Distributed Controller-Estimator for Target Tracking of Networked Robotic Systems under Sampled Interaction *

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Abstract

This paper investigates the target tracking problem for networked robotic systems (NRSs) under sampled interaction. The target is assumed to be time-varying and described by a second-order oscillator. Two novel distributed controller-estimator algorithms (DCEA), which consist of both continuous and discontinuous signals, are presented. Based on the properties of small-value norms and Lyapunov stability theory, the conditions on the interaction topology, the sampling period, and the other control parameters are given such that the practical stability of the tracking error is achieved and the stability region is regulated quantitatively. The advantages of the presented DCEA are illustrated by comparisons with each other and the existing coordination algorithms. Simulation examples are given to demonstrate the theoretical results.

Key words: Target tracking; Networked robotic systems(NRSs); Sampled interaction; Distributed controller-estimator algorithms(DCEA); Small-value norm.

1 Introduction

Numerous real-world applications have required a group of interconnected robots (networked robotic system, NRS) to accomplish one or several global tasks cooperatively, for instance, assembly of heterogeneous robots, trajectory tracking of networked mobile robots, operation of multi-fingered hands, management of intelligent highways [1–5]. Distributed algorithms have been widely invoked in these applications due to their advantages, including strong robustness, low consumptions and high efficiency [6,7]. Distributed consensus for NRSs under continuous interaction has been studied in [9, 10]. Consensus tracking of a constant value of NRSs under undirected continuous interaction has been investigated in [11]. Consensus tracking of time-varying trajectories of NRSs under continuous interaction has been studied in [12,13]. Note that the aforementioned literatures have focused on continuous interaction, which leads to higher interaction cost comparing with sampled interaction.

Since sampling operation is one of the inevitable steps to realize digital interaction in practical applications, the effect of sampled interaction for networked systems has been well studied recently, see [14,15] and references therein. Impulsive control is an efficient technology in coordination of networked systems under sampled interaction [16, 17]. Comparing with continuous control, impulsive control has shown satisfying performance and prominent superiorities, including faster transient, less cost, lower computation, more flexible design [18,19,25]. However, the existing schemes cannot be directly applied to NRSs due to their inherent characteristics, including strong nonlinearity, tight coupling, complex construction and fragility to chattering.

Motivated by the above discussions, two novel DCEA, consist of PD-like controllers and sampled-data estimators, are given for target tracking of NRSs under sampled interaction. The main contributions are summarized as follows. 1) Comparing with the coordination algorithms for NRSs under continuous interaction [10–13], we focus on sampled interaction, which can prominently reduce the interaction cost. 2) Comparing with the sampled-data coordination algorithms for single- and double-integrator networks with a constant agreement value [14–19], we develop sampled-data coordination algorithms for NRSs with a time-varying target. 3) The presented DCEA provide a theoretical guidance for

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sampled-data control and estimation of many physical networks with complex and strong nonlinear dynamics.

Notations: \mathbb{Z}^{\dagger} , \mathbb{C} and \mathbb{R} are the sets of positive integers, complex numbers and real numbers, respectively. $\|\cdot\|_1$, $\|\cdot\|_2$ and $\|\cdot\|_{\infty}$ are the 1-norm, the 2-norm and the ∞ -norm, respectively. $\mathbf{1} = \operatorname{col}[1, \cdots, 1]$ and $\mathbf{0} = \operatorname{col}[0, \cdots, 0]$ are the column vectors of proper dimensions, respectively. I_p is the identity matrix of order p. Re(\cdot) and Im(\cdot) are the real part and the imaginary part of a complex number, respectively. $\lambda_{\min}(\cdot)$, $\lambda_{\max}(\cdot)$, $\sigma(\cdot)$ and det(\cdot) denote the minimum eigenvalue, the maximum eigenvalue, the spectrum and the determinant of a matrix, respectively. We say that $\mathcal{A} \in \Omega^{p \times p}$ if the eigenvalues of \mathcal{A} lie within the open unit disc, $\forall \mathcal{A} \in \mathbb{C}^{p \times p}$. We say that $\eta_1 \in \mathcal{U}(\eta_2; \delta)$ if and only if $\|\eta_1 - \eta_2\|_2 \leq \delta$, $\forall \delta > 0, \eta_1, \eta_2 \in \mathbb{R}^p$.

2 Problem Formulation and Preliminaries

2.1 Dynamic Models and Control Problem

Following [20], a NRS is described by

$$\mathcal{M}_{i}\left(q_{i}\right)\ddot{q}_{i} + \mathcal{C}_{i}\left(\dot{q}_{i}, q_{i}\right)\dot{q}_{i} + \mathcal{G}_{i}\left(q_{i}\right) = \tau_{i} + \tau_{id}, \quad (1)$$

where $q_i(t)$, $\tau_i(t)$ and $\tau_{id}(t)$ are abbreviated to q_i , τ_i and τ_{id} , respectively, $t \in \mathcal{Q} = [t_0, +\infty)$, $t_0 \geq 0$ is the initial time, $i \in \mathcal{I} = \{1, \dots, n\}$, $q_i, \dot{q}_i, \ddot{q}_i \in \mathbb{R}^m$ are the vectors of position, velocity and acceleration, respectively, $\mathcal{M}_i(q_i), \mathcal{C}_i(\dot{q}_i, q_i) \in \mathbb{R}^{m \times m}$ are the inertia matrix and the centripetal-Coriolis matrix, respectively, $\mathcal{G}_i(q_i), \tau_i, \tau_{id} \in \mathbb{R}^m$ are the vectors of gravity, input and disturbance, respectively.

The target is described by a second-order oscillator

$$\dot{\varepsilon}_0(t) = v_0(t), \ \dot{v}_0(t) = a_0(t),$$

where $\varepsilon_0, v_0, a_0 \in \mathbb{R}^m$ denote the vectors of position, velocity and acceleration, respectively.

The control problem is to develop proper input τ_i for target tracking of robot $i(\forall i \in \mathcal{I})$ with its own states and the sampled data of its neighbours, *i.e.*, for any $i \in \mathcal{I}$,

$$\lim_{t \to \infty} q_i(t) \in \mathcal{U}\left(\varepsilon_0; \rho_1\right), \quad \lim_{t \to \infty} \dot{q}_i(t) \in \mathcal{U}\left(\upsilon_0; \rho_2\right),$$

where $\rho_1, \rho_2 > 0$ can be sufficiently small by choosing appropriate parameters for the DCEA designed later.

