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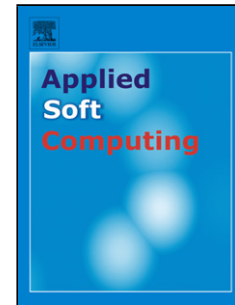
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Linear, Transfinite and Weighted Method for Interpolation from Grid Lines Applied to OCT Images

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Abstract

When performing a line scan using optical coherence tomography (OCT), the distance between the successive scan lines is often large compared to the resolution along each scan line. If two sets of such line scans are acquired orthogonal to each other, intensity values are known along the lines of a square grid, but are unknown inside each square. To view these values as an image, intensities need to be interpolated at regularly spaced pixel positions. In this paper we evaluate three methods for interpolation from grid lines: linear, transfinite and weighted. The linear method does not preserve the known values along the grid lines. The transfinite method, known from mesh generation, preserves the known values but might cause artifacts further away from the grid lines. The weighted method, which we propose, is designed to combine the desired properties of the transfinite method close to grid lines and the stability of the linear method further away. An important parameter influencing the performance of the interpolation methods is the upsampling rate. We perform an extensive evaluation of the three interpolation methods across a range of upsampling rates. Our statistical analysis shows significant difference in the performance of the three methods. We find that the transfinite interpolation works well for small upsampling rates and the proposed weighted interpolation method performs very well for all upsampling rates typically used in practice. On the basis of these findings we propose an approach for combining two OCT scans, acquired such that the lines of the second scan are orthogonal to the first.

Keywords: Interpolation, image processing, performance analysis, line scans, medical image analysis

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1. Introduction

Scanning along a set of parallel lines is a common setting in optical coherence tomography (OCT) [8], well established in ophthalmology for obtaining volumetric images of the retina. Using OCT, the retina is scanned in depth (z) and along a line (x) with a high depth and transversal resolution, resulting in a single xz cross-section of the retina (a so-called B-scan). Collecting a number of images by scanning along parallel lines results in a volumetric xyz data set. Since scanning speed of the OCT systems employed in clinic is limited, and prolonged scanning is unpleasant for the patient, the distance between the recorded B-scans is often large compared to the transverse resolution of the B-scans. Therefore, if an xy cross-section (a so-called *en face* image) is of interest, the resolution is much coarser in the y -direction and pixels are non-square. Elongated pixels appear as stripes and influence the visual appearance of the image. The stripes can disturb the interpretation of the image and make it difficult to distinguish the anatomical structures. Furthermore, anatomical structures running in parallel to scan lines, e.g. blood vessels, are not visible.

To reveal additional anatomical structures, another OCT scan may be performed along the lines orthogonal to the first scan. Nevertheless, this leaves us with two volumes which are not straightforwardly combined, as pixels are sparsely sampled along the y -direction in the first image, while the second volume has pixels sparsely sampled along the x -direction. It is our goal to compute a volume which combines the information from those two volumetric scans. Several problems emerge in connection to this. The eye might move during scanning, and this needs to be accounted for. Furthermore, the intensity might vary significantly between the scans and images. And most importantly, the question is how to combine two volumetric scans covering the same area, one with high resolution cross-sections in the xz -planes, and the other in the yz -planes.

In this paper we re-visit the interpolation problem when combining two line scans which we previously addressed in [9]. In this extension of our prior work, we apply the developed methods for merging two OCT volumes. As we have a high resolution in the z -direction, we practically sample at any height and our problem reduces to a 2D case. Furthermore, given high resolution along the scan lines we ignore the discrete sampling in this direction. Therefore, our problem is image interpolation from grid lines.

The problem is illustrated in Fig. 1 (a). The information is available along the two sets of parallel lines, and it needs to be sampled at regularly spaced pixel positions.

To the best of our knowledge, the problem of interpolating information from grid lines while preserving boundary values, has not been addressed in the context of image interpolation. In the context of mesh generation for finite element modeling a related problem is

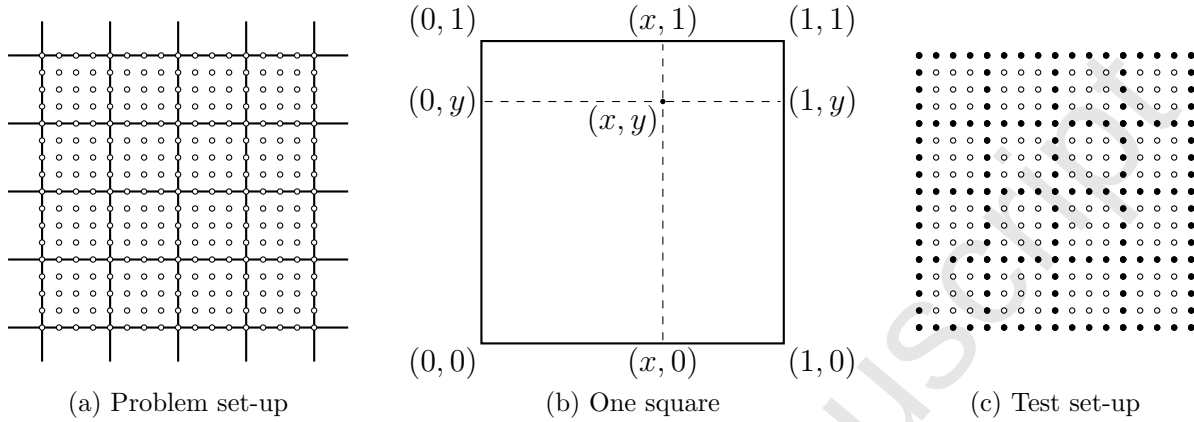


Figure 1: Interpolation from the grid lines. (a) two sets of scan lines with known intensities are shown in black, this need to be sampled at regularly spaced pixel positions illustrated as white dots. (b) shows one square region defined by four scan lines and its local coordinate system. (c) is a test set-up, black are the known and white are the unknown pixels, here shown with an upsampling rate of 4.

often solved using transfinite interpolation [6, 7], a method for constructing a smooth function over a planar domain given the values on the boundary. Transfinite interpolation has been used for solving problems where information on boundaries should be preserved. It has been used in more recent studies e.g. [14] for solving time-dependent changes of volumetric material properties in heterogeneous volumes and in [12] for solving elliptic boundary value Poisson problems in arbitrary shaped 2D domains. In this work we employ the transfinite interpolation for image interpolation from grid lines, and we compare it against an approach based on linear upscaling. Furthermore, we propose a novel weighted interpolation which preserves the desirable properties of the transfinite and the linear method.

