Change points detection in crime-related time series: an on-line fuzzy approach based on a shape space representation

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Abstract

The extension of traditional data mining methods to time series has been effectively applied to a wide range of domains such as finance, econometrics, biology, security, and medicine. Many existing mining methods deal with the task of change points detection, but very few provide a flexible approach. Querying specific change points with linguistic variables is particularly useful in crime analysis, where intuitive, understandable, and appropriate detection of changes can significantly improve the allocation of resources for timely and concise operations. In this paper, we propose an on-line method for detecting and querying change points in crime-related time series with the use of a meaningful representation and a fuzzy inference system. Change points detection is based on a shape space representation, and linguistic terms describing geometric properties of the change points are used to express queries, offering the advantage of intuitiveness and flexibility. An empirical evaluation is first conducted on a crime data set to confirm the validity of the proposed method and then on a financial data set to test its general applicability. A comparison to a similar change-point detection algorithm and a sensitivity analysis are also conducted. Results show that the method is able to accurately detect change points at very low computational costs. More broadly, the detection of specific change points within time series of virtually any domain is made more intuitive and more understandable, even for experts not related to data mining.

Keywords: change points detection, qualitative description of data, time series analysis, fuzzy logic, crime analysis

1. Introduction

The analysis of time series naturally arises in crime analysis as well as in any data-driven domain. Finding sudden changes in criminal activities is a particular task known as change points detection. In this paper, a flexible on-line change points detection method for helping crime analysts to easily and understandably monitor changes is proposed. Change points are detected in two steps: the segmentation of the time series and the querying of points with a fuzzy inference system.

1.1. Motivation

Knowledge extraction of time series can be viewed as an extension of traditional mining methods with an emphasis on the temporal aspect. Among these, *Change points detection* methods focus on finding time points at which data *suddenly* change (in contrast to *slow* changes). Many studies have shown interesting applications of change points detection in various domains. These methods are based on neural networks, regressions, or other statistical models, with an emphasis on the efficiency of these methods. However, only a few consider approaches with these two properties: a meaningful and expressive subspace representation of the time series, and a dynamic segmentation process without fixed-sized windows, linked together flexibly.

In the domain of crime analysis, such flexible and intuitive approaches for change points detection are particularly sought, especially for crime trends monitoring. Previous studies from the authors ([1], [2], [3], and [4]) emphasize on the usefulness of crime trends monitoring activities and advocate the use of appropriate methods for considering the specificities and the constraints of the crime analysis domain, that is basically dealing with uncertainties. The automated process of change points detection is considered as a major step in the production of intelligence, supporting the activity of crime analysis (also sometimes referred to crime intelligence).

Flexible change points detection methods are critical for supporting analysts in their daily tasks, especially for the monitoring of serial and high-volume crimes (e.g., burglaries). Most of the time, crime analysts have no particular background in time series analysis, but still need to analyze and monitor crime trends. These trends are drawn into the whole activities and are not always perceived by police forces. As for example, querying criminal activities about a particular increase in crime trends for targeted police interventions, as well as querying patterns of changes for the general understanding of crime phenomena are common tasks.

Finding changes in crime trends assumes two conditions: (a) the actual existence of a trend, and (b) its detection within the data. The first condition is far from obvious, but as crime analysis is founded on environmental criminology theories, a justi-

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fication for the existence of crime trends appears ([4], [5], and [6]). The second is generally simply assumed, but difficult to detect in massive data sets and needs intuitive and understandable analytical methods.

Although the proposed method is a specific answer for the domain of crime analysis, we believe that it has great potential applications in several domains. As an example, in the financial domain, it proves very useful to find and query change points in real-time, giving the investors flexible means to detect trends indicating the right moment for selling or buying stocks.

1.2. Contribution of this paper

The method proposed in this paper, for the Fuzzy Change Points Detection in crime-related time series (FCPD), aims to focus on flexibility and intuitiveness. To achieve this purpose, the method combines a segmentation step and a querying step. Moreover, the following characteristics make FCPD unique:

- a meaningful and expressive representation of the time series is used ;
- the segmentation is *dynamic*, that is, segments are set according to the underlying shapes of the time series, without using a fixed-size window parameter;
- changes are queried with linguistic terms, using a fuzzy inference system;
- the method does not rely on training sets;
- the method is on-line and iterative, i.e., change points can be detected with past values only and there is no need to compute the entire model at each new observed value. The computational cost is very low; it is related to the size of the approximating polynomials (instead of the number of the observations).

Indeed, with the use of a meaningful representation and a dynamic segmentation, change points can be more easily described and identified. The segments found in a time series, reflecting change points, are described with meaningful estimators such as the average, the slope, the curvature, etc. Then, with the use of a fuzzy inference system, a query can be specified using linguistic terms describing the geometric estimators. It becomes then easier, for instance, to query a time series about the most abrupt changes in terms of slope. In the example "IF *average* is *low* AND *slope* is *very_high*, THEN *pertinence* is HIGH", the inference system would return a high score on segments representing shapes of the given description. This approach makes the querying of change points particularly intuitive and flexible, especially for domain experts.

1.3. Structure of this paper

The remainder of this paper is structured as follows: in Section 2, a literature review in the mining of time series is provided; Sect. 3 introduces some concepts in the preparation, representation, and analysis of time series; Sect. 4 details FCPD, a step-by-step method for the fuzzy querying and detection of change points in crime-related time series; an empirical validation on synthetic and real-world data is conducted in Sect. 5; results are discussed in Sect.6; and finally in Sect. 7 a conclusion is drawn from the experiments and some tracks for future work are suggested.

2. Literature review

Change points detection has numerous application domains, as for example finance, biology, ocean engineering, medicine, and crime analysis. It is considered as a final objective in the whole process of time series analysis among classification, rules discovery, prediction, and summarization. Almost all of these mining tasks require data preparation, namely the representation of the time series, its indexing, its segmentation, and/or its visualization. In this section, we propose a review of these steps, before comparing existing methods for change points detection. An extensive review of the analysis of time series can be found in [7], as well as a general methodology in [8].

2.1. Representation of time series

Many representation models of time series have been dealt with in the literature, each claimed with relative advantages and drawbacks. Two main categories are symbolic representations and numeric representations. Symbolic representations are less sensitive to noise and are usually computationally faster. For the last decade, the community has been paying particular attention to the Symbolic Aggregate Approximation (SAX) representation ([9] and [10]), with the main advantages to reduce the original dimensionality of the data, being on-line, and having a robust distance measure. However, it does not cover all needs. In [11], a numeric representation —which differs from SAX and many others by giving a meaning to the representation— is used to perform several mining tasks. This *shape space* representation uses coefficients as shape estimators of the time series it represents, leading to an intuitive description.

2.2. Segmentation of time series

Most mining methods use subsequences (or segments) of time series as input to the analysis. Segmentation algorithms with the approach of a sliding window are simple to use but present the main drawback of being static, i.e., segmenting the time series according to a fixed and exogenous parameter (e.g., the length of the window) without considering the observed values. Other algorithms, based on a bottom-up or a top-down approach are considered as dynamic (e.g., by using some error criteria as segmentation thresholds) but need the whole data set to operate. These off-line algorithms usually perform better in terms of accuracy but have higher computational costs and are not suitable for real-time applications. A combination of the aforementioned algorithms, namely the SWAB segmentation algorithm, is presented in [12]. A study [13] provides benchmarks on these claims and as a result suggests that SWAB is empirically superior to all other algorithms discussed in the literature. As we believe there is no silver bullet, each application has its own requirements. A more flexible approach is the SwiftSeg algorithm [14], providing a dynamic and on-line approach to segmentation, with the possibility of a mix between growing and sliding window. Another interesting segmentation approach [15], specific to stock mining and described as dynamic, is based on the identification of perceptually important points (PIP).

2.3. Fuzzy analysis of time series

A small subset of temporal mining methods takes advantage of the characteristics offered by fuzzy logic and fuzzy sets. The concept of fuzzy time series has first been defined by Song and Chissom in [16] and [17], with an application in class enrollment forecasting. Soon followed multiple variations and improvements of the basic method (e.g., [18], [19], [20], or [21]), with their own types of fuzzy inference systems (FIS). Two common FIS, namely the Mamdani inference system [22] and the Takagi-Sugeno inference system [23], can be intuitively used to deal with uncertain and flexible data. In [24], an application in finance uses an FIS for pattern discovery. In [25], prediction of long shore sediments is also dealt with the use of an FIS. In parallel, a combination of FISs and neural networks have found an origin in [26]. As for examples, the prediction of time series is performed with dynamic evolving neuro-fuzzy inference systems [27], the classification of electroencephalograms [28], as well as the prediction of hydrological time series [29].

2.4. Change points detection

Change points detection in time series analysis has been thoroughly investigated, mainly using statistical models (see [30] for a general introduction). Reeves et al. [31] attempt to review and compare the major change points detection methods for climate data series.

