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Title: APPLICATION OF SIMPLE NUMERICAL TECHNIQUES FOR INCREASING THE EFFICIENCY OF A FORWARD-IN-TIME SHALLOW WATER CODE ON A SPHERE

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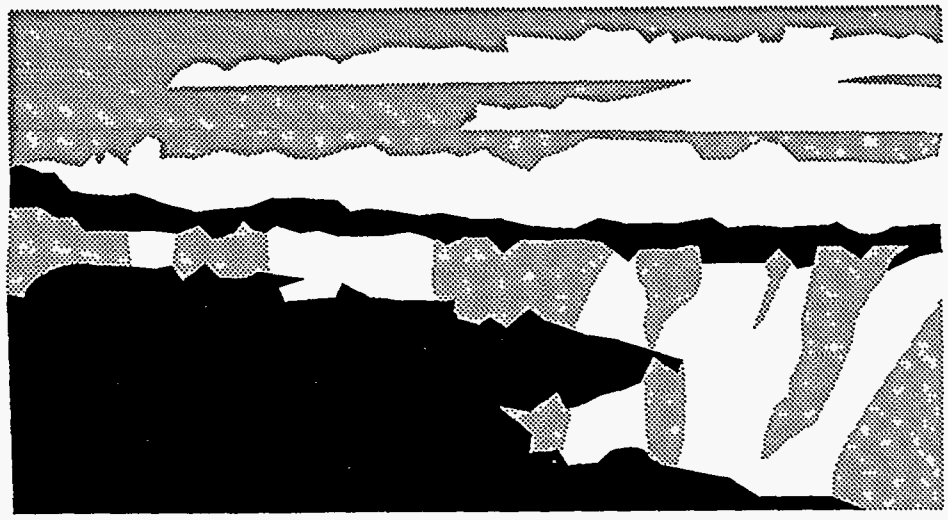

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Application of Simple Numerical Techniques for Increasing the Efficiency of a Forward-In-Time Shallow Water Code on a Sphere

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Atmospheric motions contain signals such as gravity waves and/or sound waves which usually have little influence on the actual weather. These signals do however have an effect on atmospheric models which predict the weather, the signals require the use of small time steps for numerical stability. Two explicit numerical techniques, time-splitting and temporal averaging, are described in this paper which help increase the efficiency of atmospheric models which contain fast signals. An interesting aspect of this work is that these techniques are being applied within the framework of a fully forward-in-time model. This is in contrast to findings by other researchers who have noted that the use of forward-in-time schemes in conjunction with time-splitting leads to numerical instabilities. In addition to demonstrating that the method is efficient, we will illustrate that the approach produces a solution which suffers from less dissipative errors than the original model. Additional features of the approach is that it does not suffer from temporal splitting errors and as such the method is fully second-order in time and space. Because the approach is explicit, no difficulties exist with its implementation on parallel architectures. The model is written in fortran 90 and runs on the CM5.

1. Introduction

In this paper numerical techniques for increasing the efficiency of a model capable of simulating shallow water flows on a rotating sphere are described. The original fully explicit model is based on Eulerian spatial differencing and nonoscillatory forward-in-time (NFT) temporal differencing. Finite difference methods have advantages over spectral methods when implemented on massively parallel computers with distributed memory because the computations are localized in space. However finite difference methods have more restrictive time step constraints and so computational efficiency becomes an important issue. The current model is explicit in time, and its computational time step is limited by the largest Courant number, $\alpha = c\frac{\Delta t}{\Delta x}$ with c being either a flow velocity or a wave speed, Δt is the time step, and Δx is a grid increment, on the mesh. Since the fastest wave speed is that of gravity waves (for atmospheric flows the Courant number, α_g, associated with the fastest gravity wave is usually at least 4 times bigger than the Courant number, α_a, associated with the fastest flow velocity on the mesh), and is uniform over the mesh, the largest Courant number is associated with the smallest cell dimensions. In the typical latitude-longitude mesh, these smallest dimensions are found in the cells nearest the poles.

