Quantized Model-free Adaptive Iterative Learning Bipartite Consensus Tracking for Unknown Nonlinear Multi-agent Systems

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Abstract: This paper considers the data quantization problem for a class of unknown nonaffine nonlinear discrete-time multi-agent systems (MASs) under the repetitive operations to perform bipartite consensus tracking tasks. Here, each agent's dynamic knowledge is not required to design the controller, only employing the input/output data of MAS, where the data is decoded by the logarithmic quantizer before transmitted. Moreover, both cooperative and competitive relationships among agents are considered. To perform bipartite consensus tracking tasks for MAS, a quantized distributed model-free adaptive iterative learning bipartite consensus control (QDMFAILBC) approach is proposed by using the dynamic linearization technology and the sector bound approach. The stability condition of the proposed scheme is presented by the strictly proof, and the effects of data quantization are also analyzed. It shows that data quantization does not destroy the stability of MASs, and the proposed approach can guarantee that the bipartite consensus tracking errors convergence to zero, thus the data quantization causes the convergence rate to slow down. Furthermore, the result is extended to time-varying switching topologies, and three simulations further validate the effectiveness of the designed control method.

1 Introduction

As one of the core cooperative control problems, the consensus of Multi-agent systems (MASs) has attracted considerable attention from the research. Numerous effective approaches have been developed for real-world applications in the past decade. The state-of-the-art methods of consensus issues in MASs can be found in the [1]-[3]. Among these results, there is a common assumption that the relationship among agents is collaborative. However, collaborative and antagonistic relationships are coexistence. For example, in the multi-robot systems, the relationship between a robot and its teammates is collaborative, but between it and its antagonistic robots is antagonistic. In biological systems, a pair of genes are viewed as activators when they are in cooperative interaction, and as inhibitors when in competitive interactions [4].

Both the two relationships among agents are first investigated in the works of Altafini [5] and the concept of bipartite consensus (BC) is presented, where all agents are divided into two alliances so that all agents in each alliance converge to a common value, while the two alliances are separated in reverse. To describe the coopetition interactions among agents, a signed graph is introduced by Altafini, where if two agents exist with cooperative relationships, the edge between them is positive; otherwise, the edge is negative. Inspired by the results of [5], many interesting approaches of BC have been developed such as Qin et al. [6] investigate the input saturation problem for generic linear MASs to implement the semi-global BC tasks, Cai et al. [7] consider the event triggering BC problem of linear MASs with input time delay, the robust finite-time and fixed-time BC issues of MASs are investigated in the works of Guo et al. in [8]. It is noted that the aforementioned BC control methods are researched with the assumption of known system dynamics, and these approaches are called model-based control (MBC) methods. It is well known that modern industrial processes have become more and more complex. It is challenging to identify an accurate mathematical model for each agent, especially when the scale of MASs is massive. Moreover, almost all the dynamics of the agents consist of nonlinearities. Hence, the research of BC problems for unknown nonlinear MASs without employing the knowledge of the dynamics is a changing topic.

With this concern, many researchers seek for model-free solutions for MASs, which is named as model-free control or data-driven control (DDC). The DDC concept is an important and necessary complement for modern control theory [9], and it can utilize the on-line/off-line input/output (I/O) data available or other knowledge instead of the mathematical model to design the control protocol under reasonable assumption [10]. Several excellent results of MASs have been conducted as follows. Bu et al. propose a model-free adaptive control for MASs to achieve consensus control in [11]. A based-on neutral network consensus control is proposed by Zhang et al. in [12], and Long et al. [13] employ the theories of low-gain feedback theory and Q-learning to design a data-driven consensus control approaches for MASs with saturation phenomenon. For more detailed DDC approaches of MASs related literature, the readers are referred to [14]-[18] and references therein.

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Among these mentioned approaches in [1]-[17], researches only considered the MASs to achieve consensus objective when operation time tends to infinite. Nevertheless, in some practical industrial process, the operation process is repeatable or periodic, which requires that the error is declined to the origin after finite periodic, and the next novel periodic will not existing any errors, for instance, IC welding, wafer manufacturing, chemical reactions and so on. To satisfy these industrial requirements, Iterative Learning Control (ILC) is one of the excellent schemes. The history of ILC can be traced back to the works of Arimoto et al. [19]. Then, Ahn et al. [20] successfully introduce ILC into formulating controller for MASs, and Bu et al. in [21] propose a DDC ILC approach for MASs. Inspired by [21], several effective schemes are developed. To name a few, the unknown disturbance problems for MASs to achieve time-varying formation and consensus tracking are investigated in [22] and [23], respectively. The constant learning gain and iteration-time-varying learning gain of the ILC approach for MASs are investigated in [24]. [25] studies the output saturation problem for MASs to implement terminal consensus control. Especially, [26] introduces the space dimension for the ILC and proposes a 3D-AILC for MASs to achieve a fast and precise iteration-varying formation tracking. Some other interesting results could find in [27] and [28]. However, to the best of our knowledge, consensus tracking control has not been fully studied for unknown nonlinear MASs with quantized data, especially for MASs to implement BC tracking tasks under the repeatable operation circumstances.

In many practical engineering systems, since digital communication advantages over analog communication, which can reduce the transmission burden, digital communication will be preferentially selected for devices, where signals are often quantized before being transmitted through communication. Unfortunately, it introduces quantization error, which will influence the convergence property of controlled plant such that the stability of the controlled system will be broken. Thus, the quantization problem, a significant topic of modern control, has received considerable attention. A fully distributed protocol with dynamic coupling gain is designed for MASs to implement the BC tracking task in the works of Wu et al. [29]. Ding et al. [30] propose the distributed estimator-based control algorithm to deal with parametric uncertainties, input disturbances, and networked robotic systems' quantized-data to achieve BC. A quantized adaptive finite-time BC problem for MASs is investigated in [31] by Wu et al. Liu et al. [32] propose an event-triggered fuzzy adaptive quantized control approach for stochastic nonlinear nonaffine pure-feedback MASs. Besides, the results of quantized control in recent years can be found in [33]-[34]. There are few DDC results for MASs under data quantization to the best of our observation, only Bu et al. researching this problem for a single system in [35] and [36]. Hence, it is more difficult to investigate quantization problems for MASs than a single system, and how to design an appropriate DDC for MASs with quantized data is meaningful work.

Motivated by the above discussions, the problem of quantized distributed model-free adaptive iterative learning bipartite consensus tracking control is addressed for unknown nonaffine nonlinear discrete-time MASs with quantized information under a repeatable operating environment. Here, the compact form dynamic linearization (CFDL) scheme is employed along the iteration axis by using the so-called pseudo-partial derivative (PPD) of the nonlinear system [21]-[28]. Meanwhile, the signed graph and sector bound theory are introduced, and then, a QDMFAILBC scheme is proposed only depending on the agents' quantized data. Compared with the existing results of MASs, the main contributions of this article are summarized as follows.

(1) A compact form dynamic linearization model along the iteration axis is established by using PPD approaches. Compared with MBC approaches [29]-[32], the model structure is more straightforward, reducing computation burdens. Also, it is an essential and necessary complement to modern control theory.

(2) The proposed QDMFAILBC approach is formulated by employing the incomplete I/O data because of the data quantization. Comparing with the existing DDC ILC approaches [21]-[28], the designed scheme can use fewer data to ensure the convergence of MASs, that is, it reduces the consumption of communication resources.

(3) The relationship between the convergence rate and the quantization level is estimated, which can provide a guideline for digital communication.