The Euler-Lagrange system (1) is a suitable description for many NRSs, including multiple manipulators, multi-fingered hands and networked mobile robots, to name a few [21]. Then the following properties for Euler-Lagrange systems are introduced [22].

- (P1) $\mathcal{M}_i(q_i)$ is symmetric and positive definite;
- (P2) $\mathcal{M}_i(q_i) 2\mathcal{C}_i(\dot{q}_i, q_i)$ is skew-symmetric;
- (P3) $\lambda_{im} \leq \|\mathcal{M}_i(q_i)\|_2 \leq \lambda_{iM}, \|\mathcal{C}_i(\eta, q_i)\|_2 \leq \lambda_{ic} \|\eta\|_2, \\ \|\mathcal{G}_i(q_i)\|_2 \leq \lambda_{ig}, \|\tau_{id}\|_2 \leq \lambda_{id}, \forall \eta, q_i \in \mathbb{R}^m, \text{ where } \\ \lambda_{im}, \lambda_{iM}, \lambda_{ic}, \lambda_{ig}, \lambda_{id} > 0 \text{ are positive constants.}$

2.2 Graph Theory and Lemmas

Let $\mathcal{J} = \{0, 1, \cdots, n\} \supset \mathcal{I}$, where node 0 is the target, node *i* is robot *i*. The NRS interaction is denoted by a digraph $\Im = \{\mathcal{J}, \mathcal{E}, \mathcal{W}\}$ with edge set $\mathcal{E} \subseteq \mathcal{J} \times \mathcal{J}$. An edge $\{j, i\} \in \mathcal{E}$ means node *i* can access the information of node *j* directly, but not vice versa. $\mathcal{W} = [w_{ij}]_{(n+1)\times(n+1)}$ is the adjacency matrix, where $w_{ii} = 0$; $w_{ij} > 0 \Leftrightarrow$ $\{j, i\} \in \mathcal{E}; w_{ij} = 0$ otherwise, $\forall i, j \in \mathcal{J}$. A directed path is a finite ordered sequence $\{i_1, i_2\}, \{i_2, i_3\}, \cdots$, in a digraph. A digraph contains a spanning tree means that there exists a root node that has a directed path to the other nodes. Node 0 (the target) is the root node. Let $\hat{\mathcal{W}} = [w_{ij}]_{n \times n}, \zeta = \operatorname{col}(w_{10}, \cdots, w_{n0}), \varpi_i = \sum_{\iota \in \mathcal{J}} w_{i\iota},$ $\mathcal{B} = \operatorname{diag}(\varpi_1, \cdots, \varpi_n), \mathcal{D} = \mathcal{B}^{-1}\hat{\mathcal{W}}, \forall i, j \in \mathcal{I}.$

The NRS interaction only occurs at the sampling time t_k and is thus called sampled interaction. The sampling time sequence $\{t_1, \dots, t_k, \dots\}$ satisfies $t_0 < t_1 < \dots < t_k < \dots, \lim_{k \to +\infty} t_k = +\infty, t_k - t_{k-1} = h \ (\forall k \in \mathbb{Z}^{\dagger}),$ where h > 0 is the sampling period. Then the following assumptions are made.

- (A1) $||v_0(t)||_{\infty} \leq \gamma_1, ||a_0(t)||_{\infty} \leq \gamma_2, \forall t \in \mathcal{Q}$, where $\gamma_1, \gamma_2 > 0$ are positive constants;
- (A2) The digraph \Im contains a spanning tree at each sampling time $t_k, \forall k \in \mathbb{Z}^{\dagger}$.

Definition 1 For any $\mathcal{A} \in \Omega^{p \times p}$, a matrix norm $\|\cdot\|_{\mathcal{A}}$ is called small-value norm if $\|\mathcal{A}\|_{\mathcal{A}} < 1$. Additionally, a vector norm $\|\cdot\|_{\mathcal{A}}$ defined as $\|\eta\|_{\mathcal{A}} = \|\eta \mathbf{1}^T\|_{\mathcal{A}} (\forall \eta \in \mathbb{R}^p)$ is also called small-value norm.

Lemma 1 [14] Suppose that Assumption A2 holds. \mathcal{B} is invertible, $\|\mathcal{D}^n\|_{\infty} < 1$, i.e., $\mathcal{D} \in \Omega^{n \times n}$.

Lemma 2 For any $\mathcal{A} \in \Omega^{p \times p}$, there exist corresponding small-value norms $\|\cdot\|_{\mathcal{A}}$, including a matrix norm and a vector norm. Let $\eta_1, \eta_2 \in \mathbb{R}^p$ and $\mathcal{H} \in \mathbb{R}^{p \times p}$. Then $\|\cdot\|_{\mathcal{A}}$ satisfies: 1) $\|\mathcal{H}\eta_1\|_{\mathcal{A}} \leq \|\mathcal{H}\|_{\mathcal{A}} \|\eta_1\|_{\mathcal{A}}$; 2) $\|\eta_1 + \eta_2\|_{\mathcal{A}} \leq$ $\|\eta_1\|_{\mathcal{A}} + \|\eta_2\|_{\mathcal{A}}$; 3) the vector norm $\|\cdot\|_{\mathcal{A}}$ is equivalent to $\|\cdot\|_2$ and $\|\cdot\|_{\infty}$.

