Architectures and Coder Design for Networked Control Systems ⋆

Graham C. Goodwin [a], Daniel E. Quevedo [a], Eduardo I. Silva [a]

[a] *School of Electrical Engineering & Computer Science, The University of Newcastle, NSW 2308, Australia*

Abstract

In Networked Control Systems (NCS's) achievable performance is limited by the communication links employed to transmit signals in the loop. In the present work, we characterise LTI coding systems which optimise performance for various NCS architectures. We study NCS's where the communication link is situated between plant output and controller, and NCS's where the communication link is located between controller and actuator. Furthermore, we present a novel NCS architecture, which is based upon the Youla parameterisation. We show that, which of these architectures gives best performance depends, *inter alia*, upon characteristics of a related non-networked design, plant disturbances and reference signal. A key aspect of our work, resides in the utilisation of fixed signal-to-noise ratio channel models which give rise to parsimonious designs, where channel utilisation is kept low. The results are verified with simulations utilising bit rate limited channels.

Key words: Networked control systems, noisy channels, coding schemes, minimum variance control, quantisation.

1 Introduction

In traditional control systems, one commonly assumes that the interconnection of plant and controller is *transparent*, i.e., transmitted signals are equal to received signals. This paradigm is often appropriate and underlies many successful control design methods, especially for linear time invariant (LTI) systems; see, e.g., [7]. However, in some situations, the assumption of transparent communication is not justified. Control systems where the communication link constitutes a bottleneck in achievable performance are commonly termed *Networked Control Systems* (NCS's); see, for example, articles contained in the special issue [1], and the survey papers [10, 29]. The communication link can either be dedicated or consist of a network which is shared between several users. Novel aspects introduced by the presence of non-transparent communication links in control include time delays, data-dropouts and quantisation, see also [11, 12, 21, 25]. From an analysis perspective, even basic system theoretic notions, such as closed loop stability are far from trivial in the networked control context; see, e.g., [4–6, 8, 14, 21, 26, 31].

When designing NCS's, the characteristics of the communication system should be explicitly taken account of to ensure acceptable performance levels. This raises new challenges. A key observation is that, in NCS's, there exist additional degrees of freedom in the design process, when compared to traditional control loops. As a consequence, to optimise performance, it is useful to investigate architectural issues and signal coding methods, see also [2, 3, 22, 28].

Several NCS architectures have been studied. One can distinguish configurations where the channel is located in the *up-link*, i.e., between sensors and controller input [2, 31], and where it lies in the *down-link*, i.e., between controller output and actuators [6]. More general architectures, where the processing power is distributed, have also been examined, for example, in [8, 16, 26, 28].

The goal of the present work is to compare various NCS architectures for the control of a single-input single-output (SISO) plant. We will consider a design situation, where an LTI controller is to be implemented in an NCS which employs a signal-to-noise ratio constrained communication channel. This type of channel model is commonly utilized in the signal processing literature, see, e.g., [13]. The controller is assumed to have already been

⋆ This paper was not presented at any IFAC meeting. Corresponding author G. C. Goodwin. Tel. +61-2-49217072. Fax +61-2-49601712.

Email addresses: graham.goodwin@newcastle.edu.au (Graham C. Goodwin), dquevedo@ieee.org (Daniel E. Quevedo), eduardo.silva@studentmail.newcastle.edu.au (Eduardo I. Silva).

designed for a given plant so as to achieve desired control objectives when the communication link is assumed to be transparent. However, when implementing this controller in an NCS, unavoidably performance losses occur. We will show how performance degradation, as measured by the variance of the tracking error, can be minimised through appropriate signal coding. As in other contemporary approaches to NCS design, see [4, 15, 32], we will employ design methodologies that utilise LTI system theoretic ideas. This will allow us to obtain results which quantify the achievable performance of various NCS architectures. The present paper extends work described in [9, 24].

The remainder of this work is organised as follows: In Section 2, we present the nominal control design which is to be implemented via an NCS and give details about the specific communication link to be utilised. Optimal coding for two different standard NCS architectures is studied in Section 3. In Section 4, we present a novel NCS architecture and show how to equip it with optimal coding systems. Section 5 documents a design study. Finally, conclusions are drawn in Section 6.

2 NCS Elements

2.1 Nominal design

We will examine a networked scenario, where the control loop is closed through a communication link. Specifically, we will consider the situation where an LTI SISO discrete time controller, say $C(z)$, has already been designed for a discrete time SISO LTI plant $G(z)$. This design has been carried out assuming that the plant is affected by output disturbances [1], d, that the output measurement is corrupted by noise, n, and that a given reference signal, r, should be tracked; see Figure 1. We will refer to this design as the *nominal design* and we will assume that it gives satisfactory performance when the communication links are transparent.

In the closed loop of Figure 1, the tracking error,

$$e \triangleq r - y, \tag{1}$$

is given by

$$e = S(z)\,(r - d) + T(z)n, \tag{2}$$

where $S(z) \triangleq (1+G(z)C(z))^{-1}$ and $T(z) \triangleq 1-S(z)$ are the loop sensitivity functions; see, e.g., [7]. The signals r, d and n are assumed to be independent stationary zero mean processes, with power spectral densities (PSD's) $\left|R(e^{j\omega})\right|^2$, $\left|D(e^{j\omega})\right|^2$ and $\left|N(e^{j\omega})\right|^2$, respectively.

In the particular case when $G(z)$ is stable, the nominal design can alternatively be realised in the form of the affine parameterisation of all stabilising controllers (*Youla parameterisation*) depicted in Figure 2; see, e.g., [7, 20]. In that figure, $\hat{G}(z)$ is an explicit model for the plant. The *Youla controller* $C_Y(z)$ in Figure 2 is related to $C(z)$ in Figure 1 via $C(z) = C_Y(z)(1-C_Y(z)\hat{G}(z))^{-1}$.

We will ignore differences between the plant transfer function $G(z)$ and the model $\hat{G}(z)$. As a consequence, the signal δ_0 in Figure 2 satisfies $\delta_0 = d + n$. Thus, δ_0 summarises essential information about the system to be controlled. Heuristically, this signal is a useful candidate to be transmitted through a communication link in an NCS, where channel utilisation is to be kept low. We will return to this Youla configuration later in Section 4.

