Automatica 46 (2010) 222–225

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Technical communique

Gain-scheduled, model-based anti-windup for LPV systems ☆

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ARTICLE INFO

Article history:
Received 9 June 2009
Received in revised form
16 June 2009
Accepted 24 September 2009
Available online 17 October 2009

Keywords:
Anti-windup
Saturation
Robustness
Gain scheduling
Linear parameter varying (LPV) systems

ABSTRACT

The aim of this paper is to show that a recently proposed technique for anti-windup control of exponentially unstable plants can be easily extended to solve the corresponding robust anti-windup problem for linear parameter varying systems, for which the time varying parameters are measured online. The proposed technique is minimally conservative with respect to the size of the resulting operating region (which coincides, up to an arbitrarily small quantity, with the largest set on which asymptotic stability can be guaranteed for the considered plant with the given saturation level and uncertainty characteristics), and is not limited to plants having only small uncertainties or being open-loop stable.

1. Introduction

Input saturation and plant uncertainty are two ubiquitous phenomena in any real design problem. Hence, anti-windup compensation has been proposed to counteract saturation nonlinearities (see *e.g.* the surveys (Åström & Rundqwist, 1989; Hanus, 1988) for background) and robust control to counteract plant uncertainty. However, as pointed out in Turner, Herrmann and Postlethwaite (2007), there is a surprising lack of literature about the study of robustness limitations specifically arising in anti-windup control systems, as well as about the problem of designing anti-windup compensators ensuring robust-in-the-large stability (i.e. for any uncertainty in an *a priori assigned*, possibly "large", set of uncertainties, so that small gain arguments cannot be easily invoked).

Even in the absence of uncertainty, anti-windup compensation for exponentially unstable plants is challenging, since the bounds on the input imply that the null controllable region is bounded, and then in order to achieve stability the anti-windup compensator must keep the state of the plant inside the null controllable region. Anti-windup designs for exponentially unstable linear plants have been suggested in a number of papers, including (Cao, Lin, & Ward, 2002; Crawshaw, 2003; Gomes da Silva & Tarbouriech, 2003; Wu & Lu, 2004). Linear Parameter Varying (LPV) systems can be used to embed an uncertain, possibly nonlinear, unstable and time varying plant; hence a robust anti-windup problem can be studied in this framework (see *e.g.* Cao & Lin, 2006; Wu, Yang, Packard, & Becker, 2000 and the references therein). However, in all these papers the operating region of the anti-windup control system is not determined by the intrinsic plant limitations, but depends on the *a priori* given unconstrained controller; so, especially if this controller is very aggressive, the proposed solutions lead to unduly small operating regions.

In this paper, the constructive anti-windup solution in Galeani, Teel, and Zaccarian (2007) (based on Teel (1999)) for exponentially unstable plants without uncertainties is extended to solve a robust-in-the-large anti-windup problem for LPV plants. With respect to previously available approaches, the proposed solution has the following advantages:

(1) the operating region is only restricted by the structural limitations of the saturated uncertain plant;
(2) bounded responses are ensured for all references;
(3) no bounds on the rate of variation of parameters are required.

The structurally largest possible operating regions (not achievable by any previously proposed LPV approach) are obtained by modifying the construction in Galeani et al. (2007) by the use of polyhedral Lyapunov functions (Blanchini, 1995; Blanchini & Miani, 2000). The main conceptual contribution of this paper is that the *necessary* condition of unsaturated closed-loop robust stability (Galeani & Teel, 2006; Turner et al., 2007) is also *sufficient* for LPV anti-windup, so that no performance-robustness trade-off in the sense of Galeani and Teel (2006) arises; and that gain scheduling has a

☆ Research supported in part by ENEA-Euratom and MIUR under PRIN projects. The material in this paper was partially presented at 46th IEEE Conference on Decision and Control, New Orleans, Louisiana (USA), December 12–14, 2007. This paper was recommended for publication in revised form by Associate Editor Faryar Jabbari under the direction of Editor André L. Tits.

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0005-1098/$ – see front matter
doi:10.1016/j.automatica.2009.10.011

key role in reducing the anti-windup problem to a constrained robust stabilization problem, but not in solving this last problem.