3 Target Tracking over Sampled Interaction

3.1 DISTRIBUTED CONTROLLER-ESTIMATOR ALGO-RITHMS

Let ε_i and v_i be the estimate value of ε_0 and v_0 for robot $i, \forall i \in \mathcal{I}$. The first-order DCEA is given as

$$\begin{cases} \tau_i = \mathcal{K}_{ip}(\varepsilon_i - q_i) + \mathcal{K}_{id}(\upsilon_i - \dot{q}_i), \\ \dot{\varepsilon}_i = \upsilon_i, \quad \dot{\upsilon}_i = 0, \quad t \in (t_{k-1}, t_k], \\ \Delta \varepsilon_i = \alpha \sum_{j \in \mathcal{J}} \frac{w_{ij}}{\varpi_i} \left(\varepsilon_j - \varepsilon_i\right), \quad t = t_k, \\ \Delta \upsilon_i = \beta \sum_{j \in \mathcal{J}} \frac{w_{ij}}{\varpi_i} \left(\upsilon_j - \upsilon_i\right), \quad t = t_k, \end{cases}$$
(2)

and the second-order DCEA is given as

$$\begin{cases} \tau_{i} = \mathcal{K}_{ip}(\varepsilon_{i} - q_{i}) + \mathcal{K}_{id}(\upsilon_{i} - \dot{q}_{i}), \\ \dot{\varepsilon}_{i} = \upsilon_{i}, \quad \dot{\upsilon}_{i} = 0, \qquad t \in (t_{k-1}, t_{k}], \\ \Delta \upsilon_{i} = \alpha \sum_{j \in \mathcal{J}} \frac{w_{ij}}{\varpi_{i}} (\varepsilon_{j} - \varepsilon_{i}) \\ + \beta \sum_{j \in \mathcal{J}} \frac{w_{ij}}{\varpi_{i}} (\upsilon_{j} - \upsilon_{i}), \quad t = t_{k}, \end{cases}$$
(3)

where $\alpha, \beta > 0, \mathcal{K}_{ip}, \mathcal{K}_{id}$ are positive definite matrices, $\Delta \varepsilon_i(t_k) = \varepsilon_i(t_k^+) - \varepsilon_i(t_k), \ \varepsilon_i(t_k^+) = \lim_{\sigma \to 0^+} \varepsilon_i(t_k + \sigma),$ and $\Delta v_i(t_k)$ is defined analogously. It is assumed that $\varepsilon_i(t)$ and $v_i(t)$ are left continuous at $t = t_k, \forall k \in \mathbb{Z}^{\dagger}.$

Remark 1 Note that ε_i and v_i jump at each sampling time instant t_k in the first-order DCEA, while only v_i jumps in the second-order DCEA. Thus, the trajectories of q_i and ε_i for the NRS under the second-order DCEA are smoother than that under the first-order DCEA.

3.2 Analysis of First-order Algorithm

Substituting DCEA (2) into NRS (1) gives that

$$\begin{cases} \mathcal{M}_{i}\left(q_{i}\right)\ddot{q}_{i} + \mathcal{C}_{i}\left(\dot{q}_{i}, q_{i}\right)\dot{q}_{i} + \mathcal{G}_{i}\left(q_{i}\right) \\ = \mathcal{K}_{ip}(\varepsilon_{i} - q_{i}) + \mathcal{K}_{id}(\upsilon_{i} - \dot{q}_{i}) + \tau_{id}, \\ \varepsilon_{i}\left(t_{k+1}\right) = \varepsilon_{i}(t_{k}^{+}) + h\upsilon_{i}(t_{k}^{+}), \\ \upsilon_{i}\left(t_{k+1}\right) = \upsilon_{i}(t_{k}^{+}), \\ \varepsilon_{i}(t_{k}^{+}) = \varepsilon_{i}(t_{k}) + \alpha \sum_{j \in \mathcal{J}} \frac{w_{ij}}{\varpi_{i}}\left(\varepsilon_{j}\left(t_{k}\right) - \varepsilon_{i}\left(t_{k}\right)\right), \\ \upsilon_{i}(t_{k}^{+}) = \upsilon_{i}(t_{k}) + \beta \sum_{j \in \mathcal{J}} \frac{w_{ij}}{\varpi_{i}}\left(\upsilon_{j}\left(t_{k}\right) - \upsilon_{i}\left(t_{k}\right)\right), \end{cases}$$

where $\varepsilon_{i}(t_{1}) = \varepsilon_{i}(t_{0}) + h\upsilon_{i}(t_{0}), \upsilon_{i}(t_{1}) = \upsilon_{i}(t_{0}), i \in \mathcal{I}, k \in \mathbb{Z}^{\dagger}.$

Let $e_i = q_i - \varepsilon_0$, $\dot{e}_i = \dot{q}_i - v_0$, $\bar{\varepsilon}_i = \varepsilon_i - \varepsilon_0$, $\bar{v}_i = v_i - v_0$, $\bar{\varepsilon} = \operatorname{col}(\bar{\varepsilon}_1, \cdots, \bar{\varepsilon}_n)$, $\bar{v} = \operatorname{col}(\bar{v}_1, \cdots, \bar{v}_n)$, $x = \operatorname{col}(\bar{\varepsilon}, \bar{v})$, $\forall i \in \mathcal{I}$. Then system (4) becomes

$$\begin{cases} \mathcal{M}_i(q_i)\ddot{e}_i + \mathcal{C}_i(\dot{q}_i, q_i)\dot{e}_i = \phi_i(t) - \mathcal{K}_{ip}e_i - \mathcal{K}_{id}\dot{e}_i, \\ x(t_{k+1}) = \Lambda x(t_k) + \Delta(k), \end{cases}$$

where $\Delta(k) = \operatorname{col}[\mathbf{1} \otimes \Delta_1(k), \mathbf{1} \otimes \Delta_2(k)], \otimes \operatorname{denotes the} Kronecker product, \Delta_1(k) = \varepsilon_0(t_k) - \varepsilon_0(t_{k+1}) + hv_0(t_k), \Delta_2(k) = v_0(t_k) - v_0(t_{k+1}), \phi_i(t) = \tau_{id} - \mathcal{M}_i(q_i) a_0 - \mathcal{C}_i(\dot{q}_i, q_i) v_0 - \mathcal{G}_i(q_i) + \mathcal{K}_{ip} \bar{\varepsilon}_i(t) + \mathcal{K}_{id} \bar{v}_i(t),$

$$\Lambda = \begin{bmatrix} (1-\alpha)I_n + \alpha \mathcal{D} \ h[(1-\beta)I_n + \beta \mathcal{D}] \\ 0 \qquad (1-\beta)I_n + \beta \mathcal{D} \end{bmatrix} \otimes I_m.$$