2.2 The Communication Link

The novel ingredient in an NCS, when compared to a traditional control loop, is the communication link. From a design perspective, this opens the possibility of coding signals prior to transmission and also plays a key role in the achievable performance. Accordingly, we will consider a communication link consisting of a communica-

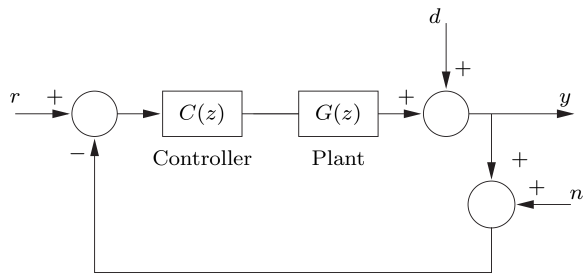

Fig. 1. Standard non-networked control system.

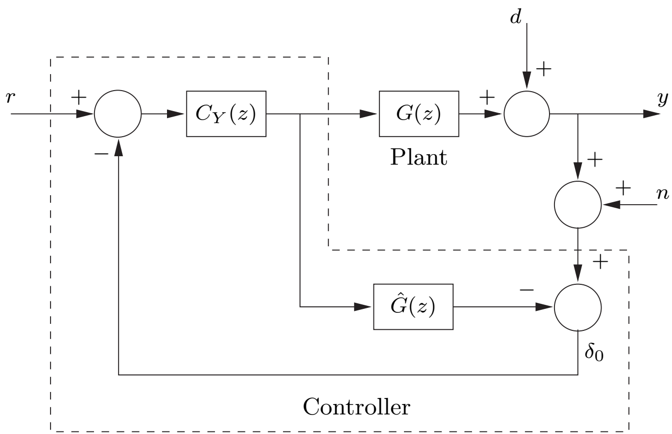

Fig. 2. Standard non-networked control system for stable plant: Youla form.

[1] We use vector space notation to denote signals. For example, y denotes $\{y(\ell)\}_{\ell\in\mathbb{N}}$. The symbol z is used to refer to both the argument of the z-transform and to the forward shift operator.

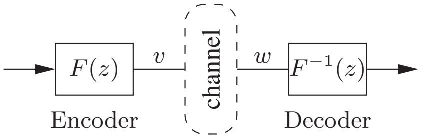

Fig. 3. Communication link.

tion channel together with an encoder-decoder pair, as shown in Figure 3.

We start by describing the channel model. There exist several ways of characterising a communication channel; see, e.g., [27,30]. We will focus on an additive signal-to-noise ratio constrained channel model, where the channel output w is related to the channel input v via

$$w = v + q, \tag{3}$$

and q is zero mean stationary discrete time random process, having PSD $\left|Q(e^{j\omega})\right|^2$ and variance

$$\Phi_q = \frac{1}{2\pi}\int_{-\pi}^{\pi} |Q(e^{j\omega})|^2 d\omega. \tag{4}$$

The signal-to-noise ratio constraint implies that Φ_q is related to the variance of v, Φ_v, by

$$\gamma = \frac{\Phi_v}{\Phi_q}, \tag{5}$$

where γ is the (finite) channel signal-to-noise ratio. Therefore, we can write

$$|Q(e^{j\omega})|^2 = \frac{\Phi_v}{\gamma}|Q_0(e^{j\omega})|^2, \tag{6}$$

where $Q_0(e^{j\omega})$ is such that $\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|Q_0(e^{j\omega})\right|^2 d\omega = 1$.

Many (source) coding schemes have been studied in the NCS literature; see, e.g., [2,3,19,23,28,31]. These methods vary in complexity and on the assumptions made regarding the information available to the encoder and decoder. In the present work, we concentrate on LTI encoder-decoders pairs that have access only to local information; see, e.g., [13]. In order not to modify the nominal design relations (as discussed in Section 2.1), we restrict the encoder-decoder pair to achieve *perfect reconstruction*, i.e., we assume that

$$F(z)F^{-1}(z) = 1 \tag{7}$$

(see Figure 3). To avoid unstable pole zero cancellations and non-causality, we restrict the encoder $F(z)$ to be stable, minimum phase and biproper. The design of the encoder-decoder pairs, i.e., of $F(z)$ and $F^{-1}(z)$ in Figure 3, will form a central theme in the subsequent analysis.

3 Optimal Coding in Standard NCS Architectures

This section examines the two most common single-channel networked control architectures, namely, where the channel is located in the down-link and where it is located in the up-link. These architectures are depicted in Figures 4 and 5, respectively.

3.1 Optimal coding for the down-link case

In the architecture of Figure 4 it holds that the tracking error (see (1)) is given by

$$e = S(z)\,(r-d) + T(z)n - S(z)F_D(z)^{-1}G(z)q,$$

where we have used the additive noise channel in (3). Using (2), we note that the variance of the component of the tracking error that arises from the communication link is given by

$$J = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|S(e^{j\omega})F_D^{-1}(e^{j\omega})G(e^{j\omega})Q(e^{j\omega})\right|^2 d\omega. \tag{8}$$

The above expression serves to quantify the impact of the communication link on the performance of the NCS. It depends upon the encoder-decoder pair used.

Based on (8), one might be tempted to simply set $F_D^{-1}(e^{j\omega}) \approx 0$, for all frequencies ω. This, however, would lead to a decoder such that $F_D(e^{j\omega}) \to \infty$, $\forall\omega$ and hence, to an unbounded signal v. This is clearly unacceptable. Thus, encoder-decoder design has to be carried out more carefully.

A key point is that Φ_q is related to Φ_v via (5). Indeed, from Figure 4, it follows that

$$\begin{aligned}\Phi_v &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|T(e^{j\omega})Q(e^{j\omega})\right|^2 d\omega \\ &\quad + \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|S(e^{j\omega})C(e^{j\omega})F_D(e^{j\omega})\Omega_D(e^{j\omega})\right|^2 d\omega,\end{aligned} \tag{9}$$

where

$$|\Omega_D(e^{j\omega})|^2 \triangleq |D(e^{j\omega})|^2 + |N(e^{j\omega})|^2 + |R(e^{j\omega})|^2 \tag{10}$$

is the PSD of $d+n+r$.

