H_∞ Control of Markov Jump Time-Delay Systems Under Asynchronous Controller and Quantizer ⋆

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Abstract

This paper aims to design an asynchronous state feedback controller for Markov jump time-delay systems. The highlight of this work lies in that the state feedback is quantized by a logarithmic quantizer, and both the controller and quantizer are asynchronous with the controlled systems. By Lyapunov-Krasovskii functional, a sufficient condition is presented to ensure that the resulting closed-loop system is stochastically mean square stable with a prescribed H_∞ performance index. Finally, an example is presented to illustrate the effectiveness and new features of proposed design method.

Key words: H_∞ performance; Markov jump systems; asynchronous control; asynchronous quantization; hidden Markov model.

1 Introduction

In practical control systems, the occurrences of abrupt structure or parameter variations produced by external causes are inevitable. To facilitate the theoretical research for such kind of systems, a special class of hybrid systems, named as Markov jump system (MJS), has emerged. Conceptually, a MJS is a dynamic system that varies among a finite or infinite collection of modes or subsystems, and the variations are governed by a Markov process subject to some transition probability matrix. In the past few decades, study on filtering/estimation and control/stabilization of MJSs has enjoyed enduring popularity and a large number of works have been published in the literature, see for instance [1, 2, 5, 7, 9, 12, 17, 19–21, 24, 26, 28, 29, 34], and the references therein. Specially, the filtering/estimation problem has been addressed in [1, 9, 24]. Stability and stabilization have been vastly investigated in [2, 5, 7, 12, 17, 19, 20, 26, 34], e.g., a novel sufficient condition for almost sure stability of MJS has been proposed in [2], LQ-optimal control and output feedback control have been addressed respectively in [5] and [12]. MJSs have been widely used in practical systems. For example, the works in [21, 28, 29] have investigated the applications of MJSs in DC motor and DC-DC buck converter.

Since communication network is introduced into control systems, the control system unavoidably suffers from communication problems, e.g., packet loss, time delay and quantization, which may deteriorate the stability and performance of the system. These new problems introduced by communication network are interesting to consider. Quantization which maps a continuous quantity into a discrete set at the cost of certain distortion, is required due to finite transmission rate of the network. The work [16] has traced the early history of quantization theory, and elaborated on the fundamentals and early results concerning quantization. Quantized control has been one of the most important issues in the area of

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NCSs. The literatures [3, 8, 10] are early works on this topic. The authors in [10] have shown that the logarithmic quantizer is the coarsest quantizer that can quadratically stabilize a linear discrete time-invariant system with a single input, which can be explained intuitively by that, the farther from the equilibrium the state is, the less precise the control action needs to be. In [14], the authors have conducted a comprehensive investigation on quantized feedback control with logarithmic quantization based on the sector bound approach. Following this, authors of [32] have considered the quantized stabilization problem for MJSs. When quantization is considered, another choice is the dynamic quantizer. Dynamic quantization has advantages in the dynamical scalability of quantization levels, such that the attraction region can be increased and the steady state limit cycle can be reduced. However, the dynamic quantization has some disadvantages as mentioned in [14], such as poor transient responses and invalidity of capacity results for practical communication channels.

As mentioned above, there exist problems such as packet loss, time delay in NCSs, which will lead to information loss. In MJSs, incomplete information (e.g. the system modes) transmission may cause asynchronization between the original system and controller/filter, which has not been taken into consideration in many of the existing works. For instance, the controllers or filters involved in [1, 12, 17, 19–21, 26, 34] are either synchronous or mode-independent. However, it is gratifying to see that there are increasing concerns about asynchronization problem in the past several years [30, 31, 35]. In [35], the controlled system and controller are asynchronous because of the time delay between their modes. In [30], an asynchronous $l_2 - l_\infty$ filter has been proposed for stochastic discrete-time MJSs, and the asynchronization is reflected in the piecewise homogeneous Markov chain of the filter. Unlike [30], the work in [31] has described the asynchronization phenomenon as a hidden Markov model, where the controller's mode depends only on the system's mode through conditional probabilities. As a matter of fact, the hidden Markov model is not new. The book [11] has carried out an exhaustive study on hidden Markov model, which helps lay a solid theoretical foundation. In [4], hidden Markov model is used to study network intrusion detection. Based on the hidden Markov model, authors of [6] have studied feedback control using an estimate $\hat{\theta}_k$ instead of the actual system mode θ_k as only partial information is available, where $\hat{\theta}_k$ depends on θ_k in terms of certain conditional probability. Furthermore, the authors of [22, 23, 25, 27] have conducted extensive studies on filtering and control for both continuous-time and discrete-time systems under the same framework of hidden Markov model $(\theta_k, \hat{\theta}_k)$, where $\hat{\theta}_k$ is emitted by a detector. As can be seen, the hidden Markov models in [31] and [6, 22, 23, 25, 27] are essentially the same. The only difference is that the asynchronization in [6, 22, 23, 25, 27] is induced by the detector. Despite all this, the asynchronous quantized state feedback control has not been fully considered, which motivates us for the current study.

