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On the application of the Exp-function method to the KP equation for *N*-soliton solutions

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ABSTRACT

We observe that the form of the Kadomstev–Petviashvili equation studied by Yu (2011) [S. Yu, *N*-soliton solutions of the KP equation by Exp-function method, Appl. Math. Comput. (2011) doi:10.1016/j.amc.2010.12.095] is incorrect. We claim that the *N*-soliton solutions obtained by means of the basic Exp-function method and some of its known generalizations do not satisfy the equation considered. We emphasize that Yu's results (except only one) cannot be solutions of the correct form of the Kadomstev–Petviashvili equation. In addition, we provide some correct results using the same approach.

As is well known, the Kadomstev–Petviashvili equation (or simply the KP equation) [1] is a completely integrable fourth-order nonlinear partial differential equation. Being two spatial dimensional analog of the one-dimensional Korteweg-de Vries equation (or briefly the KdV equation), it is of the form

$$(u_t + \alpha u u_x + \beta u_{xxx})_x + \gamma u_{yy} = 0, \tag{1}$$

where the coefficients α, β, and γ can be chosen appropriately. In the literature, one can encounter some distinct forms of the KP equation such as KPI equation (the case $\alpha = 6$, $\beta = 1$, $\gamma = -3$ of (1)) and KPII equation (the case $\alpha = 6$, $\beta = 1$, $\gamma = 3$ of (1)), etc.

In a recent study, Yu [2] investigated a third-order nonlinear partial differential equation, believing to be the KP equation, for constructing multi-soliton solutions by the basic Exp-function method [3] and its known generalizations. We make the following critics on the results of Yu [2]:

(1) The author considers the KP equation as

$$(u_t + 6uu_x + u_{xxx}) \pm u_{yy} = 0 \tag{2}$$

and claims that Eq. (2) admits the one-soliton solution (the formula (9) in [2])

$$u(x, y, t) = \frac{2b_1 k_1^2 e^{\xi_1}}{(1 + b_1 e^{\xi_1})^2}, \quad \xi_1 = k_1 x + r_1 y - \frac{r_1^2 + k_1^4}{k_1} t, \tag{3}$$

where b_1, k_1, and r_1 remain arbitrary. But, the formula (2) describes two equations which differ in the sign of their u_{yy}-terms. The author does not mention which equation in (2) admits the function (3) as a solution. Moreover, the direct substitution of (3) into (2) results in the expression

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$$\pm\frac{2b_{10}k_1r_1^2\left((k_1-1)e^{\frac{2(k_1^4+r_1^2)}{k_1}t}-4b_{10}k_1e^{k_1x+r_1y+\frac{(k_1^4+r_1^2)}{k_1}t}+b_{10}^2(1+k_1)e^{2k_1x+2r_1y}\right)e^{k_1x+r_1y+\frac{(k_1^4+r_1^2)}{k_1}t}}{\left(e^{\frac{(k_1^4+r_1^2)}{k_1}t}+b_{10}e^{k_1x+r_1y}\right)^4}, \tag{4}$$

which is not zero in the general case. Thus, the function (3) cannot be a solution of Eq. (2).

By the same token, it can be shown that the two-soliton (the formula (10) in [2]) and the three-soliton (the formula (14) solutions in [2]) obtained by the author do not satisfy Eq. (2). We have verified this fact by direct substitution as well. However, to make the discussion short, we would like to have the details reserve here.

(2) It is evident that Eq. (2) is an incorrect form of the KP equation. From an optimistic point of view, let us assume that Eq. (2) was misprinted in [2] and the author intended to study the correct form of the KP equation which reads

$$(u_t+6uu_x+u_{xxx})_x \pm u_{yy}=0. \tag{5}$$

In this case, we figured out that the function (3) obtained by the author does satisfy Eq. (5) with the "+" sign in the term u_{yy} only. On the other hand, we have also verified by direct substitution that the two-soliton (the formula (10) in [2]) and the three-soliton (the formula (14) in [2]) solutions derived by the author do not satisfy the KP equation (5). Again, we skip the details for brevity.