Substituting (6) into (9) gives

$$\Phi_v = \frac{\alpha\,\gamma}{2\pi}\int_{-\pi}^{\pi}\left|S(e^{j\omega})C(e^{j\omega})F_D(e^{j\omega})\Omega_D(e^{j\omega})\right|^2 d\omega, \tag{11}$$

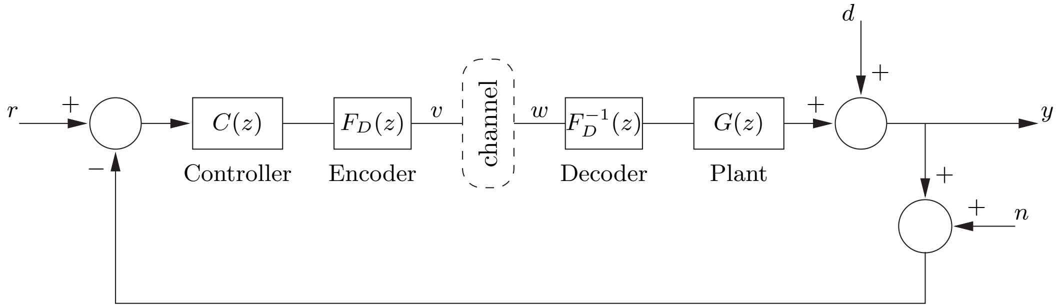

Fig. 4. Channel in the down-link.

where

$$\alpha \triangleq \left(\gamma - \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|T(e^{j\omega})Q_0(e^{j\omega})\right|^2 d\omega\right)^{-1}. \qquad (12)$$

Using (6), (8) and (11), it follows that J in (8) satisfies

$$J = \frac{\alpha}{2\pi}\int_{-\pi}^{\pi} \left|S(e^{j\omega})F_D^{-1}(e^{j\omega})G(e^{j\omega})Q_0(e^{j\omega})\right|^2 d\omega \cdot \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|S(e^{j\omega})C(e^{j\omega})F_D(e^{j\omega})\Omega_D(e^{j\omega})\right|^2 d\omega. \qquad (13)$$

Remark 1 (Bound on γ) *In (13), J is the the variance of the effect that the communication link has on the tracking error. Therefore, α in (12) has to be positive. This imposes a constraint on γ, namely $\gamma > (2\pi)^{-1}\int_{-\pi}^{\pi}\left|T(e^{j\omega})Q_0(e^{j\omega})\right|^2 d\omega$. If γ does not satisfy this constraint, then the fixed signal to noise ratio model will not hold, and instability may occur. This observation is consistent with the results reported in [6]. (Note that the results in [6] are based on a channel model with fixed noise variance, rather than fixed signal-to-noise ratio, as considered here.)*

The following theorem characterises optimal encoder-decoder pairs for the NCS in Figure 4.

Theorem 1 (Down-link Channel [24]) *Consider the NCS depicted in Figure 4 and the loss function J in (13). Then $J \geq J_D^{opt}$, where*

$$J_D^{opt} \triangleq \alpha\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|S(e^{j\omega})T(e^{j\omega})\Omega_D(e^{j\omega})Q_0(e^{j\omega})\right| d\omega\right)^2 \qquad (14)$$

and α is defined in (12). This performance bound is tight and can be achieved by all encoders which satisfy

$$|F_D(e^{j\omega})|^2 = k_D\left|\frac{G(e^{j\omega})Q_0(e^{j\omega})}{C(e^{j\omega})\Omega_D(e^{j\omega})}\right|, \ \forall\omega \in [-\pi,\ \pi], \qquad (15)$$

where $k_D > 0$ is any positive (fixed) real number.

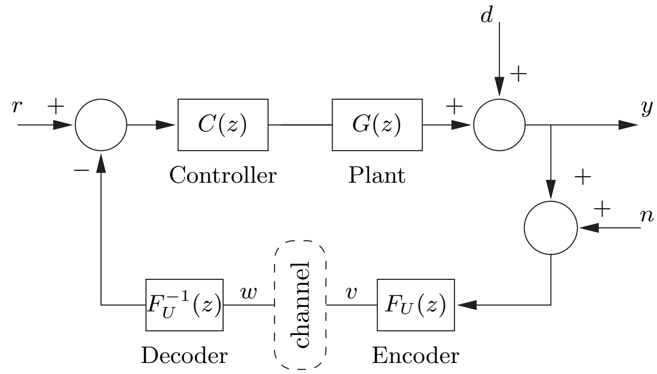

Fig. 5. Channel in the up-link.

Proof: Using the Cauchy-Schwartz inequality in (13) it follows that $J \geq J_D^{opt}$, and that equality is achieved if and only if there exists k_D constant and positive such that

$$\left|S(e^{j\omega})F_D^{-1}(e^{j\omega})G(e^{j\omega})Q_0(e^{j\omega})\right| = k_D\left|S(e^{j\omega})C(e^{j\omega})F_D(e^{j\omega})\Omega_D(e^{j\omega})\right|,$$

for all ω, i.e., if and only if (15) holds.

□□□

The class of coding systems specified via (15) achieves the best trade-off between channel utilisation and the effect of channel noise on loop performance, as measured by the variance of the tracking error. We will denote any coder $F_D(z)$ that satisfies (15) by $F_D^{opt}(z)$.

Since (15) is a restriction on the *magnitude* of $F_D^{opt}(e^{j\omega})$, one can always resort to appropriate all-pass filters to guarantee that $F_D^{opt}(z)$ is biproper, stable and minimum phase (recall Section 2.2).

Remark 2 (Parsimony of the channel model) *We can see that by using a fixed signal-to-noise ratio channel model, optimal coders do not have an infinite gain over all frequencies (see also [17]). In contrast, if the channel noise variance (rather than the signal-to-noise ratio) were to be fixed (as in [6]), then setting $F_D^{-1}(e^{j\omega}) \approx 0$, $\forall\omega$ (so that v becomes unbounded) would minimise the* *channel induced plant output variance. In such cases, for a parsimonious design which only incurs limited channel utilisation a different loss function should be used (see, e.g., [23]).*

3.2 Optimal coding for the up-link case

We next investigate optimal encoder-decoder pairs for the alternative architecture depicted in Figure 5, where the communication system is located in the up-link.