(4) Both cooperative and competitive relationships of MASs with fixed and switching topologies are considered in the QDMFAILBC approach. Most of the existing methods [21]-[34] merely consider one or two elements of it. These differences bring some difficulties in the analysis of convergence of MASs.

The rest work of this paper is structured as follows. Several necessary preliminaries are presented in Section II. Section III introduces the distributed QDMFAILBC algorithm and gives rigorous mathematical proof. Section IV extends the design to MASs with switching topologies. The three simulation experiments are shown in Section V. Finally, conclusions and future work are provided in Section VI.

2 Preliminary and Problem Formation

Signed Graph Theory

In this study, R, $R^{N\times N}$, and I denote the set of real numbers, $N\times N$ real matrices, and identity matrix with an appropriate dimension, respectively. $\|\Theta\|$ denotes the Euclidean norm of a vector $\Theta\in R$. the diagonal matrix is expressed by $diag(\bullet)$. Here, a signed digraph $\bar{\mathcal{G}}_{\mathcal{S}}=\mathcal{G}_{\mathcal{S}}\cup\{0\}$ is employed to describe the coopetition communication network among N agents and 1 virtual leader, where $\mathcal{G}_{\mathcal{S}}=(\mathcal{V},\mathcal{E},\mathcal{A})$ denotes the communication relationship between N follower agents. the set of agents is denoted as A_N . $\mathcal{V}=\{v_1,v_2,\dots,v_N\}$, $\mathcal{E}=\{(v_i,v_j)|v_i,v_j\in\mathcal{V}\}\subseteq\mathcal{V}\times\mathcal{V}$, and $\mathcal{A}=[a_{ij}]\in R^{N\times N}$ with $a_{ii}=0$ indicate the nodes set, edges set, and adjacency matrix, respectively. Moreover, if $a_{ij}\neq 0$, the agent i can receive the information from the agent j . $a_{ij}>0$ (or

$a_{ij} < 0$) represents that the relationship between agent i and j is collaborative (or antagonistic). The neighbors set of agent i, degree matrix, and the Laplacian matrix of $\mathcal{G}_{\mathcal{S}}$ are represented by $N_i = \{ j \mid j \neq i, (v_j, v_i) \in \mathcal{E} \}$, $\mathcal{D} = \{ d_1^{in}, d_2^{in}, \ldots, d_N^{in} \}$ with $d_i^{in} = \sum_{j \in N(i)} |a_{ij}|$, and $\mathcal{L} = -\mathcal{A} + \mathcal{D} \in R^{N \times N}$, separately. Furthermore, define a diagonal matrix $\mathcal{B} = diag\{ b_1, \ldots, b_N \} \in R^{N \times N}$ with $b_i \in \{0,1\}$ as an expression of the information transmission relationship between the leader and followers. $b_i = 1$ represents the agent i can directly receive the information from the leader. The direction of information transmission is directed such as (v_j, v_i) denotes the information flow from node v_i to node v_j and also a directed path could be obtained as $\{ (v_i, v_{k1}), (v_{k1}, v_{k2}), \ldots, (v_{km}, v_j) \}$. If the network $\bar{\mathcal{G}}_{\mathcal{S}}$ contains a spanning tree, the information can flow from the leader to any other followers. Meanwhile, the signed graph $\bar{\mathcal{G}}_{\mathcal{S}}$ is called structurally balanced [5]. The main character of structurally balanced graph is that existing a dichotomization of nodes $\mathcal{V}_1$ and $\mathcal{V}_2$ satisfy $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \varnothing$, where if agent i belongs to $\mathcal{V}_1$, $s_i = 1$; otherwise, $s_i = 0$.

In order to investigate time-varying switching topologies, let $\bar{\mathcal{G}}_{\mathcal{S}}(k)$ indicate a time-varying switching graph with a virtual leader, which is dependent on k, and the adjacency matrix, degree matrix and Laplacian matrix are expressed by $\mathcal{A}(k) = [a_{ij}(k)] \in R^{N \times N}$, $\mathcal{D}(k) = \{ d_1^{in}(k), d_2^{in}(k), \ldots, d_N^{in}(k) \}$ with $d_i^{in}(k) = \sum_{j \in N(i)} |a_{ij}(k)|$, and $\mathcal{L}(k) = -\mathcal{A}(k) + \mathcal{D}(k) \in R^{N \times N}$, respectively. Meanwhile, the corresponding matrix N_i and $\mathcal{B}$ become $N_i(k)$ and $\mathcal{B}(k)$. Moreover, the set of all directed graphs for the agents is expressed by $\bar{\mathcal{G}}_p = \{ \bar{\mathcal{G}}_1, \bar{\mathcal{G}}_2, \ldots, \bar{\mathcal{G}}_\kappa \}$, where $\kappa \in Z^+$ denotes the total number of possible interaction graphs.

Problem Formation

A class of SISO (simple-input-simple-output) nonaffine nonlinear discrete-time MASs with N agents is investigated and the nonlinear dynamics of the ith agent is described as:

$$
\begin{aligned}
y_i(l,k+1) = \; & f_i(y_i(l,k), \cdots, y_i(l,k-n_y), \\
& u_i(l,k), \cdots, u_i(l,k-n_u))
\end{aligned}, \tag{1}
$$

where $l = 0,1,2,\ldots$ and $k \in (1, 2, \ldots, T)$ are denoted as iteration number and the time interval. n_y, n_u are two unknown positive integers. $u_i(l,k) \in R$ and $y_i(l,k) \in R$ are control input and output of agent i with $i \in A_N$ at time instant k, respectively. $f_i(\bullet)$ denotes an unknown nonlinear function and the communication topology of MASs is expressed by $\bar{\mathcal{G}}_{\mathcal{S}}$.

In the following, some reasonable assumptions with respect to the agent's dynamics are given.

***Assumption 1*:** The partial derivative of $f_i(\bullet)$ with respect to the control input $u_i(l,k)$ is continuous.

***Assumption 2*:** Equation (1) satisfies the generalized Lipschitz condition, that is, $|\Delta y_i(l,k+1)| \leq r |\Delta u_i(l,k)|$ holds for all k, where r is a positive constant, $\Delta y_i(l,k+1) = y_i(l,k+1) - y_i(l-1,k+1)$ and $\Delta u_i(l,k) = u_i(l,k) - u_i(l-1,k) \neq 0$.

Remake 1: The Assumptions 1 and 2 are necessary assumptions of the proposed QDMFAILBC method and the reasonability of them has been discussed in [18], [21] and [22].

Lemma 1 ([26], [27]): Suppose the dynamics (1) satisfies Assumptions 1 and 2, then the following compact form dynamic linearization model can be obtained.

$$
\Delta y_i(l,k+1) = \Gamma_i(l,k) \Delta u_i(l,k), \tag{2}
$$

where $\Gamma_i(l,k)$ is named as pseudo-partial-derivative (PPD) and it is a time-varying parameter. $|\Gamma_i(l,k)| \leq r$ and input gain $|\Delta u_i(l,k)| \leq a$. r and a are small positive constant, which are dependent on the controlled plant.

Remake 2: From Equation (2) we can find that the PPD is only dependent on the I/O gain of the controlled plant, which is easy to be estimated. Here, the all nonlinear behaviors are fused on this simple parameter so that we only need to estimate the value sequences of PPD to construct the equivalent dynamic linear model of the controlled plant.

