Neighbor Oblivious and Finite-State Algorithms for Circumventing Local Minima in Geographic Forwarding

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Abstract—We propose distributed link reversal algorithms to circumvent communication voids in geographic routing. We also solve the attendant problem of integer overflow in these algorithms. These are achieved in two steps. First, we derive partial and full link reversal algorithms that do not require onehop neighbor information, and convert a destination-disoriented directed acyclic graph (DAG) to a destination-oriented DAG. We embed these algorithms in the framework of Gafni and Bertsekas [1] in order to establish their termination properties. We also analyze certain key properties exhibited by our neighbor oblivious link reversal algorithms, e.g., for any two neighbors, their t-states are always consecutive integers, and for any node, its t-state size is upper bounded by log(N). In the second step, we resolve the integer overflow problem by analytically deriving one-bit full link reversal and two-bit partial link reversal versions of our neighbor oblivious link reversal algorithms.

Index Terms—geographic forwarding, full link reversal, partial link reversal, distributed algorithm, finite bit width

I. Introduction

A. Motivation

Consider a wireless sensor network (WSN) with a single designated sink node. We shall focus particularly on an example application where the WSN is used to detect undesirable events. An alarm packet originating at a node near the location of the alarm event has to be routed to the sink node. For such purposes, geographic routing [2] is a popular protocol for packet delivery. It is scalable, stateless, and reactive, without the need for prior route discovery. In this protocol, a node forwards a packet to another node within its communication range (hence, called a neighbor node) and closer to the destination. Ties can be broken arbitrarily, for example, by using node indices. Such a protocol requires a node with a packet to be aware of its own geographical location, and that of the sink and of its neighbors. To each node, the next hop nodes that are closer to the sink are defined as greedy neighbors, and wireless "links" are oriented from the nodes to their greedy neighbors. The resulting routing graph is a directed acyclic graph (DAG).

A DAG is said to be *destination-oriented* when there is a directed path in the DAG from any node to the sink. A DAG is

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This work was supported in part by a research grant on Wireless Sensor Networks from DRDO, Government of India, and in part by the Indo-French Centre for Promotion of Advanced Research (IFCPAR), Project No. 4000-IT-A

This work was presented in part at the National Conference on Communications (NCC) 2010, IIT Madras, Chennai, India.

destination-disoriented if and only if there exists a node other than the sink that has no outgoing link [1]. The disadvantaged node with no outgoing links is said to be *stuck* (as it is unable to forward towards the sink a packet that it receives). A destination-oriented network under geographic routing may be rendered destination-disoriented due to various reasons such as node failures, node removal or sleep-wake cycling. The failure of geographic routing in the presence of stuck nodes is commonly referred to as *the local minimum condition* [3]. Numerous solutions have been proposed in the literature to pull the network out of a local minimum condition (See Section I-B for details). *However, all these solutions require knowledge of one-hop neighbors and their locations*. Maintenance of one-hop neighbor information, in general, requires periodic transmissions of *keep alive* packets.

We associate each node in the network with a unique numerical value, henceforth referred to as state. A link between a pair of neighboring nodes is oriented from the node with the higher state to the node with the lower state. Thus states (of all the nodes) determine the routing graph. It is clearly acyclic. Whenever a node updates its state, it communicates the new state to its neighbors. Thus all the nodes always have an updated view of the directions of all their links.¹ Whenever a node wants to determine whether it is stuck or not, it broadcasts a hello packet containing only its index. All the *alive* neighbors with incoming links from the tagged node acknowledge. If the tagged node does not receive any acknowledgment until a fixed timeout period, it concludes that its state is the least among its alive neighbors, i.e., it is stuck. Then, the node updates its state appropriately to reverse its links. It also broadcasts the new state to facilitate its neighbors to update the corresponding link directions. An update protocol is called *neighbor oblivious* if the updating node does not need to know the exact values of its neighbors' states. Neighbor oblivious protocols do not incur the overhead of neighbor state discovery, and thus save precious communication time and energy.

Gafni and Bertsekas [1] proposed a general class of distributed *link reversal algorithms* for converting a destination-disoriented DAG into a destination-oriented DAG. They also described two representative algorithms, *full link reversal* and *partial link reversal*, of their general class. Henceforth, we refer to their algorithms as *GB algorithms*. In the GB

¹For communication between two neighbors, they must have a consistent view of the direction of the link between them. Thus broadcast of the updated state is an intrinsic part of all routing algorithms.

algorithms, a stuck node's update depends on its one-hop neighbors' states. Thus the GB algorithms are *not neighbor oblivious*.

Our work is motivated by the question: Are there *distributed*, *finite-state*, *neighbor oblivious* protocols that can pull a network out of its local minimum condition and render it destination-oriented?²

B. Related Literature

Kranakis et al. [4] introduced geographic routing protocols for planar mobile ad hoc networks, called *compass routing* or face routing. This technique guarantees delivery in a connected network, but requires a priori knowledge of full neighborhood. Karp and Kung [2] presented greedy perimeter stateless routing (GPSR) which also ensures successful routing over planar networks. Kalosha et al. [5] addressed a beaconless recovery problem where the local planar subgraph is constructed on the fly. Chang et al. [6] presented route guiding protocol (RGP), a shortest path routing protocol to bypass voids, but that also requires communication of current states among neighbors. Yu et al. [7] discussed a void bypassing scheme when both source and sink nodes are mobile. Leong et al. [8] presented a new geographic routing protocol called greedy distributed spanning tree routing (GDSTR). GDSTR employs convex hulls which require maintaining topology information. Casari et al. [9] proposed adaptive load-balanced algorithm (ALBA), another greedy forwarding protocol for WSNs. Some other algorithms developed for mobile adhoc networks include destination sequenced distance vector (DSDV) routing [10], wireless routing protocol (WRP) [10], dynamic source routing (DSR) [11] and node elevation ad hoc routing (NEAR) [12]. All the above algorithms require neighbor information at a stuck node, and some even require more extensive topology information (e.g., [8]).