As before, we define the tracking error variance due to channel effects as J. Proceeding as in the down-link case analysed in Section 3.1, we conclude that in the NCS of Figure 5

$$J = \frac{\alpha}{2\pi}\int_{-\pi}^{\pi}\left|F_U^{-1}(e^{j\omega})T(e^{j\omega})Q_0(e^{j\omega})\right|^2 d\omega \cdot \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|S(e^{j\omega})F_U(e^{j\omega})\Omega_U(e^{j\omega})\right|^2 d\omega, \quad (16)$$

where α is as in (12) and

$$|\Omega_U(e^{j\omega})|^2 \triangleq |D(e^{j\omega})|^2 + |N(e^{j\omega})|^2 + |G(e^{j\omega})C(e^{j\omega})R(e^{j\omega})|^2 \quad (17)$$

is the PSD of $d + n + G(z)C(z)r$; compare to (10).

As in the down-link architecture, when designing the coding system, there exists a trade-off between channel noise attenuation and channel utilisation. Optimal performance can be attained by means of the following theorem:

Theorem 2 (Up-link Channel) *Consider the NCS depicted in Figure 5 and the loss function J in* (16)*. Then $J \geq J_U^{opt}$, where*

$$J_U^{opt} \triangleq \alpha\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|S(e^{j\omega})T(e^{j\omega})\Omega_U(e^{j\omega})Q_0(e^{j\omega})\right| d\omega\right)^2 \quad (18)$$

and α is defined in (12)*. This bound is tight and is attained by all encoders which satisfy*

$$|F_U(e^{j\omega})|^2 = k_U\left|\frac{G(e^{j\omega})C(e^{j\omega})Q_0(e^{j\omega})}{\Omega_U(e^{j\omega})}\right|, \ \forall\omega \in [-\pi,\ \pi], \quad (19)$$

where k_U is any (fixed) positive real number.

Proof: Similar to the proof of Theorem 1. □□□

We will denote any coder that satisfies (19) by $F_U^{opt}(z)$. As in the down-link case, all-pass filters can be used to impose additional properties on $F_U^{opt}(z)$.

3.3 Performance Comparison

For the two architectures examined so far, the only difference in the optimal performance resides in the terms $\Omega_D(e^{j\omega})$ and $\Omega_U(e^{j\omega})$ (see (14) and (18)). Further insight can be gained by analysing three different scenarios:

3.3.1 Disturbance rejection

Assume that the reference is zero. Comparison of (10) and (17) shows that, if $r = 0$, then $\Omega_D(e^{j\omega}) = \Omega_U(e^{j\omega})$ $\forall\omega \in [-\pi,\ \pi]$. Thus, in a regulation loop, both architectures, *when equipped with optimal coder-decoder pairs*, attain the same performance. It is interesting to note that, in this situation, (15) and (19) suggest choosing $F_U^{opt}(z) = C(z)F_D^{opt}(z)$. With this choice, *placing the channel in the down-link is algebraically equivalent to placing it in the up-link.*

3.3.2 Cancelling nominal design

Here we consider a special case where the controller cancels (stable) poles and zeros of the plant model [2]. We also assume that $n = 0$, $r = 0$, and that the channel noise is white, i.e., $|Q_0(e^{j\omega})| = 1$, $\forall\omega \in [-\pi, \pi]$. It then follows from (19) that, if there exists $\beta \in \mathbb{R}$ such that $|D(e^{j\omega})| = \beta|G(e^{j\omega})C(e^{j\omega})|$, $\forall\omega \in [-\pi,\ \pi]$, then the optimal encoder in the architecture represented in Figure 5 can be chosen as $F_U^{opt}(z) = 1$ (see also [24]). Thus, in this case, it is optimal to send the measured plant output without any coding. Stated differently, for the simple case under study, if uncoded signals are used, and if the physical constraints allow this to be done, then it is preferable to *place the communication link in the up-link rather than in the down-link.* (See [24] for an experimental validation of this observation).

3.3.3 Non-zero references

With non-zero reference signals, the two NCS architectures studied so far will, in general, give different performance. In particular, if $d = n = 0$, then the achievable loss functions in (14) and (18) are lower bounded by

$$J_D^{opt} = \alpha\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|S(e^{j\omega})T(e^{j\omega})Q_0(e^{j\omega})R(e^{j\omega})\right| d\omega\right)^2,$$
$$J_U^{opt} = \alpha\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|(T(e^{j\omega}))^2 Q_0(e^{j\omega})R(e^{j\omega})\right| d\omega\right)^2. \quad (20)$$

To investigate this situation further, it is worthwhile recalling that common design strategies will ensure that

[2] This is the case of, for example, simple internal model based controllers, see [20].

$T(e^{j\omega}) \approx 1$ ($\Leftrightarrow S(e^{j\omega}) \approx 0$) at those frequencies where $R(e^{j\omega})$ is significant, and that $|T(e^{j\omega})|$ is small at other frequencies [7]. As a consequence, we would anticipate, by examining (20), that $J_D^{opt} < J_U^{opt}$. We conclude that, for non-zero reference signals (and optimal coding), the architecture of Figure 4 generally outperforms that of Figure 5, i.e., if possible *the channel should be placed in the down-link.*

In the following section we will propose a novel NCS architecture, which, when equipped with optimal coder-decoder pairs, will often give enhanced performance relative to the two architectures analysed so far.

Remark 3 (Two channel architectures) *To some extent, our results can be applied to NCS architectures with communication channels in both the up-link and down-link. Indeed, if signal-to-noise ratios of these channels are sufficiently large, then the design of each coder-decoder pair can be based on the channel noise model in each respective link and expressions* (15) *and* (19).

4 Networked Youla Architecture for Stable Plants

We will use the form of the Youla parameterisation described in Section 2.1 to develop an alternative parsimonious NCS architecture for stable [3] plants. A key observation here is that the signal δ_0 in Figure 2 will often have small energy relative to other signals, especially if the reference signal dominates. The scheme depicted in Figure 6, which is an embellishment of that proposed in [9], puts this idea into practice. As stated before, we will neglect differences between the true plant $G(z)$ and the model $\hat{G}(z)$.