Then the solution of the control problem is to analyze the asymptotic behaviors of the states of system (5). Let $s_i \in \mathbb{C}$ be the *i*-th $(i = 1, \dots, n)$ eigenvalue of \mathcal{D} ,

$$\kappa_1 = b_1 c_1 \max(2\gamma_1; \gamma_2) (1 - \|\Lambda\|_{\Lambda})^{-1}, \kappa_2 = b_2 c_2 \gamma_2 (1 - \|\mathcal{P}\|_{\mathcal{P}})^{-1},$$
(6)

where $\mathcal{P} = (1-\beta)I_{mn} + \beta \mathcal{D} \otimes I_m$, $\|\cdot\|_{\Lambda}$ and $\|\cdot\|_{\mathcal{P}}$ are smallvalue norms, b_1, c_1, b_2, c_2 are positive constants such that $\|\eta\|_{\infty} \leq b_1 \|\eta\|_{\Lambda}, \|\eta\|_{\Lambda} \leq c_1 \|\eta\|_{\infty}, \|\eta\|_{\infty} \leq b_2 \|\eta\|_{\mathcal{P}},$ $\|\eta\|_{\mathcal{P}} \leq c_2 \|\eta\|_{\infty}$. Then the convergence analysis of $\bar{\varepsilon}_i$ and \bar{v}_i is given in Theorem 1 proved in Appendix A.

Theorem 1 Suppose that Assumptions A1-A2 hold. If

$$0 < \alpha, \beta < \min_{s_i \in \sigma(\mathcal{D})} \frac{2 - 2Re(s_i)}{\left|1 - s_i\right|^2},\tag{7}$$

then DCEA (2) implies that for any $i \in \mathcal{I}$,

$$\begin{cases} \lim_{t \to \infty} \|\bar{\varepsilon}_i\|_{\infty} \leq \delta_1, \\ \lim_{t \to \infty} \|\bar{\upsilon}_i\|_{\infty} \leq \delta_2, \\ \delta_1 = h\kappa_1, \\ \delta_2 = h\min(\kappa_1; \kappa_2), \end{cases}$$
(8)

where $|\cdot|$ denotes the modulus, h is the sampling period, κ_1 and κ_2 are given by (6).

Remark 2 δ_1 and δ_2 are proportional to the sampling period h, which means δ_1 and δ_2 can be sufficiently small by choosing h small enough and the formulas of $\|\cdot\|_{\Lambda}$, $\|\cdot\|_{\mathcal{P}}$ are not necessarily known for regulating δ_1 and δ_2 .

Next based on Theorem 1, the convergence of system (5) is studied. Assumption A1 and Property P3 implies

$$\|\phi_i(t)\|_2 \le \mu_{1i} + \mu_{2i} \|\dot{e}_i\|_2 + \mu_{3i}(t), \ \forall i \in \mathcal{I}, \quad (9)$$

where $\mu_{1i}, \mu_{2i} > 0$ are positive constants, $\mu_{3i}(t) = \lambda_{\max}(\mathcal{K}_{ip}) \|\bar{\varepsilon}_i\|_2 + \lambda_{\max}(\mathcal{K}_{id}) \|\bar{\upsilon}_i\|_2$. For the first subsystem of (5), consider the Lyapunov function candidate

$$\mathcal{V}_{i} = \frac{1}{2} \dot{e}_{i}^{T} \mathcal{M}_{i}\left(q_{i}\right) \dot{e}_{i} + \dot{e}_{i}^{T} \mathcal{M}_{i}\left(q_{i}\right) \tanh\left(e_{i}\right) + \frac{1}{2} e_{i}^{T} \mathcal{K}_{ip} e_{i} + \left\|\mathcal{K}_{id} \ln(\cosh(e_{i}))\right\|_{1},$$
(10)

where $tanh(\cdot), ln(\cdot), cosh(\cdot)$ are the function vectors of hyperbolic tangent, natural logarithm and hyperbolic cosine, respectively. Note that

$$\begin{aligned} &\frac{1}{4}\dot{e}_{i}^{T}\mathcal{M}_{i}\left(q_{i}\right)\dot{e}_{i}+\dot{e}_{i}^{T}\mathcal{M}_{i}\left(q_{i}\right)\tanh\left(e_{i}\right)+\frac{1}{2}e_{i}^{T}\mathcal{K}_{ip}e_{i}\\ &=\frac{1}{4}\left(\dot{e}_{i}+2\tanh\left(e_{i}\right)\right)^{T}\mathcal{M}_{i}\left(q_{i}\right)\left(\dot{e}_{i}+2\tanh\left(e_{i}\right)\right)\\ &+\frac{1}{2}e_{i}^{T}\mathcal{K}_{ip}e_{i}-\tanh^{T}\left(e_{i}\right)\mathcal{M}_{i}\left(q_{i}\right)\tanh\left(e_{i}\right)\\ &\geq\frac{1}{2}\left(\lambda_{\min}\left(\mathcal{K}_{ip}\right)-2\lambda_{iM}\right)e_{i}^{T}e_{i},\end{aligned}$$

where λ_{iM} is given in Property P3. Then by (10),

$$\mathcal{V}_{i}\left(e_{i}, \dot{e}_{i}\right) \geq \frac{1}{4} \dot{e}_{i}^{T} \mathcal{M}_{i}\left(q_{i}\right) \dot{e}_{i} + \left\|\mathcal{K}_{id} \ln(\cosh(e_{i}))\right\|_{1} \\ + \frac{1}{2} \left(\lambda_{\min}\left(\mathcal{K}_{ip}\right) - 2\lambda_{iM}\right) e_{i}^{T} e_{i}.$$