More specifically, related approaches for change points detection have been investigated in a relatively limited set of studies. For example, a statistical based approach using fuzzy clustering is described in [32, 33]. Verbesselt et al., in [34] and [35], detect breaks for additive seasonal and trends (BFAST), with a principal application is phenology. To deal with imprecise observation in time series, changes are analyzed with fuzzy variables in [36]. In [37], a contextual change detection algorithm addresses relative changes with respect to a group of time series. In [38] and [39], the utility of a framework for outliers detection of time series prediction is highlighted. In [21], the need to use linguistic values for comprehensible results is advocated, where fuzzy time series mining is used for association rules between data points (but not between segments) with fixed-size window. A qualitative description of multivariate time series with the use of fuzzy logic is presented in [40]. Yu et al. [41] propose a fuzzy piecewise regression analysis with automatic change points detection. In [42], DoS attacks are monitored with a change point approach based on the non-parametric Cumulative Sum (CUSUM) method.

3. Time series representation and fuzzy concepts

In the following subsections, a review of a time series representation using polynomials is presented and their main advantages are explained. Then a dynamic segmentation method is described. Finally, concepts of fuzzy time series and fuzzy inference systems, which are useful to analyze segments, are introduced.

3.1. A polynomial shape space representation

Let us consider a time series defined by the sequence

$$s = (y_0, y_1, \dots, y_N), \quad y_i \in \mathbb{R} \quad (i = 0, 1, \dots, N) \tag{1}$$

of $N + 1$ points measured over the equidistant points in time $x_0, x_1, \dots, x_N$. Basically, our set of points s can be modeled by a parametrized function $f(x)$, which is obtained with a linear combination of basis functions f_k:

$$f(x) = \sum_{k=0}^{K} w_k \, f_k(x)\,. \tag{2}$$

The properties of this approximation depend on the choice of the basis functions f_k and their weights. Given some appropriate basis functions, an optimal approximation can be found with the vector of weights $\mathbf{w}^* \in \mathbb{R}^{K+1}, \mathbf{w}^* = (w_0, w_1, \dots, w_k)^T$, which minimizes the approximation error in the least-squared sense. Fuchs et al., in [14], claim that these weights show interesting properties when using some specific of these $K + 1$ basis functions. Indeed, when particular conditions are met, these weights describe the shape of the considered time series intuitively. As a corollary, an efficient similarity measure can be defined based on the extracted features.

Let us now describe these particular approximating polynomials, as in [14], with

$$p(x) = \sum_{k=0}^{K} \alpha_k \, p_k(x)\,, \tag{3}$$

where $p(x)$ is the approximating polynomial, the polynomials p_k are the basis functions f_k and the coefficients α_k are the weights w_k, relating to Equation 2. These coefficients are defined as

$$\alpha_k = \frac{1}{\|p_k\|^2} \sum_{n=0}^{N} y_n \, p_k(n)\,, \tag{4}$$

where

$$\begin{aligned} p_{-1}(x) &= 0\,, \\ p_0(x) &= 1\,, \\ p_{k+1}(x) &= (x - a_k)\, p_k(x) - b_k \, p_{k-1}(x)\,. \end{aligned}$$

Then, by defining $\boldsymbol{\alpha}$ as the *vector of coefficients*

$$\boldsymbol{\alpha} = (\alpha_0, \alpha_1, \dots, \alpha_K)^T \ , \tag{5}$$

any time series can be characterized by these coefficients only, given some polynomials. The interesting property is that the i_{th} coefficient represents the i_{th} derivative of the approximated time series; i.e., the first coefficient α_0 is an optimal estimator (in the least-squared sense) for the average of the considered $N+1$ data points, α_1 an estimator for the slope, α_2 an estimator for the curvature, etc.

The parameter K of the orthogonal expansion (Eq. 3) has to be carefully chosen in accordance with the desired description of the time series. As depicted in Figure 1, setting $K = 0$ defines a single polynomial term in Eq. 3 with a maximum degree of 0 and a corresponding vector coefficient $\boldsymbol{\alpha} \in \mathbb{R}^1$, representing a time series according to its average only; setting $K = 1$ adds the estimator of slope; setting $K = 2$ adds on top the estimator of curvature; and so on. Choosing this parameter is a trade-off between computational costs and representation accuracy.

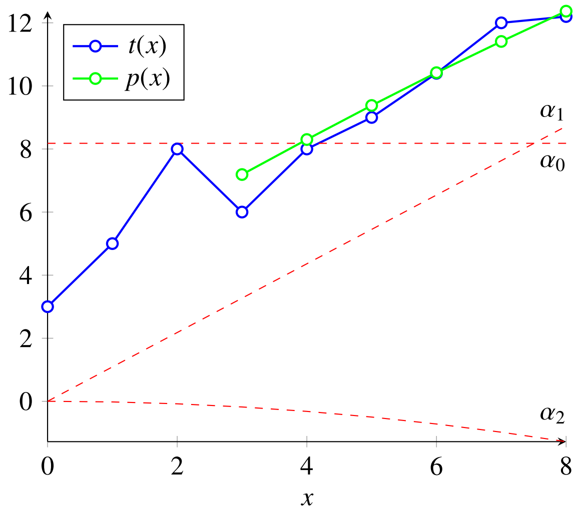


Figure 1: A time series $t(x)$ and its polynomial approximation $p(x)$ with $K = 2$, and their coefficients. The coefficients $\alpha_0 = 8.18$ (estimator for the average), $\alpha_1 = 1.09$ (estimator for the slope), and $\alpha_2 = -0.02$ (estimator for the curvature) are depicted with the first term only of their respective polynomials. It should be noticed that the approximation can only start from $K + 1$ data points.

In order to hold these desired properties, each of the given polynomials p_k defining the *orthogonal expansion* of the approximation must:

- have different ascending degrees 0,1,...,K;
- have a leading coefficient of 1;
- be orthogonal with respect to the inner product

$$\langle p_i | p_j \rangle = \sum_{n=0}^{N} p_i(x_n)\, p_j(x_n), \quad i \neq j\,. \tag{6}$$

For instance, the discrete Chebyshev's polynomials reveal these criteria. Defined in a recursive way, the first Chebyshev's terms are:

$$\begin{aligned}
p_0(x) &= 1, \\
p_1(x) &= x - \frac{N}{2}, \\
p_2(x) &= x^2 - Nx + \frac{N^2 - N}{6}, \\
p_3(x) &= x^3 - \frac{3N}{2}x^2 + \frac{6N^2 - 3N + 2}{10}x \\
&\quad - \frac{N(N-1)(N-2)}{20}\,.
\end{aligned}$$

More generally, a term of the series is defined as

$$p_{k+1}(x) = \left(x - \frac{N}{2}\right) p_k(x) - \frac{k^2((N+1)^2 - k^2)}{4(4k^2 - 1)}\, p_{k-1}(x)\,, \tag{7}$$

and their squared norms are given by

$$\|p_k\|^2 = \frac{(k!)^4}{(2k)!\,(2k+1)!} \prod_{i=-k}^{k} (N + 1 + i)\,, \qquad k = 0, 1, \ldots, K\,. \tag{8}$$

Based on these definitions, it is now possible to redefine our time series s from Eq. 1. As the vector $\boldsymbol{\alpha}$ contains the estimators up to degree K for the considered time series, it is said that *s is approximated by $\boldsymbol{\alpha}$* with the statement

$$s = (y_0, y_1, \ldots, y_N) \sim \boldsymbol{\alpha}\,, \tag{9}$$

$$\text{where } s \in \mathbb{R}^{N+1}, \quad \boldsymbol{\alpha} \in \mathbb{R}^{K+1}, \quad K \ll N.$$

3.2. Segmenting time series

Segmenting time series is useful for analyzing and comparing subsets of data points. The considered segmentation approach in this paper is *dynamic*, meaning segmentations are performed in accordance with the intrinsic shapes underlying in the data, in contrast to other segmentation approaches that only depend on an artificial window size or with equal size segments. To do so, the sequence s from Eq. 1 is split into the set of contiguous windows

$$W(s) = \bigcup_{s^{(i)} \in s} s^{(i)}\,, \quad i \in \{0, 1, \ldots, N/2\}\,, \tag{10}$$

where W is a partition of s, and $s^{(i)}$ is the i_{th} segment containing two or more elements. To be dynamic, the partitioning is done by detecting *change points* $\hat{\boldsymbol{x}}$ within our time domain $x = (x_0, x_1, \ldots, x_N)$, relating to the concept of abrupt changes detection [30].