The three interpolation methods presented here only use the known intensity information along the grid lines. We would expect the performance to improve significantly if prior knowledge about the appearance of the images is incorporated in the method. A significant work in this line has been conducted for single-image super resolution [4] or image inpainting [1], for example using image patches [17] and sparse representation [15]. We believe that those methods might be adapted to solve the problem of interpolation from the grid lines.

Another alternative to the methods presented here involves adopting an interpolation scheme for scattered data, for example radial basis function [2]. By doing so we would not utilize the regularity of the grid lines. Furthermore, methods for interpolation from scattered points require setting a parameter which roughly corresponds to the average distance between data points. The approach is therefore sensitive to parameter tuning, when applied to regularly placed data points as in our problem.

A quality measure for merging two OCT scans should relate to the ease of distinguishing the anatomical structures present in the volume, their sharpness and precision. While sharpness may be quantified, it is difficult to assess the precision of the interpolation. Central to our problem is that we need to determine information where it is lacking. This aspect is similar to image upscaling and single-image super-resolution approaches. Therefore, when it comes to evaluating the performance of interpolation algorithms we turn to the conventional approach [18, 13] which tests each method on a set of downsampled images and uses the peak signal-to-noise ratio (PSNR) metric.

During testing, we change the upsampling rates and statistically evaluate the results from the three interpolation methods. This allows us to evaluate the performance of the methods and to provide guidelines for different upsampling rates.

Based on our findings, we return to the OCT problem and develop a method for merging two OCT scans. There is a number of considerations before applying our interpolation to acquired OCT data. We need to assess the orthogonality of the two scans, ensure consistent intensity level, and correct for eye movement orthogonal to scanning plane. These adjustments are preformed as preprocessing of the OCT data. The complete method, including preprocessing steps, is presented as a case study in Sec. 5.

To summarize, the contribution of this paper is threefold. First, we suggest three methods for interpolation from grid lines, where we developed the novel weighted method such that it combines the desired properties from the two other methods. Second, we perform a rigorous statistical evaluation of the three methods for different data sets and upsampling rates, providing guidelines for the choice of method. Third, we apply the three interpolation methods on OCT images in a small case study.

2. Methods for Interpolation from Grid Lines

Fig. 1 (b) illustrates our interpolation problem focused on a single square region defined by two pairs of neighboring scan lines. This is a local coordinate system which we use when defining the three interpolation methods. The approach is then repeated for all squares in the image.

For a better explanation of the interpolation methods and their features, we bring an example in Fig. 2 (a). The values to be interpolated are here shown as a height above a squared domain, where we know the values at the boundary.

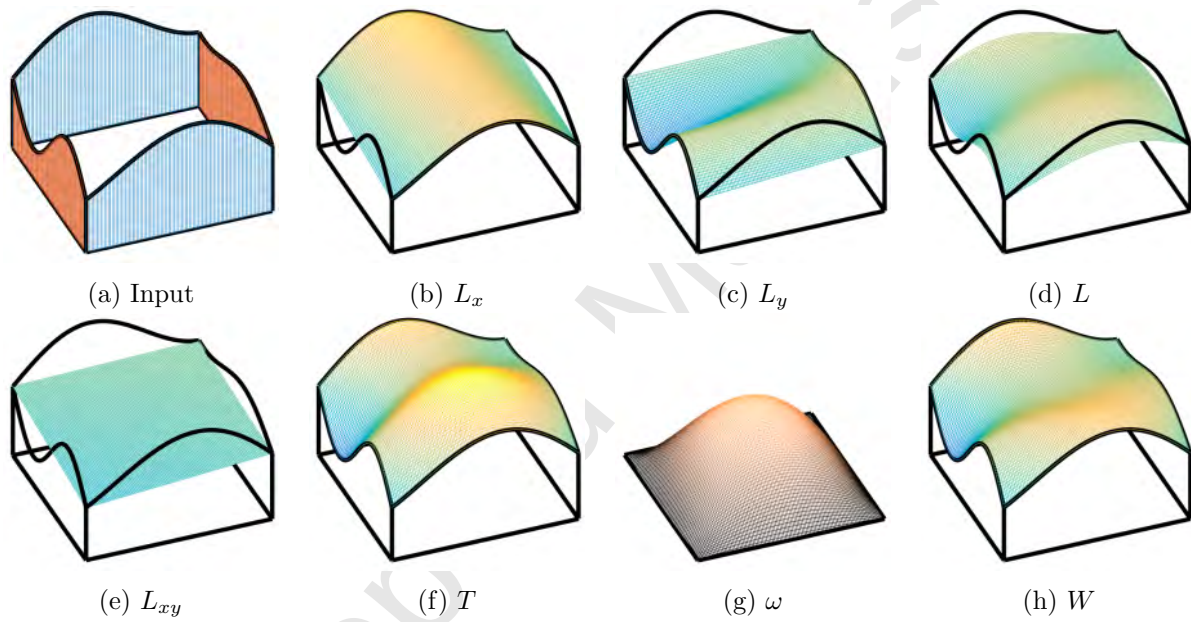


Figure 2: Interpolation over one squared domain. (a) know values from one direction in red and from the other direction in blue , (b) linear interpolation from one pair (blue) of rectangle sides, (c) linear interpolation from other pair (red) of sides, (d) mean of two linear contributions, (e) bilinear interpolation from corners, (f) transfinite interpolation, (g) weighting scheme (h) weighted interpolation.

2.1. Linear Interpolation Method

A naive approach of combining the two scans involves linearly upsampling each scan independently and averaging the results. Over one square domain we have

$$\begin{aligned} L_x(x, y) &= (1 - y)S(x, 0) + yS(x, 1), \\ L_y(x, y) &= (1 - x)S(0, y) + xS(1, y), \\ L &= \frac{1}{2}(L_x + L_y), \end{aligned}$$

where S are the known values along the boundary of the square domain, L_x and L_y are linearly upsampled boundaries in x and y direction, and L is the interpolant which we in this context denote *linear*. Construction of linear interpolation is demonstrated on Fig. 2 (a)-(d).