The paper is structured as follows: after some notation, the problem is described in Section 2 and solved in Section 3, where a design procedure is proposed. A simulation example is shown in Section 4, and conclusions are drawn in Section 5.

Notation. For $w, v \in \mathbb{R}^p$, the inequality $w > v$ means $w_i > v_i$ for $i = 1, \dots, p$. The saturation function of level one has ith component $\sigma_i(v) := \min\{1, \max\{-1, v_i\}\}$, for $i = 1, \dots, p$. Signal $q(\cdot)$ is in $\mathcal{L}_p$ if $\|q\|_p < \infty$, where $\|q\|_p := \left(\int_0^\infty \|q(\tau)\| \, d\tau\right)^{1/p}$ if $p \in [1, \infty)$, and $\|q\|_p := \sup_{\tau \in [0,+\infty)} \|q(\tau)\|$ if $p = \infty$. The class of piecewise continuous functions is $\bar{\mathcal{C}}^0$. A polyhedral set $\mathcal{P} \subset \mathbb{R}^n$ is defined as $\mathcal{P} := \{v \in \mathbb{R}^n : Fv \leq \bar{1}\}$, where $\bar{1} := [1 \cdots 1]'$ and F is a matrix. A function $\Psi : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is a *gauge function* if it is positive definite ($\Psi(x) \geq 0, \forall x \in \mathbb{R}^n$ and $\Psi(0) = 0$), homogeneous ($\Psi(\lambda x) = \lambda \Psi(x), \forall \lambda \in \mathbb{R}_{\geq 0}, \forall x \in \mathbb{R}^n$) and subadditive ($\Psi(x + y) \leq \Psi(x) + \Psi(y), \forall x, y \in \mathbb{R}^n$). Any compact, convex polyhedral set $\mathcal{P} := \{x \in \mathbb{R}^n : Fx \leq \bar{1}\}$ is a sublevel set $\bar{N}[\Psi, k] = \{x \in \mathbb{R}^n : \Psi(x) \leq k\}$ for $k = 1$ of the gauge function $\Psi(x) = \max_{1 \leq i \leq n}(F_i x)$, where F_i denotes the ith row of matrix F; conversely, any compact, convex set $\mathcal{S}$ induces a gauge function $\Psi(x) = \inf\{\mu \in \mathbb{R}_{\geq 0} : x \in \mu \mathcal{S}\}$.

2. Problem setting

Let $\mathcal{D}_0$ be a convex and compact polyhedron, and $\mathcal{W}_0 := \{w \in \mathbb{R}^\mu : w \geq 0, \sum_{i=1}^\mu w_i = 1\}$; moreover, define the classes of set bounded, piecewise continuous disturbances $\mathcal{D} = \{d(\cdot) \in \bar{\mathcal{C}}^0 : d(t) \in \mathcal{D}_0\}$ and time varying parameters $\mathcal{W} = \{w(\cdot) \in \bar{\mathcal{C}}^0 : w(t) \in \mathcal{W}_0\}$. Consider the linear parameter varying (LPV) plant

$$\dot{x} = A(w)x + B(w)u + Ed, \tag{1a}$$

$$z = C_1(w)x + D_1(w)u + G_1 d, \tag{1b}$$

$$y = C_2(w)x + D_2(w)u + G_2 d, \tag{1c}$$

where $x \in \mathbb{R}^n$ is the plant state, $u \in \mathbb{R}^m$ is the control input, y is the measured output, z is the performance output and $w(\cdot)$ is a measured signal. Each matrix of the form $M(w)$ in (1) is defined in terms of its μ *vertex values* M_i, $i = 1, \dots, \mu$, according to the relation $M(w) = \sum_{i=1}^\mu w_i(t) M_i$. An a priori fixed *unconstrained controller* is available for plant (1), given by

$$\dot{x}_c = A_c(w)x_c + B_c(w)u_c + B_r(w)r, \tag{2a}$$

$$y_c = C_c(w)x_c + D_c(w)u_c + D_r(w)r, \tag{2b}$$

designed assuming an *unconstrained interconnection*

$$u = y_c, \qquad u_c = y, \tag{3}$$

and is such that the *unconstrained closed-loop system* $\bar{\Sigma}_U$ given by (1), (2), (3) has a desirable response to external signals r, d, thus satisfying the following assumption.

