Output Feedback Strict Passivity of Discrete-time Nonlinear Systems and Adaptive Control System Design with a PFC [⋆,1]

Ikuro Mizumoto [a], Satoshi Ohdaira [a], Zenta Iwai [b]

[a] *Department of Intelligent Mechanical Systems, Kumamoto University, 2-39-1 Kurokami, Kumamoto, 860-8555, Japan*

[b] *Kumamoto Prefectural College of Technology, 4455-1 Haramizu, Kikuyo-mach, Kikuchi-gun, Kumamoto, 869-1102, Japan*

Abstract

In this paper, a passivity-based adaptive output feedback control for discrete-time nonlinear systems is considered. Output Feedback Strictly Passive (OFSP) conditions in order to design a stable adaptive output control system will be established. Further a design scheme of a parallel feedforward compensator (PFC) which is introduced in order to realize an OFSP controlled system will be provided and an adaptive output feedback control system design scheme with a PFC will be proposed.

Key words: Adaptive control; discrete nonlinear systems; strict passivity; output feedback; parallel feedforward compensator.

1 Introduction

Since many practical systems contain some kind of nonlinearities, a great deal of attention has been attracted to the control of nonlinear systems. Especially, of particular interest are passivity based controller designs for the control problem on nonlinear systems (Hill and Moylan, 1998; Byrnes *et al.*, 1991; Krstic *et al.*, 1994; Jiang and Hill, 1998; Fradkov and Hill, 1998; Byrnes and Lin, 1994; Lin and Byrnes, 1995). Although several important results have been obtained concerning passivity based controls, most of the results however were ones for continuous-time systems (Hill and Moylan, 1998; Byrnes *et al.*, 1991; Krstic *et al.*, 1994; Jiang and Hill, 1998; Fradkov and Hill, 1998). Our interest here is a discrete-time passive (or strictly passive) system. For discrete-time nonlinear systems, passivity properties has been investigated widely (Byrnes and Lin, 1994; Lin and Byrnes, 1995; Monaco and Normand-Cyrot, 1997; Navarro-Lopez *et al.*, 2002; Navarro-Lopez, 2007; Monaco *et al.*, 2008), however, only few passivity based controls have been investigated with respect to lossless or passive systems (Byrnes and Lin, 1994; Lin and Byrnes, 1995; Chellaboina and Haddad, 2002; Navarro-Lopez *et al.*, 2002; Navarro-Lopez, 2007). In Byrnes and Lin (1994); Lin and Byrnes (1995); Chellaboina and Haddad (2002) feedback lossless or passivity were developed and stabilization of discrete-time nonlinear systems were investigated for input affine systems. In Navarro-Lopez *et al.* (2002); Navarro-Lopez (2007), dissipativity and/or passivity for general discrete-time nonlinear systems have been investigated and feedback passivity has also been developed. However, most these known results are related only to properties via state feedback. Considering the control design with a simple structure in view of practical application, it seems useful and valuable to investigate the output feedback passivity properties and to establish passivity-based output feedback control for discrete nonlinear systems.

In this paper, we consider a passivity-based adaptive output feedback control for discrete-time nonlinear systems. The passivity-based control schemes can be considered one of the Lyapunov-based controls. As for the Lyapunov-based adaptive controls, several significant results have been provided for discrete-time non-linear systems(Hayakawa *et al.*, 2004). However, the developed methods were also only with state feedback forms. Unlike the former works on the passivity-based control and the Lyapunov-based adaptive control, the passivity-based adaptive control dealt with in this paper is an output feedback-based adaptive control in which only the out-

⋆ The material in this paper was partially presented at 17th IFAC World Congress, Seoul, July 6-11, 2008. Corresponding author I. Mizumoto. Tel. +81-96-342-3759. Fax +81-96-342-3729.

Email addresses: `ikuro@gpo.kumamoto-u.ac.jp` (Ikuro Mizumoto), `iwai@kumamoto-pct.ac.jp` (Zenta Iwai).

[1] This work was partially supported by KAKENHI, the Grant-in-Aid for Scientific Research (C) 18560439, from the JapanSociety for the Promotion of Science (JSPS).

put signal is utilized in the controller design. It is well known that one can easily design an output feedback based adaptive control for an output feedback strictly (exponentially) passive (OFSP) system (Jiang and Hill, 1998; Fradkov and Hill, 1998; Michino *et al.*, 2003; Mizumoto *et al.*, 2005) and the obtained control system has a strong robustness with respect to disturbances and uncertainties. Its simple structure of the adaptive controller and robustness are useful for practical applications. This is a motivation of this research.