Thus $\lambda_{\min}(\mathcal{K}_{ip}) \geq 2\lambda_{iM}$ means that \mathcal{V}_i is positive definite. By (5) and Properties P1-P2, the derivative of \mathcal{V}_i is

$$\begin{aligned} \dot{\mathcal{V}}_{i} &= \phi_{i}^{T}(t) \left(\dot{e}_{i} + \tanh\left(e_{i} \right) \right) + \dot{e}_{i}^{T} \mathcal{M}_{i} \left(q_{i} \right) \operatorname{sech}^{2} \left(e_{i} \right) \dot{e}_{i} \\ &+ \dot{e}_{i}^{T} \mathcal{C}_{i} \left(\dot{q}_{i}, q_{i} \right) \tanh\left(e_{i} \right) - e_{i}^{T} \mathcal{K}_{ip} \tanh\left(e_{i} \right) - \dot{e}_{i}^{T} \mathcal{K}_{id} \dot{e}_{i}, \end{aligned}$$

where $\operatorname{sech}(\cdot)$ is the function vector of hyperbolic secant. Since $\|\tanh(e_i)\|_{\infty} \leq 1$ and $\lambda_{\max}(\operatorname{sech}(e_i)) = 1$, by (9),

$$\begin{aligned} \dot{\mathcal{V}}_{i} &\leq -\left[\left(\lambda_{\min} \left(\mathcal{K}_{id} \right) - \varrho_{2i} \right) \| \dot{e}_{i} \|_{2} - \varrho_{3i}(t) \right] \| \dot{e}_{i} \|_{2} \\ &- \left[\left(\lambda_{\min} \left(\mathcal{K}_{ip} \right) - \varrho_{1i} \right) \| e_{i} \|_{2} - \varrho_{3i}(t) \right] \\ &\times \| \tanh(e_{i}) \|_{2}, \end{aligned}$$
(11)

where $\varrho_{1i}, \varrho_{2i} > 0$ can be easily computed from (9), $\varrho_{3i}(t) = \mu_{1i} + \mu_{3i}(t).$

Theorem 2 Suppose that Assumptions A1-A2 hold. Using DCEA (2) for (1), if $\lambda_{\min}(\mathcal{K}_{ip}) > \max(2\lambda_{iM}; \varrho_{1i})$, $\lambda_{\min}(\mathcal{K}_{id}) > \varrho_{2i}$ and (7) hold, then the control problem in this paper is solved, i.e., for any $i \in \mathcal{I}$,

$$\lim_{t \to \infty} q_i \in \mathcal{U}\left(\varepsilon_0; \delta_3\right), \quad \lim_{t \to \infty} \dot{q}_i \in \mathcal{U}\left(\upsilon_0; \delta_4\right), \\ \delta_3 = \max_{i \in \mathcal{I}} \frac{\mu_{1i} + \sqrt{m} [\delta_1 \lambda_{\max}(\mathcal{K}_{ip}) + \delta_2 \lambda_{\max}(\mathcal{K}_{id})]}{\lambda_{\min}(\mathcal{K}_{ip}) - \varrho_{1i}}, \qquad (12)$$
$$\delta_4 = \max_{i \in \mathcal{I}} \frac{\mu_{1i} + \sqrt{m} [\delta_1 \lambda_{\max}(\mathcal{K}_{ip}) + \delta_2 \lambda_{\max}(\mathcal{K}_{id})]}{\lambda_{\min}(\mathcal{K}_{id}) - \varrho_{2i}},$$

where δ_1 and δ_2 are presented in (8).

Proof. First $\lambda_{\min}(\mathcal{K}_{ip}) \geq 2\lambda_{iM}$ ensures the positivedefiniteness of \mathcal{V}_i . Thus, if $\lambda_{\min}(\mathcal{K}_{ip}) > \varrho_{1i}$ and $\lambda_{\min}(\mathcal{K}_{id}) > \varrho_{2i}$, (11) implies

$$\lim_{t \to \infty} \|e_i(t)\|_2 \leq \frac{\varrho_{3i}(t)}{\lambda_{\min}(\mathcal{K}_{ip}) - \varrho_{1i}}$$
$$\lim_{t \to \infty} \|\dot{e}_i(t)\|_2 \leq \frac{\varrho_{3i}(t)}{\lambda_{\min}(\mathcal{K}_{id}) - \varrho_{2i}}$$

By Theorem 1, for any $i \in \mathcal{I}$,

$$\lim_{t \to \infty} \mu_{3i}(t)$$

= $\lambda_{\max}(\mathcal{K}_{ip}) \lim_{t \to \infty} \|\bar{\varepsilon}_i\|_2 + \lambda_{\max}(\mathcal{K}_{id}) \lim_{t \to \infty} \|\bar{\upsilon}_i\|_2$
 $\leq h\sqrt{m} [\kappa_1 \lambda_{\max}(\mathcal{K}_{ip}) + \min(\kappa_1; \kappa_2) \lambda_{\max}(\mathcal{K}_{id})].$

Considering $\rho_{3i}(t) = \mu_{1i} + \mu_{3i}(t)$, we conclude that $\lim_{t\to\infty} \|e_i\|_2 \leq \delta_3$ and $\lim_{t\to\infty} \|\dot{e}_i\|_2 \leq \delta_4$, $\forall i \in \mathcal{I}$. This completes the proof.

Suppose that (7) holds. By Theorem 2, for any $\delta_3, \delta_4 > 0$, if there exists a positive constant $\epsilon \in (0, 1)$ such that

$$\begin{cases} \lambda_{\min} \left(\mathcal{K}_{ip} \right) \geq \max \left(2\lambda_{iM}; \frac{\mu_{1i}}{\epsilon \delta_3} + \varrho_{1i} \right), \\ \lambda_{\min} \left(\mathcal{K}_{id} \right) \geq \frac{\mu_{1i}}{\epsilon \delta_4} + \varrho_{2i}, \\ h \leq \frac{\mu_{1i}(1-\epsilon)}{\epsilon \sqrt{m} [\kappa_1 \lambda_{\max}(\mathcal{K}_{ip}) + \min(\kappa_1; \kappa_2) \lambda_{\max}(\mathcal{K}_{id})]}, \end{cases}$$
(13)

then $\lim_{t\to\infty} q_i \in \mathcal{U}(\varepsilon_0; \delta_3)$ and $\lim_{t\to\infty} \dot{q}_i \in \mathcal{U}(v_0; \delta_4)$.

Remark 3 (13) means $\mu_{1i}, \mu_{2i}, \varrho_{1i}, \varrho_{2i}$ are not necessarily known for regulating δ_3 and δ_4 . δ_3 and δ_4 can be sufficiently small by choosing $\lambda_{\min}(\mathcal{K}_{ip}), \lambda_{\min}(\mathcal{K}_{id})$ large enough and h small enough. However, smaller $\lambda_{\min}(\mathcal{K}_{ip})$ and $\lambda_{\min}(\mathcal{K}_{id})$ means h can be larger, the input cost and interaction consumption can be lower. The parameter tuning strategy can be obtained by the trial-and-error methods [26].