Let us point out two properties of linear interpolation. First, every value $L(x, y)$ is a convex combination of four values from S . As a result, L does not produce undesirable overshoot. Secondly, for a point on the boundary, the underlying known data contributes only with a half of its value, the other half coming from the values at two corners. As a result, L does not agree with the known data along the boundaries of the domain. Those two properties combined lead to smeared-out appearance when linear interpolation is used on images.

2.2. Transfinite Interpolation Method

Transfinite interpolation is used for functions given on the boundary of a domain which can be parameterized as a square. For our purposes this reduces to

$$T = L_x + L_y - L_{xy},$$

where

$$L_{xy}(x, y) = (1 - x)(1 - y)S(0, 0) + (1 - x)yS(0, 1) + x(1 - y)S(1, 0) + xyS(1, 1).$$

Here L_{xy} is the bilinear interpolant from the values at the corners of the domain, and T is the final transfinite interpolant. Those are shown in Fig. 2 (e) and (f).

The most important property of the transfinite interpolation is that it preserves the known values at the boundary of the domain. To perceive how this property is achieved by the construction of T , note in Fig. 2 that at the boundary of the domain, L_{xy} differs from the known values exactly twice as much as L does.

Second important property is that T may overshoot. This is due to the negative term in the expression. All inside points receive eight weighted contributions, and especially points close to the middle of the domain are prone to interpolation overshoot, also visible in Fig. 2 (f).

2.3. Weighted Interpolation Method

Transfinite interpolation has the desired properties (preservation of the known data) at the boundary of the square domain while the undesired properties (overshoot) are inside the domain. Linear interpolation does not overshoot, but has issues at the boundary. To combine the good properties of both methods we propose smoothly blending the linear and the transfinite interpolation. We construct a blending function which is 0 at the boundary of the square domain

$$\omega(x, y) = 16x(1 - x)y(1 - y).$$

The constant 16 is chosen such that $\omega(0.5, 0.5) = 1$.

We define our novel interpolation, which we denote *weighted*, as

$$W = \omega L + (1 - \omega)T.$$

This also evaluates to

$$W = (2 - \omega)L - (1 - \omega)L_{xy}.$$

The blending function and the weighted interpolation are shown on Fig. 2 (g) and (h). The illustrated example confirms desirable properties of the weighted interpolation. Like transfinite, the weighted interpolant matches the exact values at the boundaries of the domain. However, thanks to blending with the linear interpolant, the overshoot from inside of the domain is reduced. Finally, the smooth blending function maintains a smooth appearance of the interpolant.

3. Quantitative Measures Used for Evaluation of Interpolation Methods

Both objective and subjective tests are used [5] for evaluating the interpolation methods. Subjective tests measure a perceived image quality, while objective tests use a defined metrics for quantifying image quality or interpolation error. The choice of the tests and the quality measures depends on the intended use.

When used on OCT images, interpolation from grid lines is a step towards merging two OCT line scans. We plan to use the merged volume for automatic detection of anatomical structures and quantification of abnormalities in the eye. In this setting, the three interpolation methods will be evaluated in terms of the quality of the detection and quantification results.

In the work presented here, we bring a more meticulous and general evaluation of the interpolation methods based on measuring interpolation error for a specific upsampling rates. The ground truth is constructed by downsampling an image, which is then upsampled using

the three methods, and the results are compared against the original image. For down-sampling, we keep image columns and rows at a certain distance, which corresponds to an upsampling rate s . See Fig. 1 (c) for our test set-up.

For a certain upsampling rate, the fraction of the unknown pixel is

$$u = \frac{(s - 1)^2}{s^2}$$

For example, when $s = 2$ we keep every second row and every second column, and the fraction of unknown pixels is only 0.25.

To demonstrate the properties of the interpolation methods, we conduct tests for upsampling rates from 2 to 30. However, the high upsampling rates (above 10) are of limited practical value due to a high degeneration of image quality.

3.1. Data Sets Used for Quantification of Interpolation Methods

We evaluate the three interpolation methods on two data sets. The first contains 200 images from the Berkley Segmentation data set [11], which is widely used for evaluations of image upscaling and super-resolution algorithms [3]. The images depict scenes from nature such as landscapes, people and animals, covering a wide range of image patterns at all scales. We converted rgb Berkley data set images into grayscale prior to processing.

The second data set is ophthalmologic data in form of 72 funduscopies. Fundoscopy is an imaging technique for examination of fundus obtained using a light source and a ophthalmoscope. This was chosen because the image content is similar to OCT scans, and will allow us to assess the performance of the interpolation methods in a setting which resembles to our application. Fig. 3 shows some examples of the funduscopies.

3.2. Performance Measurement Used for Method Assessment

The interpolation quality is assessed by the pixelwise difference between the ground truth image and the interpolated image. The interpolation error can be evaluated as the root mean square error (RMSE) which is defined as

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N \left(\hat{I}(i) - I(i) \right)^2},$$

where summation runs over all pixels i from the original image I and the interpolated image $\hat{I}$ [10].



Figure 3: Three images from the fundoscopy data set. Images depict anatomical structures at the fundus of the eye.

As a measure of interpolation quality, a closely related peak signal-to-noise ratio (PSNR) is most often used [18]. The PSNR is measured in dB and is defined as [16]

$$\text{PSNR} = 20 \log_{10} \left(\frac{I_{\max}}{\text{RMSE}} \right),$$

where $I_{\max}$ is the maximum pixel intensity value, in our case 1.

4. Results of Quantitative Evaluation

First, we visually evaluate the results from the interpolations for upsampling rate 3 and 6 on a few images. Second, we compute the mean performance of each interpolation method for upsampling rates varying between 2 and 30. Last, we present the results from a statistical analysis based on the measured performance of the three methods.