4.1 Key Aspects

In the networked Youla architecture proposed, processing takes place at two locations (see Figure 6). Firstly, at the receiving side of the communication link, the Youla controller $C_Y(z)$ calculates the plant input signal. Secondly, at the plant output side, we have included a feedback loop, which incorporates a model of the signal path between communication link input, i.e., δ, and the (unperturbed) plant output (compare to the basic Youla configuration in Figure 2). This inner loop uses a model of the communication channel and assumes perfect knowledge of the reference signal [4]. The information sent through the communication link is obtained by encoding the signal δ, which is given by

$$\delta = n + d + T(z)F_Y^{-1}(z)\sigma, \tag{21}$$

where $\sigma \triangleq \hat{w} - w$ and $\hat{w}$ is the output of the channel model, i.e., $\hat{w} \approx w$. The signal σ takes into account inaccuracies in the channel model.

The signal δ summarises essential information regarding the system to be controlled. As a consequence, we can expect that, if channel utilisation is to be kept low, then the NCS of Figure 6 will often outperform the more traditional architectures described in Section 3.

Resembling the modelling of channel noise, we will adopt a finite signal-to-noise ratio model for σ, i.e., we will describe σ as a zero mean exogenous stationary process with PSD $\left|\Sigma(e^{j\omega})\right|^2$ and variance $\Phi_\sigma = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\Sigma(e^{j\omega})\right|^2 d\omega$. This variance depends upon the variance of the channel input v via $\Phi_\sigma = \lambda^{-1}\Phi_v$, where $\lambda > 0$ is fixed and finite. As in Section 2.2, we will write

$$|\Sigma(e^{j\omega})|^2 = \frac{\Phi_v}{\lambda}|\Sigma_0(e^{j\omega})|^2, \tag{22}$$

where $\Sigma_0(e^{j\omega})$ is such that $\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\Sigma_0(e^{j\omega})\right|^2 d\omega = 1$.

Remark 4 *Since σ depends only on possible differences between the channel and its model, its variance can be expected to be much smaller than that of the channel noise, i.e., $\Phi_\sigma < \Phi_q$ (see Section 2.2). Under these conditions, it follows that $\lambda > \gamma$. Accordingly, we define:*

$$\epsilon \triangleq \frac{\gamma}{\lambda} \tag{23}$$

and assume that $0 \leq \epsilon < 1$. A perfect channel model corresponds to $\epsilon = 0$.

A key point is that deterministic channel artifacts can generally be accurately incorporated into the channel model. Thus, only random channel artifacts need to be modelled via σ.

4.2 Optimal Coder Design

From Figure 6 and Equation (21), it follows that the channel input is given by

$$v = F_Y(z)(d+n) + T(z)\sigma,$$

so that, by using (22) and (23), the variance of v satisfies

$$\Phi_v = \frac{\gamma\int_{-\pi}^{\pi}\left|F_Y(e^{j\omega})\Omega_Y(e^{j\omega})\right|^2 d\omega}{2\pi\,\gamma - \epsilon\int_{-\pi}^{\pi}\left|T(e^{j\omega})\Sigma_0(e^{j\omega})\right|^2 d\omega} \tag{24}$$

where

$$|\Omega_Y(e^{j\omega})|^2 \triangleq |D(e^{j\omega})|^2 + |N(e^{j\omega})|^2; \tag{25}$$

[3] This restriction was not needed in the results of Section 3.
[4] Of course, this assumption does limit applicability of the architecture in Figure 6.

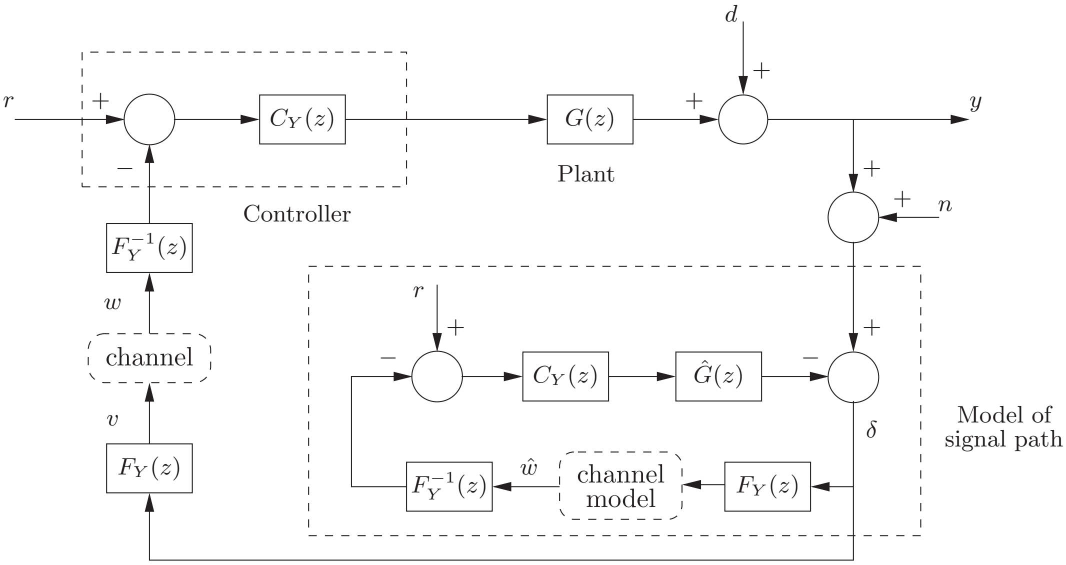

Fig. 6. Networked Youla architecture.

(compare to (10) and (17)).

In contrast to the architectures examined in Section 3, the reference signal r and the channel noise q do not contribute to the variance of v. Indeed, only the plant output disturbance d, measurement noise n and the channel model mismatch σ invoke channel utilisation. This is due to the cancelling effect of the Youla structure.