3.3 Analysis of Second-order Algorithm

Substituting DCEA (3) into (1) gives

$$\begin{cases} \mathcal{M}_i(q_i)\ddot{e}_i + \mathcal{C}_i(\dot{q}_i, q_i)\dot{e}_i + \mathcal{K}_{ip}e_i + \mathcal{K}_{id}\dot{e}_i = \phi_i, \\ x(t_{k+1}) = \Gamma x(t_k) + \Delta(k), \end{cases}$$
(14)

where $x(t_1)$, $\Delta(k)$, $\phi_i(t)$ are given after (5), and

$$\Gamma = \begin{bmatrix} (1 - \alpha h)I_n + \alpha h\mathcal{D} \ h[(1 - \beta)I_n + \beta\mathcal{D}] \\ -\alpha I_n + \alpha\mathcal{D} \ (1 - \beta)I_n + \beta\mathcal{D} \end{bmatrix} \otimes I_m.$$

Let $\theta_i = \operatorname{Re}\left(2/[1-s_i]\right), \ \vartheta_i = \operatorname{Im}\left(2/[1-s_i]\right) \text{ and } \kappa_3 = b_3c_3 \max(2\gamma_1, \gamma_2)(1-\|\Gamma\|_{\Gamma})^{-1}, \text{ where } \|\cdot\|_{\Gamma} \text{ is the small-value norm, } b_3, c_3 > 0 \text{ satisfy } \|\eta\|_{\infty} \leq b_3 \|\eta\|_{\Gamma}, \|\eta\|_{\Gamma} \leq c_3 \|\eta\|_{\infty}, \ \forall \eta \in \mathbb{R}^{2mn}.$

Theorem 3 Suppose that Assumptions A1-A2 hold. Using DCEA (3) for (1), if $\lambda_{\min}(\mathcal{K}_{ip}) > \max(2\lambda_{iM}; \varrho_{1i}),$ $\lambda_{\min}(\mathcal{K}_{id}) > \varrho_{2i}, \beta < \min_{s_i \in \sigma(\mathcal{D})} \theta_i$ and

$$0 < h < \min_{s_i \in \sigma(\mathcal{D})} \frac{2\beta^2 \left(\theta_i - \beta\right)}{\alpha \left(\vartheta_i^2 + \beta^2\right)},\tag{15}$$

then the control problem is solved, i.e., for any $i \in \mathcal{I}$,

$$\begin{cases} \lim_{t \to \infty} \|\bar{\varepsilon}_i\|_{\infty} \leq h\kappa_3, \quad \lim_{t \to \infty} \|\bar{\upsilon}_i\|_{\infty} \leq h\kappa_3, \\ \lim_{t \to \infty} q_i \in \mathcal{U}(\varepsilon_0; \delta_5), \quad \lim_{t \to \infty} \dot{q}_i \in \mathcal{U}(\upsilon_0; \delta_6), \\ \delta_5 = \max_{i \in \mathcal{I}} \frac{\mu_{1i} + h\kappa_3 \sqrt{m} [\lambda_{\max}(\mathcal{K}_{ip}) + \lambda_{\max}(\mathcal{K}_{id})]}{\lambda_{\min}(\mathcal{K}_{ip}) - \varrho_{1i}}, \\ \delta_6 = \max_{i \in \mathcal{I}} \frac{\mu_{1i} + h\kappa_3 \sqrt{m} [\lambda_{\max}(\mathcal{K}_{ip}) + \lambda_{\max}(\mathcal{K}_{id})]}{\lambda_{\min}(\mathcal{K}_{id}) - \varrho_{2i}}, \end{cases}$$
(16)

where κ_3 is defined right before Theorem 3.

Proof. First we proof $\lim_{t\to\infty} \|\bar{\varepsilon}\|_{\infty} \leq h\kappa_3$ and $\lim_{t\to\infty} \|\bar{v}\|_{\infty} \leq h\kappa_3$. Let λ be an eigenvalue of Γ . By Schur's Formula, $\det(\lambda I_{2mn} - \Gamma) = \prod_{i=1}^n [\psi_i(\lambda)]^m$, where $\psi_i(\lambda) = \lambda^2 + [(\alpha h + \beta)(1 - s_i) - 2] \lambda + 1 - \beta(1 - s_i)$. By Lemma 1, Assumption A2 implies $|s_i| < 1$, which means $1 - s_i \neq 0$. Applying the bilinear transformation $\lambda = (z+1)/(z-1)$ to $\psi_i(\lambda)$ gives that $\psi'_i(z) = (z-1)^2 \psi_i[(z+1)/(z-1)]/[\alpha h(1 - s_i)]$. Then $\psi'_i(z) = z^2 + 2\beta z/(\alpha h) + 4/[\alpha h(1 - s_i)] - 2\beta/(\alpha h) - 1$. By the stability criterion, $\psi_i(\lambda)$ is Schur stable if and only if $\psi'_i(z)$ is Hurwitz stable if and only if $2\beta/(\alpha h) > 0$ and $\beta^2 [2(\theta_i - \beta)/(\alpha h) - 1] - \vartheta_i^2 > 0$. It follows that if (15) holds, $\psi_i(\lambda)$ is Schur stable, *i.e.*, $\Gamma \in \Omega^{2mn \times 2mn}$. Note that the estimator in (14) is equivalent to $x(t_{k+1}) = \Gamma^k x(t_1) + \sum_{i=1}^k \Gamma^{k-i} \Delta(i)$. By the similar analysis in Appendix A, we can easily conclude that $\lim_{t\to\infty} \|\bar{\varepsilon}\|_{\infty} \leq h\kappa_3$, and $\lim_{t\to\infty} \|\bar{v}\|_{\infty} \leq h\kappa_3$.

For the second presentation, note that $\lim_{t\to\infty} \mu_{3i}(t) \leq h\kappa_3\sqrt{m} [\lambda_{\max}(\mathcal{K}_{ip}) + \lambda_{\max}(\mathcal{K}_{id})]$. Following the proof of Theorem 2, if $\lambda_{\min}(\mathcal{K}_{ip}) > \max(2\lambda_{iM}; \varrho_{1i})$ and $\lambda_{\min}(\mathcal{K}_{id}) > \varrho_{2i}$, then (11) implies $\lim_{t\to\infty} ||e_i(t)||_2 \leq \delta_5$, $\lim_{t\to\infty} ||\dot{e}_i(t)||_2 \leq \delta_6$. This completes the proof.

