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Published in:
Applied Soft Computing

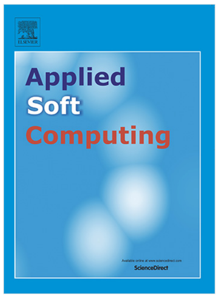
DOI (link to publication from Publisher):
10.1016/j.asoc.2018.12.035

Publication date:
2019

Document Version
Accepted author manuscript, peer reviewed version

Citation for published version (APA):
Mardani, M. M., ShaSadeghi, M., Safarinejadian, B., & Dragicevic, T. (2019). Online distributed fuzzy modeling of nonlinear PDE systems: Computation based on adaptive algorithms. *Applied Soft Computing*, *77*, 76-87. https://doi.org/10.1016/j.asoc.2018.12.035

Accepted Manuscript



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PII: S1568-4946(18)30736-1
DOI: https://doi.org/10.1016/j.asoc.2018.12.035
Reference: ASOC 5268

To appear in: *Applied Soft Computing Journal*

Received date : 13 January 2018
Revised date : 22 December 2018
Accepted date : 27 December 2018

Please cite this article as: M.M. Mardani, M. Shasadeghi, B. Safarinejadian et al., Online distributed fuzzy modeling of nonlinear PDE systems: Computation based on adaptive algorithms, *Applied Soft Computing Journal* (2019), https://doi.org/10.1016/j.asoc.2018.12.035

Online Distributed Fuzzy Modeling of Nonlinear PDE Systems: Computation based on Adaptive Algorithms

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Abstract

With the emergence of novel model-based controllers for partial differential equation (PDE) systems, identifying the mathematical model of PDE systems has become a promising and complicated research topic. This paper suggests a new method to identify an adaptive Takagi-Sugeno (TS) fuzzy PDE model for nonlinear multi-input multi-output (MIMO) first-order PDE systems. The proposed approach is performed online based on the measured input and output data of the nonlinear PDE systems. Furthermore, the identification process will be obtained for the cases that the noise is either white or colored. For the case of white noise, a nonlinear recursive least square (NRLS) approach is applied to identify the nonlinear system. On the other hand, when the colored noise is exerted to the nonlinear PDE system, the fuzzy PDE model of the nonlinear PDE system and also nonlinear colored noise are identified based on the nonlinear extended matrix methods (NEMM). Moreover, the problem of identification for both colored and white noise cases is investigated when premise variables of membership functions are known or unknown. Finally, in order to illustrate the effectiveness and merits of the proposed methods, the identification method is applied to a practical non-isothermal Plug-Flow reactor (PFR) and a hyperbolic PDE system with Ltka-Volterra type applications. As it is expected, the evolutions of the error between the state variables for the obtained TS fuzzy PDE model and the output data converge to the zero in the steady-state conditions. Thus one concludes, the proposed identification algorithm can accurately adjust both consequents and antecedents parameters of TS fuzzy PDE model.

Keywords: Nonlinear system identification, Nonlinear first-order partial differential equation (PDE) system, Takagi-Sugeno (TS) fuzzy model, Nonlinear least square (NLS), Parameter estimation

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Acronyms

ANN	Artificial neural network
EKF	Extended Kalman filter
ELS	Extended least square
EMM	Extended matrix method
ERLS	Extended recursive least square
GKCA	Gustafsone Kelssel clustering algorithm
LS	Least square
LSE	Least square estimation
MIMO	Multiple input multiple output
NEMM	Nonlinear extended matrix method
NLS	Nonlinear least square
NRLS	Nonlinear recursive least square
PDE	Partial differential equation
PFR	Plug-flow reactor
RLS	Recursive least square
TS	Takagi Sugeno

Nomenclature

x	Position
t	Time
$y_j(x,t)$	j-th system input
$y(x,t)$	System input
$z_i(x,t)$	i-th system output
$z(x,t)$	System output
F^l_{ji}	Fuzzy sets
μ^l_{ji}	Grade of membership functions
$w^l_i(y(x,t))$	Degree of activation of each rule
$h^l_i(y(x,t))$	Weighting functions
σ^l_{ji}	Adaptive parameters in antecedent parts of fuzzy rules
$e(x,t)$	Measurement white noise
$u(x,t)$	Adaptive parameters
R_e	Covariance matrix
$\hat{u}(x,t)$	Estimation of adaptive parameters
P	Covariance of the estimation
K	Kalman gain
$\nu(x,t)$	Measurement colored noise
a^l_{ji}	Set of adaptive parameters
b^l_{ji}	Set of adaptive parameters
c^l_{ji}	Set of adaptive parameters
d^l_{ji}	Set of adaptive parameters
$\Gamma(u,x,t)$	Nonlinear function
$g(u,x,t)$	Nonlinear function
$C(u,x,t)$	Jacobian matrix

1. Introduction

A significant number of physical phenomena in the real-world such as industrial process and biological systems inherently depend on spatially position as well as time (i.e., their behaviors are distributed in space) (1) (2). Whereas their dynamics depend on more than one independent variable thus they are well-known as a partial differential equation (PDE) systems (3) (4). Based on spatially distributed points, PDE systems are classified into three categories: *(1) hyperbolic (5) (2) parabolic (4) (3) elliptic (6)*. Consequently, due to infinite dimensional and spatially distributed behaviors of PDE systems, more effort is needed to design the controller, analyze the stability and also identify the PDE systems. Moreover, it is generally more difficult to directly apply the existing lumped parameter systems techniques to the distributed ones (7).

Recently, a significant number of research has been devoted to the problem of stability and stabilization of nonlinear PDE systems based on TS fuzzy PDE model (5) (8) (9). TS fuzzy PDE modeling of parabolic PDE systems is presented in (10) (11) (12) (13) and the hyperbolic ones is investigated in (14) (7) (2). In the literature of PDE systems, it is assumed that the nonlinear system equations exist and subsequently, the exact TS fuzzy model has been obtained based on sector nonlinearity approach (7) (15).

According to the control and system engineering points of view, the fundamental part of a study is achieving an accurate model for the existing linear or nonlinear physical system (16). Since enough information for obtaining a suitable mathematical model does not exist, the exact mathematical dynamic representation of real-world systems is seldom available (17). On the other hand, in real applications, we encounter with colored noise instead of white noise. Subsequently, the effect of the colored noise is as critical as the un-modeled dynamic in the system identification and modeling (18). Hence, an important problem is to model and identify the linear or nonlinear system based on input-output data. The identified model must describe the physical behavior of the original plant with an adequate level of accuracy (19) (20).

Most of the real-world systems are inherently nonlinear (21) (22). Takagi-Sugeno (TS) fuzzy models provide a powerful and systematic framework to analyze the stability and synthesize the controller for nonlinear systems (23) (19). Moreover, it can describe the complicated smooth nonlinear systems in the convex structure (24) (25) (26). Thus, lots of attention has been focused on the TS fuzzy systems during the last two decades (27) (28) (23). A TS fuzzy model represents the nonlinear system via some local linear subsystems that will be introduced in fuzzy IF-THEN rules structure. Then, by fuzzy blending of the local linear subsystems, the overall fuzzy model will be obtained. Such models have the capability to approximate a wide range of nonlinear systems (29). There exist two approaches to obtain a TS fuzzy model. The first and more attractive one is based upon the identification using validation input-output data when the system is unknown and the second one is derived from the given nonlinear system equations (29). This paper focuses on the first approach which involves a technique to find optimal values of *(1) premise and (2) consequent* parameters sets (30). The premise parameters set constructs the characteristics of fuzzy membership functions and the consequent one contains the coefficients of the local linear subsystems (19). It is generally difficult to quantify these parameters based on an expert manual validation knowledge. Hence, the parameters will be usually approximated based on the least squares estimate (LSE) (31), recursive least square (RLS), Kalman filter, extended Kalman filter (EKF) (25) and data-driven approach (32).

Numerous researches have argued about the identification of linear and nonlinear ODE systems. Ref. (18) has suggested a method to estimate the states and parameters of the linear dynamic system which is affected by the colored noise. Furthermore, it considers the minimum discrepancy measure criterion to model the colored noise. However, the problem of identifying nonlinear system has not been addressed in Ref. (18). In addition, if the sampling times get lower or sampling frequencies get higher, then due to the computational time the approach (18) cannot be applicable. Thus, constructing a fast computational algorithm is necessary. In (33), the extended recursive least square (ERLS) algorithm has been presented to estimate the parameters of discrete-time nonlinear stochastic systems. Based on the ERLS algorithm in (33), the consistency of the parameters has been guaranteed without any restrictive conditions such as *(1) the persistent excitation condition (2) the noise condition and (3) the variance functions condition*. However, the identification algorithm (33) is only valid for a class of nonlinear systems called polynomial systems. Due to the linearization processes the presented algorithms (18) and (33) identify the nonlinear system in a small vicinity of operating point or equilibrium point. Whereas TS fuzzy models create a powerful algorithm to represent the nonlinear systems, respectable amount of studies have been focused on TS fuzzy modeling of nonlinear ODE systems based on input-output data (19). Recently, several approaches are presented to identify the TS fuzzy model of a nonlinear system such as: genetic algorithm (34), artificial neural networks (ANN) (32), gravitational search-based hyper-plane clustering algorithm (26), self-organizing migration algorithm (35) (36), least square (LS) algorithm (30) and EKF (25). The propose of Ref. (34) is to present a new encoding scheme for identifying the TS fuzzy model by the non-dominated strong genetic algorithm. In addition, identification of multiple input multiple output (MIMO) systems based on MIMO TS fuzzy model is presented in (35) (37). In (38), the Kalman filter is utilized to design a state estimator for each local model of the fuzzy system. Then, the states of the overall time-varying discrete time system are estimated by aggregating the local models. A small number of researches have been focused on the problem of identifying TS fuzzy model-based on the Kalman filters. Ref. (39) uses the Kalman filter and Gustafsone Kessel clustering algorithm (GKCA) to update the information of the consequent and antecedent parts, respectively. In other words, the parameters and structure of the fuzzy model are identified in two separate steps. Thus, the accuracy of the obtained TS fuzzy model is reduced significantly. Refs. (40) (38) use EKF to adjust the parameters of the TS fuzzy model. In Refs. (40) (38), the structure of the membership functions is assumed to be triangular. However, because of the complexity of the learning algorithm, the efficiency and the applicability of the approaches (40) (38) for other types of membership functions are reduced. Ref. (25) also identifies the TS fuzzy model by utilizing the EKF algorithm. The method presented in Ref. (25) is simpler than the ones (40) (38). In addition, in the situation that the membership functions are overlapped by pairs, the approach (25) is not applicable. Ref. (41) employs the EKF to approximate the parameters of the antecedent and consequent parts of TS Type-2 fuzzy systems. In addition, the high-speed convergence and desirable accuracy of the learning algorithm in comparison with the PSO algorithm are improved significantly (41). However, according to the best knowledge of the authors, the references (25) and (41) have some main drawbacks. First, the problem of identifying the system in the presence of colored-noise has not been addressed in the literature of identifying TS fuzzy model-based on KF. Second, as we mentioned previously, large numbers of phenomena are described by PDE systems. The presented approaches are not capable to identify TS fuzzy model of such systems. Thus, it is essential to construct a symmetric approach to identify the TS fuzzy model of PDE systems. To the best knowledge of the authors, the identification of TS PDE fuzzy model of nonlinear PDE systems based on input-output data has not been addressed yet, which is the main contribution of this paper.