***Assumption 3*:** For all $k \in (1, 2, \ldots, T)$ and $l = 0,1,2,\ldots$, $\Gamma_i(l,k) > \bar{\sigma} > 0$ ($\Gamma_i(l,k) < -\bar{\sigma} < 0$) holds, where $\bar{\sigma}$ is an arbitrarily small positive constant. As the analysis in [18], [21], and [28], we also assume $\Gamma_i(l,k) > \bar{\sigma} > 0$.

Assumption 4: Suppose $\mathcal{G}_{\mathcal{S}}$ and $\mathcal{G}_p$ are strongly connected, that $\mathcal{L} + \mathcal{B}$ and $\mathcal{L}(k) + \mathcal{B}(k)$ are reducible matrices [37] with nonpositive off-diagonal elements [5].

Definition 1: The objective of the QDMFAILBC approach is to design an appropriate control protocol $u_i(l,k)$ for MASs to track the desired trajectory, where $u_i(l,k)$ is only dependent on I/O data of agent i and its neighbours. Here, each agent satisfies the following conditions.

$$
\lim_{k \to \infty} (y_i(l,k) - s_i y_0(l,k)) = 0, \tag{3}
$$

where $y_0(l,k)$ denotes the position of the virtual leader and $i \in A_N$. Let $e_i(l,k) = s_i y_0(l,k) - y_i(l,k)$ donate the BC tracking error.

***Definition 2*:** $\xi_i(l,k)$ denotes the distributed BC information measured or received of the agent i at lth iteration, which is defined as below.

$$
\begin{aligned}
\xi_i(l,k) = & \sum_{j \in N_i} |a_{ij}| (sign(a_{ij}) y_i(l,k) - y_j(l,k)) \\
& + b_i (s_i y_0(l,k) - y_i(l,k))
\end{aligned}, \tag{4}
$$

where $sign(\cdot)$ is sign function.

Remake 3: It is noted that Equation (4) only employs the measured output of agent i, $i \in S_N$ and agent i's neighbors.

Moreover, some agents even cannot directly receive the information from the virtual leader when $b_i = 0$. Hence, the proposed QDMFAILBC doesn't need the full information from the network, that is, it is a distributed method and can reduce the costs of the computation.

Generally, the type of quantizer is selected as the logarithmic type, which is described as $U = \left\{ \pm h_p : h_p = \theta^p h_0, p = 0, 1, \pm 1, \pm 2, \cdots \right\} \cup \left\{ 0 \right\}$, where $h_0 > 0$ and θ with $0 < \theta < 1$ denotes the quantization density.

***Definition 3*:** In this study, the quantizer is defined as below.

$$q(\mathrm{x}) = \begin{cases} h_p, & if\ \dfrac{1}{1+\alpha} h_p < \mathrm{x} \le \dfrac{1}{1+\alpha} h_p \\ 0, & if\ \mathrm{x} = 0 \\ -q(-\mathrm{x}), & if\ \mathrm{x} < 0 \end{cases} \quad (5)$$

where $\alpha = (1-\theta)/(1+\theta)$. It is noted that $q(-\mathrm{x}) = -q(\mathrm{x})$ and $0 < \alpha < 1$.

Remark 4: As we all know, the information transmitting from the network is dependent on cable or wireless connecting. In the industrial process, as hardware restrictions, the bandwidth of the communication channel cannot be designed with infinite length. Thereby, using the quantization technique to deal with the analog data from the sensors before transmitted is one of the important methods to reduce the communication burden of the communication network.

3 Main results

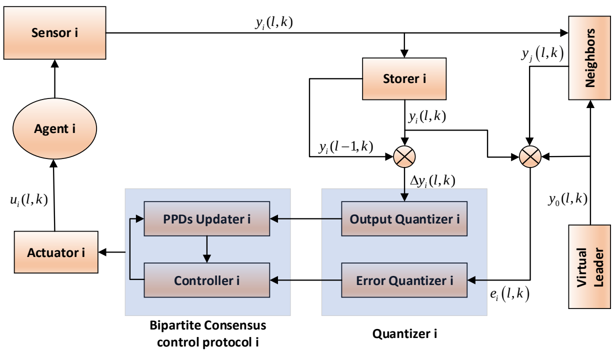

Figure 1. Diagram of agent i with an DMFAC scheme.

A diagram of the unknown dynamics MASs is governed by the proposed QDMFAILBC approach shown in Fig. 1, where we can see that the output gain $\Delta y_i(l,k) = y_i(l,k) - y_i(l-1,k)$ and the BC tracking error $e_i(l,k)$ are quantized before they are transmitted into BC control protocol, respectively.

For the above BC tracking objective (3), the QDMFAILBC scheme is designed as below.

$$\hat{\Gamma}_i(l,k) = \frac{\eta \Delta u_i(l-1,k)}{\mu + |\Delta u_i(l-1,k)|^2}(q_{\Delta i}(\Delta y_i(l-1,k+1)) - \hat{\Gamma}_i(l-1,k)\Delta u_i(l-1,k)) + \hat{\Gamma}_i(l-1,k), \quad (6)$$

$$\hat{\Gamma}_i(l,k) = \hat{\Gamma}_i(1,k), \begin{cases} |\hat{\Gamma}_i(l,k)| \le c \\ |\Delta u_i(l-1,k)| \le c \\ sign(\hat{\Gamma}_i(l,k)) \ne sign(\hat{\Gamma}_i(1,k)) \end{cases}, \quad (7)$$

$$u_i(l,k) = u_i(l-1,k) + \frac{\rho \hat{\Gamma}_i(l,k)}{\lambda + |\hat{\Gamma}_i(l,k)|^2} \xi_{qi}(l-1,k+1), \quad (8)$$

where $0 < \eta \le 1$ and $\mu > 0$ are weighing factors. ρ and λ are stability parameters, and they will be analyzed in the following part. $\hat{\Gamma}_i(l,k)$ is the estimation of $\Gamma_i(l,k)$ and its initial value is $\hat{\Gamma}_i(l,k)$. c is a gain threshold of (7) often selected as 10^{-4} or 10^{-5}. $\xi_{qi}(l,k)$ is the quantized distributed measurement output of (4), which also will be discussed in the following part.

Remark 5: It is noted that the proposed QDMFAILBC approach merely depend on input gain $\Delta u_i(l-1,k)$, quantized output gain $q_{\Delta i}(\Delta y_i(l-1,k+1)$, and the quantized distributed measurement output $\xi_{qi}(l,k)$. Beyond that, the proposed QDMFAILBC doesn't employ any other information of MASs. Comparing with the related DDC ILC approaches for MASs formulated in [25], [27], and [28], it needs fewer data of controlled MASs so that the proposed scheme can further reduce the costs of the communication and improve the practicability that it is more suitable for industrial production

In this paper, we introduce $\theta_{\Delta i}$ and θ_{ei} to donate the quantization densities of $q_{\Delta i}(\cdot)$ and $q_{ei}(\cdot)$, separately. According the sector bound approach [36], [38] and Definition 3, we can obtain

$$q_{\Delta_i}(\Delta y_i(l,k)) - \Delta y_i(l,k) = \sum\nolimits_{\Delta_i} \Delta y_i(l,k) \quad (9)$$

$$q_{e_i}(e_i(l,k)) - e_i(l,k) = \sum\nolimits_{e_i} e_i(l,k) \quad (10)$$

where $\left|\Sigma_{\Delta_i}\right| \le \delta_{\Delta_i}$, $\delta_{\Delta_i} = \dfrac{1-\theta_{\Delta_i}}{1+\theta_{\Delta_i}} < 1$, $\left|\Sigma_{e_i}\right| \le \delta_{e_i}$, and $\delta_{e_i} = \dfrac{1-\theta_{e_i}}{1+\theta_{e_i}} < 1$.