This paper is concerned with the asynchronous control problem for time-delay MJSs following the framework in [31]. This paper distinguishes itself with the existing works [6, 22, 23, 25, 27, 31] by considering quantizing the state feedback through a logarithmic quantizer before input to the controlled system. Both the controller and quantizer are asynchronous with the controlled system. Hence, there are two hidden Markov models with respect to controller and quantizer respectively. It is worth mentioning that, in [6, 22, 23, 25, 27], the controller or filter obtains systems' mode information through a detector, whereas in the present paper, the controlled system will transmit mode information actively to the controller and quantizer. Besides, time-varying time delay in state is taken into consideration in our work. By applying Lyapunov-Krasovskii functional, a sufficient condition is obtained, which guarantees that the closed-loop system is stochastically mean square stable and has a prescribed H_∞ noise attenuation performance. Furthermore, the control gain is parameterized by settling the nonlinearity in the sufficient condition. Finally, a numerical example illustrates that the proposed method is correct and effective, and also validates some heuristic conjectures.

Notation: The notations used throughout this paper are fairly standard. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the n-dimensional Euclidean space and the set of all $m \times n$ real matrices, respectively. $\mathbb{N}^+$ is the set of positive integers. $l_2[0, \infty)$ denotes the space of square summable infinite sequence. The notation $X > Y$ $(X \geqslant Y)$, where X and Y are symmetric matrices, means that $X - Y$ is positive definite (positive semidefinite). I and 0 represent the identity matrix and zero matrix, respectively. $||\cdot||$ denotes the Euclidean norm of a vector and its induced norm of a matrix. $(\cdot)^{\mathrm{T}}$ and $E(\cdot)$ denote the transpose operator and expectation operator, respectively. $diag\{\cdot\}$ denotes a block diagonal matrix. For an arbitrary matrix B and two symmetric matrices A and C, in the symmetric matrix

$$\begin{bmatrix} A & B \\ * & C \end{bmatrix}$$

where "$*$" denotes the term that is induced by symmetry. All matrices in this paper are assumed to have compatible dimensions for algebraic operations.

2 Preliminaries

This paper focuses on the following MJS with time-variant delay:

$$\mathbb{S}:\begin{cases} x(k+1) = A(\theta_k)x(k) + A_d(\theta_k)x(k-d(k)) \\ \qquad\qquad + B_1(\theta_k)u(k) + B_2(\theta_k)w(k) \\ y(k) = C(\theta_k)x(k) + C_d(\theta_k)x(k-d(k)) \\ \qquad\qquad + D_1(\theta_k)u(k) + D_2(\theta_k)w(k) \\ x(k_0) = \varsigma(k_0), k_0 = -d_2, -d_2+1, \cdots, -1, 0 \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ denotes system state and $\varsigma(k_0)$ is the initial state. It is assumed that all the state values are available. $y(k) \in \mathbb{R}^{n_y}$ is the controlled output from the system. $u(k) \in \mathbb{R}^m$ and $w(k) \in \mathbb{R}^{n_w}$ are control and deterministic disturbance inputs respectively, and $w(k) \in l_2[0,\infty)$. Note that time-variant delay $d(k)$ is introduced in (1) considering that time delay is ubiquitous in practical engineering systems and is always a source of instability [13], $d(k) \in \mathbb{N}^+$ is bounded by positive integers d_1 and d_2, and $d_1 < d_2$. The remaining matrices in the system are real and known a priori. The transitions of system $\mathbb{S}$ are in the control of the Markov parameter $\theta_k \in \mathscr{N}$ ($\mathscr{N} = \{1, 2, \cdots, n\}$), which are subject to transition probability matrix $\Theta = [\pi_{ij}]$ with π_{ij} given by