The system is said to be OFSP if there exists an output feedback such that the resulting closed loop system is strictly passive. Here we investigate the OFSP property of discrete-time nonlinear systems, and consider an output feedback-based adaptive control design problem for discrete-time nonlinear systems. To this end, we first derive a discrete-time nonlinear version of Kalman-Yakubovich-Popov (KYP) Lemma for a strictly passive system. The strict passivity of the control system plays an important role in adaptive controls. The KYP-Lemma for continuous-time nonlinear systems has been interpreted (Hill and Moylan, 1998; Jiang and Hill, 1998) and the KYP-Lemma for discrete-time nonlinear systems has been investigated for lossless and passive systems (Byrnes and Lin, 1994; Lin and Byrnes, 1995). Recently, KYP-Lemma for discrete-time nonlinear systems is further investigated in relation to QS (Quadratic Storage)-passivity (Navarro-Lopez, 2007). Here, we will pay attention to the KYP-Lemma for strictly passive discrete-time nonlinear systems in order to design an adaptive control system. After that, OFSP conditions for discrete-time nonlinear systems will be clarified, and the design of output feedback-based adaptive control will be shown. As it is well known, a passive discrete-time system must have a direct feedthrough term of input, that is, a passive system must have a relative degree of 0 (Byrnes and Lin, 1994). This indicates that the OFSP system also has to have a direct feedthrough term of the input (i.e. relative degree of 0). Since most practical systems do not have a direct feedthrough term of the input, the OFSP condition provides severe restrictions for practical applications of the considered adaptive design scheme. The introduction of a parallel feedforward compensator (PFC) will be considered in order to alleviate OFSP condition and it is shown that there exists a PFC which renders the augmented system with the PFC OFSP if the system can be stabilizable by a dynamic feedback. The inverse system of the dynamic controller can be a PFC. Further an adaptive output feedback control system design with a PFC will be developed. The OFSP condition which the system must have a relative degree of 0 possibly results in a causality problem in the controller design. A condition in which one can design the adaptive controller without causality problems will be provided as strong output feedback strict passivity, and according to the obtained conditions, an adaptive output feedback controller design scheme will be shown for a discrete-time nonlinear system with a PFC.

2 Preparation: Strict passivity

Consider the following n-th order discrete-time SISO nonlinear system with a relative degree of 0.

$$\begin{aligned} x(k+1) &= f(x(k)) + g(x(k))u(k) \\ y(k) &= h(x(k)) + J(x(k))u(k) \end{aligned} \tag{1}$$

where $x(k) \in R^n$ is a state vector, $u(k)$, $y(k) \in R$ are the input and output of the system. $f(x(k)) : R^n \to R^n$, $g(x(k)) : R^n \to R^n$, $h(x(k)) : R^n \to R$ and $J(x(k)) : R^n \to R$ are smooth in $x(k)$, and we assume that $f(0) = 0$, $h(0) = 0$.

The passivity and the strict passivity of the system (1) are defined as follows (Byrnes and Lin, 1994):

Definition 1 *(Passivity) A system* (1) *is said to be passive if there exists a non-negative function $V(x(k)) : R^n \to R$ with $V(0) = 0$, called the storage function, such that*

$$V(x(k+1)) - V(x(k)) \le y(k)u(k) \tag{2}$$

for all $u(k) \in R$, $\forall k \ge 0$.

Definition 2 *(Strict Passivity) A system* (1) *is said to be strictly passive if there exists a non-negative function $V(x(k)) : R^n \to R$ with $V(0) = 0$ and a positive definite function $S(x(k)) : R^n \to R$ such that*

$$V(x(k+1)) - V(x(k)) \le y(k)u(k) - S(x(k)) \tag{3}$$

for all $u(k) \in R$, $\forall k \ge 0$.

The property of a passive or lossless system has been studied in Byrnes and Lin (1994); Lin and Byrnes (1995). Here we first investigate the strict passivity by means of the discrete-time nonlinear version of the KYP-Lemma in order to develop the adaptive controller for discrete-time nonlinear systems.

Theorem 1 *A system* (1) *is strictly passive if and only if, there exists a non-negative function $V(x(k)) : R^n \to R$ with $V(0) = 0$ such that*

A1-1) There exist functions $l(x)$, $W(x)$ and a positive definite function $S(x)$ such that

$$V(f(x)) - V(x) = -l(x)^2 - S(x) \tag{4}$$

$$\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=f(x)} g(x) = h(x) - 2l(x)W(x) \tag{5}$$

$$g^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=f(x)} g(x) = 2J(x) - 2W(x)^2. \tag{6}$$

A1-2) $V(f(x) + g(x)u)$ is quadratic in u.

Proof: The proof can be seen in Navarro-Lopez (2007) in relation to the QS-passivity. A specific proof for the strict passivity is given in Appendix A.

Remark 1: In Navarro-Lopez *et al.* (2002); Navarro-Lopez (2007), general discrete-time nonlinear systems have been dealt with in relation to KYP lemma. However, in order to establish the basic concept of the following passivity-based output feedback controls, here in this paper, the input affine system is dealt with.

3 Output feedback strict passivity

Next, we define an output feedback strict passivity for a system (1).

Definition 3 *(Output feedback strictly passive: OFSP) A system* (1) *is said to be output feedback strictly passive (OFSP) if there exists an output feedback:*

$$u(k) = \alpha(y(k)) + \beta(y(k))v(k) \tag{7}$$

such that the resulting closed loop system is strictly passive.

Further we define a strong output feedback strict passivity as follows:

Definition 4 *(Strongly OFSP) A system* (1) *is said to be strongly OFSP if the system is OFSP with a static output feedback, i.e. there exists a static output feedback:*

$$u(k) = -\theta^* y(k) + v(k), \theta^* > 0 \tag{8}$$

such that the resulting closed loop system from $y(k)$ to $v(k)$,

$$\begin{aligned} x(k+1) &= \bar{f}(x(k)) + \bar{g}(x(k))v(k) \\ y(k) &= \bar{h}(x(k)) + \bar{J}(x(k))v(k) \end{aligned} \tag{9}$$

with

$$\bar{f}(x(k))=f(x(k))-\frac{\theta^*}{1+\theta^* J(x(k))}h(x(k))g(x(k)) \tag{10}$$

$$\bar{g}(x(k))=\frac{1}{1+\theta^* J(x(k))}g(x(k)) \tag{11}$$

$$\bar{h}(x(k))=\frac{1}{1+\theta^* J(x(k))}h(x(k)) \tag{12}$$

$$\bar{J}(x(k))=\frac{1}{1+\theta^* J(x(k))}J(x(k)) \tag{13}$$

is strictly passive and, in addition, a transformed closed loop system with

$$\bar{v}(k) = \frac{1}{1+\theta^* J(x(k))}v(k) \tag{14}$$

as input,

$$\begin{aligned} x(k+1) &= \bar{f}(x(k)) + g(x(k))\bar{v}(k) \\ y(k) &= \bar{h}(x(k)) + J(x(k))\bar{v}(k) \end{aligned} \tag{15}$$

is also strictly passive.