Lemma 2 ([27], [37]): Suppose the set of all possible time-varying irreducible substochastic $W(k)$ with non-negative diagonal element are denoted by W. Then, we have

$$\| W(Q)W(Q-1)\cdots W(1) \| \le \beta .$$

where $0 < \beta < 1$ and $W(k)$, $k = 1, 2, \ldots, Q$, are Q matrices arbitrarily selected from W.

Then, the stability analysis of the proposed QDDDBFC algorithm is presented in the following Theorem.

Theorem 1: Consider the dynamics (1) of agent satisfying Assumptions 1-3, MASs' communication topology satisfies Assumption 4, and MASs is governed by the proposed QDMFAILBC algorithm (6)-(8) to implement BC tracking task. If ρ satisfies the following condition

$$\rho < \frac{1}{\max_{i \in S_n} \sum_{j=1}^{N} a_{ij} + b_i}$$

and $\lambda > \lambda_{min} > \frac{r^2}{4}$, the tracking error $e_i(l,k)$ can be declined to the zero when l tends to infinity.

Proof: The proof comprises three steps as follows.

Step 1 (Proving the Boundedness of $\hat{\Gamma}_i(l,k)$): Define $\tilde{\Gamma}_i(l,k) = \hat{\Gamma}_i(l,k) - \Gamma_i(l,k)$ and $\Delta\Gamma_i(l,k) = \Gamma_i(l,k) - \Gamma_i(l-1,k)$. According to Lemmas 1, Equations (6) and (9), we can obtain that

$$
\begin{aligned}
\tilde{\Gamma}_i(l,k) &= \tilde{\Gamma}_i(l-1,k) - \Delta\Gamma_i(l,k) + \frac{\eta\Delta u_i(l-1,k)}{\mu + |\Delta u_i(l-1,k)|^2} \\
&\quad \times (q_{\Delta i}(\Delta y_i(l-1,k+1)) - \hat{\Gamma}_i(l-1,k)\Delta u_i(l-1,k)) \\
&= \tilde{\Gamma}_i(l-1,k) - \Delta\Gamma_i(l,k) + \frac{\eta\Delta u_i(l-1,k)}{\mu + |\Delta u_i(l-1,k)|^2} \\
&\quad \times \left(\left(\Sigma_{\Delta i} + 1 \right) \Delta y_i(l-1,k+1) - \hat{\Gamma}_i(l-1,k)\Delta u_i(l-1,k) \right) \\
&= \tilde{\Gamma}_i(l-1,k) - \Delta\Gamma_i(l,k) + \frac{\eta\Delta u_i(l-1,k)^2}{\mu + |\Delta u_i(l-1,k)|^2} \\
&\quad \times \left(\left(\Sigma_{\Delta i} + 1 \right) \Gamma_i(l-1,k) - \hat{\Gamma}_i(l-1,k) \right) \\
&= \tilde{\Gamma}_i(l-1,k) - \Delta\Gamma_i(l,k) + \frac{\eta\Delta u_i(l-1,k)^2}{\mu + |\Delta u_i(l-1,k)|^2} \\
&\quad \times \left(\left(\Sigma_{\Delta i} + 1 \right) \Gamma_i(l-1,k) - \Gamma_i(l-1,k) \right. \\
&\quad \left. + \Gamma_i(l-1,k) - \hat{\Gamma}_i(l-1,k) \right) \\
&= \tilde{\Gamma}_i(l-1,k) - \Delta\Gamma_i(l,k) + \frac{\eta\Delta u_i(l-1,k)^2}{\mu + |\Delta u_i(l-1,k)|^2} \\
&\quad \times \left(\Sigma_{\Delta i} \Gamma_i(l-1,k) - \tilde{\Gamma}_i(l-1,k) \right) \\
&= \left(1 - \frac{\eta\Delta u_i(l-1,k)^2}{\mu + |\Delta u_i(l-1,k)|^2} \right) \tilde{\Gamma}_i(l-1,k) - \Delta\Gamma_i(l,k) \\
&\quad + \frac{\eta\Delta u_i(l-1,k)^2}{\mu + |\Delta u_i(l-1,k)|^2} \Sigma_{\Delta i} \Gamma_i(l-1,k)
\end{aligned} \tag{11}
$$

Then, we have

$$
\begin{aligned}
\left| \tilde{\Gamma}_i(l,k) \right| &\le \left| 1 - \frac{\eta\Delta u_i(l-1,k)^2}{\mu + |\Delta u_i(l-1,k)|^2} \right| \left| \tilde{\Gamma}_i(l-1,k) \right| + \left| \Delta\Gamma_i(l,k) \right| \\
&\quad + \left| \frac{\eta\Delta u_i(l-1,k)^2}{\mu + |\Delta u_i(l-1,k)|^2} \right| \left| \Sigma_{\Delta i} \right| \left| \Gamma_i(l-1,k) \right|
\end{aligned} \tag{12}
$$

Since $0 < \eta \le 1$ and $\mu > 0$, we can obtain that $\eta\Delta u_i(l-1,k)^2 \le |\Delta u_i(l-1,k)|^2 \le \mu + |\Delta u_i(l-1,k)|^2$. Then, we can obtain

$$
0 < \frac{\eta\Delta u_i(l-1,k)^2}{\mu + |\Delta u_i(l-1,k)|^2} < 1 \tag{13}
$$

Hence, there must be a constant $\hbar$ to satisfy the following inequality sequences.

$$
0 < \left| 1 - \frac{\eta\Delta u_i(l-1,k)^2}{\mu + |\Delta u_i(l-1,k)|^2} \right| \le \hbar < 1 \tag{14}
$$

According to $\left|\Gamma_i(l,k)\right| \le r$, $0 < \left|\Sigma_{\Delta_i}\right| < 1$, and (14), Equation (12) becomes

$$
\begin{aligned}
\left| \tilde{\Gamma}_i(l,k) \right| &\le \hbar \left| \tilde{\Gamma}_i(l-1,k) \right| + 2r + r \\
&\le \hbar \left(\hbar \left| \tilde{\Gamma}_i(l-2,k) \right| + 3r \right) + 3r \\
&\cdots \\
&\le \hbar^{k-1} \left| \tilde{\Gamma}_i(1,k) \right| + \left(\hbar^{k-2} + \cdots + \hbar + 1 \right) 3r \\
&\le \hbar^{k-1} \left| \tilde{\Gamma}_i(1,k) \right| + \frac{\left(1 - \hbar^{k-2}\right)}{(1 - \hbar)} 3r
\end{aligned} \tag{15}
$$

Hence, $\lim_{k->\infty} \left| \tilde{\Gamma}_i(l,k) \right| = \frac{3r}{(1-\hbar)}$, that is, $\tilde{\Gamma}_i(l,k)$ is bounded. Meanwhile, since $\Gamma_i(l,k)$ is also bounded, we can obtain that $\hat{\Gamma}_i(l,k)$ is bounded.