Assumption 1. $\bar{\Sigma}_U$ is well-posed and asymptotically stable, $\forall w \in \mathcal{W}$.

Remark 1. Assumption 1 (which is a necessary condition for robust anti-windup compensation; see Galeani and Teel (2006), Turner et al. (2007)) implies *global robust asymptotic stability* of $\bar{\Sigma}_U$, but otherwise allows plant (1) to be *both unstable and affected by large uncertainties*. This is a novelty with respect to previously available literature on anti-windup, where the plant is only allowed to be *either unstable or affected by large uncertainties*, but not both. ○

When (3) is replaced by the saturated interconnection $u_c = y$, $u = \sigma(y_c)$, the arising *saturated closed-loop system* $\widehat{\Sigma}_S$ exhibits undesirable behavior; moreover, the global stability properties in Assumption 1 are lost for $\widehat{\Sigma}_S$ if (1) is exponentially unstable (since under bounded input the null controllability region is bounded in the exponentially unstable directions). In order to limit this windup effect, the following *anti-windup compensator*

$$\dot{x}_{aw} = A(w)x_{aw} + B(w)[y_c - \sigma(y_c + v_1)], \tag{4a}$$

$$v_1 = \alpha(x, x + x_{aw}, y_c, w), \tag{4b}$$

$$v_2 = C_2(w)x_{aw} + D_2(w)[y_c - \sigma(y_c + v_1)], \tag{4c}$$

$$u_c = y + v_2, \qquad u = \sigma(y_c + v_1), \tag{4d}$$

with $\alpha(\cdot)$ yet to be specified, will be considered in this paper. The arising *(saturated) anti-windup closed-loop system* $\check{\Sigma}_{SAW}$ is given by (1), (2), (4). Denote respectively by $\bar{z}, \bar{u}, \dots$, by $\hat{z}, \hat{u}, \dots$ and by $\check{z}$, $\check{u}, \dots$ the responses of $\bar{\Sigma}_U$, $\widehat{\Sigma}_S$ and $\check{\Sigma}_{SAW}$ to the same choices of $r(\cdot)$, $d(\cdot)$ and $x(0)$, $x_c(0)$ (with $x_{aw}(0) = 0$). Let $\mathcal{X}^+$ be a compact and convex subset[1] of the null controllable region of (1) under the available bounded input; moreover, for $\varepsilon > 0$ define $\mathcal{X} = (1 + \varepsilon)^{-1}\mathcal{X}^+$. When $x \in \mathcal{X}$, the anti-windup compensator pushes the response of $\check{\Sigma}_{SAW}$ towards the response of $\bar{\Sigma}_U$; when $x \in \mathcal{X}^+$ but $x \notin \mathcal{X}$, it just works to keep the state x inside $\mathcal{X}^+$. The set of *(steady state) feasible external signals*, containing those pairs of constant references and disturbances leading to equilibria within the set $\mathcal{X}$, is defined as follows.

Definition 1. The set $\mathcal{RD}(w, \mathcal{X})$ of *feasible external signals for w and $\mathcal{X}$* contains all the pairs $(r_\circ, d_\circ)$ such that the state response of $\bar{\Sigma}_U$ to $(r(t), d(t)) = (r_\circ, d_\circ)$, $\forall t \geq 0$, converges to a steady state (x^*, x_c^*) with $x^* \in \mathcal{X}$. ○

Remark 2. Compared with the corresponding definition in Galeani et al. (2007) (where all parameters were fixed and perfectly known), Definition 1 contains three main differences, all related to the need for more generality and reduced conservativeness in the present context (where very little restrictions are imposed on w by the fact the $w \in \mathcal{W}$). First, convergence of the state response of $\bar{\Sigma}_U$ is explicitly assumed (though w is not assumed to be constant or converging in Definition 1). Second, the set $\mathcal{RD}(w, \mathcal{X})$ of feasible external signals depends on w (and not only on $\mathcal{X}$). Third, both $r_\circ$ and $d_\circ$ are simultaneously accounted for in the definition of $\mathcal{RD}(w, \mathcal{X})$, so that there is a trade-off between the size of $r_\circ$ and the size of $d_\circ$ in each feasible pair $(r_\circ, d_\circ)$ (whereas in Galeani et al. (2007), $d_\circ = 0$ could always be assumed without any loss of generality due to the particular structure considered for the plant dynamics). ○

The following problem will be addressed and solved (the notation $\bar{x}, \bar{z}, \bar{u}, \dots, \check{x}, \check{z}, \check{u}, \dots$ is defined right after (4)).