Remark 2: In Definition 4, it should be $1+\theta^* J(x) \neq 0$, $\forall x \in R$, so that the system to be strongly OFSP globally.

For linear discrete-time systems, this strongly OFSP is recognized as strongly almost strictly positive real (strongly ASPR) (Mizumotoa *et al.*, 2007).

The sufficient conditions for a system (1) to be OFSP are provided by the following theorem.

Theorem 2 A system (1) is OFSP with a static output feedback (8) and a C^2 positive definite storage function if

A2-1) The system has relative degree of 0 and $J(x(k)) > 0, \forall x(k)$.

A2-2) The zero dynamics of the system:

$$x(k+1) = f^*(x(k)) \tag{16}$$

is asymptotically stable with the following C^2 positive definite function V satisfying

a) $V(f^*(x)) - V(x) = -\zeta(x)$ (17)
with a positive definite function $\zeta(x)$.

b) $V(f^*(x) + g(x)u)$ is quadratic in u.

c) There exist positive definite matrices Γ_m, Γ_M such that

$$0 < \Gamma_m \leq \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x(k))} \leq \Gamma_M \tag{18}$$

A2-3) $\frac{g(x(k))}{J(x(k))}$ is bounded.

Proof: The zero dynamics of the system(1) is obtained by (Byrnes and Lin, 1994)

$$x(k+1) = f^*(x(k)) = f(x(k)) - \frac{h(x(k))}{J(x(k))}g(x(k)) \tag{19}$$

Since $\bar{f}(x)$ in the closed loop system (9) can be represented from (10) and (19) by

$$\begin{aligned} \bar{f}(x) &= f(x) - \frac{\theta^*}{1+\theta^* J(x)}h(x)g(x) \\ &= f^*(x) + \tilde{J}(x)h(x)g(x) \end{aligned} \tag{20}$$

with

$$\tilde{J}(x) = \frac{1}{J(x)\left(1+\theta^* J(x)\right)}, \tag{21}$$

from assumption A2-2), b), $V(\bar{f}(x))$ can be expressed as

$$V(\bar{f}(x)) = V(f^*(x)) + \tilde{J}(x)h(x) \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=f^*(x)} g(x) + \frac{1}{2}\tilde{J}(x)^2 h(x)^2 g^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=f^*(x)} g(x). \quad (22)$$

Thus we have from (17),(22) that

$$\begin{aligned} &V(\bar{f}(x)) - V(x) \\ &\quad = -\zeta(x) + \tilde{J}(x)h(x) \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} g(x) \\ &\qquad -\frac{1}{2}\tilde{J}(x)^2 h(x)^2 g^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} g(x). \end{aligned} \quad (23)$$

Now, consider a function $\bar{W}(x)$ that satisfies the following relation:

$$\bar{W}(x)^2 = \bar{J}(x) - \frac{1}{2}\bar{g}^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} \bar{g}(x). \quad (24)$$

Such function $\bar{W}(x)$ is certain to exist for a sufficiently large θ^* from assumptions A2-2),c) and A2-3). Further, consider a function $\bar{l}(x(k))$ that satisfies

$$\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) = \bar{h}(x) - 2\bar{l}(x)\bar{W}(x). \quad (25)$$

Since (25) yields that

$$\bar{l}(x)^2\bar{W}(x)^2 = \frac{1}{4}\left\{\bar{h}(x)^2 - 2\bar{h}(x) \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) + \left(\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x)\right)^2\right\}, \quad (26)$$

we have from (24) and (26) that

$$\begin{aligned} &\bar{l}(x)^2\left(\bar{J}(x) - \frac{1}{2}\bar{g}^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} \bar{g}(x)\right) \\ &\quad = \frac{1}{4}\left\{\bar{h}(x)^2 - 2\bar{h}(x) \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) \right. \\ &\qquad \left. + \left(\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x)\right)^2\right\}. \end{aligned} \quad (27)$$

Thus, we obtain from (27) that

$$\begin{aligned} &\bar{h}(x) \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) \\ &\quad = -2\bar{l}(x)^2\left\{\bar{J}(x) - \frac{1}{2}\bar{g}^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} \bar{g}(x)\right\} \\ &\qquad + \frac{1}{2}\left\{\bar{h}(x)^2 + \left(\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x)\right)^2\right\}. \end{aligned} \quad (28)$$

Furthermore, taking the definitions of $\bar{g}(x)$ and $\bar{h}(x)$ in (11) and (12) in to account, we have from (28) that

$$\begin{aligned} &h(x) \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} g(x) \\ &\quad = -2\bar{l}(x)^2\left\{(1+\theta^* J(x))J(x) - \frac{1}{2}g^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} g(x)\right\} \\ &\qquad + \frac{1}{2}\left\{h(x)^2 + \left(\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} g(x)\right)^2\right\}. \end{aligned} \quad (29)$$

Therefore, we obtain from (23) and (29) that

$$\begin{aligned} &V(\bar{f}(x)) - V(x) \\ &\quad = -\zeta(x) - 2\bar{l}(x)^2 \\ &\qquad + \frac{1}{J(x)\,(1+\theta^* J(x))}\left[\frac{1}{2}\left\{h(x)^2 \right.\right. \\ &\qquad \left. + \left(\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} g(x)\right)^2\right\} \\ &\qquad + \left\{\bar{l}(x)^2 - \frac{1}{2}\frac{1}{J(x)\,(1+\theta^* J(x))}h(x)^2\right\} \\ &\qquad \left. \times g^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} g(x)\right]. \end{aligned} \quad (30)$$