Suppose that (15) holds. For any $\delta_5, \delta_6 > 0$, by Theorem 3, if there exists a constant $\epsilon \in (0, 1)$ such that

$$\begin{cases} \lambda_{\min} \left(\mathcal{K}_{ip} \right) \geq \max \left(2\lambda_{iM}; \frac{\mu_{1i}}{\epsilon \delta_5} + \varrho_{1i} \right), \\ \lambda_{\min} \left(\mathcal{K}_{id} \right) \geq \frac{\mu_{1i}}{\epsilon \delta_6} + \varrho_{2i}, \\ h \leq \frac{\mu_{1i}(1-\epsilon)}{\epsilon \kappa_3 \sqrt{m} [\lambda_{\max}(\mathcal{K}_{ip}) + \lambda_{\max}(\mathcal{K}_{id})]}, \end{cases}$$
(17)

then $\lim_{t\to\infty} q_i \in \mathcal{U}(\varepsilon_0; \delta_5)$, $\lim_{t\to\infty} \dot{q}_i \in \mathcal{U}(\upsilon_0; \delta_6)$.

Remark 4 δ_5 and δ_6 in (16) can be arbitrarily small by choosing appropriate parameters $\mathcal{K}_{ip}, \mathcal{K}_{id}, h$ without the exact knowledge of $\mu_{1i}, \mu_{2i}, \varrho_{1i}$ and ϱ_{2i} . The parameter setting follows the similar discussion in Remark 3.

Remark 5 By Theorems 1 and 3, the stability conditions of the first- and second-order DCEA (2) and (3) are (7) and (15), respectively. Note that (15) for the secondorder DCEA restrict the upper bound of h while (7) dose not impose any restrictions on the sampling period h.

Remark 6 The results in Theorems 1 and 2 for the firstorder DCEA (2) can be directly invoked to solve the sampled hetero-information case presented in [19] while the second-order DCEA (3) in Theorem 3 cannot handle with the sampled hetero-information case directly.

Remark 7 Comparing with the traditional estimatorbased coordination algorithms considering continuous interaction [5, 8, 13], the presented DCEA only transmit and update the interaction information at the sampling time, which lead to the following benefits: less requirement of target information (only use sampling data of the target), lower cost for maintaining interaction (only require sampled interaction), fewer consumption of calculation resources (only update the estimate value at sampling time).

Remark 8 The traditional estimator-based coordination algorithms for NRSs based on continuous interaction may be still effective for sampled interaction where the sampling period is sufficiently small. However, the quantitative conditions and relationships between the sampling period, feedback gains and the bound of the stability region cannot be obtained by the existing results on traditional estimator-based algorithms.

Remark 9 Comparing with the model-based adaptive algorithms of NRSs [1, 2, 10, 11], the presented DCEA use model-free PD-like control, which means the algorithms are easily realizable due to their low computation complexity, including less requirement of the information of system models, lower cost for real-time input computing.

4 Illustrative Examples

Consider a NRS containing 6 robots. Then $\mathcal{I} = \{1, \dots, 6\}$. The NRS interaction is described by a digraph \Im with an adjacency matrix $\mathcal{W} = [0 \ \mathbf{0}; \zeta \ \hat{\mathcal{W}}]$, where $\zeta = \operatorname{col}(0, 0, 1, 0, 0, 0)$ and

For simplicity, the detailed setting of $\mathcal{M}_i(q_i)$, $\mathcal{C}_i(\dot{q}_i, q_i)$ and $\mathcal{G}_i(q_i)$ in (1) follows the dynamics of the robotic manipulator with two revolute joints given in [22]. Let $\tau_{i,d}(t) = 2\operatorname{col}[\sin(t), \cos(2t)]$, $\varepsilon_0(t) = \operatorname{col}[2t + \sin(t), -2t - \cos(t)]$ and $v_0(t) = \operatorname{col}[2 + \cos(t), -2 + \sin(t)]$, $\forall i \in \mathcal{I}$. The elements of $\varepsilon_i(0)$, $v_i(0)$, $q_i(0)$ and $\dot{q}_i(0)$ are randomly selected from [-25, 25], $\mathcal{K}_{i,p} = 200I_2$, $\mathcal{K}_{i,d} = 300I_2$, $\forall i \in \mathcal{I}$.

Example 1 Let h = 0.1. For the first-order DCEA (2), the practical stability of ε_i and v_i can be achieved if $0 < \alpha, \beta < 1.1716$, which can be easily computed from (7) in Theorem 1. Fig.1 shows that the practical stability of ε_i and v_i can be achieved when $\alpha = 0.9, \beta = 1.1$ and $\alpha, \beta = 1.17$, but it cannot be achieved if $\alpha, \beta = 1.18$.

Example 2 Let $\alpha = 0.9$ and $\beta = 1.1$. The detail views in Fig.2 are the amplifications of steady-state errors for NRS (1) under DCEA (2). It is shown in Fig.2 that the

smaller the sampling period h, the smaller the stability region.

Example 3 Theorem 3 implies that for NRS (1) under DCEA (3), the practical stability of the NRS can be achieved if $\beta < 1.1716$. Let $\alpha = 1.1$ and $\beta = 0.9$. By (15), target tracking can be achieved if 0 < h < 0.4938. Fig.3 shows that the target tracking can be achieved when h = 0.45, but it cannot be obtained when h = 0.5. It follows that the sufficient conditions (7) and (15) are almost equivalent to necessary and sufficient conditions. Besides, it can be seen that in some sense, the smaller the sampling period h, the smaller the stability region.

Remark 10 By the detail views in Fig.1 and Fig.4, ε_i is discontinuous at each sampling time in the first-order DCEA while it is continuous in the second-order DCEA, which means that the trajectory of ε_i and q_i for NRS under the second-order DCEA is smoother than that under the first-order DCEA.