This paper presents a novel approach for online adaptive TS fuzzy PDE modeling of nonlinear MIMO first-order PDE systems. The proposed identification algorithm investigates the cases that the system is affected by the white noise or the colored one. The important feature of the proposed approach is adjusting the antecedent and consequence parts of the TS fuzzy PDE model of nonlinear PDE system without limiting the size of the input-output data. To cope with these difficulties, the authors create a suitable structure to identify the nonlinear PDE system with NRLS and NEMM approaches. Generally, the main contributions of the proposed approach can be classified as follows:

- *Identifying the nonlinear PDE systems based on input-output data*
- *For the cases that the colored noise affectes the nonlinear PDE system, not only the TS PDE fuzzy model of the nonlinear PDE system is identified but also the TS fuzzy model of the colored noise is identified.*
- *The TS fuzzy PDE model is defined in a suitable structure such that deploying the NRLS and NEMM approaches will be possible.*

To illustrate the efficiency of these key ideas, a practical PFR system and a hyperbolic PDE system with Lotka-Volterra type are considered. The identification is obtained for two cases: *the premise variables in membership functions are known or unknown*. The results will be indicated that the nonlinear first-order PDE system can be suitably approximated by the obtained TS fuzzy PDE first-order model. Moreover, in the case that the measurement colored noise is presented, the measurement colored noise dynamic will be correctly approximated by the TS fuzzy PDE model.

The remainder of the paper is organized as follows. In Section 2, the problem formulation regarding MIMO TS fuzzy PDE models is reviewed. Section 3 focusses on two methods. The first one investigates the nonlinear least square (NLS) method and the second one studies the extended matrix method (EMM). In Section 4, TS fuzzy PDE modeling of nonlinear PDE systems in the presence of white and colored measurement noise are discussed. Then in Section 5, the simulation results are presented to identify the nonisothermal PFR based on the identification methods. Finally, the conclusions will close the paper in Section 6.

2. Problem formulation

TS fuzzy models are known as universal approximators. Thus, any smooth nonlinear system can be approximated via a TS fuzzy model with any desired degree of accuracy (30) (29). The TS fuzzy model has been widely used to analyze and synthesize the nonlinear ODE or PDE systems. Furthermore, it is suitable for designing fuzzy ODE or PDE controllers (42) (43) (44). Therefore, fuzzy modeling and identification of nonlinear PDE processes are very essential. In this Section, a TS fuzzy MIMO first-order PDE model is presented which will be identified in Section 4. The nonlinear first-order PDE system can be represented with the following discrete-time linear MIMO first-order PDE rules.

Rule l for output i

IF $y_1(x,t)$ is F^l_{1i} **and** $\cdots$ **and** $y_n(x,t)$ is F^l_{ni},

$$\textbf{THEN}\ z^l_i(x,t) = \sum_{j=1}^{n} a^l_{ji}\frac{\delta y_j(x,t)}{\delta x} + b^l_{ji} y_j(x,t) + c^l_{ji} x y_j(x,t) + d^l_{ji} x^2 y_j(x,t) + \cdots \tag{1}$$

where $l \in \{1,\cdots,R_i\}$, $j \in \{1,\cdots,n\}$ and $i \in \{1,\cdots,q\}$ indices indicate the l-th plant rule, j-th system input ($y_j(x,t)$) and i-th system output ($z_i(x,t)$), respectively. F^l_{ji} are fuzzy sets. a^l_{ji}, b^l_{ji},

c^l_{ji}, d^l_{ji} and $\cdots$ are the set of adaptive parameters in the consequent parts of the fuzzy rules, which will be identified. x and t denote the current sampling position and time, respectively.

The fuzzy representation (1) is constructed in a multiple-input and multiple-output structure. Each output of the (1) is modeled with different numbers of fuzzy rules. This representation not only reduces the number of fuzzy rules but also facilitates the modelling procedure by decreasing the number of model parameters. Furthermore, x^s, $s \in \{1, 2, \cdots\}$ are achieved by Taylor-series expansion of the nonlinear spatially distributed elements in i-th output of the system. Subsequently, if s increases then the identified first-order PDE model is more reliable.

By aggregating the set of rules (1) and applying singleton fuzzifier, product inference engine and center average defuzzifier, the overall TS fuzzy MIMO first-order PDE model for output $z_i(x,t)$ is expressed as follows

$$\begin{aligned} z_i(x,t) = \;& \sum_{l=1}^{R_i} h^l_i(y(x,t))\Big\{ \textstyle\sum_{j=1}^{n} a^l_{ji}\frac{\partial y_j(x,t)}{\partial x} + b^l_{ji} y_j(x,t) \\ & + c^l_{ji} x y_j(x,t) + d^l_{ji} x^2 y_j(x,t) + \cdots \Big\} \end{aligned} \tag{2}$$

where $y(x,t) = [y_1(x,t)\ y_2(x,t)\ \cdots\ y_n(x,t)]^T$, and

$$\begin{aligned} w^l_i(y(x,t)) &= \prod_{j=1}^{n} \mu^l_{ji}(y_j(x,t), \sigma^l_{ji}) \\ h^l_i(y(x,t)) &= \frac{w^l_i(y(x,t))}{\sum_{s=1}^{R_i} w^s_i(y(x,t))} \end{aligned} \tag{3}$$

where $\mu^l_{ji}(y_j(x,t), \sigma^l_{ji})$ are the grade of membership functions. $w^l_i(y(x,t))$ are the degree of activation of each rule, and $h^l_i(y(x,t))$ are the weighting functions. Furthermore, σ^l_{ji} are the adaptive parameters in antecedent parts of fuzzy rules which will be determined with estimation algorithms to obtain a more efficient TS fuzzy first-order PDE model.

3. Nonlinear least square and extended matrix method

Rudolf E. Kalman developed the Kalman filter that is defined as a linear combination of measurements (45). It is well-known as an optimal linear filter and also, it is the best recursive state estimator for linear systems in the presence of zero-mean white noise in measurements and model (45). In general, the real systems are inherently nonlinear and complex. For the nonlinear systems, several kinds of nonlinear Kalman filters are formulated to aproximate the solutions, such as: linearized Kalman filter, EKF (46), uncented Kalman filter and particle filter (45). Here, we consider the EKF which is defined by linearizing the nonlinear system around each working point, and then applying the Kalman filter on the linearized model. The NRLS algorithm for nonlinear PDE systems will be presented in this Section.

3.1. Nonlinear least square approach

Consider the following nonlinear PDE system

$$\begin{aligned} u(x,t+1) &= u(x,t) \\ z(x,t+1) &= g(u,x,t) + e(x,t) \end{aligned} \tag{4}$$

where $e(x,t)$ indicates the measurement noise. We assume that the measurement noise is white, with a mean of zero and a covariance of $R_e = E(e(x,t)e^T(x,t))$. The vector $u(x,t)$ consists of adaptive parameters which will be approximated in each iteration for each spatially distributed point. $g(u, x, t)$ is the spatial and time-varying nonlinear function of vector $u(x, t)$. $z(x, t)$ indicates the output of the system. This algorithm must be started with initial conditions. We initialize it via

$$\hat{u}(x, 0) = E(u(x, 0)) \tag{5}$$

$$P(x, 0) = E\Big(\big(\hat{u}(x, 0) - u(x, 0)\big)\big(\hat{u}(x, 0) - u(x, 0)\big)^T\Big) \tag{6}$$

and in the next step Jacobian matrix is computed as follows

$$\phi(x, t + 1) = \frac{\partial g(u, x, t)}{\partial u}\bigg|_{u=\hat{u}(x,t)} \tag{7}$$

where $\hat{u}(x, t)$ is the current estimation of $u(x, t)$. Now, the Kalman filter is utilized to estimate the adaptive parameters of the linearized PDE system (4). The Kalman filter is divided into two phases: time update (a priori estimation) and measurement update (a posteriori estimation). The time update algorithm (a priori estimation) for system (4) is

$$\hat{u}(x, t + 1|t) = \hat{u}(x, t|t) \tag{8}$$

$$P(x, t + 1|t) = P(x, t|t) \tag{9}$$

and the measurement update (a posteriori estimation) is

$$K(x, t + 1) = P(x, t + 1|t)\phi(x, t + 1)\big(\phi^T(x, t + 1)P(x, t + 1|t)\phi(x, t + 1) + R_e\big)^{-1} \tag{10}$$

$$\hat{u}(x, t + 1|t + 1) = \hat{u}(x, t + 1|t) + K(x, t + 1)\big(z(x, t) - \hat{z}(x, t)\big) \tag{11}$$

$$P(x, t + 1|t + 1) = \big(I - K(x, t + 1)\phi(x, t + 1)\big)P(x, t + 1|t) \tag{12}$$

where $t + 1|t$ and $t + 1|t + 1$ denote a priori and a posteriori estimations, respectively. $\hat{u}$ indicates the estimation of u, and P is the covariance of the estimation and K is the Kalman gain. Finally, the estimated outputs $\hat{z}(x, t + 1)$ are achieved by

$$\hat{z}(x, t + 1) = g\big(\hat{u}(x, t + 1|t + 1), x, t\big) \tag{13}$$

The online process presented in (8) to (13) is updating the estimation during time for each spatially distributed point. Then, both estimation error and covariance matrix will be minimized in each time iteration.