Finally, we have

$$V(\bar{f}(x)) - V(x) = -\bar{l}(x)^2 - \bar{S}(x) \quad (31)$$

where

$$\begin{aligned} \bar{S}(x) &= \zeta(x) + \bar{l}(x)^2 \\ &\quad - \frac{1}{J(x)\,(1+\theta^* J(x))}\left[\frac{1}{2}\left\{h(x)^2 \right.\right. \\ &\quad \left. + \left(\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} g(x)\right)^2\right\} \\ &\quad + \left\{\bar{l}(x)^2 - \frac{1}{2}\frac{1}{J(x)\,(1+\theta^* J(x))}h(x)^2\right\} \\ &\quad \left. \times g^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} g(x)\right]. \end{aligned} \quad (32)$$

$\bar{S}(x(k))$ is certain to be a positive definite function with a sufficiently large θ^*. Thus we can conclude that, for a sufficiently large θ^*, there exists a positive definite C^2 function $V(x)$ with a property that $V(f(x)+g(x)u)$ is quadratic in u, functions $\bar{W}(x(k))$, $\bar{l}(x)$ and a positive definite function $\bar{S}(x(k))$ such that

$$V(\bar{f}(x)) - V(x) = -\bar{l}(x)^2 - \bar{S}(x) \tag{33}$$

$$\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) = \bar{h}(x) - 2\bar{l}(x)\bar{W}(x) \tag{34}$$

$$\bar{g}^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) = 2\bar{J}(x) - 2\bar{W}(x)^2, \tag{35}$$

that is there exists a feedback gain θ^* such that the resulting closed loop system is strictly passive. Then the system is output feedback strictly passive with a C^2 positive definite function as the storage function.

Remark 3: The conditions A2-1) and A2-2) (b) are necessary conditions to be OFSP. These conditions can be seen in Byrnes and Lin (1994) as necessary conditions on the feedback equivalent to lossless system. The conditions A2-2)(a), (c) and A2-3) are conditions for the existence of a static output feedback which renders the resulting system strictly passive. Note that the condition $0 \le \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x(k))}$ is one of the conditions with which there exists a smooth state feedback such that the resulting closed system is globally asymptotic stable (Byrnes and Lin, 1994).

Moreover, we have the following lemma concerning the strongly OFSP conditions.

Lemma 1 *Assumptions A2-1), A2-2) and A2-3) in Theorem* 2 *are satisfied with* $J(x(k)) = d > 0$ *then the system* (1) *is strongly OFSP.*

Proof: See appendix B.

The derived OFSP and/or strong OFSP conditions are very restrictive for practical systems, since most practical systems do not have relative degree of 0 and it is difficult to choose adequate sampling period with which the system is minimum-phase. In the following section, we first show that there exists a parallel feedforward compensator (PFC) which renders the augmented system with the PFC OFSP, and then the passivity-based adaptive output feedback design for the OFSP augmented system with a PFC will be proposed.

4 Realization of OFSP system

Consider the following system with $J(x) = 0$ in (1):

$$\begin{aligned} x(k+1) &= f(x(k)) + g(x(k))u(k) \\ y(k) &= h(x(k)). \end{aligned} \tag{36}$$

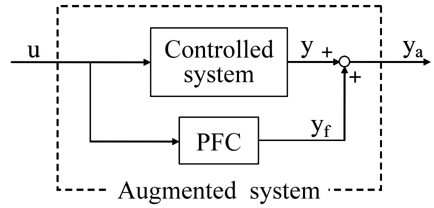

Fig. 1. Block diagram of the augmented system with a PFC

This system is not OFSP, so we consider introduction of a PFC:

$$\begin{aligned} x_f(k+1) &= A_f x_f(k) + b_f u(k) \\ y_f(k) &= c_f^T x_f(k) + du(k). \end{aligned} \tag{37}$$

which is implemented in parallel with the controlled system (36) as shown in Fig. 1, so as to make the resulting augmented system OFSP.

The resulting augmented system can be expressed by

$$\begin{aligned} x_a(k+1) &= f_a(x_a(k)) + g_a(x_a(k))u(k) \\ y_a(k) &= y(k) + y_f(k) = h_a(x_a(k)) + du(k). \end{aligned} \tag{38}$$

where

$$x_a(k) = \begin{bmatrix} x(k) \\ x_f(k) \end{bmatrix}, \; f_a(x_a(k)) = \begin{bmatrix} f(x(k)) \\ A_f x_f(k) \end{bmatrix}$$
$$h_a(x_a(k)) = y(k) + c_f^T x_f(k) = h(x(k)) + c_f^T x_f(k)$$

The PFC (37) has to be designed such that the resulting augmented system (38) is OFSP. Concerning the existence of such a PFC and design scheme of the PFC, we have the following theorem.

Theorem 3 Assume that the system (36) can be stabilized with a C^2 positive definite function V by a dynamic feedback given by

$$\begin{aligned} & x_d(k+1) = A_d x_d(k) + b_d y(k) \\ & y_d(k) = c_d^T x_d(k) + d_d y(k) \\ & u(k) = -y_d(k) \end{aligned} \tag{39}$$

Consider the inverse system of (39) with $u(k)$ as input and $y_f(t) = -y(k)$ as output expressed by

$$\begin{aligned} & x_f(k+1) = (A_d - \tfrac{1}{d_d} b_d c_d^T) x_f(k) - \tfrac{1}{d_d} b_d u(k) \\ & y_f(k) = \tfrac{1}{d_d} c_d^T x_f(k) + \tfrac{1}{d_d} u(k) \end{aligned} \tag{40}$$

and consider an augmented system with this as a PFC as shown in Fig. 1. Then the zero dynamics of the augmented system is stable with the C^2 positive definite function V.