(3) For the sake of completing the work initiated by the author, we analyzed the correct form of the KP equation (5) for multi-soliton solutions by means of the basic Exp-function method and its known generalizations. We obtained the following results:

Eq. (5) admits the one-soliton solution

$$u^{\pm}(x,y,t)=\frac{-2b_1k_1^2e^{\xi_1}}{(1+b_1e^{\xi_1})^2}, \quad \xi_1=k_1x+r_1y+\frac{k_1^4\mp r_1^2}{k_1}t, \tag{6}$$

where b_1, k_1, and r_1 remain arbitrary; and the two-soliton solution

$$u^{\pm}(x,y,t)=\frac{a_{10}e^{\xi_1}+a_{01}e^{\xi_2}+a_{11}e^{\xi_1+\xi_2}+a_{21}e^{2\xi_1+\xi_2}+a_{12}e^{\xi_1+2\xi_2}}{(1+b_1e^{\xi_1}+b_2e^{\xi_2}+b_3e^{\xi_1+\xi_2})^2}, \quad \xi_1=k_1x+r_1y+c_1t, \quad \xi_2=k_2x+r_2y+c_2t, \tag{7}$$

where

$$c_1=\frac{k_1^4\mp r_1^2}{k_1}, \quad c_2=\frac{k_2^4\mp r_2^2}{k_2}, \quad a_{01}=-2b_2k_2^2, \quad a_{10}=-2b_1k_1^2, \tag{8}$$

$$a_{21}=2b_1^2b_2k_2^2\left(-1+\frac{12k_1^3k_2^3}{k_2^2(3k_1^2(k_1+k_2)^2\pm r_1^2)\mp 2k_1k_2r_1r_2\pm k_1^2r_2^2}\right), \tag{9}$$

$$a_{11}=4b_1b_2\left(-k_2^2-k_1^2\left(1-\frac{6k_1k_2^3(k_1+k_2)^2}{k_2^2(3k_1^2(k_1+k_2)^2\pm r_1^2)\mp 2k_1k_2r_1r_2\pm k_1^2r_2^2}\right)\right), \tag{10}$$

$$a_{12}=2b_1b_2^2k_1^2\left(-1+\frac{12k_1^3k_2^3}{k_2^2(3k_1^2(k_1+k_2)^2\pm r_1^2)\mp 2k_1k_2r_1r_2\pm k_1^2r_2^2}\right), \tag{11}$$

$$b_3=b_1b_2\left(1-\frac{12k_1^3k_2^3}{k_2^2(3k_1^2(k_1+k_2)^2\pm r_1^2)\mp 2k_1k_2r_1r_2\pm k_1^2r_2^2}\right), \tag{12}$$

while b_1, b_2, k_1, k_2, r_1, and r_2 remain arbitrary. We note that the signs " $\pm$ " and " $\mp$ " in (5)–(12) should be taken into account in a vertical order.

Now, we can observe that our solution set (8)–(12) of the resulting algebraic system is distinct than the one obtained by the author [2] (the expressions (11)–(13) in there). Unfortunately, The coefficients a_{10}, a_{01}, a_{11}, a_{21}, and a_{12} in [2] are expressed in terms of the constants b_1, b_2, k_1, and k_2 only, namely, the angular wave numbers r_1 and r_2 are absent.