Problem 1. Design an augmentation to the controller (2) such that for any $x(0) \in \mathcal{X}$, $w(\cdot) \in \mathcal{W}$, $d(\cdot) \in \mathcal{D}$ and $r(\cdot)$, the following properties are satisfied:
(α) if $\sigma(\bar{u}(t)) = \bar{u}(t), \bar{x}(t) \in \mathcal{X}, \forall t$, then $\check{z}(t) = \bar{z}(t), \forall t$;
(β) $\forall (r_\circ, d_\circ) \in \mathcal{RD}(w, \mathcal{X})$, if $(r(\cdot) - r_\circ, d(\cdot) - d_\circ) \in \mathcal{L}_p$ then $(\check{z} - \bar{z})(\cdot) \in \mathcal{L}_p, \forall p \in [1, \infty]$;
(γ) if $\lim_{t\to+\infty}(\bar{x}(t), \bar{x}_c(t)) = (\bar{x}^*, \bar{x}_c^*)$ and $\bar{x}^* \in \mathcal{X}$, then $\lim_{t\to+\infty}(\check{x}(t), \check{x}_c(t)) = (\check{x}^*, \check{x}_c^*) = (\bar{x}^*, \bar{x}_c^*)$. Moreover, if $\bar{x}^* \notin \mathcal{X}$ and $\lim_{t\to+\infty} d(t) = 0$, then $\lim_{t\to+\infty}(\check{x}(t), \check{x}_c(t)) = (\check{x}^*, \check{x}_c^*)$ with $\check{x}^* \in \mathcal{X}$.

Remark 3. By item (α), the anti-windup compensator will preserve any trajectory of $\bar{\Sigma}_U$ that never saturates and never leaves $\mathcal{X}$; by item (β), any unconstrained trajectory which converges (in an $\mathcal{L}_p$ sense) to an admissible set-point will be recovered, even if saturation cause some transient performance loss (for $p = \infty$, this item implies BIBS stability of $\check{\Sigma}_{SAW}$); finally, by item (γ) any

[1] Since the boundary of the null controllable region is an invariant set, in order to be able to quickly steer the state x back inside $\mathcal{X}$, some distance between the boundary of $\mathcal{X}^+$ and the boundary of the null controllable region is desirable (see Barbu, Galeani, Teel and Zaccarian (2005, Remark 5)).

converging trajectory of $\bar{\Sigma}_U$ will correspond to a converging trajectory of $\check{\Sigma}_{SAW}$, and in particular trajectories converging outside $\mathcal{X}$ are projected on a restricted set-point, so that the state x remains in $\mathcal{X}$. ○

3. Anti-windup construction

Since the structure of the anti-windup compensator has already been described in (4), it only remains to specify how the function $\alpha(\cdot)$ can be designed. The proposed design procedure (based on Blanchini (1995), Blanchini and Miani (2000), Galeani et al. (2007)), is the following.

Procedure 1 (*Anti-windup Compensator Design*). *Step* 1. *As in Blanchini (1995), compute a polyhedral domain of attraction $\mathcal{X}^+ := \{x \in \mathbb{R}^n : Fx \leq \bar{1}\}$ and define the associated gauge function $\psi(x) := \max_i(F_i x)$ and the set $\mathcal{X} := (1+\varepsilon)^{-1}\mathcal{X}^+$ for a small $\varepsilon > 0$.*

Step 2. *As in Blanchini (1995), compute a homogeneous Lipschitz control law $\phi(x)$ making $\mathcal{X}^+$ forward invariant*

Step 3. *As in Blanchini and Miani (2000), define the pseudo-tracking control law*

$$\Phi(x, \bar{x}, \bar{u}) := \phi(\tilde{x}(x, \bar{x}))\Psi(x, \bar{x}) + (1 - \Psi(x, \bar{x}))\bar{u}.$$

where $\Psi(x, \bar{x}) := \max_i \frac{F_i(x-\bar{x})}{1-F_i\bar{x}}$, $\tilde{x}(x, \bar{x}) := \bar{x} + \frac{(x-\bar{x})}{\Psi(x,\bar{x})}$.