Then, we define the following collective stack vectors:

$$
\begin{aligned}
u(l,k) &= \left[u_1(l,k), u_2(l,k), \cdots, u_N(l,k) \right]^T \\
\xi_q(l,k) &= \left[\xi_{q1}(l,k), \xi_{q2}(l,k), \cdots, \xi_{qN}(l,k) \right]^T \\
e(l,k) &= \left[e_1(l,k), e_2(l,k), \cdots, e_N(l,k) \right]^T \\
y(l,k) &= \left[y_1(l,k), y_2(l,k), \cdots, y_N(l,k) \right]^T \\
\Delta y(l,k) &= \left[\Delta y_1(l,k), \Delta y_2(l,k), \cdots, \Delta y_N(l,k) \right]^T \\
\bar{y}_0(l,k) &= \left[y_0(l,k), y_0(l,k), \cdots, y_0(l,k) \right]^T \\
q_e(x) &= \left[q_{e1}(x), q_{e2}(x), \cdots, q_{eN}(x) \right]^T
\end{aligned}
$$

Step 2 (Proving the Convergence of $e_i(k)$): Firstly, according to $e_i(l,k) = s_i y_0(l,k) - y_i(l,k)$ and Assumption 4, Equation (4) becomes.

$$
\begin{aligned}
\zeta_i(l,k) &= \sum_{j \in N_i} \left| a_{ij} \right| (sign(a_{ij}) y_j(l,k) - y_i(l,k)) \\
&\quad + b_i (s_i y_0(l,k) - y_i(l,k)) \\
&= \sum_{j \in N_i} \left(a_{ij} y_j(l,k) - \left| a_{ij} \right| y_i(l,k) \right) + b_i e_i(l,k) \\
&= \sum_{j \in N_i} \left(a_{ij} y_j(l,k) - \left| a_{ij} \right| s_i y_0(l,k) + \left| a_{ij} \right| e_i(l,k) \right) \\
&\quad + b_i e_i(l,k) \\
&= \sum_{j \in N_i} \left(a_{ij} y_j(l,k) - a_{ij} s_j y_0(l,k) + \left| a_{ij} \right| e_i(l,k) \right) \\
&\quad + b_i e_i(l,k) \\
&= \sum_{j \in N_i} \left(\left| a_{ij} \right| e_i(l,k) - a_{ij} e_j(l,k) \right) + b_i e_i(l,k)
\end{aligned} \tag{16}
$$

Then, from (10) and (16), we can obtain the error quantized expression as below.

$$
\begin{aligned}
\xi_{qi}(l,k) &= \sum_{j \in N_i} \left(\left| a_{ij} \right| q_{ei}\left(e_i(l,k) \right) - a_{ij} q_{ej}\left(e_j(l,k) \right) \right) \\
&\quad + b_i q_{ei}\left(e_i(l,k) \right)
\end{aligned} . \tag{17}
$$

To express clearly, Equation (17) can be written as a compact form as

$$
\begin{aligned}
\xi_q(l,k) &= (\mathcal{L} + \mathcal{B}) q_e\left(e(l,k) \right) \\
&\quad + (\mathcal{L} + \mathcal{B}) \left(\varphi(l,k) + I \right) e(l,k)
\end{aligned}, \tag{18}
$$

where $\varphi(l,k)=diag\left(\Sigma_{e1},\cdots,\Sigma_{eN}\right)$. Then, from (18), we can obtain the compact form of the control law (8) as below:

$$\Delta u(l,k)=\rho\Omega(l,k)(\mathcal{L}+\mathcal{B})(\varphi(l,k)+I)e(l,k) \quad (19)$$

where

$$\Omega(l,k)=diag\left(\frac{\hat{\Gamma}_1(l,k)}{\lambda+|\hat{\Gamma}_1(l,k)|^2},\cdots,\frac{\hat{\Gamma}_N(l,k)}{\lambda+|\hat{\Gamma}_N(l,k)|^2}\right).$$

From $e_i(l,k)=s_i y_0(l,k)-y_i(l,k)$, $y_0(l,k)=constant$, (2), and (19), we can obtain that

$$\begin{aligned} e(l,k+1) &= e(l-1,k+1)-\Delta y(l,k+1) \\ &= e(l-1,k+1)-\rho\Psi(l,k)(\mathcal{L}+\mathcal{B}) \\ &\quad \times e(l-1,k+1)(\varphi(l,k)+I) \\ &= (I-\rho\Xi(l,k))e(l-1,k+1) \end{aligned} \quad (20)$$

where $\Psi(l,k)=diag\left(\vartheta_1(l,k),\cdots,\vartheta_N(l,k)\right)$, $\vartheta_i(l,k)=\frac{\Gamma_i(l,k)\hat{\Gamma}_i(l,k)}{\lambda+|\hat{\Gamma}_i(l,k)|^2}$, and $\Xi(l,k)=\Psi(l,k)(\mathcal{L}+\mathcal{B})(\varphi(l,k)+I)$.

Step 3 (Obtaining the Convergence Condition of MASs): From Assumption 3, Lemma 1, and $\lambda>\lambda_{min}>\frac{r^2}{4}$, we have

$$0<\vartheta_i(l,k)\le\frac{r\hat{\Gamma}_i(l,k)}{2\sqrt{\lambda}\,|\hat{\Gamma}_i(l,k)|}\le\frac{r}{2\sqrt{\lambda m}}<1. \quad (21)$$

Then, according to Definitions 3-4, Equation (21), $0<\left|\Sigma_{e_i}\right|<1$, and ρ satisfies

$$\rho<\frac{1}{\max_{i\in S_n}\sum_{j=1}^{N}a_{ij}(k)+b_i(k)},$$

which means that all of the diagonal entry in $L+B$ are larger than the reciprocal of ρ, we can obtain that $\rho\Xi(l,k)-I$ is an irreducible substochastic matrix with positive diagonal entries [21]-[23]. Thus, (20) can be written as below.

$$\begin{aligned} \|e(l,k+1)\| &= \|\rho\Xi(l,k)-I\|\|e(l-1,k+1)\| \\ &\le \|\rho\Xi(l,k)-I\|\|\rho\Xi(l-1,k)-I\|\|e(l-2,k+1)\| \\ &\quad \cdots \\ &\le \|\rho\Xi(l,k)-I\|\|\rho\Xi(l-1,k)-I\|\cdots \\ &\quad \|\rho\Xi(l,k)-I\|\|e(1,k+1)\| \end{aligned}, \quad (22)$$

Then, applying Lemma 2, we can obtain that

$$\lim_{k->\infty}\|e(l,k+1)\|=\lim_{k->\infty}\left(\beta^{\left\lfloor\frac{l}{Q}\right\rfloor}\|e(1,k+1)\|\right)$$

where $\lfloor\bullet\rfloor$ denotes the floor function [27]. Since $0<\beta<1$, $\lim_{k->\infty}\|e(l,k+1)\|=0$. □

4 Extension to Switching Topologies

In this part, time-varying switching topologies are investigated. The stability and convergence of MASs with quantization data to perform BC tracking tasks are analyzed.

According to the signed graph theory of the section 2, Definition 2 becomes

$$\begin{aligned} \xi_i(l,k) = &\sum_{j\in N_i}\left|a_{ij}(k)\right|(\text{sign}\left(a_{ij}(k)\right)y_j(l,k)-y_i(l,k)) \\ &+b_i(k)(s_i(k)y_0(l,k)-y_i(l,k)) \end{aligned}. \quad (26)$$

Theorem 2: When nonlinear MASs satisfies Assumptions 1-4, the (7)-(9) of the QDMFAILBC method are applied, the value of ρ satisfies

$$\rho<\frac{1}{\max_{i\in S_N,\,p=1,2,\ldots,\kappa}\sum_{j=1}^{N}a_{ij}^{p}(k)+b_i^{p}(k)},$$

and $\lambda>\lambda_{min}>\frac{r^2}{4}$ exists, the bipartite formation objective (3) can be achieved as $k\to\infty$.