Remark 1. *The following algorithm is presented to adaptively adjust the parameters of the TS fuzzy PDE model based on the NRLS approach:*

1. *Initializing the algorithm by utilizing equations* (5) *and* (6)
2. *Calculate the Jacobian matrix by considering equation* (7)
3. *Deploying equations* (8) *and* (9) *for priori estimation of the parameters (TIME UPDATE)*
4. *Calculating the Kalman gain by utilizing equation* (10)

5. *Deploying equations* (11) *and* (12) *for posterior estimation of the parameters (MEASUREMENT UPDATE)*
6. *Utilizing the parameter data obtained from step 5 to evaluate the output by equation* (13)
7. *If* $\|z(x,t) - \hat{z}(x,t)\| \leq \epsilon$ *and the identified parameters converge to the constant values* ($u(x,t) \rightarrow$ *constant), then STOP. Else Go to step 2.*

Remark 2. *In the proposed approach, it is assumed that the open-loop system is stable. The identification algorithm is addressed for the open-loop configuration which is only excited by the white noise or the colored one. Additionally, step seven of the Remark 1 indicates if the true output signals are fitted to the model output signals and the parameters of the fuzzy PDE model converge to the constant values, then the identified PDE fuzzy model is acceptable and the algorithm can be stopped. After identifying the parameters, the obtained fuzzy model of PDE system describes the behavior of the overall system.*

Remark 3. *State estimators are divided into two categories. The first category is static state estimators (i.e. their dynamic characteristics are unchangeable) and the other one is dynamic ones (i.e. they have changeable dynamic characteristics). The EKF belongs to the second category. This algorithm will be converted to the NRLS one for static systems.*

3.2. Nonlinear extended matrix method

Consider the following nonlinear PDE system

$$z(x,t+1) = g(u_1,x,t) + v(x,t) \tag{14}$$

The unmeasurable input $v(x,t)$ indicates that the measurement noise, which is assumed to be colored in this subsection (i.e. $v(x_1,t_1)$ depends on $v(x_2,t_2)$ for each $x_1 \neq x_2$ or $t_1 \neq t_2$). $u_1(x,t)$ denotes the adaptive parameters of system (14). Moreover, we consider the following nonlinear PDE model for the colored noise:

$$v(x,t) = \Gamma(u_2,x,t) + e(x,t) \tag{15}$$

where $e(x,t)$ is a white noise. $u_2(x,t)$ is the adaptive parameters of the error system (15). $\Gamma(u_2,x,t)$ is a spatial and time-varying nonlinear function of vector $u_2(x,t)$, which we want to identify its fuzzy model beside the fuzzy model of $g(u_1,x,t)$. The algorithm presented in (8) - (12) will be recursively done for the system via colored noise by considering

$$u(x,t) = \begin{bmatrix} u_1(x,t) \\ u_2(x,t) \end{bmatrix}, \qquad \phi(x,t) = \begin{bmatrix} \phi_1(x,t) \\ \phi_2(x,t) \end{bmatrix} \tag{16}$$

where

$$\phi_1(x,t) = \frac{\partial g(u_1,x,t)}{\partial u_1}\bigg|_{u_1=\hat{u}_1(x,t)}, \qquad \phi_2(x,t) = \frac{\partial \Gamma(u_2,x,t)}{\partial u_2}\bigg|_{u_2=\hat{u}_2(x,t)}$$

Thus, we initialize the above algorithm with (5) and (6). Then, based on matrices (16), we recursively apply the eqs. (8) - (12). Furthermore, the estimation of the output $\hat{z}(x,t)$ and colored noise $\hat{v}(x,t)$ which presented in equation (14) and (15), respectively, are defined as

$$\begin{aligned} \hat{v}(x,t) &= z(x,t) - \phi_1^T u_1(x,t) \\ \hat{z}(x,t) &= \phi^T(x,t)u(x,t) \\ \hat{e}(x,t) &= z(x,t) - \hat{z}(x,t) \end{aligned} \tag{17}$$

where $\hat{v}(x,t)$ and $\hat{e}(x,t)$ are used to determine $\phi(x,t)$ in (16). Finally, by applying the above algorithm, the set of adaptive parameters $\hat{u}(x,t)$ will be identified and the fuzzy model of the plant and the error system will be achieved.

4. Application of the proposed methods to fuzzy PDE modeling of nonlinear PDE systems

Resently, one of the most interesting and efficient applications of LS and EKF is TS fuzzy modeling of nonlinear ODE systems. The LS algorithm presents an offline approximation (30), while the EKF algorithm presents an online one (25). This paper obtains an online fuzzy model for nonlinear first-order PDE systems, which the identified parameters are modified during the adaptive process. The mentioned identification is achieved based on NLS in a pesudo-optimal way (i.e. optimal for linear systems).

4.1. Application of the NLS algorithm to fuzzy PDE modeling of nonlinear PDE systems

For achieving the goal, first we raise the problem of parameter identification of fuzzy model (2) by a NLS algorithm. Thus, we must build a nonlinear system like the one presented in (4) by the existing nonlinear fuzzy model (2) and then applying NLS. We consider

$$
\begin{aligned}
g_i(u,x,t) = \quad & \sum_{l=1}^{R_i} h_i^l(y(x,t))\Big\{ \textstyle\sum_{j=1}^{n} a_{ji}^l \frac{\partial y_j(x,t)}{\partial x} + b_{ji}^l y_j(x,t) + c_{ji}^l x y_j(x,t) \\
& + d_{ji}^l x^2 y_j(x,t) + \cdots \Big\} = \sum_{l=1}^{R_i} h_i^l(y(x,t)) N_i^l(x,t)
\end{aligned} \tag{18}
$$

where the parameters σ_{ji}^l, a_{ji}^l, b_{ji}^l, c_{ji}^l, d_{ji}^l, and etc. are the parameter of the fuzzy model which we want to estimate. Consider two cases: 1) the membership functions are known, 2) the membership functions are unknown, which will be investigated in the following subsections.

4.1.1. Case I: The membership functions are known

In this case, we assume that the membership functions are known. The vector of parameters for each output is obtained as follows

$$
u_i(x,t) = [a_{1i}^1 \ \cdots \ a_{ni}^{R_i} \ b_{1i}^1 \ \cdots \ c_{1i}^1 \ \cdots \ d_{ni}^{R_i} \ \cdots]^T \tag{19}
$$

and the Jacobian matrix is computed as follows

$$
\phi_i(x,t+1) = [\phi_{a_{1i}^1} \ \cdots \ \phi_{a_{ni}^{R_i}} \ \phi_{b_{1i}^1} \ \cdots \ \phi_{c_{1i}^1} \ \cdots \ \phi_{d_{ni}^{R_i}} \ \cdots]^T \tag{20}
$$

where

$$
\begin{aligned}
\phi_{a_{1i}^1} &= \tfrac{\partial g_i(u,x,t)}{\partial a_{1i}^1} = h_i^1(y(x,t)) \tfrac{\partial y_1(x,t)}{\partial x}, \\
\phi_{a_{ni}^{R_i}} &= \tfrac{\partial g_i(u,x,t)}{\partial a_{ni}^{R_i}} = h_i^{R_i}(y(x,t)) \tfrac{\partial y_n(x,t)}{\partial x}, \\
\phi_{b_{1i}^1} &= \tfrac{\partial g_i(u,x,t)}{\partial b_{1i}^1} = h_i^1(y(x,t)) y_1(x,t), \\
\phi_{c_{1i}^1} &= \tfrac{\partial g_i(u,x,t)}{\partial c_{1i}^1} = h_i^1(y(x,t)) x y_1(x,t), \\
\phi_{d_{ni}^{R_i}} &= \tfrac{\partial g_i(u,x,t)}{\partial d_{ni}^{R_i}} = h_i^{R_i}(y(x,t)) x^2 y_n(x,t).
\end{aligned}
$$

Remark 4. *In case 1, the term $g_i(u,x,t)$ yields from the linear combination of parameters which will be identified i.e. $g_i(u,x,t)$ is linear in u. In this case, the NLS algorithm will be reduced to the LS one. Furthermore, as mentioned above, since the system is linear in each output, the solution is optimal.*

4.1.2. Case II: The membership functions are unknown

In this case, the vector of parameters are

$$u_i(x,t) = [\sigma^1_{1i} \;\cdots\; \sigma^{R_i}_{ni} \;\; a^1_{1i} \;\cdots\; b^1_{1i} \;\cdots\; c^1_{1i} \;\cdots\; d^{R_i}_{ni} \;\cdots]^T \tag{21}$$

and the Jacobian matrix is computed as follows

$$\phi_i(x,t+1) = [\phi_{\sigma^1_{1i}} \;\cdots\; \phi_{\sigma^{R_i}_{ni}} \;\; \phi_{a^1_{1i}} \;\cdots\; \phi_{b^1_{1i}} \;\cdots\; \phi_{c^1_{1i}} \;\cdots\; \phi_{d^{R_i}_{ni}} \;\cdots]^T \tag{22}$$

where

$$\begin{aligned}
\phi_{\sigma^1_{1i}} &= \frac{\partial g_i(u,x,t)}{\partial \sigma^1_{1i}}\bigg|_{u_i=\hat{u}_i(x,t)} = \frac{\partial h^1_i(y(x,t))}{\partial \sigma^1_{1i}} N^1_i(x,t)\bigg|_{u_i=\hat{u}_i(x,t)},\\
\phi_{\sigma^{R_i}_{ni}} &= \frac{\partial g_i(u,x,t)}{\partial \sigma^{R_i}_{ni}}\bigg|_{u_i=\hat{u}_i(x,t)} = \frac{\partial h^{R_i}_i(y(x,t))}{\partial \sigma^{R_i}_{ni}} N^{R_i}_i(x,t)\bigg|_{u_i=\hat{u}_i(x,t)},\\
\phi_{a^1_{1i}} &= \frac{\partial g_i(u,x,t)}{\partial a^1_{1i}}\bigg|_{u_i=\hat{u}_i(x,t)} = h^1_i(y(x,t))\frac{\partial y_1(x,t)}{\partial x}\bigg|_{u_i=\hat{u}_i(x,t)},\\
\phi_{b^1_{1i}} &= \frac{\partial g_i(u,x,t)}{\partial b^1_{1i}}\bigg|_{u_i=\hat{u}_i(x,t)} = h^1_i(y(x,t))y_1(x,t)\bigg|_{u_i=\hat{u}_i(x,t)},\\
\phi_{c^1_{1i}} &= \frac{\partial g_i(u,x,t)}{\partial c^1_{1i}}\bigg|_{u_i=\hat{u}_i(x,t)} = h^1_i(y(x,t))xy_1(x,t)\bigg|_{u_i=\hat{u}_i(x,t)},\\
\phi_{d^{R_i}_{ni}} &= \frac{\partial g_i(u,x,t)}{\partial d^{R_i}_{ni}}\bigg|_{u_i=\hat{u}_i(x,t)} = h^{R_i}_i(y(x,t))x^2 y_n(x,t)\bigg|_{u_i=\hat{u}_i(x,t)}.
\end{aligned}$$