Step 4. *Define the anti-windup control law*

$$\alpha(x, x_M, y_c, w) := -y_c + \Phi(x, \pi(x_M), \pi_u(y_c, w)),$$

where, defining $B(w)^\sharp := (B(w)'B(w))^{-1}B(w)'$,

$$\pi(x_M) := \begin{cases} x_M, & \text{if } x_M \in \mathcal{X}, \\ \dfrac{x_M}{\psi(x_M)(1+\varepsilon)}, & \text{if } x_M \notin \mathcal{X}, \end{cases}$$

$$\pi_u(y_c, w) := \begin{cases} y_c, & \text{if } y_c \in \mathcal{U}, x_M \in \mathcal{X}, \\ -B(w)^\sharp A(w)\pi(x_M), & \text{otherwise}. \end{cases}$$

Theorem 1. *Under Assumption 1, an anti-windup compensator designed as in Procedure 1 solves Problem 1.*

Theorem 1 states the effectiveness of Procedure 1; for lack of space, just a sketch of its proof is given. In the coordinates (Teel, 1999) $x_M := x + x_{aw}, x_c$ and x, $\check{\Sigma}_{SAW}$ appears as the cascade of $\bar{\Sigma}_U$ (with x replaced by x_M) and

$$\dot{x} = A(w)x + B(w)\sigma(\Phi(x, \pi(x_M), \pi_u(y_c, w))) + Ed. \tag{5}$$

Since $x \in \partial\mathcal{X}^+$ implies $\Phi(x, \bar{x}, \bar{u}) = \phi(x)$, $\mathcal{X}^+$ is forward invariant for (5). Asymptotic stability of the cascade (more precisely, of any of its steady state equilibria) is ensured by the properties of the pseudo-tracking control law (Blanchini & Miani, 2000) and Assumption 1, which guarantee asymptotic stability of the two systems in the cascade. Functions $\pi(x_M)$ and $\pi_u(y_c, w)$ project unfeasible equilibria to feasible ones that the pseudo-tracking control law can effectively track. The derivation of the remaining claims in items (β) and (γ) parallels the proofs in Galeani et al. (2007), by a judicious exploitation of the properties in Blanchini and Miani (2000).

Remark 4. As discussed in Galeani and Teel (2006), without gain scheduling in (4a) and (4c) the robust-in-the-large stability of $\bar{\Sigma}_U$ and of the uncertain dynamics (5) do not guarantee robust stability of $\check{\Sigma}_{SAW}$, unless the anti-windup requirements are suitably weakened (a performance-robustness trade-off). It is worth to point out that this is not the case for gain-scheduled anti-windup. From the above sketch of proof, it is apparent that gain scheduling on $w(\cdot)$ ensures that $\check{\Sigma}_{SAW}$ has a cascade structure (in suitable coordinates); however, it does not play any role in ensuring the largest possible robust basin of attraction for the uncertain dynamics (5) (cf. Blanchini, 2000; Blanchini & Miani, 2000). Nevertheless, the cascade structure implies that the domain of attraction of $\check{\Sigma}_{SAW}$, as well as its robust-in-the-large stability, is directly determined by the domains of attraction and robust stability of the two cascaded systems; hence, since the pseudo-tracking control law ensures the intrinsic largest achievable basin of attraction for (5), global robust-in-the-large stability of $\bar{\Sigma}_U$ becomes a necessary and sufficient condition for $\check{\Sigma}_{SAW}$ to be robustly-in-the-large stable, with the intrinsic largest achievable basin of attraction. As discussed in Galeani and Teel (2006), this is not possible without gain scheduling in (4a) and (4c), because in such a case the cascade structure of $\check{\Sigma}_{SAW}$ is destroyed. ○