Gafni and Bertsekas [1] introduced a general class of link reversal algorithms to maintain routes to the destination. They also presented two particular algorithms, the full link reversal algorithm and the partial link reversal algorithm. The GB algorithms were designed for connected networks. In a partitioned network, GB algorithms lead to infinite number of state updates without ever converging. Corson and Ephremides [13] presented *lightweight mobile routing* (LMR), a variant of GB link reversal algorithms. Park and Corson [14] proposed temporally-ordered routing algorithm (TORA) for detecting and dealing with partitions in the networks. TORA is also an adaptation of GB partial link reversal algorithm and employs extended states that include current time and originator id. GB link reversal algorithms have also motivated several leader election algorithms which are an important building block for distributed computing, e.g., mutual exclusion algorithms or group communication protocols. Malpani

²One simple neighbor oblivious algorithm is to always make a stuck node increment its state by unity. This algorithm renders the network destination-oriented but requires a huge number of updates. In particular, it is neither full link reversal nor partial link reversal. Recall that each updating node broadcasts its state to determine if it stuck, and then waits for a timeout period for acknowledgments. Consequently, this simple algorithm results in significant energy expenditure and delay, and hence, is not desirable.

et al. [15] built a leader election algorithm on the top of TORA for mobile networks. Ingram et al [16] proposed a modification of the algorithm in [15] that works in an asynchronous system with arbitrary topology changes. All these link reversal algorithms employ state variables that either require infinitesimal precision (e.g., current time) or grow unbounded, thus imposing enormous memory requirements. Further, state updates in these algorithms require frequent information exchanges among neighboring nodes, and also network wide clock synchronization, thus imposing signification communication overhead. These drawbacks render the above algorithms unsuitable for large mobile networks with lightweight mobile nodes. We focus on connected mobile ad hoc networks with single destination and develop neighboroblivious and memory-savvy link reversal algorithms.

Busch and Tithapura [17] analyzed GB algorithms (full and partial link reversal) to determine the number of reversals and time until these algorithms converge. Their performance bounds apply to our algorithms also.

C. Our Contributions

We focus on connected mobile ad hoc networks with a single destination.³ We propose neighbor oblivious full and partial link reversal (NOLR) algorithms in which a stuck node does not need one-hop neighbors' states to execute its state update. However, as discussed earlier, a node still has to communicate with its neighbors in order to determine if it is stuck. But this communication only involves a *hello* packet and its acknowledgments, and thus is "lightweight". Then, we embed our NOLR algorithms into the framework of the GB algorithms. The embedding ensures that our proposed algorithms render the network destination-oriented.

In GB and NOLR algorithms, the state spaces are (countably) infinite. The reason is that in both the algorithms each node's state grows without bound with the number of link reversals. The algorithms therefore cannot be realized in a real operating environment with only a finite number of bits to represent states, when repeated link reversals may be encountered. We show that simple modifications of our NOLR algorithms result in finite-state link reversal algorithms. At each node, in addition to the initial state, the full link reversal algorithm requires only a one-bit dynamic state and the partial link reversal algorithm requires only a two-bit dynamic state.

Throughout, we assume that new nodes or links are not added to the existing network. We conclude the paper with a discussion of how addition of new nodes or links affects our algorithms.

D. Organization of the Paper

The rest of the paper is organized as follows. In Section II we provide an overview of the GB algorithms. In Section III we discuss full link reversal. We begin with the NOLR proposal, but with a countably infinite state space. Then, we make

³If routing to multiple destinations is required, for each destination, a logically separate copy of our algorithm should be run. This limitation is inherent to the class of GB link reversal algorithms (see [1], [13], [14], [15], [16]).

an observation that renders the NOLR algorithm into a finite-state algorithm without loss of correctness. In Section IV, we address partial link reversal, and pass through the same trajectory as for full-link reversal – an NOLR algorithm with infinite states followed by a finite-state version. In Section V we make some concluding remarks. The appendices contain the detailed proofs.

II. OVERVIEW OF GB ALGORITHMS

Consider a WSN with a designated destination node and nondestination nodes $\{1,2,\ldots,N\}$. The nodes are assumed to have static locations. Two nodes are neighbors if they can directly communicate, and then we say that there is a *link* between them. Link reversal schemes can be used in geographic forwarding by assigning unique states, a_1, a_2, \ldots, a_N , to the nodes. The states are totally ordered by a relation < in the sense that for any two nodes i and j, either $a_i < a_j$ or $a_j < a_i$, but not both. These states are used in assigning routing directions to links. The link between a pair of neighbors is always oriented from the node with the higher state to the node with the lower state.

In GB algorithms, the state associated with a node i is a pair of numbers (h_i,i) for full reversal and a triplet of numbers (p_i,h_i,i) for partial reversal, where h_i (called i's $height^4$) and p_i are integers. The ordering < on the tuples in each case is the lexicographical ordering. For a node i, let C_i denote i's neighbors. Also, let $h=(h_1,\ldots,h_N)$ and $p=(p_1,\ldots,p_N)$. Then, the forwarding set of node i can be written as

$$F_i(h) = \{ j \in C_i | (h_j, j) < (h_i, i) \}$$

for full reversal, and

$$F_i(p,h) = \{ j \in C_i | (p_i, h_i, j) < (p_i, h_i, i) \}$$

for partial reversal. Clearly, node i is stuck if $F_i(h) = \emptyset$ (for full reversal), or $F_i(p,h) = \emptyset$ (for partial reversal). Node i, to determine if it is stuck, broadcasts its state. All its alive neighbors with lower states acknowledge (Recall that a few of the neighbors might not be awake due to the sleep-wake cycling in place). If node i does not receive any acknowledgment until an a priori fixed timeout, it concludes that its state is the least among its neighbors, i.e., it is stuck.