Proof: According to (26), the bipartite formation tracking errors of the QDDDBFC approach in (23) becomes

$$\begin{aligned} e(l,k+1) &= (I-\rho\Psi(k)(\mathcal{L}(k)+\mathcal{B}(k))(\varphi(l,k)+I)) \\ &\quad \times e(l-1,k+1) \end{aligned} \quad 26)$$

where all the reciprocals of the diagonal entry in $\mathcal{L}(k)+\mathcal{B}(k)$, $p=1,2,\ldots,\kappa$ are larger than ρ. Hence, applying the similar analytical method of Section 3 we can obtain that $\rho\Psi(l,k)(\mathcal{L}(k)+\mathcal{B}(k))(\varphi(l,k)+I)-I$ is also irreducible substochastic matrix with non-negative diagonal entries and $\lim_{k\to\infty}\|e(l,k+1)\|=0$. □

Remark 6: In the existing consensus or formation methods for MASs, most of them are dependent on the assumption that an accurate mathematical model is available to analyze the convergence and stability of controlled systems. However, it is noted that the mathematical model is not a requirement in the QDMFAILBC algorithm. Moreover, the existing DDC ILC approaches for MASs [21]-[29] don't consider the quantization data and switching topologies problems.

Remark 7: In this study, we have proposed a BC algorithm for MASs with quantization data, switching topologies, and competition relationships, where the type of each agent is SISO. In the practical process, we can find that existing systems are multiple-input-multiple-output (MIMO), and the operating environment is more complicated such as existing the unknown disturbance, packet dropout, and sensor saturation. Hence, we will try to further design a robust BC scheme for SISO systems with a more complex operating environment and extend the proposed approach to the MIMO systems.

5 Simulation

In this Section, three simulations are given to test the correctness and efficiency of the theoretical and analysis. The first one is for MASs with fixed topology and quantization data to implement the BC tracking task, and the second one is for MASs with time-varying switching topologies and quantization data to perform BC tracking task.

To demonstrate the practicality of the proposed QDMFAILBC approach, we employ even DC motors to test both fixed and time-varying switching topologies and quantization data in the third simulation. all of the possible communication topologies of MASs are given in Fig. 2.

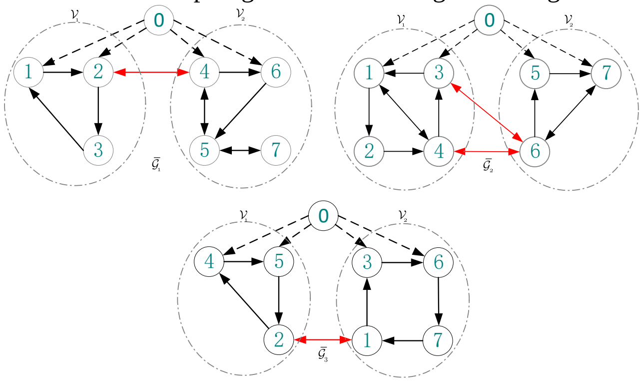

Figure 2. Communication topologies of MASs.

Fixed topology tracking example

In this simulation, we select $\bar{\mathcal{G}}_1$, which is shown in Fig. 2, as the communication topology of MASs. Here, the note 0 denotes virtual leader, which directly connects with the agents 1, 2, 4, and 6. In addition, the direction of transmitting information is fixed that the information among agents only transmits along with the arrows. We also can see that seven agents are allotted into two alliances (agents 1, 2, 3 belong to the alliance $\mathcal{V}_1$, and agents 4, 5, 6, 7 belong to the alliance $\mathcal{V}_2$), where using the black solid line to represent the collaborative relationships among agents and the antagonistic interactions among agents is expressed by red solid line.

According the communication topology $\bar{\mathcal{G}}_1$ in Fig. 2, we can get that the reciprocal of the greatest diagonal entry of $\mathcal{L}+\mathcal{B}$ is 0.5. According to the convergence condition of Theorem 2 for all $i=1,2,3,4,5,6,7$, the controller parameters are selected as $\rho=0.3$, $\mu=0.5$, $\eta=1$, $\lambda=1$, and $\sigma=10^{-4}$. Meanwhile, the initial conditions are chosen as $u_i(0,k)=0$, $y_i(0,k)=rand(-0.005,0.005)$, and $\Gamma_i(1,k)=2$ for all agents in this simulation. Moreover, the agents are governed by

$$y_1(l,k+1)=\frac{y_1^2(l,k-1)u_1(l,k-1)+(1+(k/150)u_1(l,k-1))}{1+y_1(l,k-1)y_1(l,k-2)+y_1^2(l,k-3)},$$

$$y_2(l,k+1)=\frac{y_2^2(l,k-2)u_2(l,k-2)+(1+(k/150)u_2(l,k-1))}{1+y_2(l,k-1)y_2(l,k-2)+y_2^2(l,k-3)},$$

$$y_3(l,k+1)=\frac{y_3^3(l,k-3)u_3(l,k-3)+(1+(k/150)u_3(l,k-1))}{1+2y_3^2(l,k-3)},$$

$$y_4(l,k+1)=\frac{y_4^3(l,k-2)u_4(l,k-2)+(1+(k/150)u_2(l,k-1))}{1+y_4^2(l,k-1)+y_4^2(l,k-2)},$$

$$y_5(l,k+1)=\frac{y_5^4(l,k-2)u_5(l,k-2)+(1+(k/150)u_5(l,k-1))}{1+2y_5(l,k-1)y_5(l,k-2)},$$

$$y_6(l,k+1)=\frac{y_6^4(l,k-1)u_6(l,k-2)+(1+(k/150)u_6(l,k-1))}{1+y_6^2(l,k-1)+y_6^2(l,k-2)},$$

$$y_7(l,k+1)=\frac{y_7^3(l,k-1)u_7(l,k-2)+(1+(k/150)u_7(l,k-1))}{1+2y_7(l,k-1)y_7(l,k-2)}.$$

Remark 8: From the dynamics of each agent, it is noted that each agent has a unique dynamics model, that is, MASs is heterogeneous. Furthermore, it is noteworthy that the above dynamics models are only employed to generate the I/O data for the simulation, while the designed QDMFAILBC method doesn't use any model information of the plant. During designing this algorithm, the dynamics models of MASs are unknown.

From Figs. 3-5, we can see that the BC tracking performance of MASs when $\theta_{\Delta i}=\theta_{ei}=0.9$ and $h_0=5$, where the BC tracking errors have rapidly reduced to the origin. When setting $\theta_{\Delta i}=\theta_{ei}=0.2$ and $h_0=5$, we can find that it can also achieve BC tracking, though the convergence rate is declined in Figs. 6-8. Moreover, the effect of the quantization density θ for the convergence property of the proposed QDMFAILBC is presented in Fig. 9. It clearly shows that the quantization density of each agent will affect the convergence steps of them. Generally, when the level of the quantization density is small, it strictly influences the convergence rate. As the density increases, the intensity of the influence decreases.

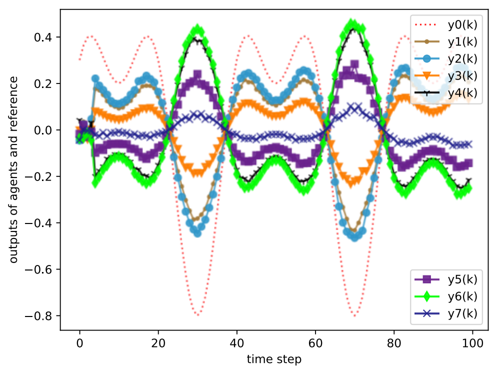

Figure 3. Tracking performance of MASs with θ=0.9 at 10th (example 1).