Let us consider a small example. Within the sequence $s = (y_0, y_1, \ldots, y_{10})$, the change points $\hat{x}$ detected are x_3 and x_7; then $\hat{\boldsymbol{x}} = (3, 7)$, $W(s) = \{s^{(1)}, s^{(2)}, s^{(3)}\}$, with $s^{(1)} = (y_0, y_1, y_2, y_3)$, $s^{(2)} = (y_4, y_5, y_6, y_7)$, $s^{(3)} = (y_8, y_9, y_{10})$. It has to be emphasized that these contiguous windows do not need to have the same size. In fact, their underlying estimators $\boldsymbol{\alpha}$ only depend on the shape of the segment. Therefore, *primitives*, or *basic* shapes are more accurately represented by these estimators, as the deviation of the predicted values (the sum of the residuals) is low within the considered segment (i.e., the estimators do not significantly change within a segment while the windows is growing). This property justifies why an adaptive way of segmenting the time series with change points is preferred to fixed-size window segmentation methods, considering the objective of this study.

As part of an iterative process, the first segmentation step starts with a window from the first point of the segment, setting $x = 0$. The corresponding orthogonal expansion is computed, holding the α coefficients. Then, specific criteria based on the coefficients are derived (such as the deviation of the predicted value or the count of sign switches of the slope) and compared to thresholds. If the thresholds are exceeded, the growing process is stopped, a change point is detected and a new segment starts with the next available data point; otherwise, the window keeps growing to the next point, the expansion is updated and the new coefficients are again compared. Figure 2 depicts an example of this segmentation.

This segmentation method is based on the *SwiftSeg* algorithm from Fuchs et al. [14]. Their study describes an on-line algorithm for updating the values of the coefficients, where the computation only depends on the last point added to the window, leading to effective computational costs; in contrast to off-line algorithms that need the entire window to update the coefficients. Combinations of growing and fixed-length window are also documented and experimented.

3.3. Fuzzy time series

The concept of fuzzy time series as first defined by Song and Chissom in [16] is here resumed. Let us consider the universe of discourse

$$U = \{u_1, u_2, \ldots, u_m\} \tag{11}$$

and the set

$$A_i = \mu_{A_i}(u_1)/u_1 + \ldots + \mu_{A_i}(u_m)/u_m\,. \tag{12}$$

A_i is a fuzzy set of U, where '/' indicates the separation between the membership grades and the elements of the universe of discourse U, '+' is the union of two elements, and the fuzzy membership function

$$\mu_{A_i}(u_j) : U \to [0, 1] \tag{13}$$

expresses the grade of membership of u_j in A_i.

Let the elements of our time series $(y_t)(t = 0, 1, \ldots, N)$, a subset of $\mathbb{R}$, be the universe of discourse replacing U on which the fuzzy sets $A_i(i = 1, 2, \ldots)$ are formed and let f_t be a collection of $\mu_{A_i}(t)(i = 1, \ldots, m)$. Then, $f_t(t = 0, 1, \ldots, N)$ is called a fuzzy time series on $y_t(t = 0, 1, \ldots, N)$.

A fuzzy relationship between one point at time (t) and its successor is represented by:

$$f_t \implies f_{t+1}\,. \tag{14}$$

We suggest a slightly more generic definition that can deal with segments. Indeed, we will consider fuzzy relationships between *any element* at time (t) and its successor, with f_t being the segment $s^{(t)}$ and f_{t+1} its segment $s^{(t+1)}$ (i.e., f_t describes the *entire* segment, instead of a specific point of the time series).

3.4. Fuzzy inference systems

Fuzzy inference systems (FIS) can model uncertain and complex human reasoning tasks. FISs use "IF antecedent THEN consequent" rules as inference mechanism, where the antecedent and the consequent of the rule are linguistic terms that can handle multi-valued logic.

Different types of fuzzy inference systems exist. Two of them are widespread in the literature, namely the Takagi-Sugeno [23] and the Mamdani [22] type. The main difference between these two is that the latter uses output membership functions to describe linguistic terms, whereas the former uses output membership functions to describe crisp values. In this paper, the Mamdani inference system is considered because of its relative simplicity.

A fuzzy inference system is defined (see Fig. 3) by a *rule base* containing the set of "IF-THEN" rules; a *database* with the fuzzy sets and their membership functions; a *decision-making unit* performing inference based on the rules; a *fuzzification interface* transforming the crisp inputs into degrees of match with linguistic values; and a *defuzzification interface* transforming the fuzzy results of the inference into numbers.

On top of this structure, the inference process is defined according to the 5 following stages:

1) fuzzification of the inputs;
2) combination of the antecedents with conjunction or disjunction functions;
3) rules firing and implication of the consequent;
4) aggregation of the consequent; and
5) defuzzification of the output.

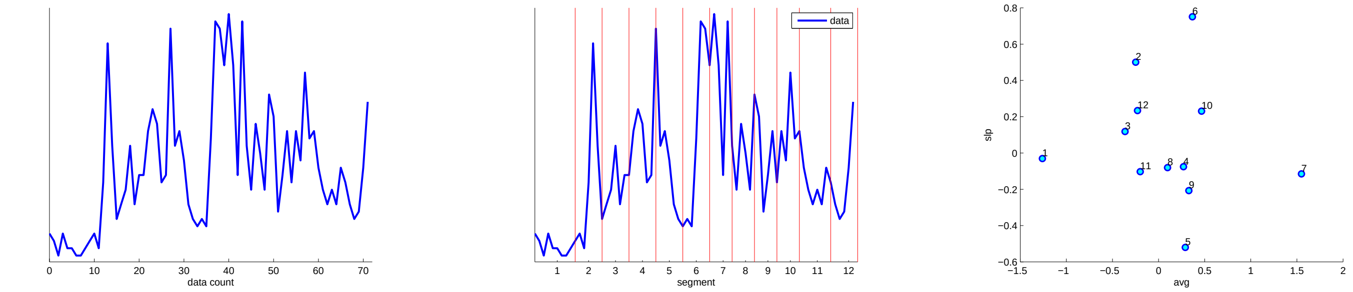
4. The proposed method for the fuzzy detection of change points in crime-related time series

In this section, a novel method for the Fuzzy Change Points Detection in crime-related time series, FCPD, is proposed. Based on the particular shape space representation and the dynamic segmentation method described in Sect. 3, a unique approach is proposed.

FCPD consists of 2 steps:

1) the *segmentation* of the time series by the means of the shape space representation (represented in pseudo-code by Algorithm 1);
2) the fuzzy *querying*, by the use of linguistic variables, of change points based on the discovered segments (represented in pseudo-code by Algorithm 1).

Figure 2: Illustration of the segmentation of a time series with its shape-space representation.



(a) A real time series of 72 samples. The time series has been normalized ($\mu = 0$, $\sigma^2 = 1$). From the pretty chaotic shape of the series, many change points are supposed to be found.

(b) Segmentation of the time series, with $K = 3$. Twelve different segments are found (represented by vertical lines). The segmentation here is based on the number of sign changes of the slope and the deviation of the predicted value.

(c) Segments depicted with their shape space representation (only the first two coefficients are shown). The segments 1 and 7 are easily identified as with the lowest and highest average (α_0) and the segments 5 and 6 with the lowest and highest slope (α_1).

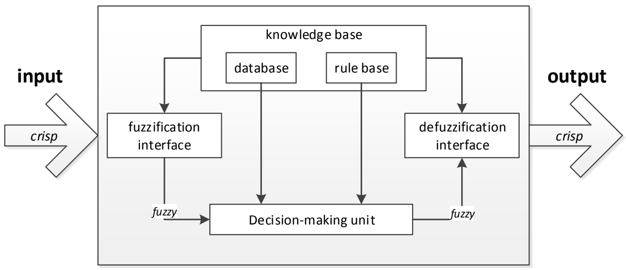


Figure 3: Structure of a fuzzy inference system.

These 2 steps are performed on-line. Algorithm 1 starts with the very first observation of a time series and grows a window at every new observation. Every time a new segment is set (defined by some error criteria), Algorithm 2 can be run to answer queries bases on the discovered segments (i.e., the outputs of Alg. 1). The FIS structure (i.e., the membership functions, the linguistic variables and the rules) and the query are defined by the user in accordance with the use case and do not change over time.

Algorithm 1: SEGMENTATION, for on-line segmenting

input : $s = (y_0, y_1, \ldots, y_n)$, the time series
K, the degree
th, the thresholds
output: $\boldsymbol{\alpha}$, the coefficients
begin
  $w \longleftarrow initWindow(s[0..K])$
  $c \longleftarrow initCoefs(w, K)$
  $i \longleftarrow K + 1$
  while $i <= n$ **do**
    $w \longleftarrow growWindow(w, s[i])$
    $c \longleftarrow updateCoefs(c, w, K)$
    if $newSegment(c, th)$ **then**
      remove $s[0..i]$ from s
      add c to $\boldsymbol{\alpha}$
      $\boldsymbol{\beta} \longleftarrow query(\boldsymbol{\alpha})$
      goto ***begin***
    $i \longleftarrow i + 1$

Step 1: Finding the segments $W(s)$ of the time series

Starting with a time series represented by $s = (y_0, y_1, \ldots, y_N)$ as in Eq. 1, the parameter K (i.e., the degree of the polynomials), and the thresholds, the segmentation process (Sect. 3.2) is iteratively applied. First, a growing window is positioned on the first element (y_0), the polynomial expansion is computed and its coefficients $\boldsymbol{\alpha}$ are extracted according to the chosen degree K. Based on these coefficients some segmentation criteria can be defined. Two of them are hereafter suggested.