The GB algorithms distributively update the states of stuck nodes so that a destination-oriented DAG is obtained. The algorithms are as follows.

Full link reversal: In this algorithm, a stuck node reverses the direction of all the incoming links. Node i updates its state as follows.

Remarks 2.1: Evidently, a node i, if stuck, leapfrogs the the heights of all its neighbors after an iteration of the above algorithm. All neighbors thereby enter the forwarding set of node i.

Algorithm 1 GB Full link reversal

```
1: loop

2: if F_i(h) = \emptyset then

3: h_i \leftarrow \max\{h_j|\ j \in C_i\} + 1

4: end if

5: end loop
```

Partial link reversal: In this algorithm, every node keeps a list of its neighbors that have already reversed their links to it. If a node is stuck, it reverses the directions of links to all those neighbors that are not in the list, and empties the list. If all its neighbors are in the list, then it reverses the directions of all the incoming links, and empties the list. Node i updates its state as follows.

Algorithm 2 GB Partial link reversal

```
1: loop
2: if F_i(p,h) = \emptyset then
3: p_i \leftarrow \min\{p_j | j \in C_i\} + 1
4: if there exists a j \in C_i with p_i = p_j then
5: h_i \leftarrow \min\{h_j | j \in C_i \text{ with } p_i = p_j\} - 1
6: end if
7: end if
8: end loop
```

Remarks 2.2: All p_i s are initialized to 0. The update rule (Line 3) ensures that for neighboring nodes p_i s are always adjacent integers. For a stuck node i, the h_i update (Lines 4-5) ensures that, i does not reverse the links to the neighbors that have updated states since i's last update.

Note that all the nodes run Algorithm 1 (or Algorithm 2 in case of partial link reversal) asynchronously, i.e., their reversals can follow any arbitrary timing and order. Gafni and Bertsekas [1] show the following properties.

Proposition 2.1: (a) Starting from any state h, or (p,h), Algorithms 1 and 2 terminate in a finite number of iterations yielding destination oriented DAGs.

- (b) Algorithm 1 results in the same destination-oriented DAG regardless of the timing and order of reversals. The same holds for Algorithm 2.
- (c) Algorithms 1 and 2 are such that only those nodes that do not initially have a greedy path to the destination update their states at any stage.

Remarks 2.3: The updates at a stuck node, in both Algorithms 1 and 2, depend on knowledge of neighbors' states (see Line 3 in Algorithm 1 and Lines 3, 4, 5 in Algorithm 2). After each link reversal, the updating node needs to broadcast its new state, and its neighbors need to gather this information in a reliable fashion (e.g., using an error detection scheme). In the following sections, we see how to avoid these exchanges, a desired level of ignorance that we call *neighbor obliviousness*.

III. FULL LINK REVERSAL

A. Neighbor Oblivious Full Link Reversal

The main idea may be summarized as follows. Suppose that the algorithm is such that a node, at any stage, knows the

 $^{^4}$ The heights ($h_i s$) are initialized to either hop counts or distances from the destination (evaluated from either actual or virtual locations [18]), with the destination's height being zero.

⁵For tuples a, b of the same dimension, a > b iff $a_i > b_i$ where i is the smallest index such that $a_i \neq b_i$.

entire *range* of all its neighbors' heights. Then it may execute a full reversal by raising its height to a value higher than the maximum in the range. Note that the updating node does not need to know the exact states of its neighbors, so valuable communication time and energy are saved.

Notation: The notation used is listed below for ease of reference.

- $[N] = \{1, 2, \dots, N\}$ is the set of nodes (or node indices).
- t_i ∈ Z₊ is the number of height updates made by node
 i; this is initialized to 0 for all i.
- $h_i(t_i) \in \mathbb{Z}_{++}$ is the height of node i after t_i updates; $h_i(0)$ refers to the initial height. The destination's height is 0.
- $a_i = (t_i, h_i(t_i), i)$ is the state of node i; t_i is referred to as its t-state.
- C_i is the set of neighbors of i, i.e., those with which i can directly communicate.
- $F_i(h) = \{j \in C_i | (h_j(t_j), j) < (h_i(t_i), i)\}$ is the forwarding set of node i, given the heights $h = (h_1(t_1), h_2(t_2), \ldots, h_N(t_N))$.
- $h_{\max} = \max\{h_1(0), \dots, h_N(0)\}.$

The algorithm is simple. Node i updates its state a_i as follows.

Algorithm 3 Neighbor oblivious full link reversal

```
1: loop
2: if F_i(h) = \emptyset then
3: t_i \leftarrow t_i + 1
4: h_i(t_i) \leftarrow h_i(t_i - 1) + h_{\max}
5: end if
6: end loop
```

Remarks 3.1: Node i, if stuck, updates its state such that the new height surpasses the heights of all its neighbors (see Line 3). Thus, it reverses all the incoming links.

Node i broadcasts a hello packet to determine if it is stuck. The lack of feedback (silence) following a broadcast suffices to determine if $F_i(h)$ is empty or not. However, node i does not need to know its neighbors' states to perform updates (see Lines 3, 4 in Algorithm 3). Other nodes also independently and asynchronously execute similar algorithms. All the nodes broadcast their new states whenever they update. Timing and order of state updates can be arbitrary. We now proceed to state and prove some of the properties of this algorithm.