There are several strategies available to increase the time step and thus improve the computational efficiency of a spherical finite difference model. For example, techniques such as the reduced grid or semi-implicit method

have been shown to improve the efficiency (Reisner et al. 1996) of the code; however, one group of techniques which have not been previously investigated are explicit techniques, such as time-split schemes and/or schemes employing temporal averaging. Unlike the semi-implicit approach, no elliptic solver need be implemented when explicit techniques are employed and hence explicit methods are ideally suited for parallel type machines. As will later be shown, efficiency gained from employing the explicit techniques is the result of computing relatively costly second-order advective procedures less frequently than that is required in the fully explicit model. Another important feature of these techniques is that they can be easily applied in model frameworks, such as an isentropic framework (Reisner and Smolarkiewicz 1994), where semi-implicit methods cannot be readily implemented due to the difficulty of solving a nonlinear elliptic equation (Piotr Smolarkiewicz, personal communication). And even if the elliptic equation can be solved, the resulting discretized equation may not necessarily produce positive values of layer thicknesses during a simulation.

Though explicit methods may be the only technique available in some cases to increase the efficiency of a code, atmospheric scientists have been reluctant to employ explicit techniques in forward-in-time models due to a stability analysis conducted by Skamarock and Klemp (1992, hereinafter referred to as SK) who found that when the split-explicit approach is used in the framework of forward-in-time differencing, computational instabilities arise—preventing the method from being applied in this framework. While the authors do not dispute the findings of SK, by incorporating the advective component of the continuity equation into the splitting procedure computational instabilities appear to be no longer excited (Janjic 1979). Presumably, the implicit diffusion associated with forward-in-time advective procedures leads to a stabilization of the scheme. Also, unlike the method proposed by SK, no explicit filtering is required in order to stabilize the time-split scheme and the scheme can be written in conservative form.

But, as will be discussed in the next section, the original splitting technique had several weaknesses. To overcome these limitations we implemented the concept of temporal averaging into our model. Recent work by Nadiga et al. (1996) has shown the use of a temporally averaged low-order predictive, fast mode-resolving solution in a final high-order long time step update as an effective strategy for alleviating time step restrictions in problems of multiple time scales, where the primary interest is in the slow-time behavior of the system. The method of temporal averaging employed is similar to that proposed by Madala (1981), except he applied the method to a scheme employing leapfrog temporal differencing. The combination of splitting and temporal averaging is in our opinion, "the best way", in which to increase both the efficiency and accuracy of a fully forward-in-time model. In the next two sections, the original model and the modified model will be described. Results for one test problem from the suite of test problems described by Williamson et al. (1992) will next be presented. The last section will give a summary of our work.

2. Original Shallow-Water Model

The equations expressing conservation of mass and momentum for a shallow fluid on a rotating sphere are as follows:

$$\frac{\partial G\Phi}{\partial t} + \nabla \bullet (\mathbf{v}\Phi) = 0, \tag{1a}$$

$$\frac{\partial GQ_x}{\partial t} + \nabla \bullet (\mathbf{v}Q_x) = GR_x, \tag{1b}$$

$$\frac{\partial GQ_y}{\partial t} + \nabla \bullet (\mathbf{v}Q_y) = GR_y, \tag{1c}$$

where $G = h_x h_y$, and h_x and h_y represent the metric coefficients of the general orthogonal coordinate system, $\Phi = H - H_o$ is the thickness of the fluid with H and H_o denoting the height of the free surface and the height of the bottom, $\mathbf{v} = G\dot{x}$ is the horizontal velocity vector, and $\mathbf{Q} = (\Phi u h_x, \Phi v h_y)$ is the momentum vector. The right-hand-side forcings are

$$R_x = -\frac{g}{h_x}\Phi\frac{\partial(\Phi + H_o)}{\partial x} + fQ_y + \frac{1}{G\Phi}\left(Q_y\frac{\partial h_y}{\partial x} - Q_x\frac{\partial h_x}{\partial y}\right)Q_y \tag{2a}$$

$$R_y = -\frac{g}{h_y}\Phi\frac{\partial(\Phi + H_o)}{\partial y} - fQ_x + \frac{1}{G\Phi}\left(Q_y\frac{\partial h_y}{\partial x} - Q_x\frac{\partial h_x}{\partial y}\right)Q_x, \tag{2b}$$

where g is the acceleration of gravity and f is the Coriolis parameter.