The first threshold is the deviation of the predicted value:

$$c_{DPU} = \begin{cases} 1, & \text{if } |p(x_t) - y_t| > th_{DPU}, \\ 0, & \text{otherwise,} \end{cases} \tag{15}$$

where $p(x_t)$ is the predicted value of the regression, $y(x_t)$ the actual value at time t (the last point of the growing window), and th_{DPU} the value of the threshold. The second is the counting of the sign switches of the slope:

$$c_{SSS} = \begin{cases} 1, & \text{if } SSS > th_{SSS}, \\ 0, & \text{otherwise,} \end{cases} \tag{16}$$

where SSS counts the number of sign switches of the slope within the window, that is, SSS is incremented if a change in the sign of the slope is observed, and th_{SSS} the value of the threshold. A new segment $s^{(i)}$ is added to $W(s)$ if these criteria are met (one single criterion can be enough), and the segment is then represented by the last $\boldsymbol{\alpha}$ computed, $\alpha^{(i)}$. Otherwise, the window is grown by adding the next point and the same steps are repeated until the end of the time series (i.e., $\boldsymbol{\alpha}$ is updated and the new criteria are again compared to the same thresholds). The result of this step is the set of continuous windows $W(s)$, their respective coefficients $\boldsymbol{\alpha}^{(i)}$ and their change points $\hat{x}^{(i)}$.

Algorithm 2: QUERY, for on-line change points querying

input : q, the query (global variable)
F, the FIS structure (global variable)
α, the coefficients
output: β, the sorted segments
begin
$FIS \longleftarrow initFIS(F)$
$scores \longleftarrow inferFIS(FIS, \alpha, q)$ 'see Fig. 3
$\beta \longleftarrow sortSegments(scores)$

Step 2: Querying the existence of particular change points with a fuzzy inference system

A query consists of the expression of geometric properties with linguistic values related to the coefficients, and the result of the query is the corresponding "relevance score" of each segment regarding the query. The query is specified with a fuzzy "IF-THEN" rule. To evaluate the query, features related to the coefficients from the subspace representation are given as input to the FIS, the query is added to the rule base, and the system infers the output. The defuzzification of the output is the answer to the original query. The membership functions of the FIS have to be specified at the beginning. In fact, linguistic variables and membership functions are part of the query, specified by the user to detect change points with regard to their applications. These parameters should not change over time, unless if the query itself changes. Setting the FIS amounts to:

a) *Choosing the input(s) and the output(s) of the fuzzy inference system in relevance of the query.*
Inputs constitute the antecedent part of the rules and outputs the consequent. As initial intuition, inputs could be the coefficients $\alpha^{(i)}$. The use of these coefficients enables the expression of linguistic terms concerning the average, the slope, or the curvature of the segments in the antecedent part of the rules.
However, to handle queries considering more aspects, other input variables can be considered to get different expressions. Relations between two *elements* (as in Eq. 14, where $f_t \implies f_{t+1}$), can also be inputs. For instance, if the element is a segment, the input of the FIS is then the coefficient variation between two segments, defined as:
$$v_{\alpha_k}(s^{(t)}, s^{(t+d)}) = \frac{\alpha_k^{(t+d)} - \alpha_k^{(t)}}{\alpha_k^{(t)}}, \tag{17}$$
where $s^{(i)}$ is the segment of index i in $W(s)$, $\alpha_k^{(i)}$ the coefficient of order k of this segment, and d the delay operator of segments, typically set to 1. For the sake of simplicity, these variations will be referred to as
$$v_{\alpha_k}^{i->i+d}. \tag{18}$$
In other words, the use of variations, instead of the coefficients input to the FIS, enable to express relative changes between two periods instead of absolute changes of value only.
Other inputs to consider are for instance the size of each segment, the variation of the size, or a set of primitive shapes. A combination of these is also possible.
The output of the FIS is more straightforward. Given the rules, the FIS outputs the degree of similarity of each input to the geometric properties specified in the query. Therefore, only one output —the relevance of the query— is assumed to be necessary in most cases. A FIS with several outputs is nonetheless possible.

b) *Defining the linguistic terms and their membership functions.*
Each input/output of the inference system is generally defined by multiple fuzzy sets. For example, (*LOW*), (*MEDIUM*), and (*HIGH*) can be fuzzy sets for the coefficients as input, whereas (*DECREASE*), (*CONSTANT*), and (*INCREASE*) are sets of variations between elements. For these terms we need membership functions that can be valued, as part of the fuzzification and defuzzification process.

c) *Defining the inference rules.*
The rules added to the inference system are the heuristics that guide the search to find the appropriate change points. These heuristics use the geometric estimators from the coefficient to express visual criteria of the researched segments. These inputs are evaluated in the antecedent of the rule and the output in the consequent. A weight can be added to each rule, giving different degrees of importance in accordance with the confidence of the heuristic.

d) *Inferring the output(s).*
Infer the output(s) of the FIS (as described in Sect. 3.4). According to the rules, the segments which are the most relevant to the query output a higher membership of the consequent.

5. Empirical Evaluation

This section provides an empirical evaluation of the proposed method, with a focus on crime data. For the sake of an overall evaluation, different types of time series with different objectives are analyzed.

First, qualitative analyses, representing illustrated case studies helping practitioners to better evaluate the method, are conducted (with a total of 4 time series):

1) the analysis of cyclic data, to illustrate the use of the proposed method in a simple environment;
2) a case study of crime trends monitoring, to support the validity and applicability of flexible change points detection and querying according to the domain of crime analysis;
3) the analysis of the *TOPIX* time series (financial real-world data), in a financial case study, to test the domain-free applicability of FCPD;

Second, quantitative analyses, each time systematically compared with two comprehensive data sets (with a total of 96 time series):

4) a comparison with a similar change points detection algorithm, BFAST, is performed on both the CICOP and the

SWX data sets for assessing the accuracy and the complexity of FCPD;

5) a sensitivity analysis on both the CICOP and SWX data sets is carried on for measuring the impact of the parameters on the results of the proposed method.

These two data sets used in the quantitative part are the following:

A) the *CICOP* data set (crime real-world data), consisting of 32 time series each with 70 observations. These time series describe monthly events for a period of 6 years (2009-2014) of serial- and itinerant-related crimes (such as burglaries);

B) the *SWX* data set (financial real-world data), consisting of 64 time series each with 120 observations. These time series describe monthly stock data for a period of 10 years (2005-2014) from the small and medium capitalizations of the SWX (Swiss Exchange Market).

For these experiments, a MATLAB version of FCPD has been implemented by the authors. Time series were normalized ($\mu = 0, \sigma^2 = 1$) before analysis. The only reason for normalizing time series is to make comparison between different data sets easier: indeed, normalizing time observations leads to consistent thresholds throughout all time series (i.e., thresholds in accordance with the mean and the variance of the time series).

5.1. Simulation with cyclic data

This first experiment aims to detect simple change points within the cycle time series. For that purpose, we generated a normalized sinusoidal time series of 2000 data points, representing a cyclical activity. We introduced two "anomalies", the first between the x-interval [500, 600] by adding noise with a standard normal distribution ($N(0, 1)$) to the observed values and the second between [1400, 1600] by replacing the observed values with $y = 0.5 + u/2$, where u is a noise factor with a standard normal distribution.

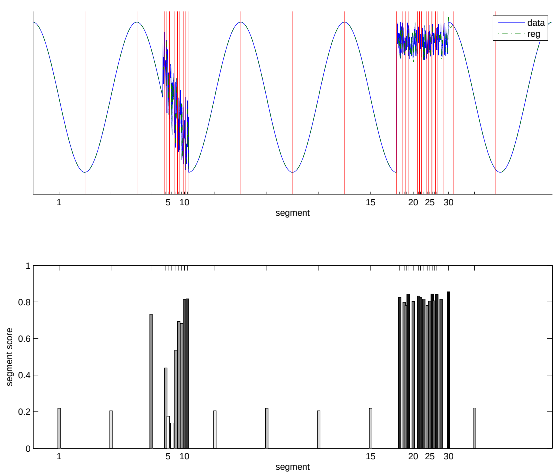


Figure 4: The cycle time series. *(Top)* Segmentation of the time series, 31 change points detected. *(Bottom)* Output of the FIS for each segment, representing the relevance of the query.