The tracking error in Figure 6 satisfies

$$e = S(z)\,(r-d) + T(z)n + T(z)F_Y^{-1}(z)q + (T(z))^2 F_Y^{-1}(z)\sigma.$$

Thus, the tracking error variance due to network effects is given by

$$J = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|T(e^{j\omega})F_Y^{-1}(e^{j\omega})Q(e^{j\omega})\right|^2 d\omega + \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|(T(e^{j\omega}))^2 F_Y^{-1}(e^{j\omega})\Sigma(e^{j\omega})\right|^2 d\omega. \quad (26)$$

Substitution of (6), (22) and (24) into (26) yields

$$J = \frac{\int_{-\pi}^{\pi}\left|F_Y(e^{j\omega})\Omega_Y(e^{j\omega})\right|^2 d\omega}{2\pi\,\gamma - \epsilon\int_{-\pi}^{\pi}\left|T(e^{j\omega})\Sigma_0(e^{j\omega})\right|^2 d\omega} \cdot \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\Gamma(e^{j\omega})T(e^{j\omega})F_Y^{-1}(e^{j\omega})\right|^2 d\omega, \quad (27)$$

where

$$|\Gamma(e^{j\omega})|^2 \triangleq |Q_0(e^{j\omega})|^2 + \epsilon|T(e^{j\omega})\Sigma_0(e^{j\omega})|^2. \quad (28)$$

It is interesting to observe that in (27) the channel model mismatch description $\Sigma_0(e^{j\omega})$ appears multiplying $T(e^{j\omega})$. If the latter is low-pass, then the networked Youla architecture will be robust with respect to differences between the channel and its model, provided that they are concentrated at high frequencies. From this viewpoint, the channel mismatch assumes a role which is akin to that of measurement noise in a non-networked situation.

We can now characterise optimising coder-decoder pairs proceeding as in previous sections of this paper:

Theorem 3 (Networked Youla Architecture) *Consider the networked Youla architecture depicted in Figure 6 and the loss function J in* (27)*. Then $J \geq J_Y^{opt}$, where*

$$J_Y^{opt} \triangleq \frac{\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}|\Gamma(e^{j\omega})T(e^{j\omega})\Omega_Y(e^{j\omega})|d\omega\right)^2}{\gamma - \frac{\epsilon}{2\pi}\int_{-\pi}^{\pi}\left|T(e^{j\omega})\Sigma_0(e^{j\omega})\right|^2 d\omega}. \quad (29)$$

This bound is tight and is achieved by all encoders $F_Y(z)$ that satisfy

$$|F_Y(e^{j\omega})|^2 = k_Y\left|\frac{\Gamma(e^{j\omega})T(e^{j\omega})}{\Omega_Y(e^{j\omega})}\right|, \ \forall\omega\in[-\pi,\ \pi], \quad (30)$$

where k_Y is any (fixed) positive real number.

Proof: Similar to the proof of Theorem 1

□□□

As in the results included in Section 3, the optimal coder, say $F_Y^{opt}(z)$, is parsimonious and depends upon closed loop properties. Also, all-pass filters can be used to ensure that $F_Y^{opt}(z)$ is proper, stable and minimum phase.

Remark 5 *It is interesting to examine the special case where $n = 0$, there is no channel mismatch ($\epsilon = 0$), q is white, and d has a PSD such that $|D(e^{j\omega})| = |T(e^{j\omega})|$. In this case, it follows from* (25), (28) *and* (30), *that the optimal coder can be chosen as $F_Y^{opt}(z) = 1$. Thus, in this situation, it is optimal to send δ through the communication channel without any prior coding (compare to the case studied in Section 3.3.2).*

4.3 Comparison to Standard NCS Architectures

As already noted, a key property of the networked Youla architecture is that the signal transmitted, namely $v = F_Y(z)\delta$, does not depend upon the reference signal. We therefore divide our subsequent comparative analysis according to the reference signal energy:

4.3.1 Large reference signal

Our first, and most important, observation is that, if r has significantly larger energy than d and n, then the *networked Youla configuration will, in general, give better performance* than the architectures of Figures 4 and 5. This is easy to see from (29) by considering the extreme case where $|N(e^{j\omega})| = |D(e^{j\omega})| = 0$ for all ω, so that $J_Y^{opt} = 0$. In contrast, under the same conditions, J_D^{opt} and J_U^{opt} will, in general, be positive (see (14) and (18)).

4.3.2 Small reference signal

The situation is different when the reference signal is small in comparison with the disturbance. Indeed, in a well designed control loop, the sensitivity $S(e^{j\omega})$ will typically have small magnitude at frequencies where $D(e^{j\omega})$ is significative. Thus,

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |T(e^{j\omega})Q_0(e^{j\omega})S(e^{j\omega})D(e^{j\omega})|d\omega < \frac{1}{2\pi}\int_{-\pi}^{\pi} |T(e^{j\omega})Q_0(e^{j\omega})D(e^{j\omega})|d\omega.$$

Inspection of (14), (18) and (29) allows one to conclude that in this case, and provided that γ is large, *placing the channel either in the down-link or in the up-link gives better performance than the networked Youla architecture.*

5 Design Study

We next illustrate the performance of the different architectures considered in this paper via a simulation study for NCS's with bit rate limited channels.

5.1 Simulation setup

The continuous time plant model is given by $G_o(s) = 2(5s + 1)^{-1}$. We choose a sampling period of $1[s]$ and use a zero order hold at the plant input, giving the discrete time model $G(z) = 0.36254(z - 0.8187)^{-1}$. References and disturbances are taken as random walks. To model such signals, we define the process ψ with PSD satisfying $\left|\Psi(e^{j\omega})\right| = \left|0.02(e^{j\omega} - 1)^{-1}\right|$. We choose the PI controller $C(z) = 2.4488(z - 0.4871)(z - 1)^{-1}$.

For each NCS architecture, we model the channel as being bit-rate limited, achieving error-free transmission, but at a limited data-rate. To force the bit-rate limitation, the channel input is passed through a b-bit uniform quantiser designed following the well-known "$4\times$ standard deviation" rule (see Ch. 4 in [13]) [5] . This design rule allows one to obtain an appropriate tradeoff between granular and overload quantization errors. We also note that, since we have chosen a sampling interval of 1[s], b equals the channel bit rate in $[bit/s]$.

For the purpose of coder design, we will approximate [6] the channel errors (namely, quantisation errors) as a zero-mean white noise sequence [13, 18, 32].