Furthermore, the derivatives of membership functions from a set of parameters σ^L_{JI} (where I, J and L indicate the particular paramaters of the set σ) are caculated as follows

$$\begin{aligned}
\frac{\partial h^l_i(y(x,t))}{\partial \sigma^L_{JI}} = \frac{\partial h^L_I(y(x,t))}{\partial \sigma^L_{JI}} &= \frac{\partial\Big(w^L_I(y(x,t))/\sum\limits_{s=1}^{R_i} w^s_i(y(x,t))\Big)}{\partial \sigma^L_{JI}}\\
&= \frac{\frac{\partial w^L_I(y(x,t))}{\partial \sigma^L_{JI}}\sum\limits_{s=1}^{R_i} w^s_i(y(x,t)) - \frac{\partial \sum_{s=1}^{R_i} w^s_i(y(x,t))}{\partial \sigma^L_{JI}} w^L_I(y(x,t))}{(\sum\limits_{s=1}^{R_i} w^s_i(y(x,t)))^2}\\
&= \frac{\frac{\partial w^L_I(y(x,t))}{\partial \sigma^L_{JI}}\Big((\sum\limits_{s=1}^{R_i} w^s_i(y(x,t))) - w^L_I(y(x,t))\Big)}{(\sum\limits_{s=1}^{R_i} w^s_i(y(x,t)))^2}\\
&= \frac{\frac{\partial \prod_{j=1}^{n} \mu^L_{jI}(y_j(x,t),\sigma^L_{jI})}{\partial \sigma^L_{JI}}\Big((\sum\limits_{s=1}^{R_i} w^s_i(y(x,t))) - w^L_I(y(x,t))\Big)}{(\sum\limits_{s=1}^{R_i} w^s_i(y(x,t)))^2}
\end{aligned} \tag{23}$$

Note that, it is necessary to determine the derivative of $\partial\mu^L_{JI}/\partial\sigma^L_{JI}$, which is related to the type of the MFs μ^L_{JI}. It can be calculated if the type of the MF and its expression is pre-defined. Moreover, it is not essential that the MFs μ^L_{JI} are differentiable. The piecewise differentiable ones are acceptable. It is well-known that the derivative of piecewise MFs cause jump discontinuity.

Since the singular points are null thus the sufficient condition to calculate the above derivative is that they are piecewise differentiable. From the numerical implementation point of view, we can consider it as a derivative of the right hand point (or left hand point or average of the left and right hand points) nearby the discontinous point.

4.2. Application of the NEMM to fuzzy PDE modeling of nonlinear PDE systems

In addition to the NRLS estimation algorithm, several modified recursive schemes are presented to identify the output of the nonlinear system and its error dynamic in the presence of colored noise (18). Some popular kind of these schemes are: extended least square algorithm, instrumental variable and EMM algorithms (47).

Assume that the colored noise affects the nonlinear PDE model. The proposition of this Section is to estimate the adaptive parameters in fuzzy PDE model for approximating the nonlinear first-order PDE systems in the presence of colored noise. This fuzzy model consists of two parts: the first one is to estimate the parameter of the fuzzy model (2) and the second one is to estimate the fuzzy error model of colored noise. The nonlinear first-order PDE model via the colored measurement noise will be approximated by the following fuzzy rules:

Rule l for output i: IF $y_1(x,t)$ is F_{1i}^l **and** $\cdots$ **and** $y_n(x,t)$ is F_{ni}^l, **THEN**

$$\begin{aligned} z_i^l(x,t+1) &= N_i^l(u_1,x,t) + v_i(x,t) \\ v_i^l(x,t+1) &= \lambda_i^l(u_2,x,t) + e_i(x,t) \end{aligned} \tag{24}$$

where

$$\begin{aligned} N_i^l(u_1,x,t) &= \sum_{j=1}^{n} a_{ji}^l \frac{\partial y_j(x,t)}{\partial x} + b_{ji}^l y_j(x,t) + c_{ji}^l x y_j(x,t) + d_{ji}^l x^2 y_j(x,t) + \cdots \\ \lambda_i^l(u_2,x,t) =& k_{1i}^l + m_{1i}^l e(x,t) + n_{1i}^l x e(x,t) + o_{1i}^l x^2 e(x,t) + \cdots + k_{2i}^l \\ &+ m_{2i}^l e(x,t) + n_{2i}^l x v(x,t) + o_{2i}^l x^2 v(x,t) + \cdots \end{aligned}$$

and $k_{1i}^l, k_{2i}^l, m_{1i}^l, m_{2i}^l, n_{1i}^l, n_{2i}^l, o_{1i}^l, o_{2i}^l$ and etc. are the adaptive parameters which will be identified during the estimation algorithm. The overall fuzzy model can be calculated as follows:

$$z_i(x,t+1) = g_i(u_1,x,t) + v_i(x,t) = \sum_{l=1}^{R_i} h_i^l(y(x,t)) N_i^l(x,t) + v_i(x,t) \tag{25}$$

$$v_i(x,t+1) = \Gamma_i(u_2,x,t) + e_i(x,t) = \sum_{l=1}^{R_i} h_i^l(y(x,t)) \Lambda_i^l(x,t) + e_i(x,t) \tag{26}$$

Note that as shown in (24), for each rule we assume that the colored noise share the same fuzzy set with the PDE fuzzy model in the premise parts. Thus, the membership functions of output $z_i(x,t+1)$ are the same as the colored noise membership functions $v_i(x,t+1)$. This scenario can be also investigated in two points of views (*case I.* known and *case II.* unknown membership functions). For each case, the adaptive parameters $u_1^i(x,t)$ and $u_2^i(x,t)$ are considered as follows:

4.2.1. Known membership functions

$$\begin{aligned} u_1^i(x,t) =& [a_{1i}^1 \ \cdots \ b_{1i}^1 \ \cdots \ c_{1i}^1 \ \cdots \ d_{ni}^{R_i} \ \cdots]^T \\ u_2^i(x,t) =& [k_{1i}^l,\ m_{1i}^1,\ \cdots m_{1i}^{R_i}, n_{1i}^1,\ \cdots n_{1i}^{R_i}, o_{1i}^1,\ \cdots o_{1i}^{R_i},\ \cdots, k_{2i}^l, \\ & m_{2i}^1,\ \cdots m_{2i}^{R_i}, n_{2i}^1,\ \cdots n_{2i}^{R_i}, o_{2i}^1,\ \cdots o_{2i}^{R_i},\ \cdots] \end{aligned} \tag{27}$$

4.2.2. Unknown membership functions

$$
\begin{aligned}
u_1^i(x,t) =& [\sigma_{1i}^1 \ \cdots \ \sigma_{ni}^{R_i} \ a_{1i}^1 \ \cdots \ b_{1i}^1 \ \cdots \ c_{1i}^1 \ \cdots \ d_{ni}^{R_i} \ \cdots]^T \\
u_2^i(x,t) =& [k_{1i}^l, \ m_{1i}^1, \ \cdots m_{1i}^{R_i}, n_{1i}^1, \ \cdots n_{1i}^{R_i}, o_{1i}^1, \ \cdots o_{1i}^{R_i}, \cdots, k_{2i}^l, \\
& m_{2i}^1, \ \cdots \ m_{2i}^{R_i}, n_{2i}^1, \ \cdots n_{2i}^{R_i}, o_{2i}^1, \ \cdots o_{2i}^{R_i}, \cdots]
\end{aligned}
\tag{28}
$$

Finally, by utilizing the same procedure as investigated in subsection 3.2, the adaptive parameters of the fuzzy model will be achieved and the TS fuzzy model of nonlinear first-order PDE system will be obtained. Moreover, the nonlinear dynamic of colored measurement noise will be identified by TS PDE fuzzy model based on the proposed algorithm.

Remark 5. *In this subsection (subsection 4.2), we approximate the fuzzy model of the colored noise. Thus according to* (26)*, the behavior of the colored noise is assumed to be nonlinear. The proposed approach can be reduced to a more simplest case in which the dynamic of the colored noise has linear behavior. Under the mentioned conditions, the equation* (26) *will be described by a linear system. Subsequently, it will be identified based on NEMM by modifying matrix* $u_2^i(x,t)$ *in* (27) *or* (28)*.*

Remark 6. *Recall that, the proposed fuzzy representation of nonlinear first-order PDE system is completely general. It can describe hyperbolic, parabolic and elliptic categories of first order PDE systems. Furthermore, there was no restriction on the convection matrix. As a result, the general first-order PDE model will be considered with the proposed kind of fuzzy representation in* (18) *and* (25)*. On the other hand, some of the approaches presented to analyze the stability and performance of nonlinear PDE systems are based on these restrictions on fuzzy model (7). Hence, if we want to apply these restrictions, it is enough to choose the convection coefficients in* (18) *and* (25) *as* $a_{ji}^l = a_{ji}$*. Consequently, based on the method which we select for identification, by applying some modifications, the parameter of the considered fuzzy model will be approximated.*

Remark 7. *The main advantages and disadvantages of the proposed approach are investigated in this remark.*

- *The main advantages of the proposed approach can be classified as follows: (1). A novel framework is proposed to identify the TS fuzzy PDE model-based on input-output data. (2). In the presence of destructive effects such as colored-noise, the TS fuzzy PDE model can be identified. (3). The proposed identification procedure is simple, which is suitable for the complicated nature of the nonlinear first order hyperbolic PDE systems. (4). For the first time, the NRLS, and NEMM are extended for identifying PDE systems.*

- *Apart from advantages of the proposed approach, when the effect of diffusion matrix is negligible, then the PDE system is described by a first order hyperbolic PDE system. The proposed identification procedure is valid for the first order PDE systems. Identifying the higher order PDE systems needs more efforts which was not investigated through this manuscript.*

Remark 8. *By deploying spatially distributed sensing elements, the proposed approach can be easily implemented in the real-world applications by the micro-electro-mechanical systems (MEMS) technology. Due to the recent improvements in the MEMS, the problem of applying large array of micro-sensors is applicable. Furthermore, the proposed identification approach prepares the atmosphere for further improvements.*

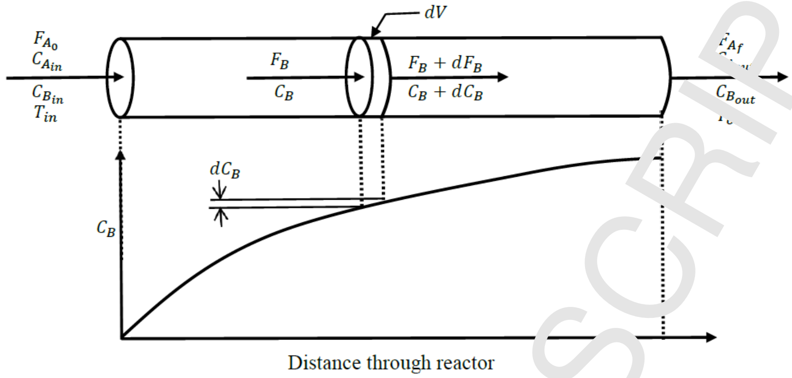


Figure 1: Nonisothermal plug-flow reactor.