For the segmentation step, we set the maximum degree of the polynomial regression K equals to 5 and a single threshold for the switches of the sign slope (th_{SSS}) to the value of 1. As a result, 31 change points were detected (top of Fig. 4). We then used as input to the FIS the coefficient matrix for the average (i.e., the coefficients of degree 0 for each segment). The input is described with 3 different linguistic terms, namely *negative*, *zero*, and *positive*, respectively represented by a Z-shaped, a Gaussian, and a S-shaped membership function as depicted in Fig. 5. The output membership is a triangular-shaped function, using *low*, *medium*, and *high* as fuzzy sets to denote the relevance of the queried geometric properties (Fig. 6). For the inference part, we chose the *min* function for the implication, the *max* function for the aggregation, and the *centroid* function for the defuzzification. The query for identifying change points in the cycle is then modeled through the following rules:

a) IF (*average* is not *zero*), THEN (*score* is *high*)

b) IF (*average* is *zero*), THEN (*score* is *low*).

These two basic rules use the average of each segment to determine the degree of change within the cycle. The output of the FIS (bottom of Fig. 4) describes a score within the $[0, 1]$ interval for each segment of the time series. High scores are produced for the values where some noise was added (segments 3 to 11, and segments 16 to 30), which confirms the expected results.

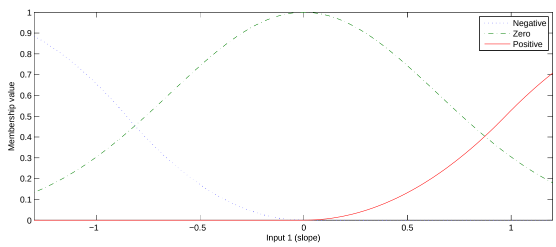


Figure 5: Input membership functions of the cycle time series.

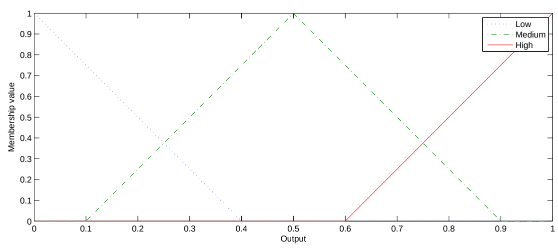


Figure 6: Output membership functions of the cycle time series.

5.2. Case study: crime trends monitoring

The objective of crime trends monitoring is to automatically detect change points within the development of crime. This case study shows how crime analysts can monitor sudden changes in the number of crimes, allowing them better to allocate resources (e.g., by sending dedicated patrols when a rise is detected). We want to emphasize that the proposed method is used on-line, meaning that we do not need the entire time series to perform these analyses and the results only depend on past values.

For this purpose, we illustrate the detection of change points with two time series from the CICOP data set. The first time series describes evening burglaries of individual houses or flats, with 72 monthly data points for a period of 6 years (top of Fig. 7). The second time series represents ATM break-ins, with the same sampling (top of Fig. 8). A more detailed description of this data set can be found in [3]. The segmentation settings are identical for both time series of the data set: values are normalized, two disjunctive thresholds are set ($th_{DPU} = 0.05$ and $th_{SSS} = 2$; only one threshold need to be exceeded to set a new segment), and K is set to 5. The input of the FIS is the coefficient *variation* between two consecutive segments ($v_{\alpha_k}^{i->i+1}$, as in Eq. 18) of the average coefficient, with 5 fuzzy sets (the membership functions are shown in Fig. 9).

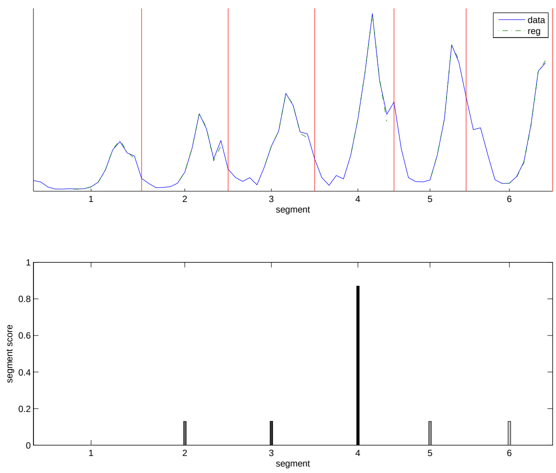


Figure 7: Evening burglaries time series from the CICOP data set. *(Top)* Segmentation of the time series, with 6 change points detected. *(Bottom)* Output of the FIS for each segment, representing the relevance of the query (i.e., changes in trend).

The same three inference rules for both time series were given as heuristics to querying changes in the trend:

a) IF (*var_average* is *large_decrease*), THEN (*score* is *high*)
b) IF (*var_average* is *large_increase*), THEN (*score* is *high*)
c) IF (*var_average* is *constant*), THEN (*score* is *low*).

The highest score for both time series is observed in segment 4. Indeed, both time series are suggesting a high variation of the average after segment 3; in the next segments, the trends remain pretty stable and the score remains low.

5.3. Case study: change points detection on the financial TOPIX time series

To assess the general applicability of the method in a different domain, we used the TOPIX time series to detect and query

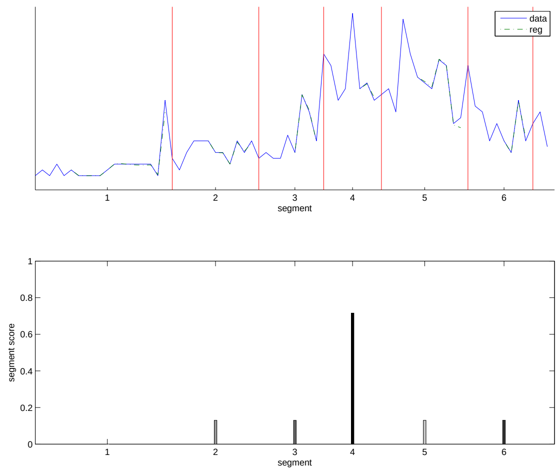


Figure 8: ATM break-ins time series from the CICOP data set. *(Top)* Segmentation of the time series, with 6 change points detected. *(Bottom)* Output of the FIS for each segment, representing the relevance of the query (i.e., changes in trend).

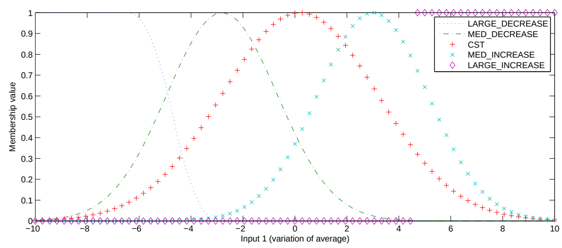


Figure 9: Input membership functions of both evening burglaries and ATM burglaries time series.

change points. The weekly values consist of 522 data points (years 1985 to 1994) from the TOPIX index (*TPX:IND*, i.e., the Tokyo Stock Exchange Price Index). We also compare our results with the work of Yamanishi and Takeuchi in [38] and [39], which have used the same time series for change points and outliers detection.

The purpose of this case study is to detect change points considered as the steepest slopes of the considered time-frame. In the financial domain, it proves very useful to find change points in real-time, giving the investors a trend indicating the right moment for selling or buying stocks.

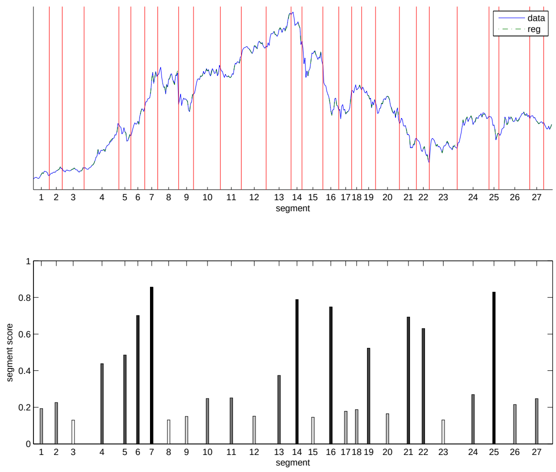


Figure 10: The TOPIX time series. *(Top)* Segmentation of the time series, with 27 change points detected. *(Bottom)* Output of the FIS for each segment, representing the relevance of the query (i.e., finding the steepest slopes).

For the segmentation process, we set K to 5 and two independent thresholds (th_{SSS} = 2 and th_{DPU} = 0.05 respectively for the switches of the sign slope and the deviation of the predicted value). Figure 10 (top) shows the 27 change points detected.

The querying step is used for identifying steep slopes. As input to the FIS, the slope coefficient is used. The fuzzy sets describing the input are *negative*, *zero*, and *positive* (Fig. 11). The output membership is the same as for the cycle time series (Fig. 6). Three inference rules were given as heuristics to describe a steep slope shape:

a) IF (*slope* is *negative*), THEN (*score* is *high*)
b) IF (*slope* is *positive*), THEN (*score* is *high*)
c) IF (*slope* is *zero*), THEN (*score* is *low*).