Proof: The closed loop system with $u(k) = -y_d(k)$ as the control input can be represented by

$$\begin{aligned}\begin{bmatrix} x(k+1) \\ x_d(k+1) \end{bmatrix} &= \begin{bmatrix} f(x(k)) \\ A_d x_d(k) \end{bmatrix} \\ &\quad + \begin{bmatrix} -g(x(k))(c_d^T x_d(k) + d_d y(k)) \\ b_d y(k) \end{bmatrix} \\ &= \begin{bmatrix} f(x(k)) - g(x(k)) c_d^T x_d(k) \\ A_d x_d(k) \end{bmatrix} \\ &\quad + \begin{bmatrix} -d_d g(x(k)) y(k) \\ b_d y(k) \end{bmatrix} \end{aligned} \tag{41}$$

This is a stable system from assumption.

Now, let's consider the inverse system of (39) with $u(t)$ as the input and $y_f(k) = -y(k)$ as the output. The inverse system can be given in (40). The augmented system with the inverse system (40) as a PFC is then expressed by

$$\begin{bmatrix} x(k+1) \\ x_f(k+1) \end{bmatrix} = \begin{bmatrix} f(x(k)) \\ (A_d - \frac{1}{d_d} b_d c_d^T) x_f(k) \end{bmatrix} + \begin{bmatrix} g(x(k)) \\ -\frac{1}{d_d} b_d \end{bmatrix} u(k) \tag{42}$$

$$\begin{aligned} y_a(k) &= h(x(k)) + y_f(k) \\ &= y(k) + \frac{1}{d_d} c_d^T x_f(k) + \frac{1}{d_d} u(k) \end{aligned} \tag{43}$$

The zero dynamics of this augmented system is then obtained by

$$\begin{aligned}\begin{bmatrix} x(k+1) \\ x_f(k+1) \end{bmatrix} &= \begin{bmatrix} f(x(k)) \\ (A_d - \frac{1}{d_d} b_d c_d^T) x_f(k) \end{bmatrix} \\ &\quad + \begin{bmatrix} g(x(k)) \\ -\frac{1}{d_d} b_d \end{bmatrix} (-c_d^T x_f(k) - d_d y(k)) \\ &= \begin{bmatrix} f(x(k)) - g(x(k)) c_d^T x_f(k) \\ A_d x_f(k) \end{bmatrix} \\ &\quad + \begin{bmatrix} -d_d g(x(k)) y(k) \\ b_d y(k) \end{bmatrix} \end{aligned} \tag{44}$$

This zero dynamics have the same structure as in (41). Thus the zero dynamics of the resulting augmented system is stable with C^2 positive definite function V.

This theorem indicates that if a controller which stabilizes the system with a C^2 positive definite function V satisfying the conditions given in Theorem 2, then designing the PFC as a inverse system of the system stabilizing controller, one can obtain a OFSP augmented system.

5 Adaptive output feedback controller design

Assumption 4 (1) $g(x(k))$ is bounded for all $x(k)$.
(2) There exists a known PFC given in (37) such that the resulting augmented system is rendered OFSP with a static output feedback, that is the augmented system (38) satisfies the OFSP conditions in the Theorem 2.

The objective here is to design an adaptive output feedback control system under Assumption 4.

Under Assumption 4, (2), from Theorem 2 and Lemma 1, there exists a static output feedback:

$$u^*(k) = -\theta^* y_a(k) + v(k) \tag{45}$$

for the augmented system (38), such that the resulting closed loop system with the transformed signal $\bar{v}(k) = (1+\theta^* d)^{-1} v(k)$ as the input:

$$\begin{aligned} x_a(k+1) &= \bar{f}_a(x_a(k)) + g_a(x_a(k))\bar{v}(k) \\ y_a(k) &= \bar{h}_a(x_a(k)) + d\bar{v}(k) \end{aligned} \tag{46}$$

$$\bar{f}_a(x_a(k)) = f_a(x_a(k)) - \frac{\theta^*}{1+\theta^* d} h_a(x_a(k)) g_a(x(k)) \tag{47}$$

$$\bar{h}_a(x_a(k)) = \frac{1}{1+\theta^* d} \bar{y}(k), \ \bar{y}(k) = h_a(x_a(k)) \tag{48}$$

is strictly passive with a C^2 positive definite storage function.

Thus, if one can design a control input by

$$u^*(k) = -\theta^* y_a(k), \tag{49}$$

then a stable control system is obtained. However for a system with uncertainties, of course, θ^* is unknown, and because of the existence a direct feedthrough term of the input, the input (49) can not be implemented due to causality problems.

To overcome these problems, we first consider the following equivalent input obtained from (38):

$$u^*(k) = -\frac{\theta^*}{1+\theta^* d} \bar{y}(k) = -\tilde{\theta}^* \bar{y}(k), \tilde{\theta}^* = \frac{\theta^*}{1+\theta^* d}. \tag{50}$$

Then for this ideal control input, we design the control input adaptively as follows:

$$u(k) = -\tilde{\theta}(k)\bar{y}(k) \tag{51}$$

where the feedback gain $\tilde{\theta}(k)$ is adaptively adjusted by the following parameter adjusting law:

$$\tilde{\theta}(k)=\tilde{\theta}(k-1)+\gamma y_a(k)\bar{y}(k), \gamma>0. \tag{52}$$

In this case, the augmented output $y_a(k)$ can be obtained from (38) by

$$y_a(k)=\frac{\left(1-d\tilde{\theta}(k-1)\right)\bar{y}(k)}{1+d\gamma\bar{y}(k)^2} \tag{53}$$

without causality problems. It should be noted that if the controller is designed based on the input (49), then causality problems will appear.