4.1. Interpolated Results

The differences in performance of the three interpolation methods are subtle, and to visualize the results we bring a small detail of an image from the Berkley segmentation data set in Fig. 4 (a), and we also show the grid lines for upsampling rate 3 and 6 in Fig. 4 (b) and (c). The interpolated results for this image, the two sets of grid lines and the three interpolation methods are shown on Fig. 5. We also bring the pixelwise error between the interpolated images and the original image. It can be seen that the error is zero along the grid lines for the transfinite and the weighted interpolation, while this is not the case for the linear interpolation. Furthermore, for all methods, the interpolation quality in form of

the PSNR decreases when the upsampling rate is increased. For this example, the weighted interpolation outperforms the transfinite when using upsampling rate 6.

4.2. Performance Analysis

Fig. 6 shows the PSNR values for the three interpolation methods for 10 images from the Berkley segmentation data set, interpolated with upsampling rate 6. The images were chosen randomly using pseudorandom integers generated in our testing script. We see a big variance in performance across the images, compared to relatively small variance between the three interpolation methods. However, weighted interpolation obtains the best performance for 8 out of 10 images, while the transfinite method is best for the other two images. The linear method is not the best for any of the images, but is still superior to transfinite in 5 out of 10 images.

To evaluate an overall performance of the interpolation methods, we computed the mean PSNR for the whole Berkley segmentation data set, for each interpolation method and for a range of upsampling rates. We conducted a similar experiment for the funduscopy data set. Fig. 7 shows a plot of the obtained values with upsampling rates varying between 2 and 19. We notice the same performance pattern for both data sets. The transfinite method has the largest mean PSNR for smallest upsampling rates, while the linear method has the largest mean PSNR for highest upsampling rates. In the interval around the point where the transfinite and the linear method cross, the weighted method achieves the highest mean PSNR.

We conducted experiments for upsampling rate up to 30, and we confirm that the linear method achieves best mean PSNR for high upsampling rates. We find this being of limited practical value, as for upsampling rates higher than 18 we interpolate over 89 % of the pixels in the image.

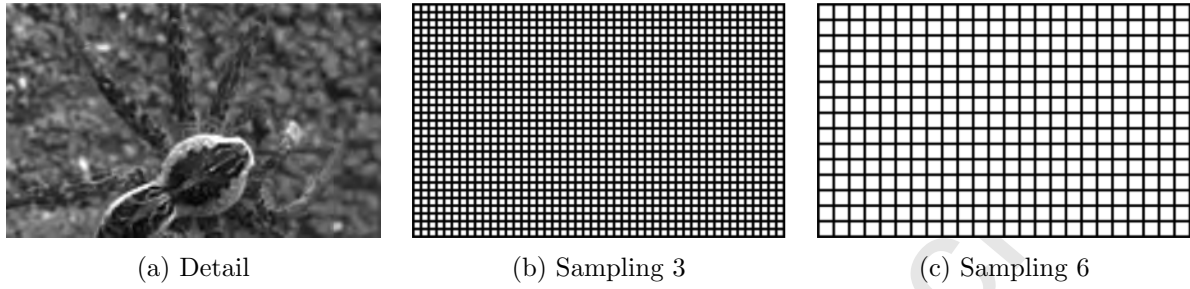


Figure 4: Testing example. (a) zoom on a detail in one of the images from the Berkley data set. (b) and (c) are grid lines with an upsampling rate of 3 and 6.

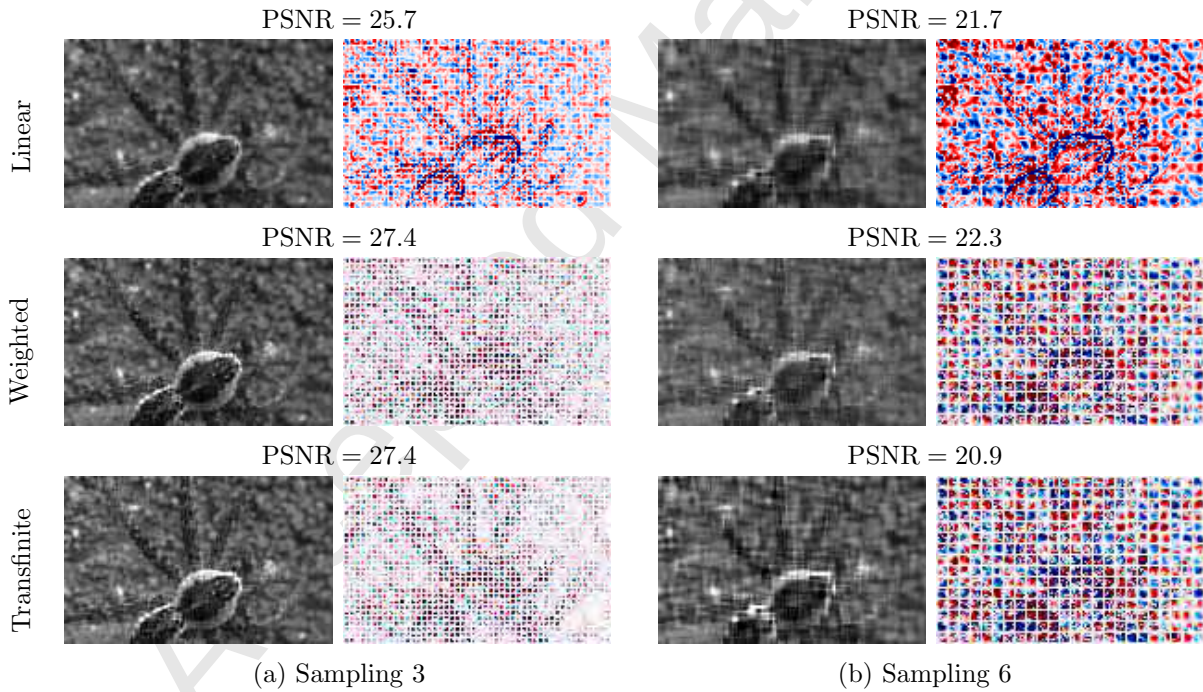


Figure 5: The interpolated results for the detail and the grid lines shown on Fig. 4. The interpolation methods are presented with linear on top, weighted in the middle and transfinite in the bottom row. Columns (a) and (b) bring the results for an upsampling rate of 3 and 6. The PSNR value for each interpolated image is listed above it. Next to each interpolated image is the pixelwise difference between the interpolated image and the original image, with red and blue color indicating positive and negative difference respectively, and white indicating zero difference.