Segments #7, #25, #14, and #16 have the highest score with FCPD (see Table 1). We want to emphasize that these results depend on the segmentation method, as the slope of the segment is the average of the slopes of the data points belonging to the segment.

In their experiment [38], Yamanishi and Takeuchi highlight 4 significant changes, occurring in our resulting segments #8, #2, #14, and #25. These results are very similar to ours (i.e., segment #7 from FCPD occurs in segment #8 with Yamanishi et al., segment #24 in segment #25, segment #14 is identical, and segment #16 has no direct correspondence).

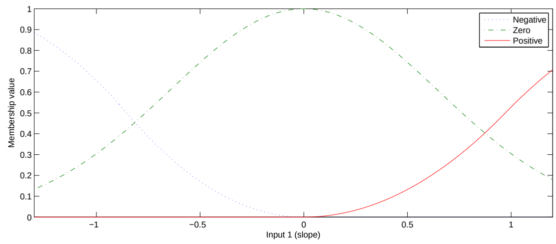


Figure 11: Input membership functions of the TOPIX time series.

Table 1: Top 4 segments in the TOPIX time series identified as the *significant changes* with the proposed method. Segment intervals are specified by values on the x-axis.

Rank	Number	Interval	Score
1	7	[112,125]	0.856
2	25	[458,468]	0.829
3	14	[259,270]	0.789
4	16	[291,307]	0.748

Besides, we also want to illustrate that FCPD is not limited to change points detection. Indeed, the shape space representation can be used to perform other types of analysis based on the meaningful distance computed with the shape space representation, such as clustering. In our example, we attempt to discover basic/primitive shapes in the time series. For that, we apply the K-means algorithm to the slope and curvature coefficients of the segments (i.e. the output of step 1 of the proposed method, Sect.4), with the number of clusters set to 4 (with the objective to delineate both negative and positive clusters of slopes and curvatures). The centroids are depicted in Fig. 12. The closest segments to the centroids, identified as the 4 potential primitive shapes, are shown in Fig. 13. One should notice that in this case the slope variable contains more information on the shape than the curvature variable.

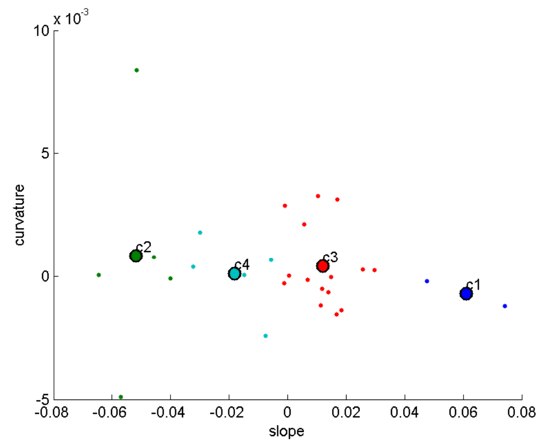


Figure 12: Clustering of the segments from the TOPIX time series. The segments are described with their slope and their curvature.

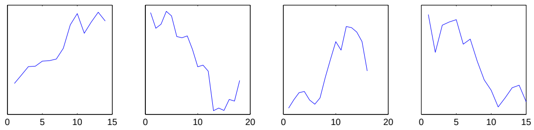


Figure 13: Primitive shapes of the TOPIX time series, resulting from the clusters

5.4. Comparison with the BFAST algorithm

Our proposed method is compared to the "break for additive seasonal and trend" (BFAST) algorithm ([34] and [35]) in terms of similarity of the three most important change points detected by each method. BFAST has originally been developed for detecting changes in phenology, more precisely for climatic variations from remote sensor data. The method is however not specific to a particular data type. It uses the seasonal-trend decomposition procedure based on Loess (STL) and an estimation of breaks based on [43] with least-squares. It basically performs two steps, each being off-line, requiring access to past and future values in the computation window: first, the seasonal component is computed and removed from the observed data and second, the breakpoints are estimated.

We used the implementation of the R package *bfast* with standard parameters (*h = 0.15*, *max.iter = 1*, *season* = "*harmonic*", *breaks = 3*) to find a maximum of 3 significant change points in both the CICOP and the SWX data sets.

The settings in FCPD are the same for both data sets, as the nature of these two data sets are similar and the objectives are identical. We set a threshold for the deviation of the predicted value (th_{DPU}) of 0.11 and K to 5. Fig. 14 depicts the uniform membership functions for the input variables and Fig. 15 for the output variable. The rules are simply relating the degree of variation to the degree of change, considering both the average and the slope:

a) IF (*var_average* or *var_slope* is *very_large_decrease*), THEN (*score* is *very_high*)
b) IF (*var_average* or *var_slope* is *large_decrease*), THEN (*score* is *high*)
c) IF (*var_average* or *var_slope* is *medium_decrease*), THEN (*score* is *medium*)
d) IF (*var_average* or *var_slope* is *small_decrease*), THEN (*score* is *low*)
e) IF (*var_average* or *var_slope* is *constant*), THEN (*score* is *very_low*)
f) IF (*var_average* or *var_slope* is *small_increase*), THEN (*score* is *low*)
g) IF (*var_average* or *var_slope* is *medium_increase*), THEN (*score* is *medium*)
h) IF (*var_average* or *var_slope* is *large_increase*), THEN (*score* is *high*)
i) IF (*var_average* or *var_slope* is *very_large_increase*), THEN (*score* is *very_high*)

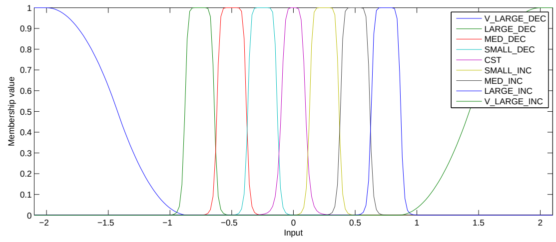


Figure 14: Input membership functions for the FIS of the CICOP and SWX data set.

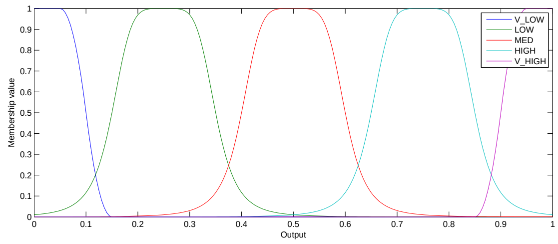


Figure 15: Output membership functions for the FIS of the CICOP and SWX data set.

For the comparison, the 3 most significant points detected by BFAST are compared to the 3 most significant points detected by FCPD, and the offset in the occurrence of the detection (measured in number of data points from the time domain) is reported. The offsets are the difference between each significant points from BFAST and FCPD (as illustrated in Fig. 16).

For the 32 time series of the CICOP data set (the data set actually contains 60 time series, but we only used the time series in which at least one change was detected by BFAST for a consistent comparison), the offsets are shown in Fig. 17 (left).

For the 64 time series of the SWX data set (the data set consists of 70 time series, but we only considered time series where all values were defined for the concerned period), the offsets are shown in Fig. 17 (right).

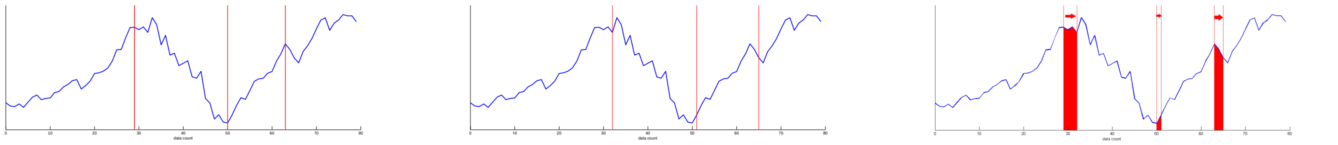
Table 2 summarizes this comparison. It shows that the average offset for the CICOP data set is about 4.02 data points, and about 8.15 for the SWX data set.

Table 2: Offset statistics for the 3 most significant changes (O1, O2, O3), compared with BFAST, in absolute values.

	O1	O2	O3	μ	σ
CICOP	4.66	3.89	3.50	**4.02**	0.48
SWX	7.09	5.88	11.49	**8.15**	2.41

The average observed absolute offsets of 4 data points for CICOP and 8 data points for SWX with BFAST are considered as pretty good results in terms of similarity, especially when we know that FCPD is on-line and therefore only past values are used to detect change points, which is not the case with BFAST. Because of that, in the context of crime trends monitoring, BFAST cannot be used in a real environment. The distributions of the offsets from Figure 17 show that very few time

Figure 16: Illustration of the comparison of the 3 most significant changes between the BFAST algorithm and the proposed method.