Remark 5. The proposed result is tight for the LPV system (1), since (up to an arbitrarily small error due to the approximation by a polyhedral set) the largest domain of attraction under the given bounds on the control input, the disturbances and the uncertainties is obtained by Procedure 1 thanks to the algorithms in Blanchini (1995). However, conservativeness is introduced if the LPV system is used to "hide" a nonlinear, uncertain system under a "linear" structure, since, for example, in the original nonlinear system some "parameter variations" of the LPV system are not possible (similar remarks apply with respect to the fact that no bound on the rate of variation of $w(\cdot)$ is imposed in our analysis). All the above still holds if, instead of polyhedral functions, a different family of universal Lyapunov function is used, provided that the pseudo-tracking control law is accordingly modified; such modifications allow for extra flexibility in the computation of the domains of attraction. ○

Remark 6. Since the null controllability region of exponentially unstable plants with bounded inputs is also bounded in the exponentially unstable directions (Sontag, 1984), the anti-windup compensator must ensure that the "unstable part" of the state is kept inside its bounded null controllable region at all times. With this goal in mind, it was assumed in Galeani et al. (2007), Teel (1999) that suitable coordinates exist such that $x := [x_s' \ x_u']'$, with the "unstable part" x_u measured and unaffected by the "stable part" x_s and the disturbance d. Under these assumptions, the anti-windup compensator, using a state feedback from x_u, needs to restrict x_u as little as possible; otherwise, at the price of introducing an additional "safety boundary" (and thus restricting $\mathcal{X}$ even more), d can be allowed to act on x_u and an observer can be used to estimate x_u. Parameter variations modify the null controllable region, and then the robust null controllable region is bounded in all directions, and then the whole state of the plant must be kept bounded (compare the example in Avanzini and Galeani (2005) with respect to Barbu et al. (2005)). As a consequence, the natural adaptation of the assumptions in Galeani et al. (2007), Teel (1999) to the present context requires the whole state x to be measured and subject to the disturbance d; however, exactly as in Galeani et al. (2007), Teel (1999), an observer could be used at the price of introducing an additional safety boundary (and thus restricting $\mathcal{X}$ even more) to cope with estimation errors. ○