$$\Pr\{\theta_{k+1} = j | \theta_k = i\} = \pi_{ij}, \tag{2}$$

and there exist constraints $\pi_{ij} \in [0,1]$ and $\sum_{j=1}^{n} \pi_{ij} = 1$ for $\forall i, j \in \mathscr{N}$.

We attempt to design a mode-dependent controller using quantized state feedback law for $\mathbb{S}$ which is described as follows:

$$\mathbb{C} : v(k) = K(\lambda_k)x(k), \tag{3}$$

$$\mathbb{Q} : u(k) = \mathscr{Q}(\xi_k, v(k)), \tag{4}$$

where $v(k) \in \mathbb{R}^m$ is the unquantized feedback law with control gain $K(\lambda_k)$. Subsequently, $u(k)$ is obtained by quantizing $v(k)$ through quantizer $\mathscr{Q}(\cdot,\cdot)$ which is composed of m mode-dependent logarithmic quantizers, i.e.,

$$\mathscr{Q}(\xi_k, v(k)) = \Big[f_1(\xi_k, v_1(k)) \ \cdots \ f_m(\xi_k, v_m(k)) \Big]^{\mathrm{T}}. \tag{5}$$

The variables λ_k and ξ_k denote received mode information from the controlled system. Due to unreliable transmission, λ_k and ξ_k are not identical to θ_k. Without loss of generality, assume that they take values in the same finite set $\mathscr{N}$ as θ_k does. On one hand, λ_k and ξ_k exert influence on the variations of $\mathbb{C}$ and $\mathbb{Q}$ among different modes, on the other hand, they are affected by the mode of system $\mathbb{S}$ through conditional probability matrices $\Omega = [\omega_{ip}]$ and $\Sigma = [\sigma_{iq}]$, $i, p, q \in \mathscr{N}$. The conditional probability ω_{ip} (or σ_{iq}) implies the possibility that the controller $\mathbb{C}$ runs in mode p (or the quantizer $\mathbb{Q}$ in mode q) given the mode information i of system $\mathbb{S}$, i.e.,

$$\Pr\{\lambda_k = p | \theta_k = i\} = \omega_{ip}, \ \Pr\{\xi_k = q | \theta_k = i\} = \sigma_{iq}. \tag{6}$$

It is quite clear that $\omega_{ip} \in [0,1]$, $\sum_{p=1}^{n} \omega_{ip} = 1$ and $\sigma_{iq} \in [0,1]$, $\sum_{q=1}^{n} \sigma_{iq} = 1$, $\forall i, p, q \in \mathscr{N}$. It should be noted that we are able to obtain the conditional probability matrices Ω and Σ by Monte Carlo method in real applications. We should mention that, usually, a mode-dependent controller or quantizer depends on the original system's mode directly, e.g. [33]. However, the meaning of "mode-dependent" in this paper is slightly different from the common usage. The controller $\mathbb{C}$ and quantizer $\mathbb{Q}$ depend on θ_k indirectly through conditional probabilities (6).

Remark 1 *Note that the jumps of controller $\mathbb{C}$ are under the control of Markov parameter λ_k directly and influenced by Markov parameter θ_k (subject to transition probability matrix Θ) indirectly through conditional probability matrix Ω. Hence a hidden Markov model $(\theta_k, \lambda_k, \Theta, \Omega)$ is formed, which describes the asynchronization between $\mathbb{S}$ and $\mathbb{C}$ well. Similarly, the asynchronization between $\mathbb{S}$ and $\mathbb{Q}$ is described by a hidden Markov model $(\theta_k, \xi_k, \Theta, \Sigma)$.*

Remark 2 *Note that similar to the discussions in [22, 23, 31] concerning hidden-Markov-model-based asynchronization description, taking different values in $(\theta_k, \lambda_k, \Theta, \Omega)$, our results will reduce to the following three special cases:*

(1) Synchronous case: in this case, $\Omega = I$, the system and the controller will achieve perfect synchronization.