5. Examples

In this section, the proposed online distributed fuzzy modeling approach is applied on two examples: PFR (7) and a nonlinear hyperbolic PDE system with Lotka-Volterra type (2).

5.1. Plug-Flow Reactor

In this section, the proposed approach to identify the nonlinear first-order PDE systems is applied on nonisothermal PFR (48). In ideal PFR, no back mixing will be occured when the reactants pass through the vessel (49). Also, all of the reaction mixture elements have a spatial reaction time which is precisely the same as the reactor residence time. The following chemical reaction is occurred in this reactor

$$A \longrightarrow \tilde{b}B$$

where $\tilde{b}$ is the stoichiometric coefficient. Thus, as shown in Figure 1, among the spatially distributed points x, the composition of the reaction mixture will be changed. This reaction is a kind of endothermic one and the jacket is used to heat the reactor, hence the system is open-loop stable and dissipative.

The dynamic model of the reactor will be obtained from the energy and mass balance by considering the negligible diffusion and constant heat capacity and density (49):

$$\begin{aligned}
\frac{\partial T}{\partial t} &= -v\frac{\partial T}{\partial x} - \frac{k_0 \Delta H}{\rho_p C_p} C_A \cdot e^{\frac{-E}{RT}} + \frac{4h}{\rho_p C_p d}(T_J - T) \\
\frac{\partial C_A}{\partial t} &= -v\frac{\partial C_A}{\partial x} - k_0 C_A \cdot e^{\frac{-E}{RT}} \\
\frac{\partial C_B}{\partial t} &= -v\frac{\partial C_B}{\partial x} + bk_0 C_A \cdot e^{\frac{-E}{RT}}
\end{aligned} \tag{29}$$

subject to the following initial and boundary conditions

$$\begin{aligned}
&T(0,t) = T_{in}, \qquad && C_A(0,t) = C_{A_{in}}, \quad && C_B(0,t) = 0 \\
&T(x,0) = T_0(x), \qquad && C_A(x,0) = C_{A_0}(x), \quad && C_B(x,0) = 0
\end{aligned}$$

Table 1: Model parameters and their definitions for simulation.

Parameters	Defnition of each parameter	Numerical values
v	Velocity of the fluid phase	$0.025\ m/s$
L	Length of the reactor	$1\ m$
E	Activation energy	$11250\ cal/mol$
k_0	Pre-exponential factor	$10^6\ s^{-1}$
$C_{A_{in}}$	Concentration of the inlet stream	$0.02\ mol/L$
R	Ideal gas	$1.986\ cal/(mol.K)$
T_{in}	Temperature of the inlet stream	$340K$
δ	$((-\Delta H)C_{A_{in}})/(\rho_p C_p T_{in})$	0.25
b	$4h/\rho_p C_p d$	$0.2\ s^{-1}$
μ	E/RT_{in}	16.6667
β_2	$k_0 e^{-\mu}$	0.0581
β_1	$\delta\beta_2$	0.0145

where C_A and C_B are the reactant concentration and product one, respectively. T and T_J indicate the reactor temperature and the jacket temperature, respectively. F_B is the partial flow of product B. Furthermore, ΔH denotes the enthalpy of the reaction. h indicates the wall heat transfer coefficient. d illustrates the reactor diameter. ρ_p shows the density, and C_p is specific heat capacity. Besides the other parameters, their definitions and their numerical values are given in Table 1.

From (29) we conclude that, if C_A and T are known, then C_B will be computed. Hence, only the two first equations are considered. The dimension-less model will be obtained from the following change of variables

$$\chi_1 \triangleq \frac{T - T_{in}}{T_{in}}, \quad \chi_2 \triangleq \frac{C_{A,in} - C_A}{C_{A,in}}, \quad \phi_j \triangleq \frac{T_j - T_{in}}{T_{in}}$$

The equilibriume porfile of the dimension-less model is computed as follows

$$\chi_{1e}(x) = 0, \quad \chi_{2e}(x) = 1 - exp(-\frac{\beta_2 L}{v}x), \quad \phi_{je} = -\frac{-\beta_1}{b}exp(-\frac{\beta_2 L}{v}x)$$

Consider the follwing state transformation and input vector

$$y(x,t) = \begin{bmatrix} \chi_1(x,t) - \chi_{1e}(x) \\ \chi_2(x,t) - \chi_{2e}(x) \end{bmatrix}, \quad u(x,t) = \phi_j(x,t) - \phi_{je}(x)$$

Now, the unforced system in the presence of noise can be rewritten as

$$\begin{aligned} \frac{\partial y_1(x,t)}{\partial t} &= -v/L\frac{\partial y_1(x,t)}{\partial x} + \beta_1 f_0(y(x,t),x) - by_1 + e_1(x,t) \\ \frac{\partial y_2(x,t)}{\partial t} &= -v/L\frac{\partial y_2(x,t)}{\partial x} + \beta_2 f_0(y(x,t),x) + e_2(x,t) \end{aligned} \tag{30}$$

where $e_i(x,t)$, $i \in \{1,2\}$ denote the white noises and

$$f_0(y(x,t),x) = (1 - \chi_{2e}(x))\Big[exp\Big(\frac{\mu y_1(x,t)}{1 + y_1(x,t)}\Big) - 1\Big] - y_2(x,t)\Big[exp\Big(\frac{\mu y_1(x,t)}{1 + y_1(x,t)}\Big)\Big]$$

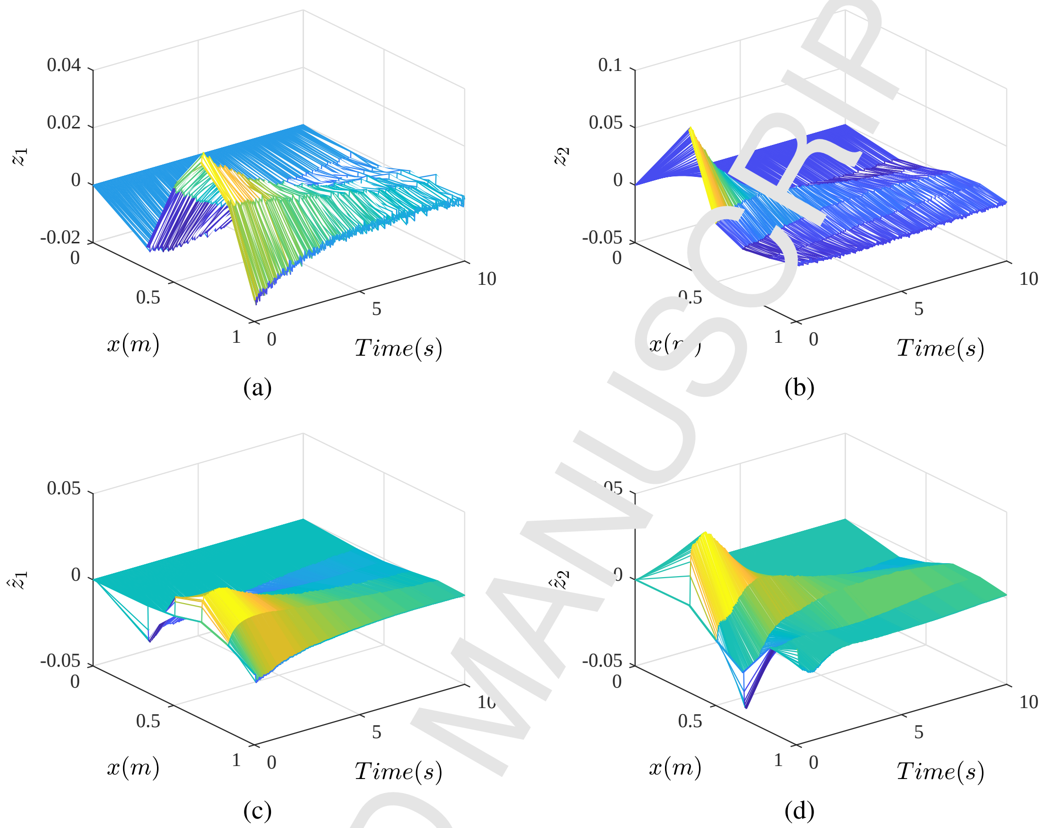


Figure 2: (a) First (b) second, output of the system with validation data as input. (c) First (d) second, output of the estimated fuzzy model.

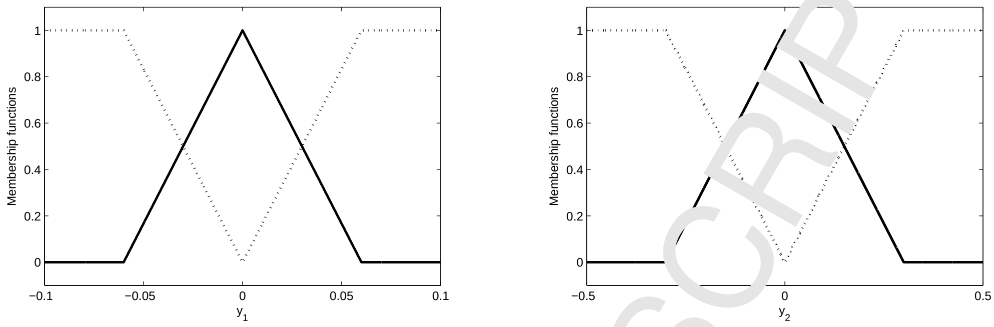
The behavior of the open-loop nonlinear PDE system (30) is shown in Figures 2 (a) and (b). This practical application is considered to demonstrate the fuzzy modeling performance of a first-order PDE system via input-output data. This example is investigated for both cases which are introduced in Section 4. Fuzzy modeling of PFR in the presence of white and colored noises are presented in Sections 5.1.1 and 5.1.2, respectively.

5.1.1. Fuzzy modeling of plug-flow reactor in the presence of white noise

Case I: The membership functions are known

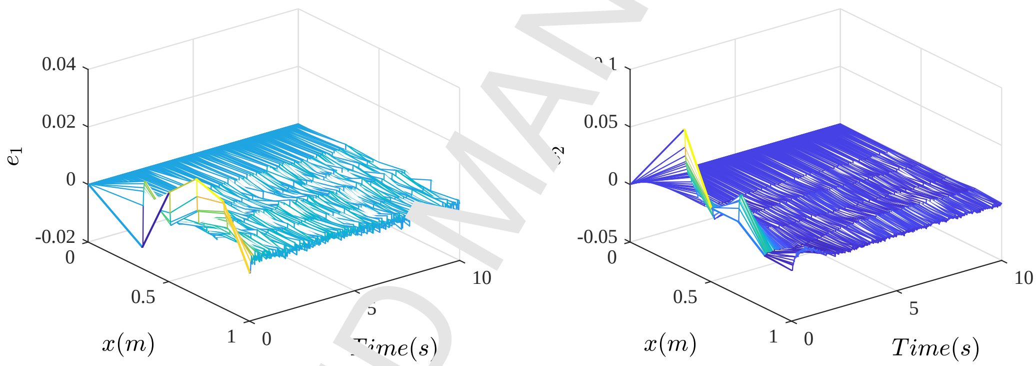
In this case, triangular membership functions as shown in Figure 3, are considered and the parameters in the consequence of membership functions are computed according to the NLS algorithm.