5.1 Stability analysis

The obtained closed loop system with the input (51) is expressed by

$$\begin{aligned} x_a(k+1) &= \tilde{f}_a(x_a(k))+g_a(x_a(k))\Delta u(k) \\ y_a(k) &= \tilde{y}(k)+d\Delta u(k), \end{aligned} \tag{54}$$

where

$$\tilde{f}_a(x_a(k))=f_a(x_a(k))-\tilde{\theta}^*\bar{y}(k)g_a(x_a(k)) \tag{55}$$

$$\tilde{y}(k)=\left(1-d\tilde{\theta}^*\right)\bar{y}(k) \tag{56}$$

$$\Delta u(k)=-\Delta\tilde{\theta}(k)y(k), \Delta\tilde{\theta}(k)=\tilde{\theta}(k)-\tilde{\theta}^*. \tag{57}$$

From the definition of $\tilde{\theta}^*$, we have

$$\begin{aligned} \tilde{f}_a(x_a(k)) &= f_a(x_a(k))-\frac{\theta^*}{1+\theta^* d}\bar{y}(k)g_a(x_a(k)) \\ &= \bar{f}_a(x_a(k)) \end{aligned} \tag{58}$$

$$\begin{aligned} \tilde{y}(k) &= \left(1-\frac{\theta^* d}{1+\theta^* d}\right)\bar{y}(k)=\frac{1}{1+\theta^* d}\bar{y}(k) \\ &= \bar{h}_a(x_a(k)). \end{aligned} \tag{59}$$

This means that the system (54) is strictly passive with C^2 positive definite storage function.

Thus, there exists a C^2 positive definite function V_1, functions $l_1(x_a(k))$,$W_1(x_a(k))$, and a positive definite function $S_1(x_a(k))$ such that

C1)
$$V_1(\bar{f}_a(x_a))-V_1(x_a)=-l_1(x_a)^2-S_1(x_a)$$
$$\left.\frac{\partial V_1(\alpha)}{\partial\alpha}\right|_{\alpha=\bar{f}_a(x_a)} g(x)=\bar{h}_a(x_a)-2l_1(x_a)W_1(x_a)$$
$$g_a^T(x_a)\left.\frac{\partial^2 V_1(\alpha)}{\partial\alpha^2}\right|_{\alpha=\bar{f}_a(x_a)} g_a(x_a)=2d-2W_1(x_a)^2$$

C2) $V_1(\bar{f}_a(x_a)+g_a(x_a)\Delta u)$ is quadratic in Δu.

Therefore, considering the difference of $V_1(x_a(k))$, it is easy to show that we have

$$\begin{aligned} &V_1(x_a(k+1))-V_1(x_a(k)) \\ &\quad=y_a(k)\Delta u(k)-S_1(x_a(k)) \\ &\qquad-\left(l_1(x_a(k))+W_1(x_a(k))\Delta u(k)\right)^2. \end{aligned} \tag{60}$$

Now, consider the following positive definite function V:

$$V(k)=V_1(x_a(k))+V_2(k) \tag{61}$$

$$V_2(k)=\frac{1}{2\gamma}\Delta\tilde{\theta}(k-1)^2. \tag{62}$$

Define a difference $\Delta V(k)$ as

$$\begin{aligned} \Delta V(k) &= V(k+1)-V(k) \\ &= \Delta V_1(x_a(k))+\Delta V_2(k) \end{aligned} \tag{63}$$

$$\Delta V_1(x_a(k))=V_1(x_a(k+1))-V_1(x_a(k)) \tag{64}$$

$$\Delta V_2(k)=V_2(k+1)-V_2(k). \tag{65}$$

The difference $\Delta V_2(k)$ is represented by

$$\Delta V_2(k)=\frac{1}{2\gamma}\left(\Delta\tilde{\theta}(k)^2-\Delta\tilde{\theta}(k-1)^2\right). \tag{66}$$

Since we have from (52) that

$$\Delta\tilde{\theta}(k-1)=\Delta\tilde{\theta}(k)-\gamma y_a(k)\bar{y}(k), \tag{67}$$

we obtain

$$\Delta V_2(k)=-\Delta u(k)y_a(k)-\frac{1}{2}\gamma y_a(k)^2\bar{y}(k)^2. \tag{68}$$

Consequently, the difference ΔV can be evaluated from (60) and (68) by

$$\begin{aligned} \Delta V(k) &= -S_1(x_a(k))-\left(l_1(x_a(k))+W_1(x_a(k))\Delta u(k)\right)^2 \\ &\quad-\frac{1}{2}\gamma y_a(k)^2\bar{y}(k)^2 \\ &\le -S_1(x(k))\le 0. \end{aligned} \tag{69}$$

From this result, we can conclude that all the signals in the control system are uniformly bounded. Further, from (69), we have $\lim_{k\to\infty} x_a(k)=0$. Thus we obtain $\lim_{k\to\infty} y(k)=0$.

Finally, we have the following theorem.