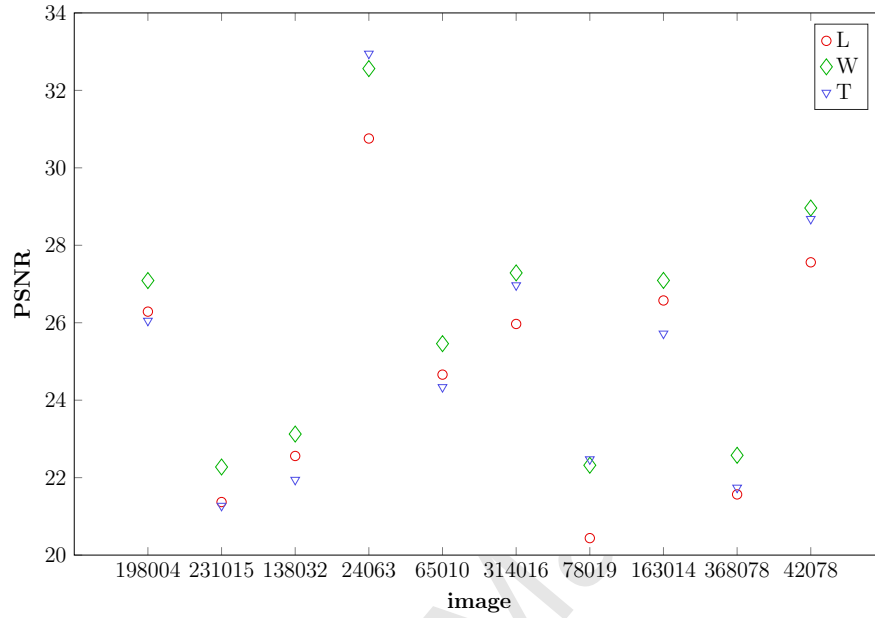


Figure 6: A set of 10 randomly chosen images from the Berkley segmentation data set, and the resulting PSNR for three interpolation methods, linear (L), transfinite (T) and weighted (W). Upsampling rate is 6.

As already shown in Fig. 6, the variance of performances is large between the images and small between the methods. To confirm our findings presented in Fig. 7, we performed a statistical test of the interpolation performance measured by the PSNR value. We set up a regression model to investigate the correlation between the PSNR value and the categorical variables for image and for method, for each sampling rate. F-values indicate that the method is the main descriptor. Moreover, we found that a significant difference between the three methods exists. Therefore, we tested the methods pairwise to check for difference between them at each upsampling rate and moreover, to find out which method performs best. The results are listed in Table 1(a) for the Berkley data set and in Table 1(b) for the funduscopy data set. The results show the same trend, and we use notation a/b when referring to the two data sets. It is seen that the transfinite interpolation performs best for upsampling rates below $3/4$ and the weighted interpolation performs best for sampling rates above $4/5$ and below $15/20$. The linear method performs best for upsampling rates above $20/22$.

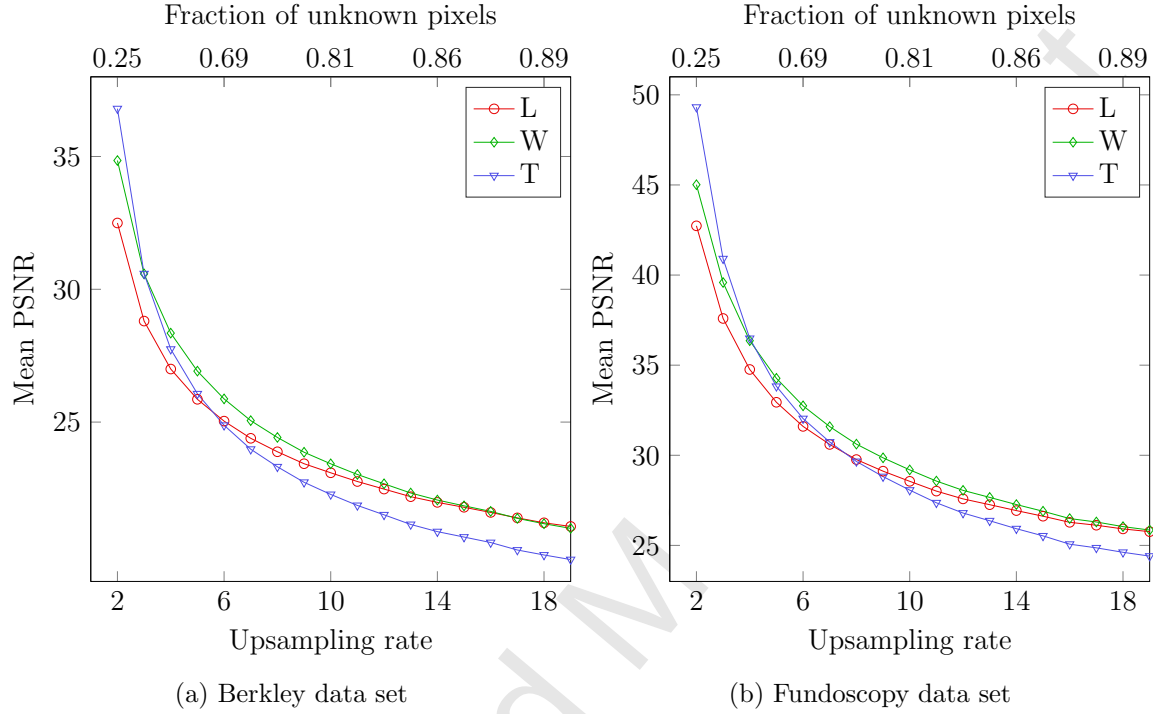


Figure 7: Comparison of the linear (L), transfinite (T) and weighted (W) interpolation. The mean PSNR value for (a) Berkley and (b) fundoscopy data set at different upsampling rate. The horizontal axis above the graphs indicates the fraction of unknown pixels.

Table 1: Results from statistical analysis of interpolation performance for three methods at different upsampling rates, for the Berkley data set (a) and the fundoscopy data set (b). The methods are linear (L), weighted (W) and transfinite (T), and upsampling rates are shown in intervals between 2 and 30. Number 1 indicates the method that performs best for a given upsampling rate, while 3 indicates the method that performs worst. The star indicates that no significant difference was found between the two methods for the given upsampling rate.