Proposition 3.1: (a) The height of a node i in t-state t_i is explicitly given by

$$h_i(t_i) = h_i(0) + t_i h_{\text{max}}.$$

- (b) For any node i, and $t_i \in \mathbb{Z}_+$, we have $t_i h_{\max} < h_i(t_i) \le (t_i + 1) h_{\max}$.
- (c) For any two neighbors i and j, and $t_i, t_j \in \mathbb{Z}_+$ we have the following implication

$$t_i > t_j \Rightarrow h_i(t_i) > h_j(t_j).$$

$$^6\mathbb{Z}_{++} := \mathbb{Z}_+ \setminus \{0\}.$$

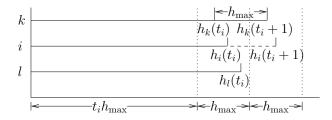


Fig. 1. An illustration of Algorithm 3 at a stuck node i. Note that $t_l=t_i$ while $t_k=t_i+1$. When node i updates its state, it reverse the links to both l and k.

- (d) For any two neighbors i and j, at any stage of the algorithm, we have $0 \le |t_i t_j| \le 1$.
- (e) For any node $i, t_i \leq N$ at any stage of the algorithm. Proof: See Appendix B.

Remarks 3.2: 1) For any node, the size of the state (i.e., the number of bits required to represent the state) grows with the number of state updates. However, Proposition 3.1(e) implies that, for any node, the number of updates is upper bounded by N, and hence the t-state size is upper bounded by $\log(N)$. Notice that heights are functions of t-states (Proposition 3.1(a)), and hence need not be stored separately.

2) Proposition 3.1(c) implies that the forwarding set of node *i* can be alternatively defined as

$$F_i(a) = \{ j \in C_i | a_i < a_i \},$$

where $a = (a_1, \ldots, a_N)$ are the nodes' states.

Proposition 3.2: In Algorithm 3, a stuck node reverses the directions of all the incoming links.

Proof: Consider a stuck node i. For any node $j \in C_i$, $h_j(t_j) \ge h_i(t_i)$. So, by virtue of Propositions 3.1(c)-(d), we have either $t_j = t_i$ or $t_j = t_i + 1$. See Figure 1 for an illustration.

- (i) Consider $t_j = t_i$. This is the case of node l in Figure 1. In this case, when node i makes an update, it moves to t-state $t_j + 1$. Hence the link is from i to j after the update.
- (ii) Consider $t_j = t_i + 1$. This is the case of node k in Figure 1. In this case, observe that when node j updated its t-state from t_i to $t_j = t_i + 1$, node i's t-state must have been t_i . Further, it must have been the case that either $h_j(t_i) < h_i(t_i)$, or $h_j(t_i) = h_i(t_i)$ and j < i. Thus we have either $h_j(t_j) < h_i(t_i) + h_{\max}$, or $h_j(t_j) = h_i(t_i) + h_{\max}$ and j < i. Hence when node i makes an update, since $h_i(t_i + 1) = h_i(t_i) + h_{\max}$, the link is now from i to j. This concludes the proof.

Proposition 3.3: Algorithm 3 can be embedded within the GB algorithms framework. Thus it inherits the properties in Proposition 2.1.

B. Two Bits Full Link Reversal

In practice, states are stored using finite bit-width representations. While the size of the states can depend on the number of nodes in the network, it should not grow with the number of iterations of the algorithm. The *t*-states which are the counts of the number of reversals, though bounded (see Proposition 3.1(e)), grow as the algorithm runs. There could be

1000s of nodes in the network, and in resource limited nodes in wireless sensor networks, memory is also at a premium. Therefore, GB and NOLR algorithms need to be modified for implementation in practical systems.

We now give a modification of Algorithm 3 that uses only two bits for the t-state and does not update heights. To do this we exploit the fact that, for any two neighbors i and j, the link direction is entirely governed by $t_i, t_j, h_i(0)$ and $h_j(0)$. More precisely, the link is directed from i to j if and only if either $t_i > t_j$, or $t_i = t_j$ and $(h_i(0), i) > (h_j(0), j)$. Thus t-states along with the initial heights suffice to determine link orientations. Moreover, since at any stage t_i and t_j are either the same or adjacent integers (Proposition 3.1(d)), we need only two bits to describe their order. Specifically, if we define, for all i,

$$\tau_i = t_i \mod 4$$
,

and a cyclic ordering

on candidate values of τ_i , we obtain

$$t_i > t_j \Longleftrightarrow \tau_i > \tau_j$$
.

For node i, τ_i is referred to as its τ -state. Following the above discussion, we can redefine the forwarding set of node i as

$$F_i(\tau) = \{ j \in C_i | \tau_j < \tau_i \text{ or }$$

$$(\tau_j = \tau_i \text{ and } (h_i(0), j) < (h_i(0), i)) \},$$

where $\tau = (\tau_1, \dots, \tau_N)$. In the two bit full link reversal algorithm node i updates its state as follows.

Algorithm 4 Two bit full link reversal

```
1: loop

2: if F_i(\tau) = \emptyset then

3: \tau_i \leftarrow (\tau_i + 1) \mod 4

4: end if

5: end loop
```

Following are the key properties of this algorithm.

Proposition 3.4: (a) In Algorithm 4, a stuck node reverses the directions of all the incoming links.

(b) Algorithm 4 exhibits the properties in Proposition 2.1.

Proof: (a) Consider a stuck node i. Following Proposition 3.1(d) and the definition of τ -states, for any node $j \in C_i$, we have either $\tau_j = \tau_i$ or $\tau_j = (\tau_i + 1) \mod 4$.

- (i) Consider $\tau_j = \tau_i$. In this case, when node i makes an update, it moves to τ -state $(\tau_j + 1) \mod 4$ which is greater than τ_i . Hence the link is from i to j after the update.
- (ii) Consider $\tau_j = (\tau_i + 1) \mod 4$. In this case it must be that $(h_j(0), j) < (h_i(0), i)$; were it not the case, node j at τ -state τ_i would not have done an update. Thus when node i updates its τ -state to $(\tau_i + 1) \mod 4 = \tau_j$, the link is now from i to j. This concludes the proof of part (a).
- (b) Let us consider a network and let all the nodes run Algorithm 4. Also consider another copy of the network (with the same initial link orientations) where all the nodes execute Algorithm 3 as follows. The same node as in the original

network does the first update. Then we are left with the same set of stuck nodes as in the original network because updates lead to full link reversals in both the networks. The next update is also done by the same node as in the original network, thus again resulting in the same set of stuck nodes. Likewise, subsequent updates also follow the same timing and order as in the original network. Since the nodes' updates in the latter network satisfy the properties in Proposition 2.1, so do the updates in the original network.