The behaviours of the state variables of the overall fuzzy first-order PDE model are displayed in Figures 2 (c) and (d). In Figure 2, the x-axis, y-axis, and z-axis indicate the position through the length of the reactor, the time variable, and the amplitude of the evolutions of the state variables, respectively. Comparing Figure 2 (a) with (c), and Figure 2 (b) with (d). For the case the membership functions are known, It can be observed that the NLS algorithm can accurately iden-



(a) (b)

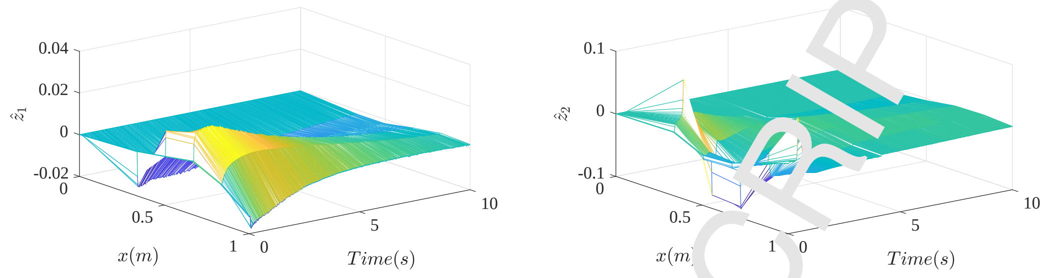
Figure 3: Antecedents in case I. (a) $F^1_{i,j}$ and $F^2_{i,j}$ ($i, j \in \{1, 2\}$) denoted by dots and dashed lines, respectively. (b) $F^3_{i,j}$ and $F^4_{i,j}$ indicated by dots and dashed lines, respectively.



(a) (b)

Figure 4: Final modeling error $z(x,t) - \hat{z}(x,t)$, (a) first output (b) second output.

tify the TS fuzzy model of nonlinear hyperbolic PDE system. Furthermore, the error dynamic between the computed fuzzy model and the exact nonlinear model is illustrated in Figures 4 (a) and (b). The results indicate that the proposed method presented in subsection 4.1.1 can suitably estimate the states of the nonlinear PFR system. Precisely, based on the proposed NRLS identification method for PDE systems, by utilizing the current and the past measurement data in each sampling period, the system parameters are identified. To do this, in each iteration, the nonlinear system is linearized around the estimated parameters (See equation (7)). Then, the Kalman gain $K(x, t + 1)$, which is the set of modifications coefficient, is calculated such that the performance index is minimized (See equation (10)). Next, by utilizing the Kalman gain, and calculating the error between the measured $z(x, t)$ and the estimated $\hat{z}(x, t)$ outputs, the parameters of the system $\hat{u}(x, t + 1)$ are estimated and the covariance matrix $P(x, t + 1)$ is calculated (See equations 11, and 12). Finally, the system output $\hat{z}(x, t + 1)$ is predicted by deploying the estimated parameters (See equation 13). Due to the minimization of the performance index, the error between the system output and identified output converges to zero over time. This issue can be seen in Figure 4.



(a) (b)

Figure 5: Estimated fuzzy model, (a) first output (b) second output.

Case II: The membership functions are unknown
In this case, the membership functions are assumed to be Gaussian

$$\mu_{ij}^{l}(Gaussian) = e^{-\frac{(y-\alpha_{ij}^{l})^2}{(\eta_{ij}^{l})^2}}$$

Hence, $\sigma_{ij}^{l} = [\alpha_{ij}^{l}, \eta_{ij}^{l}]^T$. The derivative of the particular membership function $\mu_{IJ}^{L}(Gaussian)$ is achieved as follows:

$$\frac{\partial \mu_{IJ}^{L}(Gaussian)}{\partial \alpha_{IJ}^{L}} = \frac{2(y-\alpha_{IJ}^{L})}{(\eta_{IJ}^{L})^2}\mu_{IJ}^{L}(Gaussian)\Big|_{u_I=\hat{u}_I(x,t)},$$
$$\frac{\partial \mu_{IJ}^{L}(Gaussian)}{\partial \eta_{IJ}^{L}} = \frac{2(y-\alpha_{IJ}^{L})^2}{(\eta_{IJ}^{L})^3}\mu_{IJ}^{L}(Gaussian)\Big|_{u_I=\hat{u}_I(x,t)}$$

The assumed membership functions have nonlinear behaviours. Thus, by applying the NLS algorithm presented in subsection 4.1.2, the fuzzy rules and membership functions will be achieved. In each iteration, the Kalman gains are obtained such that the error between the real output and estimated output is minimized. Thus, with an acceptable speed, the estimated outputs converge to the output variables over the time. The evolutions of states of the identified overall fuzzy model are illustrated in Figures 5 (a) and (b). Furthermore, the evolutions of the error signals between the nonlinear system (30) and the overall fuzzy identified model are shown in Figures 6 (a) and (b). As shown in Figure 6, the error signal is converged to zero over the time. From the steady-state behaviour of error signals, we can conclude that the system is suitably approximated.

5.1.2. *Fuzzy modeling of plug-flow reactor in the presence of colored noise*

In this section, the simulation results will be extended for the case that the colored noise affects the nonlinear first-order PDE system. Thus, the approach proposed in Section 4.2 has been tested to verify the effectiveness of the proposed approach in the absence of colored noise. It is assumed that the PFR system is affected by the following colored noise:

$$\begin{aligned}\frac{\partial y_1(x,t)}{\partial t} &= -v/L\frac{\partial y_1(x,t)}{\partial x} + \beta_1 f_0(y(x,t),x) - by_1 + v_1(x,t)\\ \frac{\partial y_2(x,t)}{\partial t} &= -v/L\frac{\partial y_2(x,t)}{\partial x} + \beta_2 f_0(y(x,t),x) + v_2(x,t)\end{aligned} \tag{31}$$

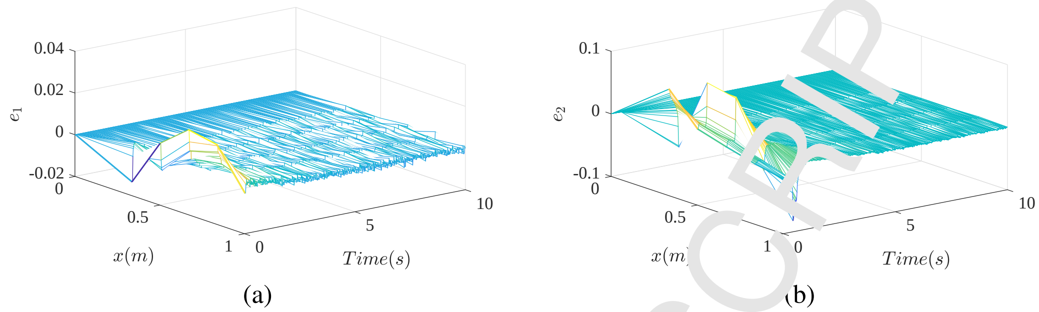


Figure 6: Final modeling error $z(x,t) - \hat{z}(x,t)$, (a) first output (b) second output.

where v_1 and v_2 are colored noises with the following nonlinear models

$$
\begin{aligned}
v_1(x,t) &= v_1^2(x,t) + 0.9x^2 sin(v_1(x,t)) + e_1^3(x,t) + x^2 e_1^2(x,t) + e_1(x,t) + 0.001 \\
v_2(x,t) &= v_2^2(x,t) + 0.9x^2 v_2(x,t) + e_2^3(x,t) + x^2 e_2^2(x,t) + e_2(x,t) + 0.001
\end{aligned} \tag{32}
$$

and also, $e_1(x,t)$ and $e_2(x,t)$ are white noise signals. The behaviour of the colored noise signals (32) are displayed in Figures 8 (a) and (b), respectively.

If we apply the NRLS approach presented in subsection 4.1.1 directly to this example, then the error signal between the exact nonlinear system and the fuzzy model will converge to infinity, which clearly indicates the unreliable results. By considering the NEMM identification method which is proposed in subsection 4.2.1 for PFR system in the presence of measurement colored noise, we can identify the fuzzy model for both the PFR system and also the dynamic of measurement colored noise one (32). Hence, similar triangular membership functions are introduced (the same as Figure 3) and the parameters of the fuzzy model are approximated via NEMM. Then, evaluations of online estimated overall fuzzy model for the first and second outputs are shown in Figures 7 (a) and (b), respectively. To illustrate the efficiency of the proposed approach, the error signals between the identified overall fuzzy model based on the NEMM algorithm presented in subsection 4.2.1 and the nonlinear system (31) are simulated in Figures 7 (c) and (d), respectively. Moreover, Figures 8 (c) and (d) illustrate the estimation of the colored noise. Thus, the simulation results in Figure 8 indicate that the proposed NEMM algorithm can correctly approximate the behaviour of colored noise (32) besides nonlinear first-order PDE system (31).