(a) Berkley data set								(b) Fundoscopy data set							
	2	3	4-5	6	7-14	15-20	21-30		2-3	4	5-6	7-8	9-19	20-22	23-30
L	3	3	3	2*	2	1*	1	L	3	3	3	2*	2	1*	1
W	2	1*	1	1	1	1*	2	W	2	1*	1	1	1	1*	2
T	1	1*	2	2*	3	3	3	T	1	1*	2	2*	3	3	3

5. Case Study: Optical Coherence Tomography of the Optic Nerve Head

Our investigation in methods for interpolation from grid lines was motivated by optical coherence tomography (OCT) images of the eye retina. We wanted to produce an *en face* image of the optic nerve head (ONH), the point in the eye where the optic nerve fibers leave the retina.

Based on the interpolation methods, we developed a general approach for creating *en face* images from two OCT scans through volume merging. In this section we present our approach with focus on ONH images as a case study. Note that the method can be applied to retinal OCT images in general.

5.1. Data Acquisition

When using OCT, the retina of the eye is scanned in depth and along a set of lines. Two sets of line scans are acquired. During acquisition, the operator manually initiates the scanning so that the ONH is approximately at the center of the scan. The first scan is acquired along the lines running from left to right, so each line gives a cross-section image of the ONH in the anatomical transverse plane. The second scan is acquired along the lines orthogonal to the first one, i.e. running from the top to the bottom, so acquired images are in the anatomical sagittal plane. In our notation, the first scan is acquired along the lines parallel with the x -axis, and results in a set of images in xz -plane. We will call this a x -direction scan. Similarly, the second scan, resulting in a set of yz cross-section images, is denoted a y -direction scan.

For our main example, each set of the data consists of 97 line scans (so-called B-scans) and each line scan contains 768 depth-measurements (so-called A-scans). This means that the x -direction scan represents a volume containing ONH by $768 \times 97 \times 496$ voxels, while the y -direction scan represents the same volume by $97 \times 768 \times 496$ voxels. Likewise, the voxel size for the x -direction scan is $5.7 \times 30 \times 3.9 \mu\text{m}$ and for the y -direction scan the voxel size is $30 \times 5.7 \times 3.9 \mu\text{m}$.

5.2. Preprocessing

As mentioned earlier, interpolation from the grid lines is only one step in the OCT volume-merging approach. To be brought in a format appropriate for interpolation, OCT volumes need to undergo three preprocessing steps. While this is not a main focus of this paper, in order to make the case study self-contained, we briefly cover the preprocessing steps.

The first preprocessing step is an alignment of the two volumes in the xy -plane. While scanning, the operator will attempt to cover the same region of the ONH within the two scans. However, since this is done manually, there might be a slight misalignment between the scans. To obtain an exact alignment of the volumes in the xy -plane, we generate 2D

images by summing all voxels in the z -direction, and resample the images so that pixels are square. To these images we apply an intensity-based image registration with the mean square difference as similarity measure. The result is a xy -plane translation of one image in respect to the other, giving us a corrected relative placement of the x -direction and the y -direction scan. Note that the registration also reveals a rotation between the x -direction and y -direction scan, which is important to check for, since our approach assumes orthogonality.

The second preprocessing step involves defining a grid for interpolation. After alignment, we know the exact placement of the two sets of line scans in the xy -plane. While the sampling resolution along the scan lines is high, it is still finite, and we need to resample the images into a format suitable for interpolation. This is illustrated in Fig. 8. We first identify the crossing points between line scans. This defines a coarse square grid, with the outermost points outlining the region of interest. Given a certain upsampling rate, we subdivide the coarse grid into a fine grid. The two scan volumes are then resampled according to the points in the fine grid. Here we use linear interpolation. Thanks to the high resolution along the scan lines, we can resample the images with only a slight loss of information.

The number of crossing points varies depending on the overlap between the scans. For the data used in this case study, all line scans overlap and the coarse grid is of size 97×97 . The size of the fine grid depends on the upsampling rate. For the examples shown in the case study we use upsampling rate of 5, so the fine grid has $5(97 - 1) + 1 = 481$ points in each direction.

The third preprocessing step corrects for eye movement in the z -direction. During a single scan, OCT scanner will track eye features and correct for eye movement in xy -plane. However, eye movement in the z -direction is not corrected for, and it can be quite large, especially between two scans. We estimate the displacement between two A-scans (depth-measured signal in the z -direction) using cross-correlation. This is done only in the coarse grid (crossing points), while displacement for the fine grid is obtained by interpolation. The columns of the scans are then re-sampled and cropped if needed.

Finally, for our method to work, overall intensity levels need to be consistent across scans. We assessed that the data used in this study required no additional adjustment of intensities.

5.3. Interpolation

The preprocessing provides two aligned volumes covering exactly the same part of the ONH. The first volume, obtained from the x -direction scan, has $481 \times 97 \times 496$ voxels, while the second volume, obtained from the y -direction scan, has $97 \times 481 \times 496$ voxels. We now merge the volumes by processing one z -slice (i.e. xy -plane for a given z) at a time. Our prototype implementation of volume merging produces results for all three interpolation methods, so we can visually compare the outputs.