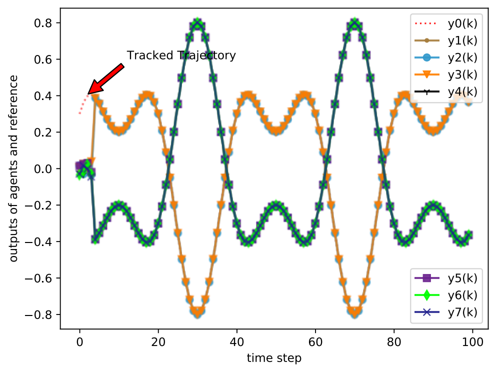

Figure 4. Tracking performance of MASs with θ=0.9 at 370th (example 1).

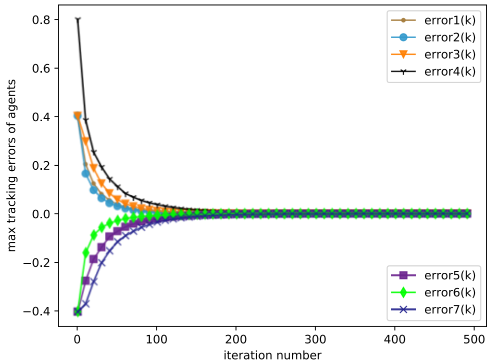

Figure 5. Tracking errors of MASs with θ=0.9 (example 1).

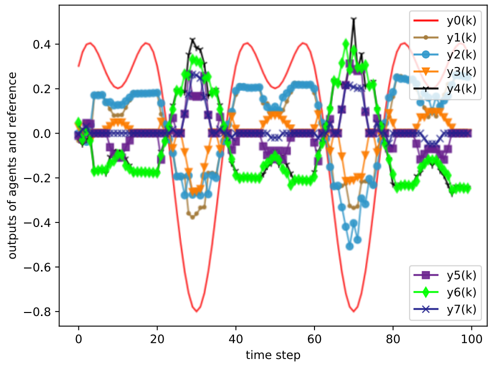

Figure 6. Tracking performance of MASs with θ=0.2 at 10th (example 1).

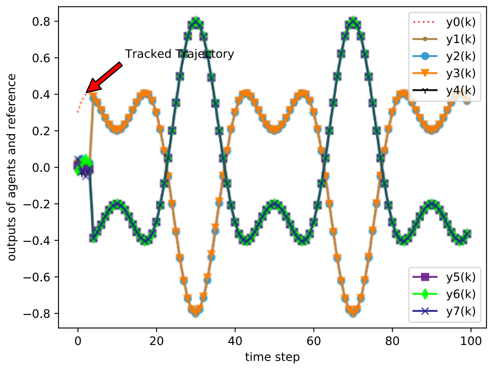

Figure 7. Tracking performance of MASs with θ=0.2 at 370th (example 1).

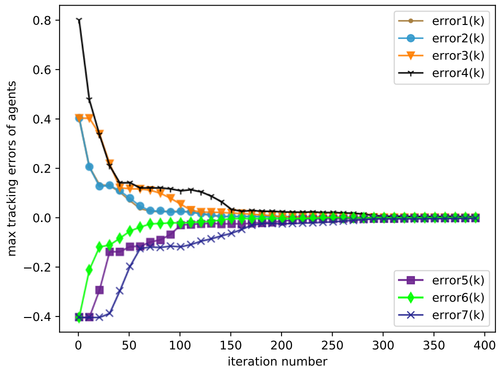

Figure 8. Tracking errors of MASs with θ=0.2 (example 1).

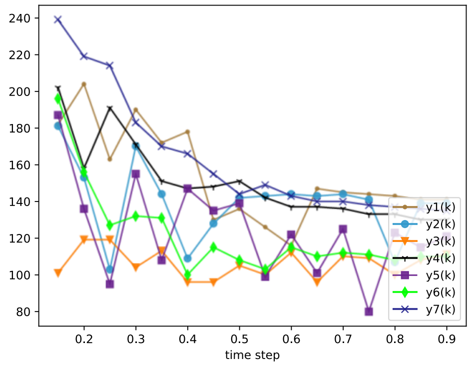

Figure 9. Convergence speed of MASs with different θ (example 1).

Remark 9: According to the results of simulation 1, it is noted that the proposed QDMFAILBC scheme can deal with the quantization problem for MASs with fixed topology to implement BC tracking tasks. Compared with the existing DDC ILC results, it is more closed to the industrial environment. It is the first time to consider the quantization problem for unknown dynamics heterogeneous discrete-time MASs to perform BC tracking tasks to the best of our knowledge.

Time-varying topologies tracking example

In this simulation, the time-varying switching topologies issues is discussed. All of the possible communication topologies is shown in Fig. 2 and related parameters are selected as same as the example 1 ($\theta_{zi}=\theta_{\Delta i}=\theta_{\varepsilon i}=0.2$) to accomplish this simulation. Moreover, a piecewise function of how to change the topology of MASs is given as below:

$$\begin{cases} \bar{\mathcal{G}}_1, & 0 \le k \le 450 \\ \bar{\mathcal{G}}_2, & 450 < k \le 900 \\ \bar{\mathcal{G}}_3, & 900 < k \le 1600 \end{cases},$$

The tracking performances are shown in Figs 10-11 under difficult interaction steps. The tracking errors of agents are shown in Fig 12. It can see that time-varying switching topologies doesn't affect the stability of MASs to implement BC tracking tasks. Especially, when some agents defer to a hostile alliance there can immediately track a new trajectory.

From Figs. 11-12, we also can obtain the same result with Example 1 that the convergence rate is affected by the parameters θ. Moreover, we further investigate the effect of the sensor saturation phenomenon of MASs with switching topologies in Figs. 13-14. Comparing with Figs. 12 and 14, we can see that the convergence speed of Fig. 12 is faster than Fig. 14. It can conclude that the sensor saturation doesn't change the convergence property of the proposed QDMFAILBC approach, though it causes the convergence rate to slow down. From Figs 9-14, we can see that the bipartite formation tracking tasks can be accomplished when MASs suffer the data quantization, sensor saturation even agents exchanging their alliances.

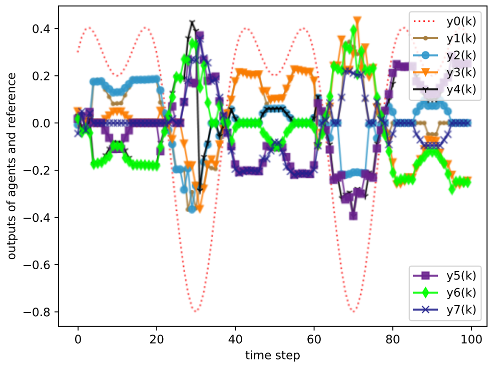

Figure 10. Tracking performance of MASs with θ=0.2 at 10th (example 2).

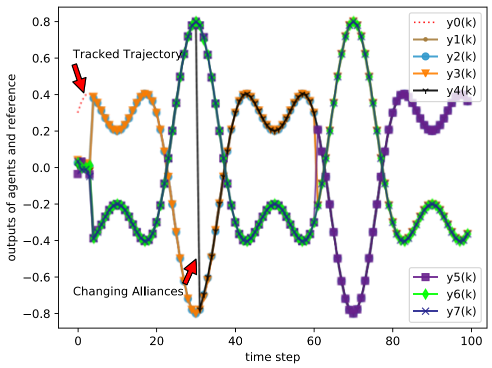

Figure 11. Tracking performance of MASs with θ=0.2 at 370th (example 2).

