Stability of time-delay systems, Keqin Gu, Vladimir L. Kharitonov, Jie Chen, Birkhäuser, Boston, 2003, ISBN 0-8176-4212-9.

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Book reviews

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1 Introduction

The importance of time-delays in control is now well recognized in a wide range of applications (transportation systems, communication networks, teleoperation systems...). However, even if some fundamental results were obtained 40 years ago during the early space explorations (Bellman and Cooke, 1963; Morse, 1976; Kamen, 1978; Manitius and Triggiani, 1978), time-delay systems have been actually the subject of intensive research works since the nineties. This is probably due to the new emerging applications in engineering (such as network controlled systems) combined with new theoretical results, that allowed to solve some open problems (decoupling problems, stabilization, robustness...) and to give less and less conservative results. Of course applications motivate the need of theory, which in return makes the control applications possible.

As a preamble, let briefly recall that a time-delay system may be described by different models such as functional differential equations (Hale, 1977), infinite dimensional systems over operators (Curtain and Pritchard, 1978), or ring models (Morse, 1976). Tools for each previous class of systems may then allow to derive results for time-delay systems.

2 Book overview

This book is concerned with the stability of time-delay systems (described by functional differential equations) and aims at focusing on recent methods leading to computable criteria and being able to deal with uncertainties. It emphasizes efficiently the interest of LMIs (Linear Matrix Inequalities) to provide useful criteria for (robust) stability, as well as the conservatism induced by each considered method.

The book is divided in three parts and an introduction. Part I is dedicated to the Frequency-Domain approach and part II to the Time-Domain approach; both contain three chapters. The last one concerns the input-output stability and is a single chapter. Notes with references are given at the end of each chapter to mention classical and recent results not detailed in the book. This tempting structure makes the reader impatient for reading a complete overview of stability analysis. Nevertheless, one might be disappointed by the selection of topics which, as mentioned in the preface, merely reflects the authors' viewpoints and preferences. Indeed the given recent results are mainly the authors' contributions, and one might miss important developments on neutral delay systems, time-varying delays or H_∞ control. Also, the relative weakness of the book concerns the lack of application (non academic) examples, that might have enabled to illustrate and compare the exposed methods. Furthermore, even though some academic examples are presented, they are few detailed.

On the other hand, the strength of the book lies in the rigorous and well-organized presentation of the theoretical developments in time-domain and frequency-domain approaches. This intended separation improves the reading and understanding of the book, even for non specialists. The robustness aspects in stability analysis (w.r.t uncertainties) and the use of the small gain theorem is a great advantage of the book. This makes this book an essential reference in stability of time-delay systems.

Compared to other recent books, let me mention the collection books (Dugard and Verriest, 1998; Niculescu and Gu, 2004) which contain recent results in various theoretical and application fields concerning time-delay systems, but consequently may be found heterogeneous, and the monograph of Niculescu (2001) where, in addition to stability analysis, many application examples are described and some results are given for closed-loop systems (Popov theory, passivity ...).

3 Book description and analysis

A concise summary and analysis of each chapter is presented in what follows.

Chapter 1 is an introduction to time-delay systems. Following few "physical" examples, the concept of stability is defined, as well as the considered linear representations of time-delay systems.

Part I (Frequency-domain approach) begins by the case of commensurate delays in chapter 2. The stability is in this case analyzed through the system characteristic equation, or quasipolynomial $a(s, e^{-\tau s})$. The core of this chapter is to give necessary and sufficient conditions for stability independent of delay, or else, i.e. to get the delay margin (stability dependent of the delay):

$$\bar{\tau} = min\{\tau \geq 0 \mid a(s, e^{-j\tau\omega}) = 0, \text{ for some } \omega \in \mathbb{R}\}$$

This study is shown to be equivalent to considering the zero crossings at positive frequencies (which are in finite number). A so-called direct method leads to a sufficient condition which corresponds to a special case of the Rouché's theorem, and which is close to the Tsypkin's theorem and the small gain condition. Frequency-sweeping tests are then given, as extended small gain conditions. The drawback of such conditions is the unavoidable frequency gridding; therefore, alternative stability tests of the quasipolynomial are given from eigenvalues of constant matrices.
This chapter contains a very interesting presentation of the frequency-domain philosophy in the case of time-delay systems.

Chapter 3 tackles the generalization to the case of incommensurate delays, considering the delay operator as an uncertainty (i.e. $\Delta(s) = e^{-\tau s}$). First the small gain theorem is recalled in the robust stability framework, and used to give a sufficient condition for stability independent of delay. Then, the structured singular value is proved to be the efficient tool to deal with stability of multiple delay systems. A very detailed and exhaustive presentation emphasizes the difference between sufficient conditions and, necessary and sufficient ones. Indeed the particular case of the null frequency ($s = 0$) is shown to be the key point where sufficient conditions may lose necessity. The computational complexity of stability independent of delay is proved to be NP hard. Then, different sufficient stability conditions are given, the conservatism of which is classified. An extension to neutral systems is briefly described. In conclusion, if the use of structured singular values as stability criteria may induce some conservatism, the advantage is that it can be generalized to time-varying systems, which is the benefit of the frequency domain approach.
While the delay operators are modelled as uncertainties, this chapter emphasizes the interest of robust control theory in the case of time-delay systems.