5.2. A hyperbolic PDE system with Lotka-Volterra type

Consider a nonlinear distributed system with Lotka-Volterra type, which are usually used in modeling of biological distributed systems and networks, competing species interaction and predatorprey (2). The distributed dynamical model of the system is represented as follows:

$$
\begin{aligned}
\frac{\partial \hat{y}_1(x,t)}{\partial t} &= -\nu_1(x)\frac{\partial \hat{y}_1(x,t)}{\partial x} + \beta_1(x)\hat{y}_1 + r_1(x)\hat{y}_1\hat{y}_2 + b(x)u + v_1(x,t) \\
\frac{\partial \hat{y}_2(x,t)}{\partial t} &= -\nu_2(x)\frac{\partial \hat{y}_2(x,t)}{\partial x} + \beta_2(x)\hat{y}_2 + r_2(x)\hat{y}_1\hat{y}_2 + v_2(x,t)
\end{aligned} \tag{33}
$$

where the state variables $\hat{y}_1(x,t)$ and $\hat{y}_2(x,t)$ indicate the predator and the prey, respectively. $u(x,t)$ is the distributed controller. $v_1(x,t)$ and $v_2(x,t)$ are colored noise. $r_1(x)$, $r_2(x)$, $\nu_1(x)$, $\nu_2(x)$, $\beta_1(x)$ and $\beta_2(x)$ are system parameters. Deploying the following change of variables

$$
y(x,t) = \begin{bmatrix} y_1(x,t) \\ y_2(x,t) \end{bmatrix} = \begin{bmatrix} \hat{y}_1(x,t) - \hat{y}_{1d} \\ \hat{y}_2(x,t) - \hat{y}_{2d} \end{bmatrix}^T \tag{34}
$$

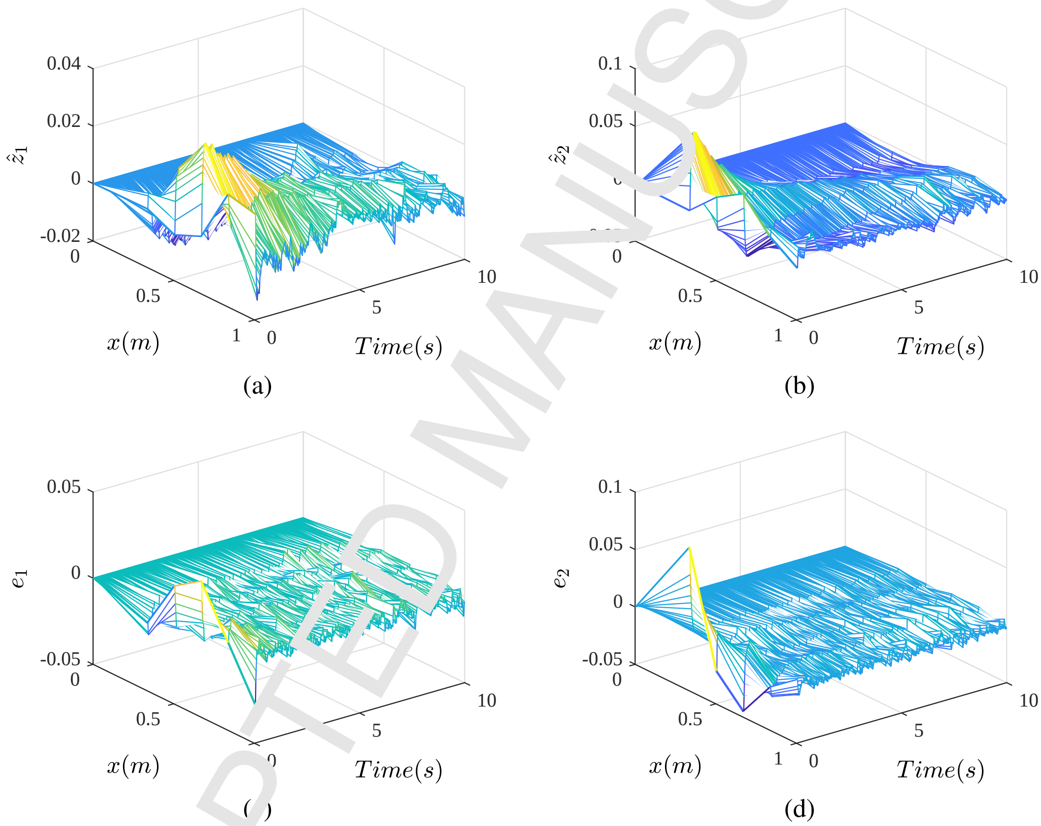


Figure 7: Estimated fuzzy model of the nonlinear system, (a) first output (b) second output and final modeling error $z(x,t) - \hat{z}(x,t)$, (c) first output (d) second output.

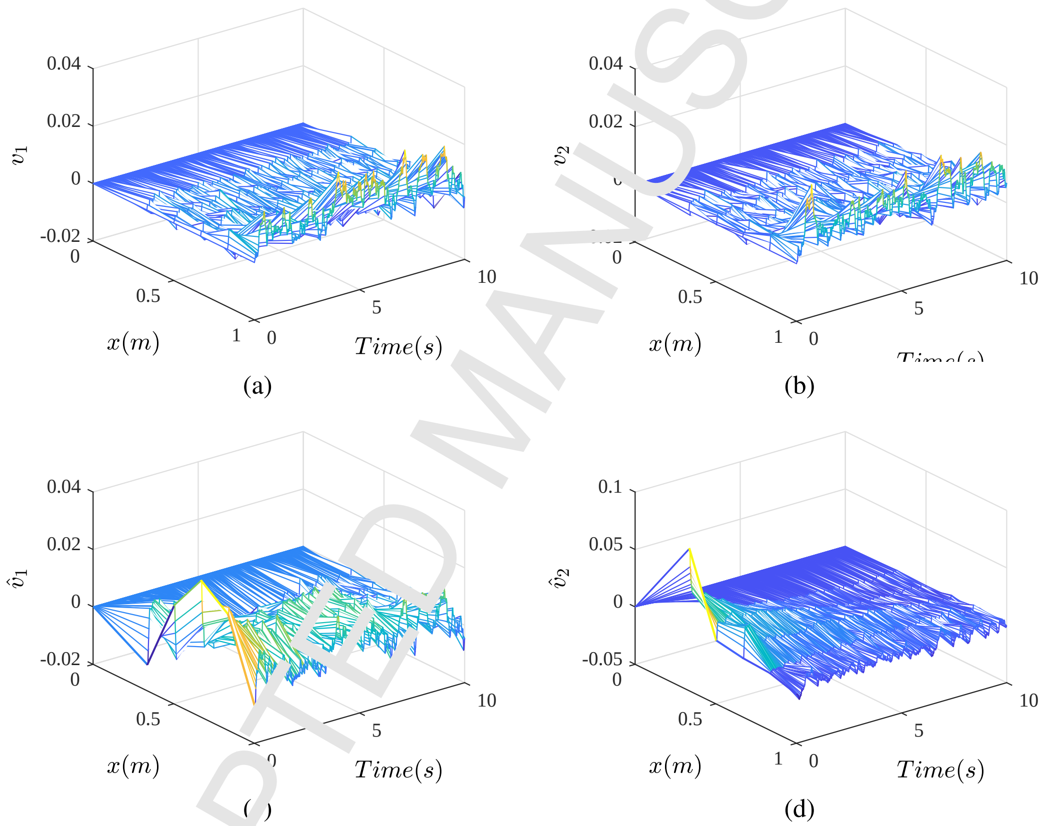


Figure 8: The colored noise signals (32) for (a) first output v_1 (b) second output v_2, and its estimation for (c) first output $\hat{v}_1$ (d) second output $\hat{v}_2$.

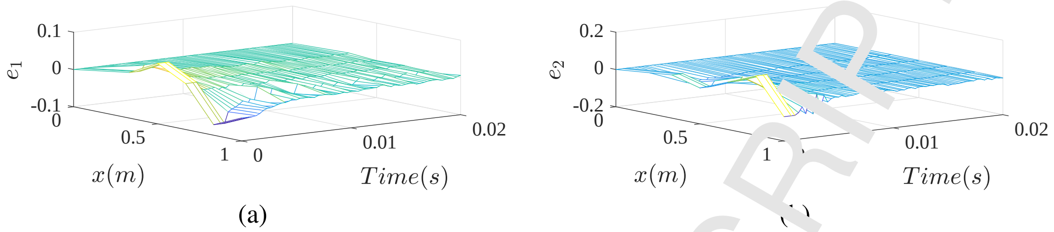


Figure 9: The error between the obtained fuzzy model and the nonlinear PDE system, (a) first output (b) second output.

one can conclude

$$\begin{aligned} \frac{\partial y_1(x,t)}{\partial t} &= -v_1(x)\frac{\partial y_1(x,t)}{\partial x} + \beta_1(x)y_1 + r_1(x)f_0(x,t) + b(x)u + v_1(x,t) \\ \frac{\partial y_2(x,t)}{\partial t} &= -v_2(x)\frac{\partial y_2(x,t)}{\partial x} + \beta_2(x)y_2 + r_2(x)f_0(x,t) + v_2(x,t) \end{aligned} \tag{35}$$

where

$$f_0(x,t) = y_1(x,t)y_2(x,t) + \hat{y}_{1d}y_2(x,t) + \hat{y}_{2d}y_1(x,t)$$

where the desired values of $\hat{y}_1(x,t)$ and $\hat{y}_2(x,t)$ are denoted by $\hat{y}_{1d}$ and $\hat{y}_{2d}$, respectively. The numerical values of the system parameters are as follows:

$$\begin{array}{lll} \beta_2(x) = 0.5\cos(2x), & r_1(x) = 1, & r_2(x) = -1, \\ v_1(x) = 0.1, & v_2(x) = 0.2, & \beta_1(x) = 0.8\sin(2x), \\ b(x) = 1, & \hat{y}_{1d} = 2, & \hat{y}_{2d} = 1.1 \end{array}$$

with the following initial and boundary conditions

$$\begin{array}{ll} y_1(0,t) = 0 & y_2(0,t) = 0 \\ y_1(x,0) = 0.1\sin(\pi x) & y_2(x,0) = 0.2\sin(\pi x) \end{array} \tag{36}$$

Additionally, the dynamical model of the colored noise is assumed to be similar to the (32). Since the open-loop system is stable, the identification problem of open-loop system is investigated. Whereas the colored noise affects the hyperbolic PDE system, the NEMM is used to identify the PDE system as well as the colored noise. The error signal between the obtained TS fuzzy model and the real nonlinear PDE system is illustrated in Fig. 9.

6. Conclusions

From this paper, one can conclude that a general structure for identifying the TS fuzzy PDE model of nonlinear MIMO first-order PDE systems in the presence of white and colored noises was proposed. Against the existing approaches on TS fuzzy PDE modeling of nonlinear PDE systems, the identification method in this paper was based on input-output data. For PDE systems with white noise, we can conclude that the NRLS method was able to identify the fuzzy PDE model. When the colored noise affects the PDE system, the NEMM method was proposed to identify the fuzzy model of the MIMO nonlinear PDE system with the measurement colored noise. In the case that the colored noise affected the nonlinear PDE system, not only the TS fuzzy PDE model of nonlinear PDE system, but also the nonlinear distributed model of colored noise was identified. Furthermore, the identification of known and unknown membership functions was investigated. Additionally, in the cases that the membership functions were unknown, it was illustrated that the proposed approach has the ability to identify the TS fuzzy PDE model of nonlinear PDE system. The proposed approach was successfully tested on the nonisothermal PFR and the applicability of the proposed approach was clearly indicated.

For the future works, the authors suggest to extend the proposed identification method for the high order class of PDE systems. Additionally, the authors suggest to introduce a new identification algorithm such that the optimal solution will be achieved.

References

[1] J. Zhou, Optimal control of a stochastic delay partial differential equation with boundary-noise and boundary-control, Journal of Dynamical and Control Systems 20 (4) (2014) 503–522.

[2] H.-N. Wu, J.-W. Wang, H.-X. Li, Design of distributed h fuzzy controllers with constraint for nonlinear hyperbolic pde systems, Automatica 48 (10) (2012) 2535–2543.