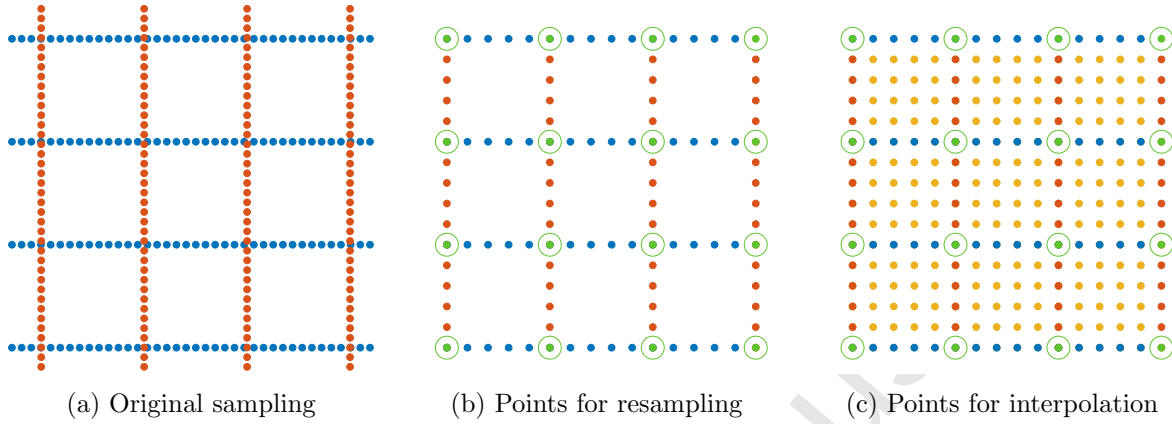


Figure 8: Defining an interpolation grid. (a) Original sampling, blue indicating x -direction scan and red indicating y -direction scan. (b) Crossing points defining the coarse square grid are green, and points for resampling along the x -direction and the y -direction scan lines are blue and red respectively. (c) Orange points are added, such that green, blue, red and orange points together yield the fine grid used for interpolation.

5.4. Case Study Results

We present the results of merging two OCT volumes on one xy -cross section image, *en face* image, on Fig. 9. We show the x -direction scan, the y -direction scan and the interpolated results for the three methods: linear, transfinite and weighted.

In both the x -direction and y -direction scan the anatomical structures parallel to scan lines are less pronounced. Furthermore, one can notice stripes in the scan direction, which might potentially influence the interpretation of the images. The results for all three interpolation methods show the benefit of combining two orthogonal set of line scans into a higher resolution *en face* image with squared pixels. Anatomical structures such as blood vessels become more distinct when two scans are combined. Furthermore, on the merged images in Fig. 9 we notice the reduction of horizontal and vertical stripes.

Given our results from the performance analysis of the three interpolation methods, we saw that the weighted method performed best for upsampling rate 5 used for our case study. Therefore, we show on Fig. 10 the results for *en face* images from three different patients obtained with the weighted interpolation method.

6. Discussion

Our experiments and the statistical evaluation of the three interpolation methods are in alignment with the previously demonstrated properties of the methods. Prior to experiments, we knew that the transfinite methods performs best close to the grid lines containing known

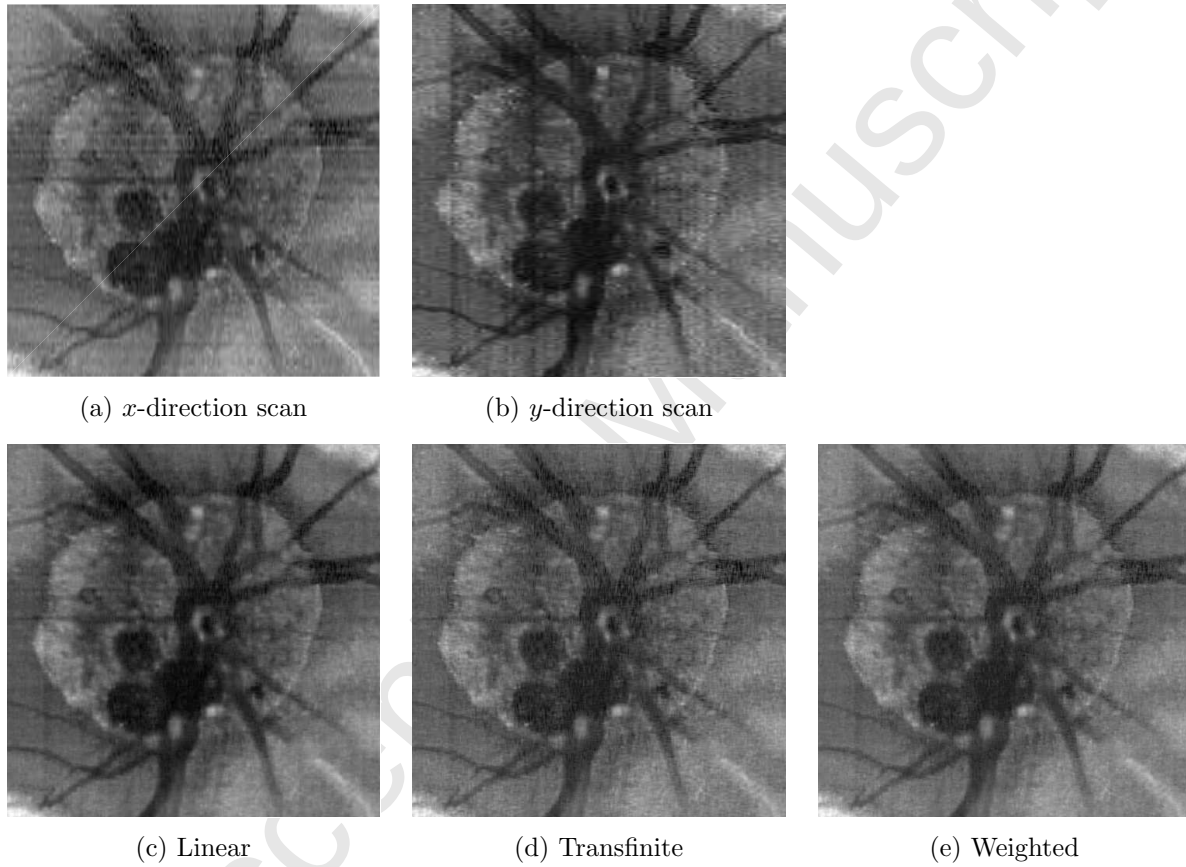


Figure 9: Results of merging two OCT scans of ONH, shown on one *en face* image. (a) x -direction scan, (b) y -direction scan, (c) result produced using linear interpolation, (d) result of transfinite interpolation and (e) result of weighted interpolation.