C. One Bit Full Link Reversal

Recall that in full reversal, a stuck node reverses the directions of all its incoming links. Algorithm 4 executes this using initial heights and a two bit state. We now describe a simpler way to achieve this using initial heights and a single flag bit at each node. More precisely, with each node i, we associate a binary state δ_i that is initialized to zero. For any two neighbors i and j with $(h_i(0),i) > (h_j(0),j)$, the corresponding link is directed from i to j if $\delta_i = \delta_j$, and from j to i if $\delta_i \neq \delta_j$. In other words, at any stage, the forwarding set of node i is

$$F_i(\delta) = \{ j \in C_i | ((h_j(0), j) < (h_i(0), i) \text{ and } \delta_j = \delta_i) \text{ or } ((h_j(0), j) > (h_i(0), i) \text{ and } \delta_j \neq \delta_i) \},$$

where $\delta = (\delta_1, \dots, \delta_N)$.

We propose the following one bit full link reversal algorithm. Node i updates its states as follows.

Algorithm 5 One bit full link reversal

```
1: loop

2: if F_i(\delta) = \emptyset then

3: \delta_i \leftarrow (\delta_i + 1) \mod 2

4: end if

5: end loop
```

Remarks 3.3: For a stuck node i, the updated δ -state is same as the δ -states of neighbors with higher heights but complements the δ -states of neighbors with lower heights. Thus, all its links become outgoing.

Algorithm 5 has similar properties as Algorithm 4.

Proposition 3.5: (a) In Algorithm 5, a stuck node reverses the directions of all the incoming links.

(b) Algorithm 5 exhibits the properties in Proposition 2.1.

Proof: (a) Consider a stuck node i and an arbitrary node $j \in C_i$. Then, either $(h_i(0),i) < (h_j(0),j)$ and $\delta_i = \delta_j$, or $(h_i(0),i) > (h_j(0),j)$ and $\delta_i \neq \delta_j$. In either case, when node i flips δ_i , the link between i and j is reversed.

(b) The proof is identical to that of Proposition 3.4(b).

IV. PARTIAL LINK REVERSAL

Recall that the link reversals are intended to yield a destination oriented DAG. However, link reversals are accompanied by state updates and information exchanges, and can potentially lead to more nodes being stuck. Thus, a stuck node could execute a partial link reversal (i.e., need not reverse all its incoming links) so that the link graph converges quickly to a destination oriented graph. We focus on the partial link reversal scheme proposed by Gafni and Bertsekas [1] (see Algorithm 2).

A. Neighbor Oblivious Partial Link Reversal

As in neighbor oblivious full link reversal, the algorithm is such that a node, at any stage, knows the entire *range* of all neighbors' heights but not the exact values. Then, the node raises its height to an appropriate value to effect only a partial link reversal. Again, as in Section III, the updating node does not need to know the exact states of its neighbors, so valuable communication time and energy are saved.

Notation: The new notation is collected below.

- $a_i = (t_i, h_i(t_i), (-1)^{t_i}i)$ is the state of node i; t_i is referred to as its t-state.
- $F_i(h) = \{j \in C_i | (h_j(t_j), (-1)^{t_j}j) < (h_i(t_i), (-1)^{t_i}i)\}$ is the forwarding set of node i for heights $h = (h_1(t_1), \dots, h_N(t_N))$.
- $\{z(0), z(1), \dots\}$ is a sequence satisfying

$$z(t) = \begin{cases} 0 & \text{if } t = 0, \\ 2^{t-1}(2h_{\max} + 1) & \text{if } t \ge 1. \end{cases}$$

In the neighbor oblivious partial link reversal algorithm node i updates its state as follows.

Algorithm 6 Neighbor oblivious partial link reversal

```
1: loop

2: if F_i(h) = \emptyset then

3: t_i \leftarrow t_i + 1

4: h_i(t_i) \leftarrow z(t_i) - h_i(t_i - 1)

5: end if

6: end loop
```

Remarks 4.1: Assume that node i is stuck. The height update (Line 4) along with the definition of sequence $\{z(0), z(1), \ldots\}$ ensure that i's updated height surpasses the heights of those neighbors that have not updated states since i's last update, but still falls short of the heights of other neighbors. A similar behavior is ensured by the third components the states (e.g., $(-1)^{t_i}i$ in a_i) when two neighbors have identical initial heights.

As discussed before, node i broadcasts a hello packet to determine if it is stuck. However, it does not need to know its neighbors' states to perform updates (see Lines 3, 4 in Algorithm 6). Also, whenever it updates its state, it broadcasts its new state to facilitate its neighbors updating the corresponding link directions. Other nodes also independently and asynchronously execute similar algorithms. In particular, multiple nodes can update at the same time. The following properties of this algorithm are similar to those of Algorithm 3.