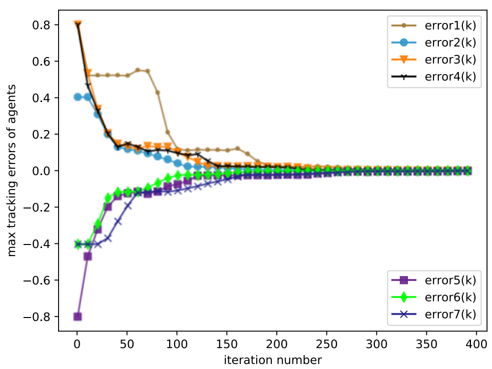

Figure 12. Tracking errors of MASs with θ=0.2 (example 2).

This result further verifies the correctness of Theorem 2 that the proposed QDMFAILBC approach can guarantee that the tracking error of each agent converges to the zero when MASs is subject to quantization and switching topologies influences.

Realistic DC linear Motors

In this example, we utilize seven permanent magnet DC linear motors to verify the effectiveness and practicability of proposed QDMFAILBC approach. Here, the mathematical model of this DC motor has been studied in [39], [40], which is identified as following model [21], [28].

$$\begin{cases} \dot{x}(t) = v(t) \\ v(t) = \dfrac{u(t) - f_{friction}(t) - f_{ripple}(t)}{m} \\ y(t) = v(t). \end{cases}$$

where $v(t)$, $x(t)$ express the position (m) and the speed (m/s), respectively. The m denotes the combined mass of translator and load and $u(t)$ denotes the developed force (N). $f_{friction}(t)$ and $f_{ripple}(t)$ are the friction force (N) and the ripple force (N), respectively. Meanwhile, the model of friction and ripple forces are expressed by following equations.

$$f_{friction}(t) = \left(f_c + (f_s - f_c) e^{-\left(\dot{x}/\dot{x}_\delta \right)^\delta} + f_v \dot{x} \right) sign(\dot{x}),$$

$$f_{ripple}(t) = b_1 \sin(w_0 x(t))$$

where $sign(\bullet)$ is sign function, f_c denotes the minimum level of Coulomb friction, f_s denotes the level of static friction, δ is an additional empirical parameter. $\dot{x}_\delta$ and f_v are lubricant and load parameters. In this example, these parameters are selected as: $m = 0.59kg$, $\dot{x}_\delta = 0.1$, $\delta = 1$, $f_c = 10N$, $f_s = 20N$, $f_v = 10N \cdot s \cdot m^{-1}$, $b_1 = 8.5N$, $w_0 = 314s^{-1}$. The desired velocity is given as

$$y(t) = 0.5 \sin(t\pi/30) + 0.3 \cos(t\pi/10),\ t \in [0,100].$$

Applying Euler Formula to discretize above model and selecting sampling time as $h = 0.001$, we have $T = 1000$.

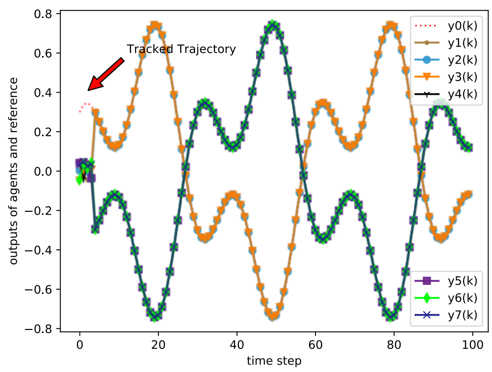

Figure 13. Tracking performance of MASs with with fixed topology and θ=0.2 at 370th (example 3).

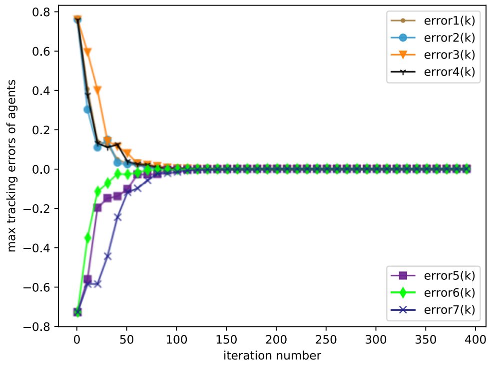

Figure 14. Tracking errors of MASs with fixed topology and θ=0.2 (example 3).

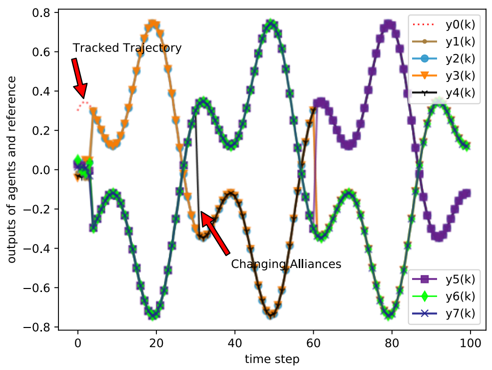

Figure 15. Tracking performance of MASs with switching topologies and θ=0.2 at 370th (example 3).

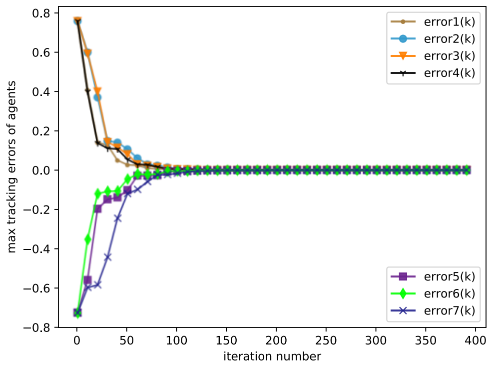

Figure 16. Tracking errors of MASs with switching topologies and θ=0.2 at 370th (example 3).

Here, we use the same parameters and the communication topology of Example 1 to test the efficiency of the proposed QDMFAILBC scheme for MASs with fixed topology. The performances of seven DC motors are shown in Fig. 13 at 370th iteration, and the corresponding tracking error of each motor is presented in Fig. 14. The time-varying switching topologies problem of seven DC motors is investigated in Figs. 14-15. It is noteworthy that the BC tracking objective of the seven-motor system with fixed or switching topologies can be well achieved by applying the QDMFAILBC approach.

Remark 10: From above simultaneous, we can see that the proposed approach can ensure the MASs to track the objective trajectory when MASs suffers quantization data, corresponding relationships, and switching topologies under repeatable operating environment.

6 Conclusion

In this paper, the problems of data quantized and coopetition interactions among agents have been investigated for unknown nonlinear discrete-time multiagent systems to perform bipartite consensus tracking tasks under a repeatable operation circumstance. To formulate an appropriate control scheme, a time-varying linear data model along the iteration axis has been established, and a QDMFAILBC scheme has been proposed. Moreover, both fixed and switching topologies are considered. Compared with the existing data-driven iteration leader control approaches, it not only considers the cooperative and competitive relationships among agents but also it can employ incomplete feedback data to update control protocol, that is, it can further reduce the costs of the communication. The results of theoretical analysis and simulations demonstrate the effectiveness of the proposed scheme. In our future efforts, we will consider the unknown disturbance, packet dropout, and the delay problems of MASs to perform bipartite formation tracking tasks.

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