5.2 Results

Simulations were conducted for the three NCS architectures discussed earlier. In all simulations n was taken to be zero, whilst d and r are considered either zero or modelled by ψ. In the networked Youla architecture, the channel model corresponds to the real channel, i.e., $\epsilon = 0$. In addition, the channel bit rate is varied between $b = 2[bit/s]$ and $b = 8[bit/s]$. The extreme case of $b = 1[bit/s]$ was also considered, but the results are not shown. In that case, the noise model for quantisation breaks down and the quantitative predictions made by the model are poor. However, the same qualitative results hold.

5.2.1 The benefits of optimal coding

We will restrict attention to the networked down-link case. Similar results and conclusions apply to the other architectures.

Figure 7 shows the *empirical* [7] sample variance of e, considering both an optimally-coded and uncoded down-

[5] To design the quantiser, we model the standard deviation of the signal to be quantised as the corresponding standard deviation when no quantisation is present.

[6] We stress that the simulations use a quantized input channel and that the fixed signal-to-noise ratio channel model is used only for coder/decoder design.

[7] In the sequel, the terms "empirical" and "measured" refer to calculations made using the quantised simulation results. All simulations results are based on 10^4 time samples.

link, for a regulation loop where $r = 0$ and $d = \psi$. The figure clearly shows that networked closed loop performance can be significantly (up to an order of magnitude) improved through coding. It can be seen that the benefits of coding are significant for bit rates up to $5[bit/s]$. For higher bit rates, the performance of the networked down-link loop (with and without coding) is almost identical to that of the nominal non-networked design. This is a consequence of the associated high signal-to-noise ratio and hence the very small value of α in (13).

To study the problem further, we will next examine the analytical expressions for the optimal performance predicted by the approximate channel noise model, i.e., J_D^{opt} (see (14)). In Figure 8 we compare these results with the empirical variance of the effects of the channel on the regulation error, when optimal coding is used. It can be seen that, in this case, the theoretical results match the measurements closely. Furthermore, a study of the quantisation noise PSD revealed that, at least for the cases considered, the white quantisation noise assumption is remarkably realistic in all cases. It gives accurate

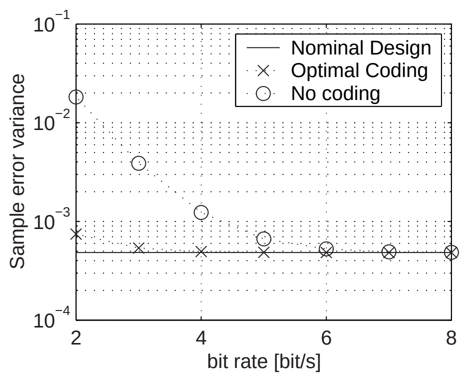

Fig. 7. Comparison between optimally coded and non-coded networked down-link with $r = 0$ and $d = \psi$.

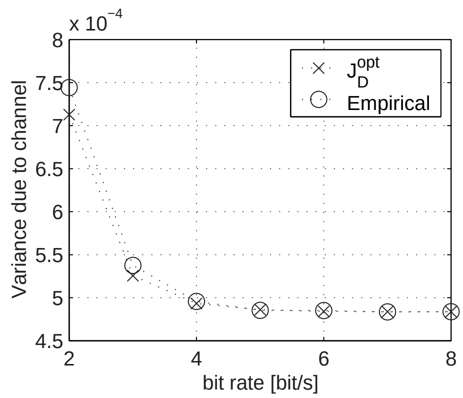

Fig. 8. Comparison between the analytical performance index values and the empirical values with $r = 0$ and $d = \psi$ for the networked down-link (considering optimal coding).

predictions in the networked down-link architecture and in the up-link case. It also provides qualitatively meaningful predictions in the case of the Youla architecture, but with slightly lower quantitative accuracy.

5.2.2 Comparison between NCS architectures

We next present a comparison between the different architectures. In particular, we will investigate the statements made in Sections 3.3 and 4.3 for the empirical results concerning bit rate limited NCS's.

- *Zero disturbance:* As stated in Sections 3.3.3 and 4.3.1, in this case the approximate noise model analysis predicts that the optimally coded down-link architecture should perform better than the optimally coded up-link architecture. Furthermore, the Youla architecture should provide a performance identical to that of the nominal design (recall that we consider $\epsilon = 0$). This is confirmed by the empirical results shown in Figure 9.
- *Zero reference:* As predicted by the approximate channel noise model in Section 3.3.1, if no reference is present and optimal coding systems are used, then placing the channel in the down- or in the up-link should give the same performance. However, if one chooses to use the Youla architecture, then the performance should be worse (recall Section 4.3.2). These assertions are confirmed in Figure 10. We also see that, as stated earlier, a sufficiently large bit-rate leads to NCS's that perform very close to the nominal non-networked design.

6 Conclusions

This paper has studied the role played by architectures and signal coding on the performance of NCS's. We have developed analytic expressions which quantify the impact of the communication link on overall performance, and we have utilised these expressions to design optimal LTI source coding schemes. The predictions made by the fixed signal-to-noise ratio channel model have been verified by empirical simulations using a bit-rate limited channel.

Open problems in this area include the MIMO case. In that setting, decentralized control and/or network architectures will certainly play a major role. Other extensions lie in the consideration of more sophisticated quantization schemes using noise shaping or predictive coding ideas (see, e.g., [13]).

References

[1] J. Baillieul and P. Antsaklis. Control and communication challenges in networked real-time systems. *Proc. IEEE*, 95(1):9–27, January 2007.

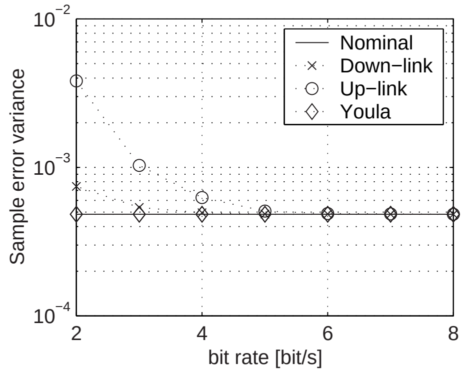

Fig. 9. Sample error variance considering optimal coding in each architecture with $d = 0$ and $r = \psi$.