The integration in time of the discretized approximations to (1) is represented as follows

$$\Phi_i^{n+1} = MPDATA(\Phi_i^n, \alpha_{i\pm 1/2e_I}^{n+1/2}, G_i) \tag{3a}$$

$$Q_{x_i}^{n+1} = MPDATA(Q_{x_i}^n + 0.5\Delta t R_{x_i}^n, \alpha_{i\pm 1/2e_I}^{n+1/2}, G_i) + 0.5\Delta t R_{x_i}^{n+1} \tag{3b}$$

$$Q_{y_i}^{n+1} = MPDATA(Q_{y_i}^n + 0.5\Delta t R_{y_i}^n, \alpha_{i\pm 1/2e_I}^{n+1/2}, G_i) + 0.5\Delta t R_{y_i}^{n+1} \tag{3c}$$

where the basic element of (3) is the nonoscillatory forward-in-time algorithm MPDATA (Smolarkiewicz and Grabowski 1990), and $\alpha_{i\pm 1/2e_I} < 2\alpha_g$. The Courant numbers are updated by either using linear extrapolation or nonlinear interpolation (Smolarkiewicz and Margolin 1993). Of note, in (3b) and (3c) the forces are intergrated along a trajectory (Smolarkiewicz and Margolin 1993) to ensure a fully second-order approximation to (1). For a complete description of the steps required to update (3) see Smolarkiewicz and Margolin (1993).

The use of two-time-level integration schemes is a departure for Eulerian global atmospheric models where three-time-level or leapfrog schemes are

traditionally used. However two-time-level schemes are widely preferred in most other fields of computational fluid dynamics. Some of the advantages of NFT schemes include a larger computational time step, reduced memory usage, and less numerical dispersion. In addition, the nonoscillatory property is crucial for preserving the sign of the layer thickness and of thermodynamic scalars, and further controls the nonlinear stability of the computations. The model is implemented on a rotating sphere, and allows for arbitrary bottom topography as well as a free surface on top of the layer.

The model has been ported to the CM-5. It runs in data parallel mode, with the horizontal dimensions being spread across processors. In a typical problem, the speed (measured in Megaflops) depends on problem size. For 32 nodes, a problem with a 64x128 mesh yields performance equivalent to 0.5 CRAY YMP processors, whereas a problem with 256x512 nodes runs at a speed equivalent to 1.5 CRAY YMP processors.

3. Modified Shallow-Water Model

Our original splitting technique leads to the following set of discretized equations

$$Q_{x_i}^{n+1} = MPDATA(Q_{x_i}^{n}, \alpha_{i\pm 1/2 e_I}^{n+1/2}, G_i) \tag{4a}$$

$$Q_{y_i}^{n+1} = MPDATA(Q_{y_i}^{n}, \alpha_{i\pm 1/2 e_I a}^{n+1/2}, G_i) \tag{4b}$$

with $2\alpha_g \leq \alpha_{i\pm 1/2 e_I} < \alpha_a$. Depending on the ratio, $n_r = \frac{\alpha_g}{\alpha_a}$, a chosen number, n_s with $n_s < n_r$, of iterations of the following

$$\Phi_i^{(n+1)_s} = MPDATA(\Phi_i^{n_s}, \alpha_{s_{i\pm 1/2 e_I}}^{(n+1/2)_s}, G_i) \tag{4c}$$

$$Q_{x_i}^{(n+1)_s} = Q_{x_i}^{n_s} + \Delta t_g R_{x_i}^{(n+1)_s} \tag{4d}$$

$$Q_{y_i}^{(n+1)_s} = Q_{y_i}^{n_s} + \Delta t_g R_{y_i}^{(n+1)_s} \tag{4e}$$

are usually conducted, with $\alpha_{s i\pm 1/2 e_I} < 2\alpha_g$ and the superscript in (4c)-(4e) denoting that advancement in time is occurring within the iterations of the subcycle. Thus, in the temporal advancement of (4) the momentum fields due to advection procedures, (4a) and (4b), are only updated once per large time step; whereas, assuming a temporal splitting error, the layer thickness due to advective procedures, (4c), and the momentum fields due to the associated forces, (4d) and (4e), are typically updated several times within the iterations of the subcycle.

Though the modified code is about twice as fast as the original code, several problems do exist with the above approach. First, a costly higher order advective routine is used in the subcycle; second, the forcing terms are only treated to first-order; and third, a splitting error is inherently present in (4). A simple way of circumventing these problems is to use the concept of temporal averaging (Nagida et al. 1996 and Madala 1981) to filter out small-scale noise found within the α_s's

and the forcing terms. Additionally, instead of employing a two-pass version of $MPDATA$ within (4), an efficient one-pass version of $MPDATA$ or $DONOR$ can be used, in what will be subsequently labeled as a predictor step, to obtain the filtered quantities. Once predicted, the first-order quantities can be used in the following corrector step

$$\Phi_i^{n+1} = MPDATA(\Phi_i^n, \bar{\alpha}_{i\pm 1/2e_I}^{n+1/2}, G_i) \tag{5a}$$