Proposition 4.1: (a) The height of a node i is explicitly

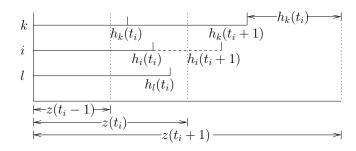


Fig. 2. An illustration of Algorithm 6 at a stuck node i. Note that $t_l=t_i$ while $t_k=t_i+1$. Node k has reversed its link to i after i's last update but node l has not. When node i updates its state, it reverse the link to l but not the one to k.

given by

$$h_i(t_i) = \begin{cases} \sum_{l=1}^{t_i/2} z(2l-1) + h_i(0) & \text{if } t_i \text{ is even,} \\ \sum_{l=1}^{(t_i-1)/2} z(2l) - h_i(0) & \text{if } t_i \text{ is odd.} \end{cases}$$

- (b) For any node i, and $t_i \in \mathbb{Z}_{++}$, we have $z(t_i-1) < h_i(t_i) < z(t_i)$.
- (c) For any two neighbors i and j, and $t_i, t_j \in \mathbb{Z}_+$ we have the following implication

$$t_i > t_j \Rightarrow h_i(t_i) > h_j(t_j).$$

- (d) For any two neighbors i and j, at any stage of the algorithm, we have $0 \le |t_i t_j| \le 1$.
- (e) For any node $i, t_i \leq N$ at any stage of the algorithm. Proof: See Appendix C.

Remarks 4.2: 1) As in the case of Algorithm 3, for any node, the number of state updates is upper bounded by N, and hence the state size is upper bounded by $\log(N)$.

2) Propositions 4.1(c) implies that the forwarding set of node *i* can be alternatively defined as

$$F_i(a) = \{ j \in C_i | \ a_j < a_i \},\$$

where $a = (a_1, \ldots, a_N)$ are the nodes' states.

Proposition 4.2: In Algorithm 6, a stuck node i reverses the directions of only those of its links that have not been reversed since i's last update. If every link to node i has been reversed after i's last update, it performs two successive updates to reverse the directions of all its links.

Proof: Since node i is stuck, for any node $j \in C_i$,

$$(h_i(t_i), (-1)^{t_i}i) > (h_i(t_i), (-1)^{t_i}i).$$

By virtue of Propositions 4.1(c)-(d), we also have either $t_j = t_i$ or $t_j = t_i + 1$. See Figure 2 for an illustration.

(i) Consider $t_j=t_i$. This is the case of node l in Figure 2. We claim that node j has not reversed its link to i since i's last update. If $t_i=0$, this claim is trivially valid. If $t_i\geq 1$, we will show that the progression of updates when both nodes' t-states were t_i-1 was: node j updated, then node i updated. As a consequence, again, our claim will be valid. To see the progression of updates, observe that if $h_j(t_j)=h_i(t_i)$, then $(-1)^{t_j}j>(-1)^{t_i}i$. Thus, by sign flipping, at t-states t_i-1

 t_j-1 , $(-1)^{t_j-1}j<(-1)^{t_i-1}i$. Also, by the form of the updates at t_i-1 , $h_j(t_j-1)=h_i(t_i-1)$. So the link was from node i to node j and it must be j that updated first. On the other hand, if $h_j(t_j)>h_i(t_i)$, then

$$h_j(t_j - 1) = z(t_j) - h_j(t_j)$$

 $< z(t_i) - h_i(t_i)$
 $= h_i(t_i - 1).$

Again we conclude that the link was from i to j, and it must be j that updated first. This establishes the claimed progression of states.

Continuing with the case, when node i now makes an update, it moves to t-state $t_j + 1$. Hence the link is from i to j after the update.

(ii) Consider $t_j=t_i+1$. This is the case of node k in Figure 2. We claim that node j has reversed its link to i after i's last update. Were it not the case, node i's t-state immediately prior to its last update would have been $t_i-1=t_j-2$ which contradicts Proposition 4.1(d).

Moreover, when node j's t-state was $t_j - 1 = t_i$, it must have been the case that

$$(h_i(t_i-1),(-1)^{t_i-1}j)<(h_i(t_i),(-1)^{t_i}i).$$

If $h_j(t_j-1)=h_i(t_i)$, then $(-1)^{t_j-1}j<(-1)^{t_i}i$. Thus, by sign flipping, at t-states $t_i+1=t_j$, $(-1)^{t_j}j>(-1)^{t_i+1}i$. Also, $h_j(t_j)=h_i(t_i+1)$. So, even after node i makes an updates and moves to t-state t_i+1 , the link continues to be from j to i. If $h_j(t_j-1)< h_i(t_i)$, then

$$h_j(t_j) = z(t_j) - h_j(t_j - 1)$$

> $z(t_i + 1) - h_i(t_i)$
= $h_i(t_i + 1)$.

Again, even after node i makes an updates and moves to t-state $t_i + 1$, the link continues to be from j to i. This proves the first part of the proposition.

Finally, suppose that every neighbor of node i has reversed its link to i after i's last update. Then, as shown above, $t_j = t_i + 1$ for all $j \in C_i$. Again as argued above, if node i updates its state, it does not reverse any of its links, i.e., it is still stuck. Thus it performs one more update. After this update its t-state is $t_i + 2$ which exceeds t_j for all $j \in C_i$. So all its links are reversed.

Remarks 4.3: For a stuck node, if all its neighbors have reversed the corresponding links after its last update, it takes two iteration to reverse all the incoming links. This is unlike Algorithm 2 which needs only one iteration.

Proposition 4.3: Algorithm 6 can be embedded within the GB algorithms framework. Thus it inherits the properties in Proposition 2.1.

B. Two-Bit Partial Link Reversal

In Algorithm 6, nodes' t-states grow as they update. We now give a modification of Algorithm 6 that uses only two bits for t-state and does not update heights. To do this we exploit the fact that for any two neighbors i and j, the link

direction is entirely governed by $t_i, t_j, h_i(0)$ and $h_j(0)$. More precisely, the link is directed from i to j if and only if either $t_i > t_j$, or $t_i = t_j$ and $(-1)^{t_i}(h_i(0), i) > (-1)^{t_j}(h_j(0), j)$. Thus t-states along with the initial heights suffice to determine link orientations. Moreover, since at any stage t_i and t_j are either same or adjacent integers (Proposition 4.1(d)), we need only two bits to describe their order. Specifically, if we define τ -states for all the nodes as in Section III-B, we obtain

$$t_i > t_j \Longleftrightarrow \tau_i > \tau_j$$
.