(2) Clustering case: in this case, the Markov parameter θ_k is grouped into several clusters, and in each cluster, the conditional probability for λ_k depends only on which cluster θ_k belongs to. One extreme case is that there is only one cluster and then the conditional probability matrix Ω has identical rows. This point has been elaborated in [22, 23].

(3) Mode-independent case: in this case, $\lambda_k \in \{1\}$, $\Omega = [1 \ \cdots \ 1]^{\mathrm{T}}$, the mode information θ_k plays no role, which is equivalent to the only one cluster case mentioned in (2).

For notational brevity, subscripts i, j, p, q will be hereinafter employed to replace these Markov parameters θ_k, θ_{k+1}, λ_k and ξ_k in system $\mathbb{S}$, controller $\mathbb{C}$ and quantizer $\mathbb{Q}$, for example, $A(\theta_k)$ is abbreviated as A_i.

A mode-dependent logarithmic quantizer $f_q(a)$ ($q \in \mathscr{N}$)

is defined as follows:

$$f_q(a) = \begin{cases} r_{ql}, & \frac{1}{1+\delta_q} r_{ql} < a \leq \frac{1}{1-\delta_q} r_{ql} \\ 0, & a = 0 \\ -f_q(-a), & a < 0 \end{cases} \tag{7}$$

where the scalar a is the input of the quantizer, $r_{ql} = \rho_q^l r_0$ are the outputs, $l = 0, \pm 1, \pm 2, \cdots$, and $0 < \rho_q < 1$, $r_0 > 0$. Without loss of generality, r_0 is invariant for all $q \in \mathscr{N}$, i.e., r_0 is mode-independent [10]. The parameters δ_q and ρ_q are related with each other by

$$\delta_q = \frac{1-\rho_q}{1+\rho_q}. \tag{8}$$

As to logarithmic quantizer, two important facts should be recalled, one of them is the quantization density which can be calculated by the formula $-2/ln(\rho_q)$. Therefore we can compare the coarseness of logarithmic quantizers by observing the values of ρ_q or δ_q, the smaller ρ_q or bigger δ_q is, the coarser the quantizer is. Another fact is that the quantizer (7) is bounded by a sector with $(1+\delta_q)a$ and $(1-\delta_q)a$ as its boundaries, which is specified in [14]. As a result, the quantization error is bounded by $-\delta_q a \leq f_q(a) - a \leq \delta_q a$, which can be written as

$$f_q(a) - a = \Delta_q a, \ \Delta_q \in [-\delta_q, \delta_q]. \tag{9}$$

Notice that, for $\forall q \in \mathscr{N}$, the quantizer $f_q(a)$ is time invariant, and hence (9) holds for all k. By means of (5) and (9), we can obtain that

$$\mathscr{Q}_q(v(k)) = (I + H_q(k))v(k), \tag{10}$$

where

$$H_q(k) = diag\{\Delta_{1q}(k), \cdots, \Delta_{mq}(k)\} \tag{11}$$

with $\Delta_{sq}(k) \in [-\delta_{sq}, \delta_{sq}]$, $s = 1, 2, \cdots, m$. Combining system $\mathbb{S}$, controller $\mathbb{C}$, quantizer $\mathbb{Q}$ and (10), we obtain the dynamics of the closed-loop system as follows:

$$\mathbb{S}_{cl}: \begin{cases} x(k+1) = \bar{A}(\theta_k \lambda_k \xi_k, k)x(k) + A_{di}x(k-d(k)) \\ \qquad\qquad + B_{2i}w(k) \\ y(k) = \bar{C}(\theta_k \lambda_k \xi_k, k)x(k) + C_{di}x(k-d(k)) \\ \qquad\qquad + D_{2i}w(k) \end{cases} \tag{12}$$

where

$$\bar{A}(\theta_k \lambda_k \xi_k, k) = A_i + B_{1i}(I + H_q(k))K_p,$$
$$\bar{C}(\theta_k \lambda_k \xi_k, k) = C_i + D_{1i}(I + H_q(k))K_p.$$

In the following, $\bar{A}(\theta_k \lambda_k \xi_k, k)$ and $\bar{C}(\theta_k \lambda_k \xi_k, k)$ will be denoted as $\bar{A}_{ipq}(k)$ and $\bar{C}_{ipq}(k)$ respectively.