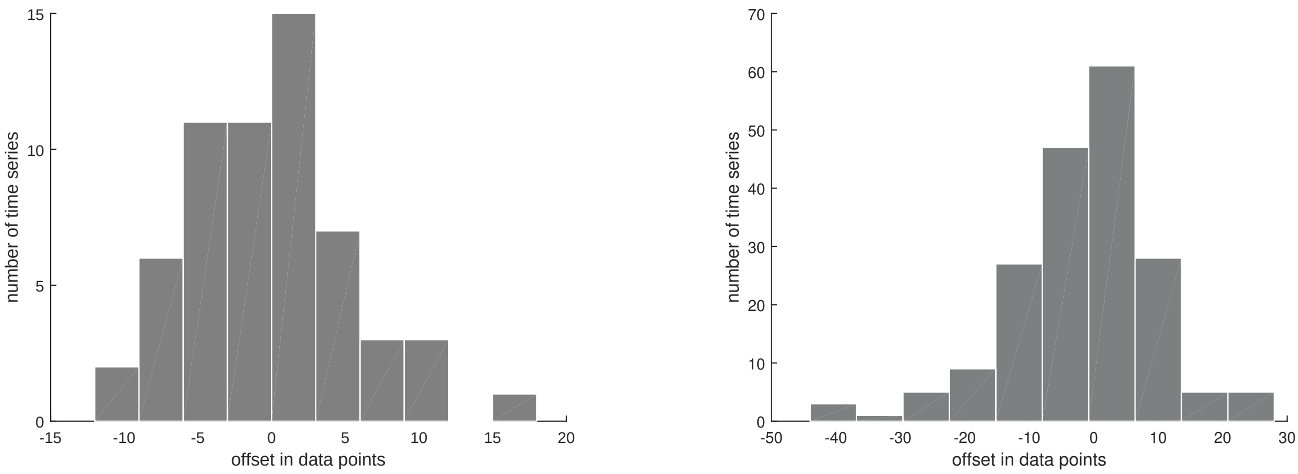


(a) The 3 most important change points detected by BFAST.

(b) The 3 most important change points detected by FCPD.

(c) Calculation of the occurrence offset (red areas, from left to right): the 1st offset is reported as +3 data points in the x-axis for FCPD, the 2nd as +1, and the 3rd as +2.

Figure 17: Distribution (histogram) of the offsets of the 3 most significant segments of the CICOP and the SWX data sets. A negative offset indicates that the FCPD algorithm detected the point before the BFAST algorithm.



(a) Distribution of the CICOP data set.

(b) Distribution of the SWX data set.

series present an offset exceeding the absolute value of 10 in data points, and also suggest that the offsets are equally distributed in terms of lag or of lead.

As an attempt to mimic the BFAST behavior and for comparison only, we used an automated method to fine-tune the FIS parameters, that is the settings of the membership functions, the linguistic variables and the rules. The MATLAB implementation of an adaptive-network-based fuzzy inference system (ANFIS, see [26]) was used with both the CICOP and the SWX data set to compare the results from Table 2, also with 5 membership functions for the same inputs. A single FIS was trained with both data sets. The consequent average offsets are higher than the "manual" version, that is 7.00 for CICOP and 11.72 for SWX.

This difficulty to extract more suitable parameters can have multiple causes. First, selecting the target of the ANFIS method (i.e., the supervised observations for the learning part) is far from obvious because many ways can be used to defined it. In our experiment, we decided to encode the score "1" when the discovered segment was within a region of ±2 data points of a BFAST detected point, and "0" otherwise. Second, both data set might not have enough observations for ANFIS to accurately learn the parameters from a machine learning perspective. And last, most of the ANFIS implementations only support Takagi-Sugeno inference types, as a result having less flexibility in the parameters and a different defuzzification method.

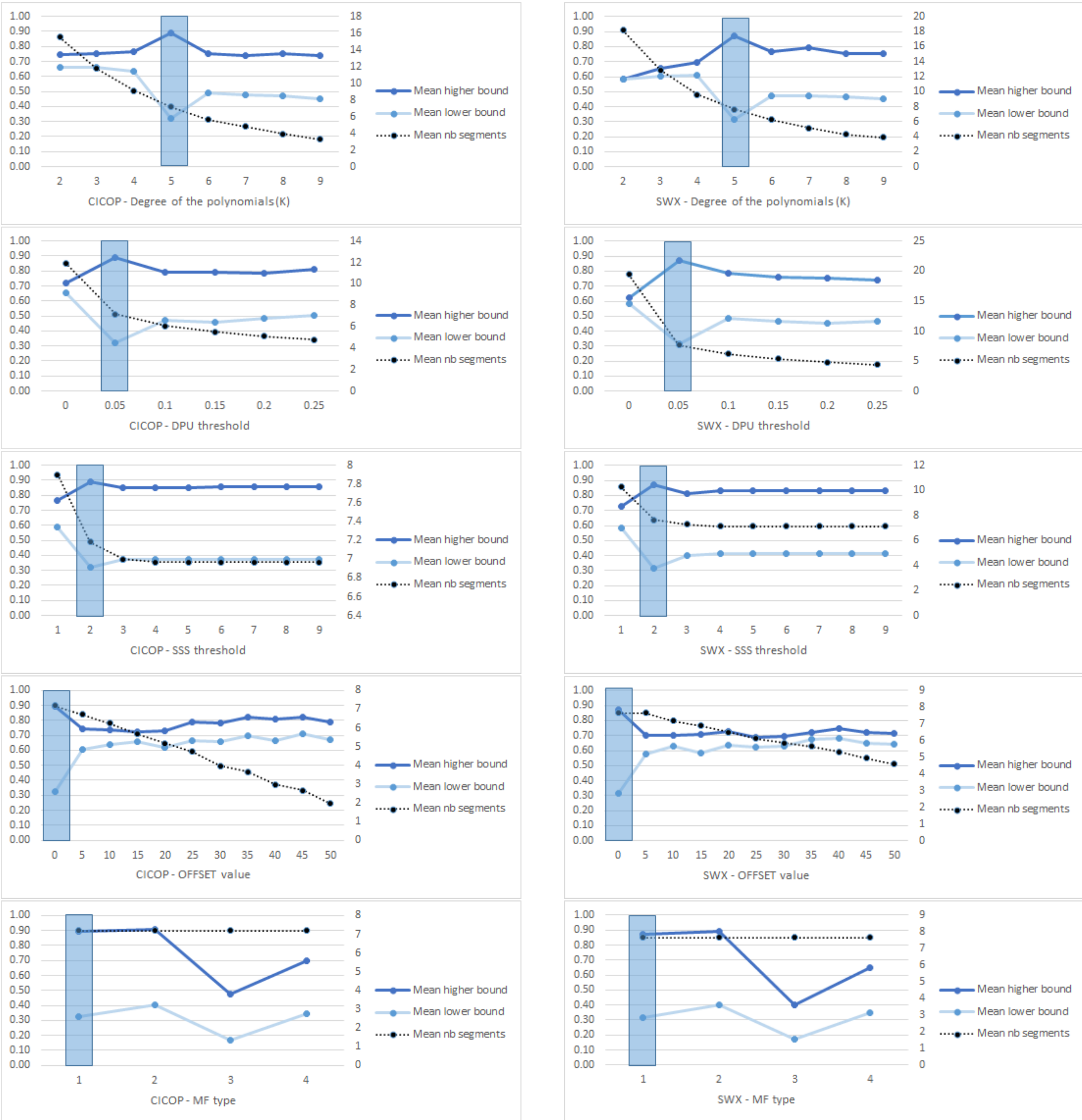
Besides, FCPD presents a huge advantage in terms of complexity. For comparison only, the most computational consuming step of BFAST, that is, the detection of breaks based on [43], is of $O(N^2)$; whereas in FCPD, for the regression, the complexity is of $O(K^2)$, where N is the number of observation and K the degree of the regression ($K \ll N$). As illustration, the running time for the SWX data set is 42 seconds for BFAST and 4 seconds for FCPD on the same computer.

5.5. *Sensitivity analysis*

In this part the sensitivity of the proposed method in regard to its parameters is evaluated. The variation of the score of a query are observed with regard to changes in the parameters of the segmentation step (i.e., K, the degree; th_{DPU} and th_{SSS}, the thresholds) and the querying step (i.e., the membership functions of the inputs). Default parameters are the ones used in the crime trends monitoring case study (Subsection 5.2). These variations are measured by computing the mean upper bound (i.e., the mean of the best 3 scores of the data set), the mean lower bound (i.e., the mean of the worst 3 scores of the data set), and the mean number of segments on both the CICOP and the SWX data sets.

These singular changes are introduced either on the parameter K, either on the threshold th_{DPU}, either on the threshold th_{SSS}, either in the introduction of an offset in the x-axis, or in the input membership functions (Fig. 18).