(4) We attempted to derive a three-soliton solution of Eq. (5) in the form

$$u(x,y,t)=\frac{v_1(\xi_1,\xi_2,\xi_3)}{v_2(\xi_1,\xi_2,\xi_3)}, \tag{13}$$

where $\xi_i=k_ix+r_iy+c_it,\ i=1,2,3$, and

$$\begin{aligned}v_1(\xi_1,\xi_2,\xi_3) = {} & a_{100}\exp(\xi_1)+a_{010}\exp(\xi_2)+a_{001}\exp(\xi_3)+a_{110}\exp(\xi_1+\xi_2)+a_{101}\exp(\xi_1+\xi_3)+a_{011}\exp(\xi_2+\xi_3)\\ & +a_{120}\exp(\xi_1+2\xi_2)+a_{102}\exp(\xi_1+2\xi_3)+a_{012}\exp(\xi_2+2\xi_3)+a_{210}\exp(2\xi_1+\xi_2)+a_{201}\exp(2\xi_1+\xi_3)\\ & +a_{021}\exp(2\xi_2+\xi_3)+a_{111}\exp(\xi_1+\xi_2+\xi_3)+a_{211}\exp(2\xi_1+\xi_2+\xi_3)+a_{121}\exp(\xi_1+2\xi_2+\xi_3)+a_{112}\\ & \times\exp(\xi_1+\xi_2+2\xi_3)+a_{221}\exp(2\xi_1+2\xi_2+\xi_3)+a_{122}\exp(\xi_1+2\xi_2+2\xi_3)+a_{212}\exp(2\xi_1+\xi_2+2\xi_3),\end{aligned}$$

$$\begin{aligned}v_2(\xi_1,\xi_2,\xi_3) = {} & (1+b_1\exp(\xi_1)+b_2\exp(\xi_2)+b_3\exp(\xi_3)+b_4\exp(\xi_1+\xi_2)+b_5\exp(\xi_1+\xi_3)+b_6\exp(\xi_2+\xi_3)\\ & +b_7\exp(\xi_1+\xi_2+\xi_3))^2,\end{aligned}$$

but the calculation procedure becomes tedious and unmanageable even with the computer help.

However, it is interesting that, applying the ansatz (13) to the equation considered in [2], the author solved the resulting algebraic system without presenting a dispersion relation. But, by induction, we do expect a dispersion relation of the form

$$c_i = \frac{k_i^4 \mp r_i^2}{k_i}, \quad i = 1,2,3 \tag{14}$$

if we apply the ansatz (13) to Eq. (5) (or to Eq. (2)). Moreover, it is unfortunate that the coefficients a_{ijk} for the three-soliton solution in [2] are expressed in terms of the constants b_1, b_2, b_3, k_1, k_2, and k_3 only, namely, the angular wave numbers r_1, r_2, and r_3 are invisible.

(5) It is obvious that the uniform formula provided by Yu [2] (the expression (18) in there) for N-soliton solutions for the KP equation is misleading. Meanwhile, it is also worth to bring two other important studies [4,5] on the KP equations (for multi-wave solutions) to the reader's attention here. These works represent good examples to get a feel about what we expect on the structure of multi-wave solutions of the KP equations.

(6) It seems that the one-soliton solution (6) and the two-soliton solution (7) are more general than the ones obtained in [4] because they contain the extra constants b_1 and b_2. However, the coefficients (8)–(12) of the two-soliton solution are very complicated. In fact, Wazwaz [4] obtained the coefficients in more systematic way using Hirota's direct method. Hence, in terms of Hirota's direct method, it is easier to generalize the coefficients for $N(\geqslant 3)$-soliton solutions.

Conclusion

We would like to mention here that the basic Exp-function method and its modified versions are almost impossible to handle without a computer algebra system. One should always study a problem in eagle-eyed solving mode, and check the results with the computer interactively. Recently, Parkes [6–8] and Kudryashov [9–13] have been continuously alerting the authors for equivalent, disguised, and incorrect results. Unfortunately, a number of cases have appeared in the literature when solutions derived by the basic Exp-function method are irrelevant [14–17]. It is observed that the application of the method can be redundant [18]. The method has also attracted a considerable amount of criticism [19–22]. We think that Yu [2] made another contribution to the list of poor applications of the basic Exp-function method.

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