12}\left[\frac{57}{7}N^{1/2}-m-\frac{32}{15}\right] \tag{4}$$

and

$$\left[\frac{\sigma_{\mathrm{H}}^2}{\sigma_{\mathrm{AVN}}^2}\right]_{\mathrm{av}}=\frac{4\cdot 2^{2t}}{N\left[\frac{57}{7}N^{1/2}-m-\frac{32}{15}\right]}. \tag{5}$$

References

[1] K.M.M. Prabhu and S.B. Narayanan, "Fixed-point error analysis of fast Hartley transform", *Signal Processing*, Vol. 19, No. 3, March 1990, pp. 191–198.