Theorem 5 *Under the Assumption 4, all the signals in the resulting closed loop control system with control input in (51) are uniformly bounded, and* $\lim\limits_{k\to\infty} y(k)=0$ *is achieved.*

6 Conclusions

In this paper, we considered a passivity based adaptive output feedback control design for discrete-time nonlinear systems. We first clarified a discrete-time nonlinear version of Kalman-Yakubovich-Popov (KYP) Lemma for a strictly passive system, and then investigated the OFSP property of discrete-time nonlinear systems. Furthermore, conditions in which one can design an adaptive controller without causality problems were provided as strong output feedback strict passivity, and according to the obtained conditions, an adaptive output feedback controller design scheme was shown for a discrete-time nonlinear system.

A Appendix A: Proof of Theorem 1

(Necessity): If the system (1) is strictly passive, then there exist a non-negative function $V(x(k))$ and a positive definite function $S(x(k))$ such that

$$V(x(k+1)) - V(x(k)) \leq y(k)u(k) - S(x(k)) \tag{A.1}$$

Considering functions $l(x(k))$ and $W(x(k))$ to satisfy

$$\begin{aligned} &V(x(k+1)) - V(x(k)) \\ &= y(k)u(k) - S(x(k)) - (l(x(k)) + W(x(k))u(k))^2, \end{aligned} \tag{A.2}$$

we have

$$\begin{aligned} V(f(x)+g(x)u) = {} & V(x)+h(x)u+J(x)u^2-S(x)-l(x)^2 \\ & -2l(x)W(x)u-W(x)^2u^2. \end{aligned} \tag{A.3}$$

Setting $u(k) = 0$, (4) is obviously satisfied. Further, from $(A.3)$ we have

$$\begin{aligned} \frac{\partial V(f(x)+g(x)u)}{\partial u} &= \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=f(x)+g(x)u} g(x) \\ &= h(x) + 2J(x)u - 2l(x)W(x) \\ &\quad -2W(x)^2u, \end{aligned} \tag{A.4}$$

$$\begin{aligned} \frac{\partial^2 V(f(x)+g(x)u)}{\partial u^2} &= g^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=f(x)+g(x)u} g(x) \\ &= 2J(x) - 2W(x)^2. \end{aligned} \tag{A.5}$$

Setting $u = 0$ yields (5) and (6). A1-2) is obvious.

(Sufficiency): From A1-2), $V(f(x)+g(x)u)$ can be expressed as

$$V(f(x)+g(x)u) = A(x) + B(x)u + C(x)u^2 \tag{A.6}$$

Applying the Taylor expansion formula at $u(k) = 0$, we have from A1-1) that

$$\begin{aligned} A(x) &= V(f(x)+g(x)u)|_{u=0} = V(f(x)) \\ &= V(x) - l(x)^2 - S(x), \end{aligned} \tag{A.7}$$

$$\begin{aligned} B(x) &= \left.\frac{\partial V(f(x)+g(x)u)}{\partial u}\right|_{u=0} \\ &= \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=f(x)} g(x) \\ &= h(x) - 2l(x)W(x), \end{aligned} \tag{A.8}$$

$$\begin{aligned} C(x) &= \frac{1}{2}\left.\frac{\partial^2 V(f(x)+g(x)u)}{\partial u^2}\right|_{u=0} \\ &= \frac{1}{2} g^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=f(x)} g(x) \\ &= J(x) - W(x)^2. \end{aligned} \tag{A.9}$$

Thus we obtain

$$\begin{aligned} &V(f(x)+g(x)u) \\ &= V(x)+h(x)u+J(x)u^2-S(x)-l(x)^2 \\ &\quad -2l(x)W(x)u-W(x)^2u^2 \\ &= V(x)+yu-S(x)-(l(x)+W(x)u))^2. \end{aligned} \tag{A.10}$$

This yields that

$$V(x(k+1))-V(x(k)) \leq y(k)u(k)-S(x(k)). \tag{A.11}$$

Finally we can conclude that the system (1) with assumptions A1-1) and A1-2) is strictly passive.

B Appendix B: Proof of Lemma 1

Consider a system (1) with $J(x(k)) = d$ satisfying assumptions A2-1) to A2-3) in Theorem 2:

$$\begin{aligned} x(k+1) &= f(x(k)) + g(x(k))u(k) \\ y(k) &= h(x(k)) + du(k). \end{aligned} \tag{B.1}$$

From Theorem 2, there exists a static output feedback (8) such that the resulting closed loop system:

$$\begin{aligned} x(k+1) &= \bar{f}(x(k)) + \bar{g}(x(k))v(k) \\ y(k) &= \bar{h}(x(k)) + \bar{d}v(k) \end{aligned} \tag{B.2}$$

with

$$\begin{aligned} \bar{f}(x(k)) &= f(x(k)) - \frac{\theta^*}{1+\theta^* d} h(x(k))g(x(k)) \\ \bar{g}(x(k)) &= \frac{1}{1+\theta^* d} g(x(k)) \\ \bar{h}(x(k)) &= \frac{1}{1+\theta^* d} h(x(k)), \bar{d} = \frac{1}{1+\theta^* d} d \end{aligned}$$

is strictly passive with a C^2 positive definite storage function. Thus from Theorem 1, there exist a C^2 positive definite function $V(x(k))$, functions $\bar{l}(x(k))$, $\bar{W}(x(k))$ and a positive definite function $\bar{S}(x(k))$ such that

$$V(\bar{f}(x)) - V(x) = -\bar{l}(x)^2 - \bar{S}(x) \tag{B.3}$$

$$\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) = \bar{h}(x) - 2\bar{l}(x)\bar{W}(x) \tag{B.4}$$

$$\bar{g}^T(x) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x)} \bar{g}(x) = 2\bar{d} - 2\bar{W}(x)^2 \tag{B.5}$$

and $V(\bar{f}(x) + \bar{g}(x)v)$ is quadratic in v. In other words, the following equality is satisfied.