Next, some definitions will be introduced, which are essential for the derivation of our main results in this paper.

Definition 1 [18] *The closed-loop system $\mathbb{S}_{cl}$ with $w(k) \equiv 0$ is said to be stochastically mean square stable, if for any initial condition $(x(k_0), \theta_0)$, the following condition holds*

$$E\{\sum_{k=0}^{\infty} \| x(k) \|^2 | \, x(k_0), \theta_0\} < \infty. \tag{13}$$

Definition 2 [18] *The closed-loop system $\mathbb{S}_{cl}$ with $w(k) \in l_2[0, \infty)$ is said to have an H_∞ noise attenuation performance γ, if under zero initial state, the following condition is satisfied:*

$$\sum_{k=0}^{\infty} E\{\| y(k) \|^2\} < \gamma^2 \sum_{k=0}^{\infty} \| w(k) \|^2, \tag{14}$$

where γ is a positive scalar.

Given the above, the objective of this paper is to develop a possible scheme of quantized state feedback law (i.e. $\mathbb{C}$ and $\mathbb{Q}$) with given logarithmic quantizer $\mathbb{Q}$ for system $\mathbb{S}$ in the hope that the resulting closed-loop system $\mathbb{S}_{cl}$ is stochastically mean square stable and has a prescribed H_∞ noise attenuation performance γ.

Remark 3 *Note that the quantized feedback control has been addressed in [32], whereas our work is quite different from [32] mainly in the following three points:*

(1) Time delay in state is considered in our work.
(2) With the controller designed in our work, the closed-loop system is not only stochastically mean square stable, but also has a prescribed H_∞ performance which was not considered in [32].
(3) Most importantly, the asynchronization is described in different ways in [32] and our work. In [32], the controller/quantizer are synchronous with the controlled system, but controller and quantizer are synchronous with each other, hence in this point, our framework is more general. Besides, in [32] the asynchronization is described by conditional probability $\Pr\{\theta_k = i, \lambda_{k+1} = p_2 | \lambda_k = p_1\}$ which implies that the controlled system's mode of time instant k and the controller/quantizer's mode of time instant $k+1$ jointly depend on the controller/quantizer's mode of time instant k. However, in our framework, the modes of the controller or quantizer depend on the mode of controlled system, which is described by (6).

3 Main Results

This section will firstly present a sufficient condition concerning stochastic mean square stability with an H_∞

noise attenuation performance γ, and then propose a design approach of an asynchronous quantized controller.

Theorem 1 *The closed-loop system $\mathbb{S}_{cl}$ is stochastically mean square stable with an H_∞ noise attenuation performance γ, if there exist a matrix $K_p \in \mathbb{R}^{m\times n_x}$, positive definite matrices $P_i \in \mathbb{R}^{n_x}$, $R \in \mathbb{R}^{n_x}$, $F_{ipq} \in \mathbb{R}^{n_x}$, and a positive definite diagonal matrix $W_{iq} \in \mathbb{R}^m$, for $\forall i,p,q \in \mathscr{N}$, such that the following conditions hold:*

$$\sum_{p=1}^{n}\sum_{q=1}^{n}\omega_{ip}\sigma_{iq}F_{ipq} < P_i, \tag{15}$$

$$\begin{bmatrix} \Phi_{ipq} & \mathscr{K}_p^{\mathrm{T}} & G_i\Lambda_q W_{iq} \\ * & -W_{iq} & 0 \\ * & * & -W_{iq} \end{bmatrix} < 0, \tag{16}$$

where

$$\Phi_{ipq} = \begin{bmatrix} -\tilde{P}_i^{-1} & 0 & \bar{A}_{ip}^* & A_{di} & B_{2i} \\ * & -I & \bar{C}_{ip}^* & C_{di} & D_{2i} \\ * & * & dR - F_{ipq} & 0 & 0 \\ * & * & * & -R & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\bar{A}_{ip}^* = A_i + B_{1i}K_p,\ \bar{C}_{ip}^* = C_i + D_{1i}K_p,$$