Figure 18: (From top to bottom) Effects of a change in the parameter K, th_{DPU}, th_{SSS}; on the introduction of an offset (*OFFSET*); and on the input membership functions of the query (*MF_TYPE*). The effects are measured on the mean lower bound and the mean upper bound of the CICOP data set (left-hand) and the SWX data set (right-hand). Lower bounds and upper bounds are calculated as the average of the 3 worst/best scores. Score values are the mean of all time series of the data set. The blue regions (K = *5*, th_{DPU} = *0.05*, th_{SSS} = *2*, *OFFSET* = *0*, and *MF_TYPE* = *1*) are the reference values for the comparisons.



The membership functions are depicted in Fig. 19. For membership functions *TYPE_3* and *TYPE_4*, the rules have been consequently adapted (the number of membership having decreased, they need to be adapted according to the output variables):

a) IF (*var_average* or *var_slope* is *large_decrease*), THEN (*score* is *very_high*)
b) IF (*var_average* or *var_slope* is *small_decrease*), THEN (*score* is *medium*)
c) IF (*var_average* or *var_slope* is *constant*), THEN (*score* is *very_low*)
d) IF (*var_average* or *var_slope* is *small_increase*), THEN (*score* is *medium*)
e) IF (*var_average* or *var_slope* is *large_increase*), THEN (*score* is *very_high*)

To better understand these results, let us take an example with a change of the degree on the CICOP data set (top-left of Fig.18). The reference value, denoted by the blue region, is shown as $K = 5$, meaning that 3 best/worst scores are defined as *reference* segments. Then, by modifying the value of K only, the score of these 6 reference segments will be compared with their mean lower bounds and mean upper bound. More generally, we can interpret these measures by saying that the bigger the difference between the lower and upper bound is, the higher the method is sensitive to the considered parameter. The effect on the mean number of segments should also be taken into account.

The first interesting observation is that the method does not seem particularly sensitive to the change of a singular effect. Indeed, the difference between the mean lower and mean upper bounds are relatively constant in most settings and for both data sets. We however denote a slightly higher difference with the thresholds.

Second, if we consider the mean upper bound, it remains high under most conditions, excepted for changes in the input membership functions. However, the mean lower bound seem to be pretty high. This could be explained by the mean number of segments, when it comes close to 6, i.e., the total number of change points considered only for the upper and lower bound.

Besides these singular changes, let us consider the interdependence of the parameters, that is between the segmentation step and the query step. The parameter K and the segmentation thresholds impact the segmentation results. If more segments are considered (i.e., by decreasing K or the thresholds, as seen in Fig. 18), the coefficients will describe the segments more precisely, but very local changes will be reported with respect to the query. On the other hand, settings parameters that create fewer segments (i.e., by increasing K or the thresholds), the coefficients will not be able to precisely describe the segment, resulting in inappropriate changes reported with respect to the query. Interdependence between the degree K and the thresholds $thDPU$ and $thSSS$ are shown in Fig. 20. The score is computed as the mean of the top 1 segment for both the CICOP and SWX data set. The variability seems to be relatively low, in the sense that modifying one parameter does not have a marked effect on the other.

6. Discussion

As mentioned in the previous sections, the proposed method presents three main advantages: (a) an intuitive and meaningful representation of the time series, (b) a dynamic and on-line segmentation method, and (c) a flexible and understandable querying system.

These claims are supported by our experiments: the two case studies illustrate the flexibility and the feasibility of the intuitive querying of change points; the comparison with the BFAST algorithm show similar results in terms of accuracy; the sensibility analysis show that the parameters of the segmentation part can be consistently determined; and in terms of computational complexity, FCPD is much more efficient than BFAST.

7. Conclusions

A method for the detection of change points within crime-related time series was described and tested with different data sets. The combination of a meaningful representation, a dynamic segmentation, and a fuzzy inference system delivers the possibility, even for experts not related to data mining, to intuitively find change points by describing geometric properties in linguistic terms. More broadly, the considerable flexibility of the method makes possible the use of the method in any application domain, with a great potential in crime analysis.

Future work suggest further investigation on the use of mining methods to automatically discover the most appropriate membership functions of the inference system in order to mimic the behavior of existing algorithms. This alternative could present a gain in the accuracy of the detected change points, however, the opposing view is a loss in the understanding of the inference system.

Also, an implementation of a crime trends monitoring process in a real environment should be tested and the results assessed in real time by crime analysts.

Acknowledgements

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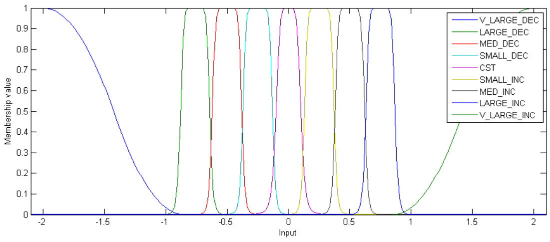
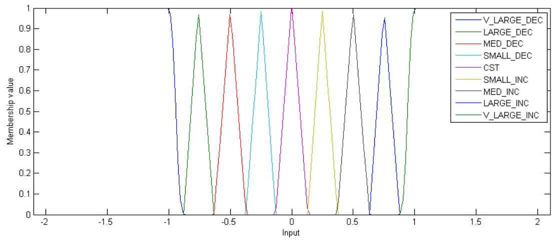
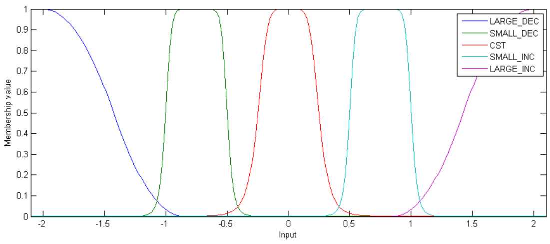
Figure 19: Input membership functions used for the sensitivity analysis.



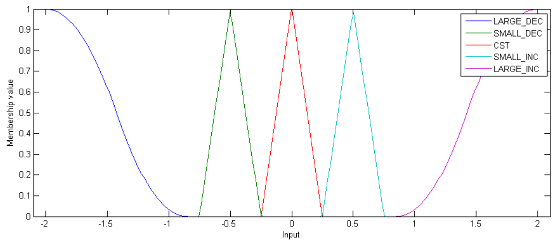
(a) Set of membership functions denoted as *TYPE_1*



(b) Set of membership functions denoted as *TYPE_2*

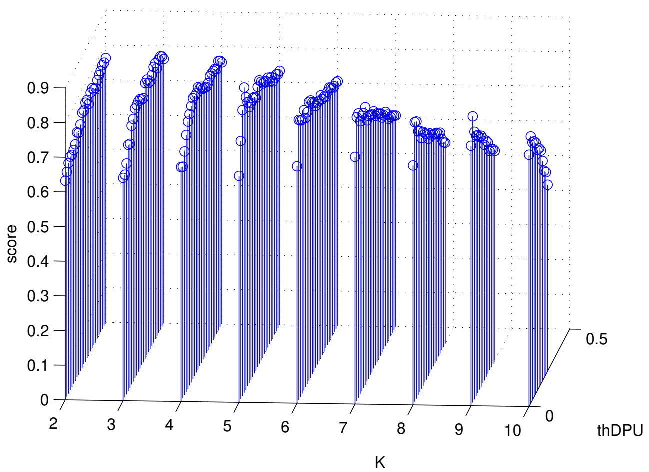


(c) Set of membership functions denoted as *TYPE_3*

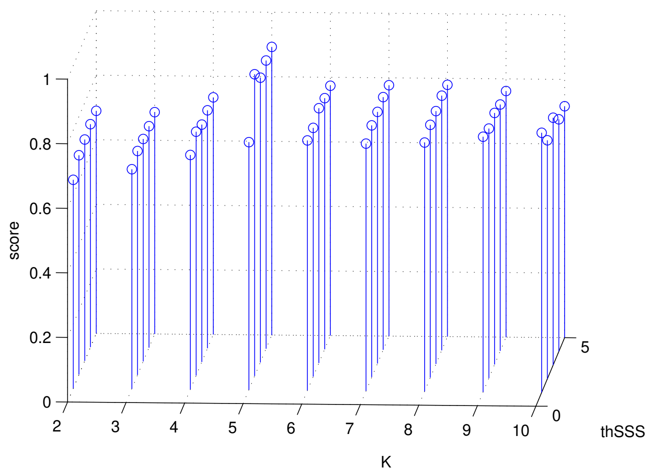


(d) Set of membership functions denoted as *TYPE_4*

Figure 20: Interdependence between K and the thresholds $thDPU$ and $thSSS$. The scores are the mean for both the CICOP and the SWX data sets of the top 1 segment. Missing values indicate that no segmentation was found.



(a) Interdependence between K and $thDPU$.



(b) Interdependence between K and $thSSS$.

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