$$\begin{aligned} &V(x(k+1)) - V(x(k)) \\ &\quad = y(k)v(k) - \bar{S}(x(k)) - \left(\bar{l}(x(k)) + \bar{W}(x(k))v(k)\right)^2. \end{aligned} \tag{B.6}$$

Considering the transformed input:

$$\bar{v}(k) = \frac{1}{1+\theta^* d} v(k), \tag{B.7}$$

$(B.6)$ can be represented by

$$\begin{aligned} &V(x(k+1)) - V(x(k)) \\ &\quad = y(k)\,(1+\theta^* d)\,\bar{v}(k) - \bar{S}(x(k)) \\ &\qquad - \left(\bar{l}(x(k)) + \bar{W}(x(k))\,(1+\theta^* d)\,\bar{v}(k)\right)^2. \end{aligned} \tag{B.8}$$

Thus we have

$$\begin{aligned} &\bar{V}(x(k+1)) - \bar{V}(x(k)) \\ &= y(k)\bar{v}(k) - \tilde{S}(x(k)) - \left(\tilde{l}(x(k)) + \tilde{W}(x(k))\bar{v}(k)\right)^2 \end{aligned} \tag{B.9}$$

where

$$\bar{V}(x(k)) = \frac{1}{1+\theta^* d} V(x(k)) \tag{B.10}$$

$$\tilde{S}(x(k)) = \frac{1}{1+\theta^* d} \bar{S}(x(k)) \tag{B.11}$$

$$\tilde{l}(x(k)) = \frac{1}{\sqrt{1+\theta^* d}} \bar{l}(x(k)) \tag{B.12}$$

$$\tilde{W}(x(k)) = \sqrt{1+\theta^* d}\,\bar{W}(x(k)). \tag{B.13}$$

This means that the system with the transformed input $\bar{v}$:

$$\begin{aligned} x(k+1) &= \bar{f}(x(k)) + g(x(k))\bar{v}(k) \\ y(k) &= \bar{h}(x(k)) + d\bar{v}(k) \end{aligned} \tag{B.14}$$

is strictly_passive with a C^2 positive definite storage function $\bar{V}$.

References

Byrnes, C.I., A. Isidori and J. C. Willems (1991). Passivity feedback equivalence and the global stabilization of minimum phase nonlinear systems. *IEEE Trans. on Automatic Control* **36**(11), 1228–1240.

Byrnes, C.I. and W. Lin (1994). Losslessness feedback equivalence and the global stabilization of discrete-time nonlinear systems. *IEEE Trans. on Automatic Control* **39**(1), 83–98.

Chellaboina, V. and W.M. Haddad (2002). Stability margins of discrete-time nonlinear-non-quadratic optimal regulations. *Int. J. of Systems Science* **33**(7), 577–584.

Fradkov, A. and D. J. Hill (1998). Exponential feedback passivity and stabilizability of nonlinear systems. *Automatica* **34**(6), 697–703.

Hayakawa, T., W.M. Haddad and A. Leonessa (2004). A lyapunov-based adaptive control framework for discrete-time non-linear systems with exogenous disturbances. *Int. J. Control* **77**(3), 250–263.

Hill, D. J. and P. Moylan (1998). The stability of nonlinear dissipative systems. *IEEE Trans. on Automatic Control* **21**(10), 708–711.

Jiang, Z.-P. and D. J. Hill (1998). Passivity and disturbance attenuation via output feedback for uncertain nonlinear systems. *IEEE Trans. on Automatic Control* **43**(7), 992–997.

Krstic, M., I. Kanellakopoulos and P. V. Kokotovic (1994). Passivity and parametric robustness of a new class of adaptive systems. *Automatica* **30**(11), 1703–1716.

Lin, W. and C.I. Byrnes (1995). Passivity and absolute stabilization of a class of discrete-time nonlinear systems. *Automatica* **31**(2), 263–267.

Michino, R., I. Mizumoto, Z. Iwai and M. Kumon (2003). Robust high gain adaptive output feedback control for nonlinear systems with uncertain nonlinearities in control input term. *Int. J. of Control, Automation, and Systems* **1**(1), 19–27.

Mizumoto, I., R. Michino, M. Takahashi, M. Kumon and Z. Iwai (2005). Adaptive output feedback control of uncertain MIMO nonlinear systems with unknown orders. *Proc. of 16th IFAC World Congress, Prague, July* p. DVD.

Mizumotoa, I., T. Chen, S. Ohdairaa, M. Kumona and Z. Iwaia (2007). Adaptive output feedback control of general mimo systems using multirate sampling and its application to a cart-crane system. *Automatica* **43**(12), 2077–2085.

Monaco, S. and D. Normand-Cyrot (1997). On the conditions of passivity and losslessness in discrete time. *Proc. of the Europian Control Conference 1997.*

Monaco, S., D. Normand-Cyrot and Femando Tiefensee (2008). From passivity under sampling to a new discrete-time passive concept. *Proc. of the 47th IEEE Conference 0n Decision and Control* pp. 3157–3162.

Navarro-Lopez, E. M. (2007). Qss-dissipativity and feedback qs-passivity of nonlunear discrete-time systems. *Dynamics of continuous, Discrete and Impulsive Systems, Series B* **14**(1), 47–63.

Navarro-Lopez, E. M., H. Sira-Ramirez and E. Fossas-Colet (2002). Dissipativity and feedback dissipativity properties of general nonlinear discrete-time systems. *Europian Journal of Control* **8**, 265–274.