$$\mathscr{K}_p = \begin{bmatrix} 0 & 0 & K_p & 0 & 0 \end{bmatrix},\ G_i = \begin{bmatrix} B_{1i}^{\mathrm{T}} & D_{1i}^{\mathrm{T}} & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\Lambda_q = diag\{\delta_{1q}, \delta_{2q}, \cdots, \delta_{mq}\},$$

$$\tilde{P}_i = \sum_{j=1}^{n}\pi_{ij}P_j,\ d = d_2 - d_1 + 1,$$

where these variables, all the system matrices in $\mathbb{S}$ and transition probability matrix Θ, conditional probability matrices Ω, Σ, quantization parameters Λ_q, boundaries of time delay d_1 and d_2 are all pre-given.

Proof. We will firstly deduce several inequalities from (15) and (16) to carry forward the proof. From (15) it is easy to know

$$\mathscr{F}_i \triangleq \sum_{p=1}^{n}\sum_{q=1}^{n}\omega_{ip}\sigma_{iq}F_{ipq} - P_i < 0. \tag{17}$$

Using Schur Complement to (16), we have

$$\Phi_{ipq} + \mathscr{K}_p^{\mathrm{T}}W_{iq}^{-1}\mathscr{K}_p + G_i\Lambda_q W_{iq}\Lambda_q G_i^{\mathrm{T}} < 0. \tag{18}$$

Considering (11) with $\Delta_{sq}(k) \in [-\delta_{sq}, \delta_{sq}]$, $s = 1, 2, \cdots, m$, and noting W_{iq} is a positive definite diagonal matrix,

$$\Phi_{ipq} + \mathscr{K}_p^{\mathrm{T}}W_{iq}^{-1}\mathscr{K}_p + G_iH_q(k)W_{iq}H_q(k)G_i^{\mathrm{T}} < 0 \tag{19}$$

holds. By means of Lemma 1 in [15], it leads to from (19)

$$\Phi_{ipq} + \mathscr{K}_p^{\mathrm{T}}H_q(k)G_i^{\mathrm{T}} + G_iH_q(k)\mathscr{K}_p < 0, \tag{20}$$

namely,

$$\begin{bmatrix} -\tilde{P}_i^{-1} & 0 & \bar{A}_{ipq}(k) & A_{di} & B_{2i} \\ * & -I & \bar{C}_{ipp}(k) & C_{di} & D_{2i} \\ * & * & dR - F_{ipq} & 0 & 0 \\ * & * & * & -R & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{21}$$

which implies

$$\begin{bmatrix} -\tilde{P}_i^{-1} & \bar{A}_{ipq}(k) & A_{di} \\ * & dR - F_{ipq} & 0 \\ * & * & -R \end{bmatrix} < 0 \tag{22}$$

holds. Then, by applying Schur Complement operations to (21) and (22), respectively, we obtain that

$$\begin{cases} \Phi_{ipq}^{\dagger} \triangleq \mathscr{C} - \mathscr{D}_{ipq}^{\mathrm{T}}\mathscr{G}_i^{-1}\mathscr{D}_{ipq} < \hat{F}_{ipq}, \\ \Phi_{ipq}^{*} \triangleq \mathscr{A} + \mathscr{B}_{ipq}^{\mathrm{T}}\tilde{P}_i\mathscr{B}_{ipq} < \tilde{F}_{ipq}, \end{cases} \tag{23}$$

where

$$\mathscr{A} = diag\{dR, -R\},\ \mathscr{B}_{ipq} = \begin{bmatrix} \bar{A}_{ipq}(k) & A_{di} \end{bmatrix},$$

$$\mathscr{C} = diag\{dR, -R, -\gamma^2 I\},\ \mathscr{D}_{ipq} = \begin{bmatrix} \bar{A}_{ipq}(k) & A_{di} & B_{2i} \\ \bar{C}_{ipq}(k) & C_{di} & D_{2i} \end{bmatrix},$$

$$\mathscr{G}_i = diag\{-\tilde{P}_i^{-1}, -I\},$$

$$\tilde{F}_{ipq} = diag\{F_{ipq}, 0\},\ \hat{F}_{ipq} = diag\{F_{ipq}, 0, 0\}.$$