$$Q_{x_i}^{n+1} = MPDATA(Q_{x_i}^n, \bar{\alpha}_{i\pm 1/2e_I}^{n+1/2}, G_i) + DONOR(\bar{R}_x^{n+1/2}, 0.5\bar{\alpha}_{i\pm 1/2e_I}^{n+1/2}, G_i) \tag{5b}$$

$$Q_{y_i}^{n+1} = MPDATA(Q_{y_i}^n, \bar{\alpha}_{i\pm 1/2e_I}^{n+1/2}, G_i) + DONOR(\bar{R}_y^{n+1/2}, 0.5\bar{\alpha}_{i\pm 1/2e_I}^{n+1/2}, G_i) \tag{5c}$$

where the bar quantities are calculated by the following

$$\bar{\psi}^t = \frac{1}{\Delta t}\int_t^{t+\Delta t} \psi dt \tag{6}$$

with ψ representing either α_s or the forcing terms calculated in the predictor step and the $DONOR$ cell step in (5) being required to ensure second-order accuracy of the forcing terms (Smolarkiewicz and Margolin 1993). Hence, not only does our approach eliminate the problems found using the simple splitting technique but allows for more flexibility as well. For example, within the predictor step a non-conservative advective procedure such as semi-lagrangian or upwind could be used to obtain the average quantities for use in the conservative corrector step. It may even be possible to employ a three-time level scheme in the predictor step. Also, note that splitting is not required in the predictor step, we have chosen to use splitting in the predictor step only for numerical efficiency. We have not yet conducted a stability analysis of the current scheme, but numerical analysis of the linearized form of the scheme (Nadiga et al. 1996) in which only temporal averaging is present suggests that as long as $\bar{\alpha}$ is somewhat less than suggested by linear stability analysis of a multi-dimensional upwind advective procedure the scheme is stable. The extent to which $\bar{\alpha}$ is restricted is a rather complicated formula which depends on both n_s and n_r.

4. Model Setup and Results

Shallow water is a useful testbed for evaluating and comparing numerical methods that can be extended to fully three-dimensional global circulation models (GCMs). As previously mentioned, shallow water layers can be stacked, and with the addition of a hydrostatic equation and slight recoding of (1)-(2), can be extended to model fully three-dimensional flows (cf. Bleck, 1984). Such models are termed isentropic in meteorology. As part of DOE's CHAMMP program, Williamson et al. (1992) have published a suite of seven test problems for shallow water. One of these, the Rossby Haurwitz (RH) wave is particularly challenging, since it represents an almost steady state flow that is a delicate balance of large and small spatial scales. (When the free surface is replaced by a rigid lid, the

solution is an exact steady state.) The RH wave is also interesting because it represents the advection of a wave which may be significantly damped if low-order forward-in-time methods are used (Smolarkiewicz and Margolin, 1993).

A grid resolution of 2.8° is used in the simulation of the RH wave. This resolution allows for a time step of 40 s to be used in (4c)-(4e) with a time step of 600s being chosen for use in the corrector and in (4a)-(4b). The resulting maximum α_a and α_g were approximately 0.2 and 1.0 respectively. Of note, the large time step could have been increased to achieve an $\alpha_a \approx 0.7$, but linear stability analysis suggests that the small step would need to be decreased in order for the scheme to be stable. Regardless, the time step being used in advective procedures of the modified model is still an order of magnitude larger than that used in the original model. Hence, less numerical dissipation of the RW wave should occur in the simulation employing the modified scheme. Because analytic solutions are not known for the RH wave, we have run high resolution simulations as a standard for comparing the results from the original model against the modified model. All simulations were run for a period of 7 days.

Since visual differences between the reduced grid and nonreduced grid are not large, and the solutions have been published elsewhere (Smolarkiewicz and Margolin 1993, Fig. 2), we show only the L_2 error measure (see Fig. 1) with respect to the height of the shallow fluid as a function of time for the original and modified model. The L_2 error measure demonstrates that the buildup of errors associated with numerical dissipation is, as expected, more severe in the simulation which employed the smaller time step. In addition to being more accurate the total CPU time used by the modified model is about one-third of the original model. For the specified value of n_s the increase in efficiency is independent of grid resolution.