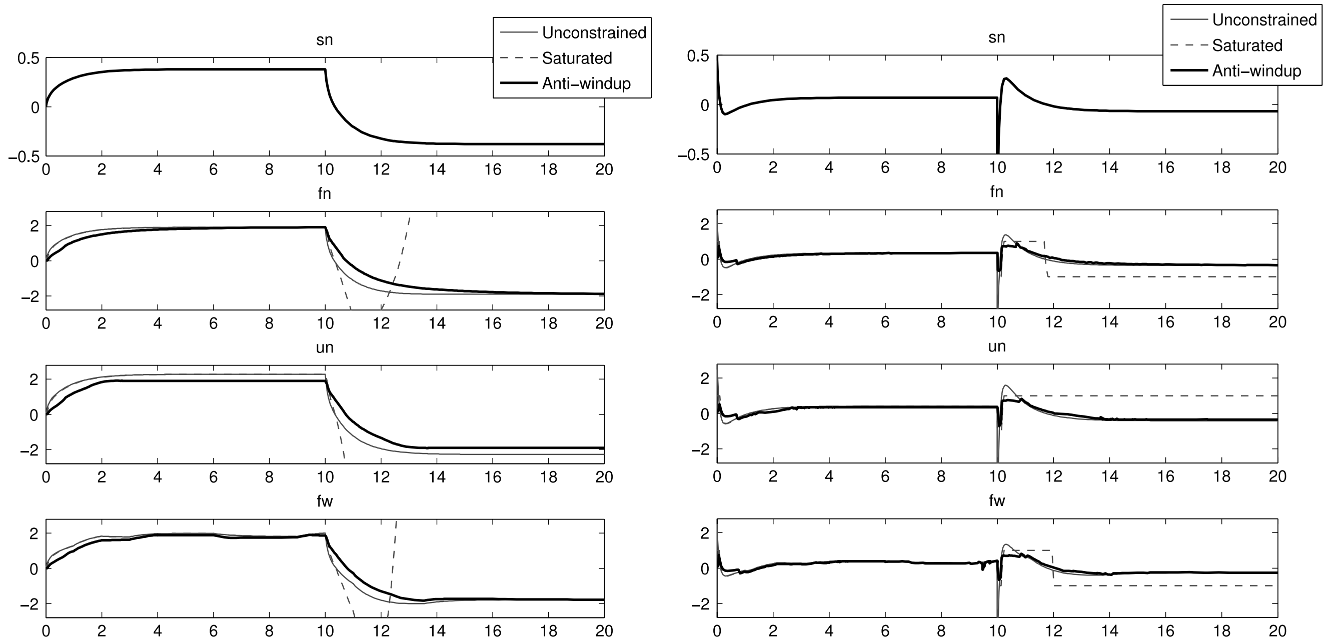
4. Simulation examples

Consider the plant described by matrices

$$A(w_1) = w_1 \begin{bmatrix} 1.8 & -1 \\ -0.2 & 0.8 \end{bmatrix} + (1 - w_1) \begin{bmatrix} 2.2 & -1 \\ 0.2 & 1.2 \end{bmatrix}$$

$$B(w_2) = w_2 \begin{bmatrix} 9.8 \\ -6.8 \end{bmatrix} + (1 - w_2) \begin{bmatrix} 10.2 \\ -7.2 \end{bmatrix}$$

with $C_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $E = D_1 = D_2 = 0$, and an unconstrained, a priori given LQR controller, ensuring robust asymptotic stability in the absence of saturation, designed for the nominal



(a) Performance output signal. (b) Control signal.

Fig. 1. Comparison among the closed-loop responses for small reference, large feasible reference, unfeasible reference (for nominal, constant parameter values) and large feasible reference (for perturbed, time varying parameter values).

plant with $R = 1$ and $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. The proposed design procedure has been applied with $\varepsilon = 0.012$, and A_0, B_0 corresponding to the parameter values $(w_1, w_2) = (0.5, 0.5)$.

The reported Fig. 1 shows the performance output and control input for the unconstrained closed-loop system ($\bar{\Sigma}_U$), for the saturated closed-loop system ($\hat{\Sigma}_S$), and for the saturated anti-windup system ($\breve{\Sigma}_{SAW}$). In the three upper subplots of each figure, the nominal parameter values ($w_1, w_2 = 0.5, 0.5$) are considered, and the letters *sn*, *fn* and *un* respectively denote the response to a small reference, to a feasible reference close to the largest feasible reference r_{MAX}, and to an unfeasible reference. In the last subplot (identified by the letters *fw*), a time varying parameter signal $w(\cdot)$ is considered, coupled with a large feasible reference close to r_{MAX}.

In each case, the state of $\hat{\Sigma}_S$ leaves the null controllable region and then the output diverges, whereas the forward invariance of the set $\mathcal{X}^+$ guaranteed by the anti-windup compensator preserves the response of $\breve{\Sigma}_{SAW}$ from diverging. Moreover, whenever the reference is feasible (cases *sn* and *fn*), the output of $\breve{\Sigma}_{SAW}$ converges to the output of $\bar{\Sigma}_U$; when the reference is not feasible (case *un*), the output of $\breve{\Sigma}_{SAW}$ still converges, and reaches a value close to the output of $\bar{\Sigma}_U$ (but such that the state x remains inside the set $\mathcal{X}$).

Finally, notice also from case *fw* that, as specified in the anti-windup problem definition, the output of $\breve{\Sigma}_{SAW}$ is close to tracking the output of $\bar{\Sigma}_U$ during the first seconds, while $w(\cdot)$ keeps varying; after $w(\cdot)$ stops varying, the output of $\bar{\Sigma}_U$ eventually converges to a constant value and is reached by the output of $\breve{\Sigma}_{SAW}$.

5. Conclusions

The construction of a gain-scheduled anti-windup compensator for LPV systems has been proposed in this paper; its main feature consists in the ability of handling exponentially unstable plants subject to large uncertainties meanwhile ensuring the largest possible domain of attraction, only dependent on the plant's intrinsic limits.

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