Next, we introduce the following Lyapunov-Krasovskii functional:

$$V(k) = \sum_{t=1}^{2}V_t(k), \tag{24}$$

where

$$V_1(k) = x^{\mathrm{T}}(k)P_{\theta_k}x(k),$$

$$V_2(k) = \sum_{\beta=-d_2+1}^{-d_1+1}\sum_{\alpha=k-1+\beta}^{k-1}x^{\mathrm{T}}(\alpha)Rx(\alpha).$$

Letting $\nabla V(k)$ be the forward difference of $V(k)$, we will figure out $E\{\nabla V(k)\}$ which is composed of two parts,

$E\{\nabla V_1(k)\}$ and $E\{\nabla V_2(k)\}$. We readily find that

$$
\begin{aligned}
E\{\nabla V_1(k)\} =&E\{V_1(k+1)-V_1(k)|x(k),\theta_k=i\}\\
=&E\{x^{\mathrm{T}}(k+1)\tilde{P}_i x(k+1)\}-x^{\mathrm{T}}(k)P_i x(k).
\end{aligned}
\tag{25}
$$

We introduce notations $\zeta_1(k)=[x^{\mathrm{T}}(k)\ x^{\mathrm{T}}(k-d(k))]^{\mathrm{T}}$ and $\zeta(k)=[\zeta_1^{\mathrm{T}}(k)\ w^{\mathrm{T}}(k)]^{\mathrm{T}}$. On the basis of system $\mathbb{S}_{cl}$, we have

$$
\begin{aligned}
&E\{x^{\mathrm{T}}(k+1)\tilde{P}_i x(k+1)\}\\
=&E\Big\{\sum_{p=1}^{n}\sum_{q=1}^{n}\omega_{ip}\sigma_{iq}\zeta^{\mathrm{T}}(k)\begin{bmatrix}\mathscr{B}_{ipq}^{\mathrm{T}}\\ B_{2i}^{\mathrm{T}}\end{bmatrix}\tilde{P}_i\begin{bmatrix}\mathscr{B}_{ipq} & B_{2i}\end{bmatrix}\zeta(k)\Big\}.
\end{aligned}
\tag{26}
$$

On the other hand,

$$
\begin{aligned}
&E\{\nabla V_2(k)\}=E\{V_2(k+1)-V_2(k)\}\\
=&E\Big\{\sum_{\beta=-d_2+1}^{-d_1+1}\sum_{\alpha=k+\beta}^{k}x^{\mathrm{T}}(\alpha)Rx(\alpha)\\
&-\sum_{\beta=-d_2+1}^{-d_1+1}\sum_{\alpha=k-1+\beta}^{k-1}x^{\mathrm{T}}(\alpha)Rx(\alpha)\Big\}\\
=&E\Big\{\sum_{\beta=-d_2+1}^{-d_1+1}\{x^{\mathrm{T}}(k)Rx(k)\\
&-x^{\mathrm{T}}(k-1+\beta)Rx(k-1+\beta)\}\Big\}.
\end{aligned}
\tag{27}
$$

We can obtain that

$$
\sum_{\beta=-d_2+1}^{-d_1+1}x^{\mathrm{T}}(k)Rx(k)=x^{\mathrm{T}}(k)dRx(k)
\tag{28}
$$

and

$$
\begin{aligned}
&\sum_{\beta=-d_2+1}^{-d_1+1}x^{\mathrm{T}}(k-1+\beta)Rx(k-1+\beta)\\
=&\sum_{\beta=k-d_2}^{k-d_1}x^{\mathrm{T}}(\beta)Rx(\beta)\\
\geq&x^{\mathrm{T}}(k-d(k))Rx(k-d(k)).
\end{aligned}
\tag{29}
$$

Then the following inequality holds

$$
\begin{aligned}
&E\{\nabla V_2(k)\}\\
\leq&E\{x^{\mathrm{T}}(k)dRx(k)-x^{\mathrm{T}}(k-d(k))Rx(k-d(k))\}\\
=&E\{\zeta_1^{\mathrm{T}}(k)\mathscr{A}\zeta_1(k)\}.
\end{aligned}
\tag{30}
$$