5. Summary

A combination of splitting and temporal averaging has been shown to increase both the efficiency and accuracy of a global shallow-water code employing forward-in-time differencing. Unlike existing split-explicit schemes, such as proposed by SK, the current scheme is fully second-order in time and space and does not suffer from splitting errors. The scheme requires little additional coding logic with only a slight reworking of the main routine being required for implementation. Being that the code is explicit and employs finite-difference techniques, parallelization of the code is trivial. Likewise, the method can easily be incorporated into the framework of the reduced grid. We believe the combination of the two approaches is a usefull framework for for general circulation models of the atmosphere and ocean based on finite difference approximations.

As discussed in SK a time-split scheme which works in a hydrostatic framework should as well work in a nonhydrostatic framework and vica versa. Indeed we have extended the technique to a nonhydrostatic compressible framework and found it works as well. Currently we are investigating techniques to efficiently split out vertically propagating sound waves. This will allow us to extend the nonhydrostatic model to global scales.

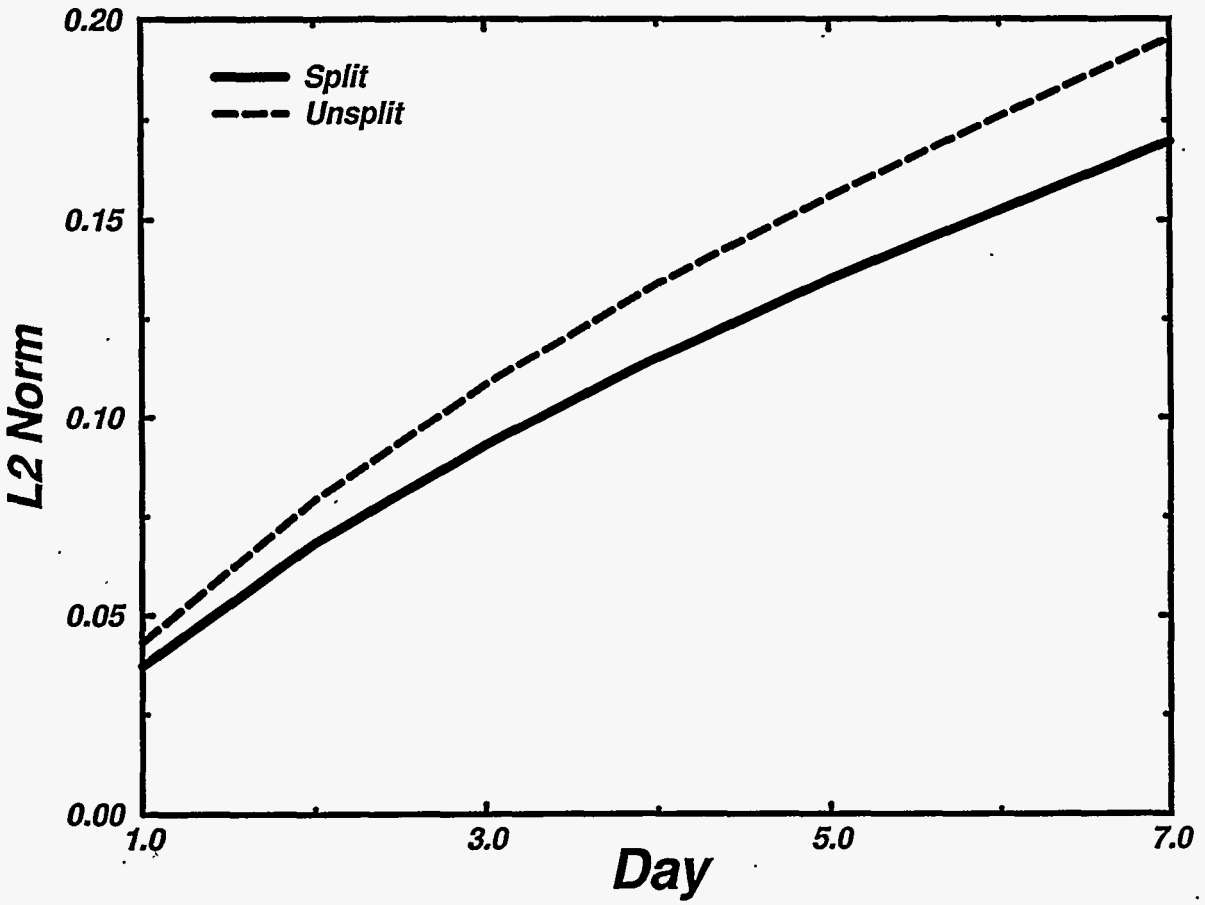

Fig. 1 Time evolution of the L_2 error measure for the model configuration with 2.8° resolution for the Rossby-Haurwitz wave.

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