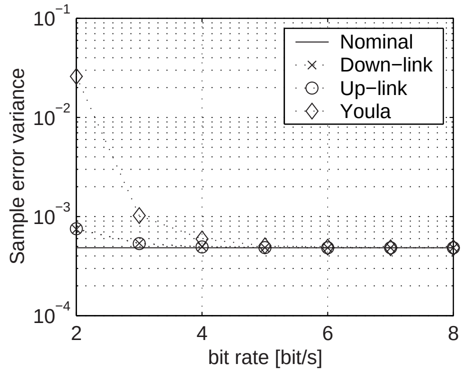

Fig. 10. Sample error variance considering optimal coding in each architecture with $r = 0$ and $d = \psi$.

[2] C. De Perisi and A. Isidori. Stabilizability by state feedback implies stabilizability by encoded state feedback. *Syst. & Contr. Lett.*, 53:249–258, 2004.

[3] C. Canudas de Wit, F. Rodríguez, J. Fornés, and F. Gómez-Estern. Differential coding in networked controlled linear systems. In *Proc. Amer. Contr. Conf.*, Minneapolis, MN, 2006.

[4] N. Elia. Remote stabilization over fading channels. *Syst. & Contr. Lett.*, pages 237–249, 2005.

[5] F. Fagnani and S. Zampieri. Quantized stabilization versus performance. *IEEE Trans. Automat. Contr.*, 44(9):1534–1548, September 2004.

[6] J. Freudenberg, J. Braslavsky, and R. Middleton. Control over signal-to-noise ratio constrained channels: Stabilization and performance. In *Proc. IEEE Conf. Decis. Contr.*, pages 191–196, 2005.

[7] G. C. Goodwin, S. F. Graebe, and M. E. Salgado. *Control System Design.* Prentice-Hall, 2001.

[8] G. C. Goodwin, H. Haimovich, D. E. Quevedo, and J. S. Welsh. A moving horizon approach to networked control system design. *IEEE Trans. Automat. Contr.*, 49(9):1427–1445, September 2004.

[9] G. C. Goodwin, D. E. Quevedo, and E. I. Silva. Filter banks in networked control. In *Proc. 17th Int. Symp. Mathematical Theory of Networks and Systems*, Kyoto, Japan, 2006.

[10] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu. A survey of recent results in networked control systems. *Proc. IEEE*, 95(1):138–162, January 2007.

[11] D. Hristu-Varsakelis and W. Levine (Eds.). *Handbook of Networked and Embedded Systems.* Birkhäuser, Boston, 2005.

[12] H. Ishii and B. A. Francis. *Limited Data Rate in Control Systems with Networks.* Springer, 2002.

[13] N. Jayant and P. Noll. *Digital Coding of Waveforms.* Prentice Hall, Englewood Cliffs, NJ, 1984.

[14] K. Li and J. Baillieul. Robust quantization for digital finite communication bandwidth (DFCB) control. *IEEE Trans. Automat. Contr.*, 49(9):1573–1584, September 2004.

[15] F. Lian, J. Moyne, and D. Tilbury. Modelling and optimal controller design of networked control systems with mutliple delays. *Int. J. Contr.*, 76(6):591–606, 2003.

[16] Q. Ling and M. D. Lemmon. Power spectral analysis of networked control systems with data dropouts. *IEEE Trans. Automat. Contr.*, 49(6):955–960, June 2004.

[17] J. Lu and R. E. Skelton. Integrating instrumentation and control design. *Int. J. Contr.*, 72(9):799–814, June 1999.

[18] D. Marco and D. L. Neuhoff. The validity of the additive noise model for uniform scalar quantizers. *IEEE Trans. Inform. Theory*, 51(5):1739–1755, May 2005.

[19] A. S. Matveev and A. V. Savkin. Multirate stabilization of linear multiple sensor systems via limited capacity communication channels. *SIAM Journal on Control and Optimization*, 44(2):584–617, 2005.

[20] M. Morari and E. Zafiriou. *Robust process control.* Prentice Hall Inc., Englewood Cliffs, New Jersey 07632, 1989.

[21] G. N. Nair, F. Fagnani, S. Zampieri, and R. J. Evans. Feedback control under data rate constraints: An overview. *Proc. IEEE*, 95(1):108–137, January 2007.

[22] D. E. Quevedo and G. C. Goodwin. An improved architecture for networked control systems. In *Proc. 16th IFAC World Congress, Prague, Czech Republic*, 2005.

[23] D. E. Quevedo, G. C. Goodwin, and J. S. Welsh. Design issues arising in a networked control system architecture. In *Proc. IEEE Conf. Contr. Appl.*, Taipei, Taiwan, 2004.

[24] D. E. Quevedo, J. S. Welsh, G. C. Goodwin, and M. McLeod. Networked PID control. In *Proc. IEEE Conf. Contr. Appl.*, Munich, Germany, 2006.

[25] L. Schenato, B. Sinopoli, M. Franceschetti, K. Poolla, and S. S. Sastry. Foundations of control and estimation over lossy networks. *Proc. IEEE*, 95(1):163–187, January 2007.

[26] P. L. Tang and C. W. de Silva. Compensation for transmission delays in an ethernet-based control network using variable-horizon predictive control. *IEEE Trans. Contr. Syst. Technol.*, 14(4):707–718, July 2006.

[27] S. Tatikonda and S. Mitter. Control over noisy channels. *IEEE Trans. Automat. Contr.*, 49(7):1196–1201, July 2004.

[28] S. Tatikonda and S. Mitter. Control under communication constraints. *IEEE Trans. Automat. Contr.*, 49(7):1056–1068, July 2004.

[29] Y. Tipsuwan and M.-Y. Chow. Control methodologies in networked control systems. *Contr. Eng. Pract.*, 11:1099–1111, 2003.

[30] D. Tse and P. Viswanath. *Fundamentals of Wireless Communication.* Cambridge University Press, UK, 2005.

[31] W. S. Wong and R. W. Brocket. Systems with finite communication bandwidth constraints–Part II: Stabilization with limited information feedback. *IEEE Trans. Automat. Contr.*, 44(5):1049–1053, 1999.

[32] L. Xiao, M. Johansson, H. Hindi, S. Boyd, and A. Goldsmith. Joint optimization of communication rates and linear systems. *IEEE Trans. Automat. Contr.*, 48(1):148–153, 2003.