Chapter 4 is concerned with the case of uncertain time-delay systems. An exhaustive characterization of the zeros of a quasipolynomial is presented, in particular their location in the complex plane. Uncertain quasipolynomials (without delay uncertainties) are then described by a polytopic family, leading to the Edge Theorem. A stability analysis of such "polynomials" is also provided in the framework of multivariate polynomials in s and $e^{-s\tau}$.
I found this chapter less interesting than the others. First, we miss computable necessary and sufficient conditions, which can be found in (Niculescu and Gu, 2004). Next, while it focuses on quasipolynomials, one might expect a robust stability analysis following the point of view of the previous chapter (three), i.e. the use of the small gain theorem. In particular uncertainties on the system parameters (including the delay) may have been described using additive or multiplicative uncertainties, allowing to check the robust stability through the Rouché's theorem.

Part II (Time-Domain approach) gives importance to computable criteria using LMIs, even in the presence of uncertainties. Chapter 5 presents the simpler case of systems with single state delay. Using the Lyapunov-Razumikhin (LR) theorem, sufficient stability conditions are given for both the delay-independent and the delay-dependent cases. The latter uses a well-known model transformation to get a delay free system plus distributed delays. However this induces some additional dynamics (the stability of which has to be checked) as well as some conservatism (in the delay margin exhibition). The application of the Lyapunov-Krasvokii (LK) theorem is developed to obtain results (for the delay independent and delay dependent cases) which are less restrictive than the LR's ones. However this is the price to pay for the application of LR's stability condition in the case of time-varying delays. It is then proved that the use of complete quadratic LK functionals can lead to necessary and sufficient condition for stability (but not numerically checkable). A discretization method (which loses necessity) is proposed by the authors to derive computable criteria. The interest of this approach is clearly to reduce conservatism, as, in particular when the system may be instable for a zero delay, it enables the stability analysis when the delay belongs to an interval $[\tau_{min},\ \tau_{max}]$.
The basis of the Time-Domain approach is presented very clearly here but the recent results on the LK approach mainly concern the authors' contribution (even if others are mentioned in the Notes section). A comparison with other approaches (for instance the descriptor one (Fridman, 2001) or the strong delay-independant stability one (Bliman, 2001)) would have been interesting.

Chapter 6 is devoted to the robust stability analysis. Uncertainties are considered on the system matrices (which belong to a compact set) mainly in polytopic or norm bounded forms. What may be surprising is that the case of time-varying delays is tackled here. The extensions of LR and LK's results are given in the general formulation of uncertainties but lead to an infinite number of matrix inequalities. In particular cases (polytopic or norm bounded uncertainties) the reduction to a finite number of LMIs is detailed.
Even though the presentation is not complete (due to the large number of contributions in this field), this brings out the key points to be considered when one aims at generalizing results to uncertain systems.

Chapter 7 deals with the extension of the previous results to systems with multiple and distributed delays. In particular the LK approach is used to obtain delay independent and delay dependent stability criteria. Then, following chapter 5, the existence of a complete quadratic LK function is proved to be a necessary and sufficient condition for stability of systems with multiple and distributed delays. The discretization method is then developed in both cases. The difficulty lies in the fact that the discretization mesh must be compatible with incommensurate delays, and then may be nonuniform. This procedure may be quite huge if a fine discretization mesh is required, which, on the presented simple examples, does not seem necessary.

My short remark on these three chapters of Part II is that one may be a bit confused when studying the case of time-varying delays. Indeed, while this takes place in in chapter 6, it is quite strange that, in this chapter, some results do concern constant delays as well. I would have preferred it if this was considered in a separate chapter or, at least, if comments and analysis were unified in another section.

Part III (chapter 8) concludes the book with Input-Output Stability analysis, mainly using the small gain theorem. First the method of comparison systems (embedding the time-delay system in a delay-free one with delay feedback) allows to get a sufficient condition of input-output stability in terms of LMIs, either using the bounded real lemma on the delay-free system with feedback uncertainty or using the LK approach using model transformation. Then a time-domain approach is used to solve a scaled small gain problem. An application of these theoretical studies concerns the approximation of time-varying or distributed delays, which is modelled as feedback uncertainty.
The interest of this chapter is to make the connection with the H_∞ approach, while remaining consistent with the objective of stability analysis only.

Finally Appendices A and B contains some useful facts on matrices and Linear Matrix Inequalities.

In conclusion, in spite of some above criticisms, I found this book well written and very interesting to read. It contains many results on stability analysis of time-delay systems and I strongly recommend it to researchers interested by analysis and control of such systems.

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