As before, for node i, τ_i is referred to as its τ -state. Following the above discussion, we can redefine the forwarding set of node i as

$$F_i(\tau) = \{ j \in C_i | \ \tau_j < \tau_i \text{ or } (\tau_j = \tau_i$$
 and $(-1)^{\tau_i} (h_i(0), i) > (-1)^{\tau_j} (h_j(0), j) \},$

where $\tau = (\tau_1, \dots, \tau_N)$. We are thus led to the following two bit version of the partial link reversal algorithm. Node i updates its states as follows.

Algorithm 7 Two bit partial link reversal

1: loop
2: if
$$F_i(\tau) = \emptyset$$
 then
3: $\tau_i \leftarrow (\tau_i + 1) \mod 4$
4: end if
5: end loop

Following are the key properties of this algorithm.

Proposition 4.4: (a) In Algorithm 7, a stuck node *i* reverses the directions of only those of its links that have not been reversed since *i*'s last update. If every link to node *i* has been reversed after *i*'s last update, it performs two successive updates to reverse the directions of all its links.

(b) Algorithm 7 exhibits the properties in Proposition 2.1.

Proof: (a) Following Proposition 4.1(d) and the definition of τ -states, for any node $j \in C_i$, we have either $\tau_j = \tau_i$ or $\tau_j = (\tau_i + 1) \mod 4$.

(i) Consider $\tau_j=\tau_i$. We claim that node j has not reversed its link to i since i's last update. If neither i nor j has ever made an update, this claim is trivially valid. If both of them have made updates, by Proposition 4.1(d), it cannot be that one of them made two updates without the other updating. So both must have been at $(\tau_i-1) \mod 4$ at some point of time. We will show that the progression of updates when both nodes' τ -states were $\tau_i-1 \mod 4$ was: node j updated, then node i updated. As a consequence, again, our claim is valid. To see the progression of updates, observe that

$$(-1)^{\tau_j}(h_i(0), j) > (-1)^{\tau_i}(h_i(0), i).$$

Thus, by sign flipping, at the nodes' immediately prior τ -states, the inequality was in reverse direction. So the link was from node i to node j and it must be j that updated first.

Continuing with the case, when node i makes an update, it moves to τ -state $(\tau_j+1) \mod 4$. Hence the link is from i to j after the update.

(ii) Consider $\tau_j = (\tau_i + 1) \mod 4$. We claim that node j has reversed its link to i after i's last update. Were it not the case, node i's τ -state immediately prior to its last update would have been $(\tau_i - 1) \mod 4 = (\tau_j - 2) \mod 4$ which contradicts the fact that at any stage τ_i and τ_j assume either same or adjacent values.

Moreover, when node j's τ -state was $(\tau_j - 1) \mod 4 = \tau_i$, it must have been the case that

$$(-1)^{\tau_i}(h_i(0),j) < (-1)^{\tau_i}(h_i(0),i).$$

Thus, by sign flipping, at τ -states $(\tau_i + 1) \mod 4 = \tau_i$,

$$(-1)^{\tau_j}(h_j(0),j) > (-1)^{\tau_j}(h_i(0),i).$$

So, even after node i makes an update and moves to τ -state $(\tau_i + 1) \mod 4$, the link continues to be from j to i.

Finally, suppose that every neighbor of node i has reversed its link to i after i's last update. Then, by the arguments above, $\tau_j = (\tau_i + 1) \mod 4$ for all $j \in C_i$. Also, if node i updates its state once, it does not reverse any of its links, i.e., it is still stuck. Thus it performs one more update. After this update its τ -state is $(\tau_i + 2) \mod 4$ which exceeds τ_j for all $j \in C_i$. So all its links are reversed.

(b) The proof is identical to that of Proposition 3.4(b).

V. CONCLUSION

We proposed neighbor oblivious link reversal (NOLR) schemes to get a destination oriented network out of the local minimum condition in geographic routing. Our algorithms fall within the general class of GB algorithms [1]. We then argued that both the algorithms, GB and NOLR, may suffer the problem of state storage overflow. This led us to modify the NOLR algorithms to obtain one bit full link reversal and two bit partial link reversal algorithms. The finite state algorithms inherit all the properties of NOLR algorithms which in turn inherit the properties of GB algorithms, and are pragmatic link reversal solutions to convert a destination-disoriented DAG to a destination-oriented DAG.

The property $|t_i - t_j| \le 1$ at every stage for all pairs of neighboring nodes is crucial for getting the finite state version of our NOLR algorithms. If addition of new nodes or links to the existing graph is allowed, this property could be violated. If full t-states (instead of only τ -states) are maintained, then since Algorithms 3 and 6 belong to the class of GB algorithms, they continue to exhibit the properties in Proposition 2.1. However, Algorithm 3 does not execute a full link reversal, and similarly, Algorithm 6 does not execute a partial link reversal. Furthermore, the finite state algorithms are not robust to addition of new nodes or links because the newly added nodes may not be able to take up a state consistent with the above property, or the DAG may be burdened by cycles.

APPENDIX A PROOFS OF PROPOSITIONS 3.3 AND 4.3

For all $i \in [N]$, let A_i be the set of feasible states of node i. Define $v = (a_1, a_2, \dots, a_N)$. Let V be the set of all such

N-tuples. For each $v \in V$, let $S(v) \subset [N]$ denote the set of stuck nodes.

$$S(v) = \{i \in [N] | a_i > a_i \text{ for all } j \in C_i\}.$$

We consider iterative algorithms of the form

$$v \leftarrow \overline{v} \in M(v),$$

where $M(\cdot)$ is a point-to-set mapping; $M(v) \subset V$ for all $v \in V$. In the following we show that the proposed neighbor oblivious link reversal algorithms satisfy the assumptions of GB algorithms.