[3] A. Belmiloudi, Dynamical behavior of nonlinear impulsive abstract partial differential equations on networks with multiple time-varying delays and mixed boundary conditions involving time-varying delays, Journal of Dynamical and Control Systems 21 (1) (2015) 95–146.

[4] A. Tsouli, A. Boutoulout, Controllability of the parabolic system via bilinear control, Journal of Dynamical and Control Systems 22 (1) (2016) 35–44.

[5] J. L. Pitarch, M. Rakhshan, M. M. Mardani, M. Sadeghi, Distributed saturated control for a class of semilinear pde systems: A sos approach, IEEE Transactions on Fuzzy Systems 26 (2) (2018) 749–760. doi:10.1109/TFUZZ.2017.2688379.

[6] R. van den Berg, Partial differential equations in modeling and control of manufacturing systems, Ph.D. thesis, MS thesis, Eindhoven Univ. Technol., Eindhoven, The Netherlands, 2004 [Online]. Available: http://se. wtb. tue. nl/~ lefeber/MscTheses/vandenBerg. pdf (2004).

[7] J.-W. Wang, H.-N. Wu, H.-X. Li, Distributed fuzzy control design of nonlinear hyperbolic pde systems with application to nonisothermal plug-flow reactor, IEEE Transactions on Fuzzy Systems 19 (3) (2011) 514–526.

[8] M. M. Mardani, M. Shasadeghi, B. Safarinejadian, Design of distributed sampled-data fuzzy controller for a class of non-linear hyperbolic pde systems: input delay approach, IET Control Theory & Applications 12 (6) (2018) 728–737.

[9] Z.-P. Wang, H.-N. Wu, H.-X. Li, Sampled-data fuzzy control for nonlinear coupled parabolic pde-ode systems, IEEE transactions on cybernetics 47 (9) (2017) 2603–2615.

[10] H.-N. Wu, Z.-P. Wang, Observer-based h_∞ sampled-data fuzzy control for a class of nonlinear parabolic pde systems, IEEE Transactions on Fuzzy Systems 26 (2) (2018) 454–473.

[11] Z.-P. Wang, H.-N. Wu, h_∞ sampled-data fuzzy control for non-linear parabolic distributed parameter systems with control inputs missing, IET Control Theory & Applications 11 (10) (2017) 1530–1541.

[12] Z.-P. Wang, H.-N. Wu, H.-X. Li, Estimator-based h_∞ sampled-data fuzzy control for nonlinear parabolic pde systems, IEEE Transactions on Systems, Man, and Cybernetics: Systemsdoi:10.1109/TSMC.2018.2818761.

[13] H.-N. Wu, H.-D. Wang, L. Guo, Disturbance rejection fuzzy control for nonlinear parabolic pde systems via multiple observers, IEEE Transactions on Fuzzy Systems 24 (6) (2016) 1334–1348.

[14] J. Qiu, S. X. Ding, H. Gao, S. Yin, Fuzzy-model-based reliable static output feedback control of nonlinear hyperbolic pde systems, IEEE Transactions on Fuzzy Systems 24 (2) (2016) 388–400.

[15] J.-W. Wang, H.-N. Wu, H.-X. Li, Distributed proportional–spatial derivative control of nonlinear parabolic systems via fuzzy pde modeling approach, IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics) 42 (3) (2012) 927–938.

[16] J. Merger, A. Borzì, Dynamics identification in evolution models using radial basis functions, Journal of Dynamical and Control Systems 23 (2) (2017) 317–335.

[17] I. Souflas, A. Pezouvanis, K. M. Ebrahimi, Nonlinear recursive estimation with estimability analysis for physical and semiphysical engine model parameters, Journal of Dynamic Systems, Measurement, and Control 138 (2) (2016) 024502–0245025.

[18] R.-M. Chen, S. Chang, State and parameter estimation for dynamic systems with colored noise: an accuracy approach based on the minimum discrepancy measure, IEEE transactions on automatic control 36 (7) (1991) 813–823.

[19] F. Matía, G. N. Marichal, E. Jiménez, Fuzzy Modeling and Control: Theory and Applications, Springer, 2014.

[20] B. Safarinejadian, N. Vafamand, Kalman randomized joint ukf algorithm for dual estimation of states and parameters in a nonlinear system, Journal of Electrical Engineering and Technology 10 (3) (2015) 1212–1220.

[21] S. Qi, D. Zhang, L. Guo, L. Wu, Adaptive dynamic surface control of nonlinear switched systems with prescribed performance, Journal of Dynamical and Control Systems 24 (2) (2018) 269–286.

[22] M. M. Mardani, N. Vafamand, M. Shokrian Zeini, M. Shasadeghi, A. Khayatian, Sum-of-squares-based finite-time adaptive sliding mode control of uncertain polynomial systems with input nonlinearities, Asian Journal of Control 20 (4) (2018) 1658–1662.

[23] H. Ghorbel, A. El Hajjaji, M. Souissi, M. Chaabane, Robust tracking control for takagi–sugeno fuzzy systems with unmeasurable premise variables: Application to tank system, Journal of Dynamic Systems, Measurement, and Control 136 (4) (2014) 041011.

[24] M. M. Mardani, N. Vafamand, M. H. Khooban, T. Dragicevic, F. Blaabjerg, Design of quadratic d-stable fuzzy controller for dc microgrids with multiple cpls, IEEE Transactions on Industrial Electronicsdoi:10.1109/TIE.2018.2851971.

[25] A. J. Barragán, B. M. Al-Hadithi, A. Jiménez, J. M. Andújar, A general methodology for online ts fuzzy modeling by the extended kalman filter, Applied Soft Computing 18 (2014) 277–289.

[26] C. Li, J. Zhou, B. Fu, P. Kou, J. Xiao, T–s fuzzy model identification with a gravitational search-based hyperplane clustering algorithm, IEEE Transactions on Fuzzy Systems 20 (2) (2012) 305–317.

[27] M. M. Mardani, N. Vafamand, M. H. Khooban, T. Dragicevic, F. Blaabjerg, Non-fragile controller design of uncertain saturated polynomial fuzzy systems subjected to persistent bounded disturbance, Transactions of the Institute of Measurement and Control (2018) 0142331218774110doi:10.1177/0142331218774110.

[28] P. Tabarisaadi, M. M. Mardani, M. Shasadeghi, B. Safarinejadian, A sum-of-squares approach to consensus of nonlinear leader-follower multi-agent systems based on novel polynomial and fuzzy polynomial models, Journal of the Franklin Institute 354 (18) (2017) 8398–8420.

[29] K. Tanaka, H. O. Wang, Fuzzy control systems design and analysis: a linear matrix inequality approach, John Wiley & Sons, 2004.

[30] B. M. Al-Hadithi, A. Jiménez, F. Matía, A new approach to fuzzy estimation of takagi–sugeno model and its applications to optimal control for nonlinear systems, Applied Soft Computing 12 (1) (2012) 280–290.

[31] J.-S. Jang, Anfis: adaptive-network-based fuzzy inference system, IEEE transactions on systems, man, and cybernetics 23 (3) (1993) 665–685.

[32] A. Tewari, M.-U. Macdonald, Knowledge-based parameter identification of tsk fuzzy models, Applied Soft Computing 10 (2) (2010) 481–489.

[33] H. Chen, Extended recursive least squares algorithm for nonlinear stochastic systems, in: American Control Conference, 2004. Proceedings of the 2004, Vol. 5, IEEE, 2004, pp. 4758–4763.

[34] M. Askari, A. Markazi, A new evolving compact optimised takagi–sugeno fuzzy model and its application to nonlinear system identification, International Journal of Systems Science 43 (4) (2012) 776–785.

[35] M. Luo, F. Sun, H. Liu, Joint block structure sparse representation for multi-input–multi-output (mimo) t–s fuzzy system identification, IEEE Transactions on Fuzzy Systems 22 (6) (2014) 1387–1400.

[36] J. Vaščák, M. Pala, Adaptation of fuzzy cognitive maps for navigation purposes by migration algorithms, International Journal of Artificial Intelligence 8 (2012) 20–37.

[37] Y.-J. Liu, S. Tong, Adaptive fuzzy identification and control for a class of nonlinear pure-feedback mimo systems with unknown dead zones, IEEE Transactions on Fuzzy Systems 23 (5) (2015) 1387–1398.

[38] D. Simon, Kalman filtering for fuzzy discrete time dynamic systems, Applied soft computing 3 (3) (2003) 191–207.

[39] K. Chafaa, M. Ghanai, K. Benmahammed, Fuzzy modelling using kalman filter, IET Control Theory & Applications 1 (1) (2007) 58–64.

[40] D. J. Simon, Training fuzzy systems with the extended kalman filter, Fuzzy sets and systems 132 (2) (2002) 189.

[41] M. A. Khanesar, E. Kayacan, M. Teshnehlab, O. Kaynak, Extended kalman filter based learning algorithm for type-2 fuzzy logic systems and its experimental evaluation, IEEE Transactions on Industrial Electronics 59 (11) (2012) 4443–4455.

[42] S.-J. Ho, B.-S. Chen, Robust fuzzy estimator-based stabilization design for nonlinear parabolic partial differential systems with different boundary conditions, IEEE Transactions on Fuzzy Systems 24 (1) (2016) 208–222.

[43] X. Zhao, Y. Yin, L. Zhang, H. Yang, Control of switched nonlinear systems via t–s fuzzy modeling, IEEE Transactions on Fuzzy Systems 24 (1) (2016) 235–241.

[44] H. Bustince, E. Barrenechea, M. Pagola, J. Fernandez, Z. Xu, B. Bedregal, J. Montero, H. Hagras, F. Herrera, B. De Baets, A historical account of types of fuzzy sets and their relationships, IEEE Transactions on Fuzzy Systems 24 (1) (2016) 179–194.

[45] D. Simon, Optimal state estimation: Kalman, H infinity, and nonlinear approaches, John Wiley & Sons, 2006.

[46] V. Yu, A. Headley, D. Chen, A constrained extended kalman filter for state-of-charge estimation of a vanadium redox flow battery with crossover effects, Journal of Dynamic Systems, Measurement, and Control 136 (4) (2014) 0410131–0410137.

[47] J. P. Norton, An introduction to identification, Courier Corporation, 2009.

[48] H. Shang, J. F. Forbes, M. Guay, Feedback control of hyperbolic distributed parameter systems, Chemical Engi-

neering Science 60 (4) (2005) 969–980.
[49] I. Aksikas, J. J. Winkin, D. Dochain, Optimal lq-feedback regulation of a nonisothermal plug flow reactor model by spectral factorization, IEEE Transactions on Automatic Control 52 (7) (2007) 1179–1193.