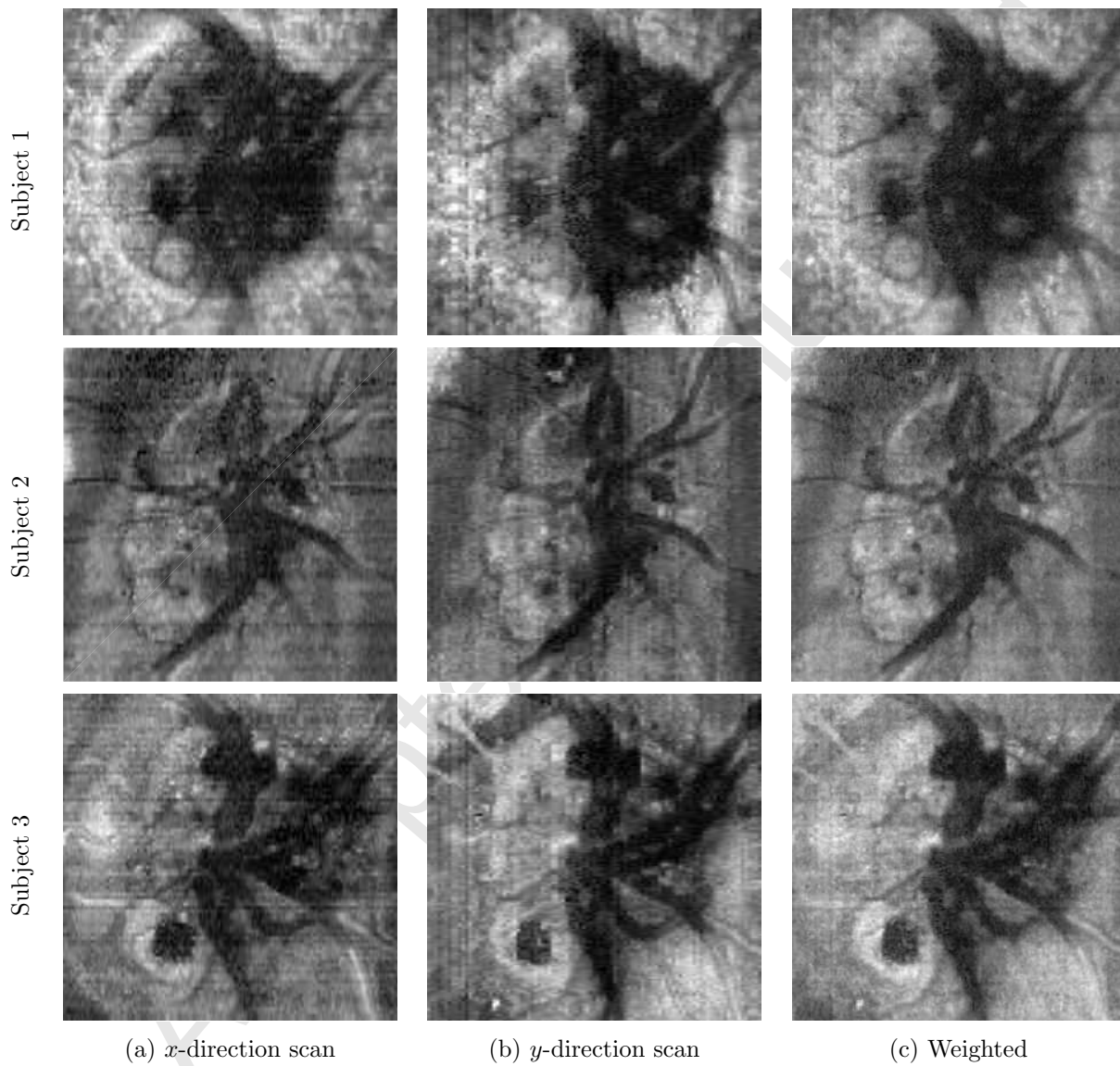


Figure 10: Three different *en face* OCT images of ONH produced using weighted method for interpolation from the grid lines. The upsampling rate is 5. Images in column (a) are from x -direction scans, column (b) are from y -direction scans and column (c) are the interpolated results.

information, while the linear method performs best in areas further away from the grid lines. Therefore we expected the transfinite method to achieve superior results for small upsampling rates where grid lines cover a large fraction of the image. Our results confirm this hypothesis. Likewise, we show that linear method is superior at high upsampling rates.

We designed the weighted method to combine the good properties of the linear and transfinite method. Our results confirm that weighted method has superior performance for a large interval of upsampling rates, and especially where the transfinite and linear method perform equally badly. This happens around upsampling rate of 7 and the interval where weighted method is superior extends from 4 to more than 15. The upsampling rate for our case study application of merging OCT images is 5, and we expect the suggested weighted interpolation method to be superior.

We measured image peak signal-to-noise ratio (PSNR) to quantify the interpolation quality. It is important to note that PSNR is only one indicator of the quality of interpolation. Further investigation that measures image quality in terms of sharpness should be performed for finding the most suitable method. Likewise, if the images are to be used for visual inspection, a perceived quality of the images should be measured.

Visually observed quality of the images is central in our case study, where we applied the three interpolation methods on optical coherence tomography (OCT) images of the optic nerve head (ONH). All three interpolation methods successfully reduce the disturbing stripe artifacts present in the raw images, and improve visualization of anatomic structures e.g. blood vessels. The difference in the performance of the three interpolation methods is subtle, but based on our previous results we conclude that the weighted interpolation should be used for OCT images.

In our approach of merging OCT scans the interpolation step relies on the assumption that the scan lines are orthogonal, and the intensities of the two scans are consistent. Bringing the input data into a format suitable for interpolation, and assessing the validity of assumptions is therefore crucial for the performance of the method. For this reason it is difficult to assess the quality of interpolation step on the acquired data, as it is highly dependent on the preprocessing.

The true value of our interpolation will be evident when we include it in a targeted application. We plan on developing a method for quantifying the volume of optic disc drusen, which will be tested within a larger clinical study. We hope that our interpolation method will improve the results of an automatic detection algorithm.

7. Conclusion

Our work on interpolation from grid lines has three main contributions. First, we introduce the problem of interpolation from grid lines and suggest three possible solutions: a linear, a

transfinite and a weighed interpolation. Second, we systematically test the three methods and conclude that the transfinite method is superior for very small upsampling rates, while the weighted method should be chosen for a broad range of upsampling rates. Lastly, through a case study, we demonstrate the use of the proposed interpolation methods and the benefits of merging two line scans.

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