First, we consider Algorithm 3. Recall that $a_i = (t_i, h_i(t_i), i)$ in this case.

(A.1): Define $g_i: V \to A_i, i = 1, \dots, N$ as

$$g_i(v) = \begin{cases} (t_i + 1, h_i(t_i) + h_{\max}, i) & \text{if } i \in S(v), \\ (t_i, h_i(t_i), i) & \text{if } i \notin S(v). \end{cases}$$

The set M(v) is then given by

$$M(v) = \begin{cases} \{v\} & \text{if } S(v) = \emptyset, \\ \{\overline{v} = (\overline{a}_1, \dots, \overline{a}_N) | \ \overline{v} \neq v \text{ and } \\ & \text{either } \overline{a}_i = a_i \text{ or } \overline{a}_i = g_i(v) \text{ for all } i \in [N]\} \\ & \text{if } S(v) \neq \emptyset. \end{cases}$$

(A.2): From (A.1), it is clear that for each $v = (a_1, ..., a_N)$ and i = 1, ..., N, the functions $g_i(\cdot)$ satisfy

$$g_i(v) > a_i \text{ if } i \in S(v),$$

and $g_i(v) = a_i \text{ if } i \notin S(v).$

Furthermore, for each $i=1,\ldots,N,$ $g_i(v)$ depends only on a_i and $\{a_j|\ j\in C_i\}$; the latter states determine if $i\in S(v)$ or otherwise.

(A.3): Consider a node i and a sequence $\{v^k\} \subset V$ for which $i \in S(v^k)$ for an infinite number of indices k. If r is one of these indices, $g_i(v^r) - a_i^r \geq (1, h_{\max}, 0)$, otherwise $g_i(v^r) - a_i^r = 0$. Hence the sequence

$$\left\{ a_i^0 + \sum_{r=0}^k [g_i(v^r) - a_i^r] \right\}$$

is unbounded in A_i . Next, we consider Algorithm 6. Recall that $a_i = (t_i, h_i(t_i), (-1)^{t_i}i)$ in this case. We define $g_i : V \to A_i$ as

$$g_i(v) = \begin{cases} (t_i + 1, z(t_i + 1) - h_i(t_i), (-1)^{t_i + 1}i) & \text{if } i \in S(v), \\ (t_i, h_i(t_i), (-1)^{t_i}i) & \text{if } i \notin S(v). \end{cases}$$

Again, it is easy to check that Assumptions (A.1)-(A.3) hold. Gafni and Bertsekas [1] show that if the communication graph is connected and an algorithm satisfies Assumptions (A.1)-(A.3), then Proposition 2.1 holds for the algorithm. This concludes the proof of Propositions 3.3 and 4.3.

APPENDIX B PROOF OF PROPOSITION 3.1

- (a) This follows immediately from the height update rule (Line 4 in Algorithm 3).
- (b) This follows from (a) and $0 < h_i(0) \le h_{\text{max}}$.
- (c) The implication holds because $h_i(t_i) > t_i h_{\max}$ and $h_i(t_i) \leq (t_i + 1) h_{\max}$ (see (b)).

- (d) Without loss of generality, assume $t_i \geq t_j$. We claim that $t_i \leq t_j + 1$. We prove the claim via contradiction. Suppose $t_i > t_j + 1$. Node i must have reached this state through $t_j + 1$ because t_i is initialized to zero and is incremented by one each time node i updates its state. When node i's t-state was $t_j + 1$, from (c) $h_i(t_j + 1) > h_j(t_j)$, and therefore it had an outgoing link to node j. Thus, i would not have updated its t-state to $t_j + 2$ or higher. This contradicts our supposition, and proves the claim.
- (e) Observe that any one hop neighbor of the destination never updates its heights; it always has an outgoing link to the destination. Consequently, for any such node, say node i, $t_i=0$ at any stage of the algorithm. Now, assume that for a node j, $t_j>N$ at some stage. Then, there is pair of neighbors k and k such that k-k such that k node k node k such that k node k node k such that k node k n

APPENDIX C PROOF OF PROPOSITION 4.1

(a) We first obtain a recursion on $h_i(t_i)$ using the height update rule (Line 4 in Algorithm 6). For any $t_i \geq 2$,

$$h_i(t_i) = z(t_i) - h_i(t_i - 1)$$

= $2z(t_i - 1) - (z(t_i - 1) - h_i(t_i - 2))$
= $z(t_i - 1) + h_i(t_i - 2).$

Successive applications of this recursion leads to expression for the case when t_i is even. If we also use that $h_i(1) = z(1) - h_i(0)$, we get the expression for the case when t_i is odd.

(b) We prove the inequalities by induction on t_i . For $t_i = 1$,

$$0 < h_i(1) < z(1)$$
.

Now, assume that $0 < h_i(t_i) < z(t_i)$ for some $t_i \in \mathbb{Z}_{++}$. From the height update rule (Line 4 in Algorithm 6),

$$h_i(t_i + 1) = z(t_i + 1) - h_i(t_i)$$

= $2z(t_i) - h_i(t_i)$
> $z(t_i)$,

where the inequality holds because $h_i(t_i) < z(t_i)$. Also, $0 < h_i(t_i)$ implies that $h_i(t_i+1) < z(t_i+1)$. This completes the induction, and shows that the inequalities hold for all $t_i \in \mathbb{Z}_{++}$.

- (c) The implication holds because $h_j(t_j) < z(t_j)$, $h_i(t_i) > z(t_i-1)$ and z(t) is increasing in t.
- (d) The proof is identical to that of Proposition 3.1(d).
- (e) The proof is identical to that of Proposition 3.1(e).

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