5 Conclusion

In this paper, the target tracking problem of NRSs under directed sampled interaction has been considered for a time-varying target. The concepts and properties of the small-value norms for Schur stable matrices have been introduced for the analysis of the practical stability of the presented first- and second-order DCEA. Several sufficient criteria on interaction topology, the sampling period, and the other control parameters for target tracking have been obtained. Besides, the quantitative relationship between the stability region of the tracking errors and the control parameters, including the sampling period, the control gain and the interaction topology, has been presented. Finally, a few examples have been delivered to verify the theoretical results.

A Proof of Theorem 1

For the first presentation, let λ be an eigenvalue of Λ . By Schur's Formula, det $(\lambda I_{2mn} - \Lambda) = \prod_{i=1}^{n} [\varphi_i(\lambda)]^m$, where $\varphi_i(\lambda) = (\lambda + \alpha - 1 - \alpha s_i)(\lambda + \beta - 1 - \beta s_i)$. Then the eigenvalues of Λ satisfy $\varphi_i(\lambda) = 0$. Invoking the bilinear transformation $\lambda = (z+1)/(z-1)$ for $\varphi_i(\lambda)$ gives $\varphi'_i(z) = (z-1)^2 \varphi_i [(z+1)/(z-1)]$. Then $\varphi'_i(z) = [\alpha(1-s_i)z - \alpha(1-s_i) + 2][\beta(1-s_i)z - \beta(1-s_i) + 2]$. By invoking Lemma 1, Assumption A2 implies $1-s_i \neq 0$. $\varphi'_i(z) = 0$ implies $z_1 = 1 - 2/(\alpha[1 - s_i])$ and $z_2 = 1 - 2/(\beta[1 - s_i])$. It follows that $\operatorname{Re}(z_1) < 0$ and $\operatorname{Re}(z_2) < 0$ if (7) holds, *i.e.*, $\varphi'_i(z)$ is Hurwitz stable if and only if polynomial $\varphi'_i(z)$ is Hurwitz stable. Therefore, (7) implies that Λ has all eigenvalues within the unit disc, *i.e.*, $\Lambda \in \Omega^{2mn \times 2mn}$.

For the second presentation, Assumption A1 means

that for any $k \in \mathbb{Z}^{\dagger}$,

$$\begin{cases} \|\Delta_1(k)\|_{\infty} \le 2h\gamma_1, \|\Delta_2(k)\|_{\infty} \le h\gamma_2, \\ \|\Delta(k)\|_{\Lambda} \le hc_1 \max(2\gamma_1; \gamma_2) \end{cases}$$
(A.1)

where we have invoked $\varepsilon_0(t_{k+1}) - \varepsilon_0(t_k) = \int_{t_k}^{t_{k+1}} v(w) dw$ and $v_0(t_{k+1}) - v_0(t_k) = \int_{t_k}^{t_{k+1}} a(w) dw$ to obtain (A.1). Following [23], $\Lambda \in \Omega^{2mn \times 2mn}$ means $\lim_{k \to \infty} \Lambda^k = 0$. System (5) gives that $x(t_{k+1}) = \Lambda^k x(t_1) + \sum_{i=1}^k \Lambda^{k-i} \Delta(i)$, where $k \in \mathbb{Z}^{\dagger}$, $\Lambda^0 = I_{2mn}$. It thus follows from $\lim_{k \to \infty} \Lambda^k = 0$ that

$$\begin{split} \lim_{t \to \infty} \|x(t)\|_{\Lambda} &= \lim_{k \to \infty} \left\| \sum_{i=1}^{k} \Lambda^{k-i} \Delta(i) \right\|_{\Lambda} \\ &\leq hc_1 \max(2\gamma_1; \gamma_2) \lim_{k \to \infty} \sum_{i=0}^{k-1} \|\Lambda\|_{\Lambda}^i \\ &\leq hc_1 \max(2\gamma_1; \gamma_2) (1 - \|\Lambda\|_{\Lambda})^{-1}, \end{split}$$

where Lemma 2 has been used to obtain the above result. It follows that $\lim_{t\to\infty} ||x(t)||_{\infty} \leq h\kappa_1$, which means $\lim_{t\to\infty} \|\bar{e}_i\|_{\infty} \leq h\kappa_1$ and $\lim_{t\to\infty} \|\bar{v}_i\|_{\infty} \leq h\kappa_1$, $\forall i \in \mathcal{I}$. Besides, the estimator in (5) also implies that $\bar{v}(t_{k+1}) = \mathcal{P}^k \bar{v}(t_1) + \sum_{i=1}^k \mathcal{P}^{k-i} \Delta_2(i), \forall k \in \mathbb{Z}^{\dagger}$, where \mathcal{P} is defined right behind (6). Thus, by the similar analysis, $\mathcal{P} \in \Omega^{mn \times mn}$ and $\lim_{t\to\infty} \|\bar{v}_i\|_{\infty} \leq h\kappa_2, \forall i \in \mathcal{I}$. This completes the proof.

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Fig. 1. States ε_i [Pictures (a1,b1,c1)] and v_i [Pictures (a2,b2,c2)] of the first-order DCEA (2) under the sampling period h = 0.1 and different α and β . (a1,a2) $\alpha = 0.9, \beta = 1.1$; (b1,b2) $\alpha, \beta = 1.17$; (c1,c2) $\alpha, \beta = 1.18$.



Fig. 2. States e_i [Pictures (a1,b1,c1)] and \dot{e}_i [Pictures (a2,b2,c2)] of NRS (1) using the first-order DCEA (2) under $\alpha = 0.9$, $\beta = 1.1$ and different sampling periods h, $\forall i \in \mathcal{I}$. (a1,a2) h = 0.05; (b1,b2) h = 0.1; (c1,c2) h = 0.5.

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Fig. 3. States e_i [Pictures (a1,b1,c1)] and \dot{e}_i [Pictures (a2,b2,c2)] of NRS (1) using the second-order DCEA (3) under $\alpha = 1.1$, $\beta = 0.9$ and different sampling periods $h, \forall i \in \mathcal{I}$. (a1,a2) h = 0.1; (b1,b2) h = 0.45; (c1,c2) h = 0.5.



Fig. 4. States ε_i [Picture (a)] and v_i [Picture (b)] of the second-order DCEA (3) under h = 0.1, $\alpha = 1.1$, and $\beta = 0.9$, $\forall i \in \mathcal{I}$.

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