Noting that $w(k)\equiv 0$ in the definition of stochastic mean square stability, we combine (25), (26), (30) and then obtain that

$$
\begin{aligned}
&E\{\nabla V(k)\}=E\{\nabla V_1(k)\}+E\{\nabla V_2(k)\}\\
\leq&E\Big\{\sum_{p=1}^{n}\sum_{q=1}^{n}\omega_{ip}\sigma_{iq}\zeta_1^{\mathrm{T}}(k)\Phi_{ipq}^{*}\zeta_1(k)-x^{\mathrm{T}}(k)P_i x(k)\Big\}\\
<&E\Big\{\zeta_1^{\mathrm{T}}(k)\Big(\sum_{p=1}^{n}\sum_{q=1}^{n}\omega_{ip}\sigma_{iq}\tilde{F}_{ipq}\Big)\zeta_1(k)-x^{\mathrm{T}}(k)P_i x(k)\Big\}\\
=&E\Big\{x^{\mathrm{T}}(k)\mathscr{F}_i x(k)\Big\}\\
\leq&\phi E\{x^{\mathrm{T}}(k)x(k)\},
\end{aligned}
\tag{31}
$$

where '<' holds as a result of (23), ϕ denotes the largest eigenvalue of $\mathscr{F}_i$, for all $i\in\mathscr{N}$, and (17) implies that $\phi<0$, then

$$
\begin{aligned}
&E\Big\{\sum_{0}^{\infty}x^{\mathrm{T}}(k)x(k)\Big\}<\frac{1}{\phi}E\Big\{\sum_{0}^{\infty}\nabla V(k)\Big\}\\
&=\frac{1}{\phi}E\{V(\infty)-V(0)\}\leq-\frac{1}{\phi}E\{V(0)\}<\infty
\end{aligned}
\tag{32}
$$

which is consistent with (13) in Definition 1, thus the stochastic mean square stability is proved.

Next we will pay attention to H_∞ noise attenuation performance, therefore we investigate the following performance index under zero initial condition:

$$
\begin{aligned}
&J=\sum_{k=0}^{\infty}E\{y^{\mathrm{T}}(k)y(k)-\gamma^2 w^{\mathrm{T}}(k)w(k)\}\\
\leq&\sum_{k=0}^{\infty}E\{y^{\mathrm{T}}(k)y(k)-\gamma^2 w^{\mathrm{T}}(k)w(k)+\nabla V(k)\}\\
\leq&\sum_{k=0}^{\infty}E\Big\{\sum_{p=1}^{n}\sum_{q=1}^{n}\omega_{ip}\sigma_{iq}\zeta^{\mathrm{T}}(k)\Phi_{ipq}^{\dagger}\zeta(k)-x^{\mathrm{T}}(k)P_i x(k)\Big\}
\end{aligned}
\tag{33}
$$

where (25), (26) and (30) contribute to the second "≤". In a similar line with (31), we get

$$
J<\sum_{k=0}^{\infty}E\Big\{x^{\mathrm{T}}(k)\mathscr{F}_i x(k)\Big\}<0,
\tag{34}
$$

which implies that (14) is satisfied. Thus the proof is completed. □

Remark 4 *A sufficient condition concerning stochastic mean square stability with an H_∞ noise attenuation performance γ is derived for system $\mathbb{S}_{cl}$ in Theorem 1. Using the similar technique as in [23, 27, 31], we introduce the matrix F_{ipq}, thus* (16) *is separated from conditional probabilities ω_{ip} and σ_{iq}. Otherwise, the matrix dimension will be high and even grow higher with the increase* *of number of elements in the set $\mathscr{N}$ when linearizing this matrix inequality. Hence, the introduction of F_{ipq} will facilitate the controller design.*

However, in view of the complexity of condition (15) and (16), we are incapable to parameterize the controller gain K_p directly by Theorem 1, which motives us to present the following result.

Theorem 2 *The closed-loop system $\mathbb{S}_{cl}$ is stochastically mean square stable with an H_∞ noise attenuation performance γ, if there exist a positive scalar $\bar{\gamma}$, matrices $\bar{K}_p \in \mathbb{R}^{m\times n_x}$, $L \in \mathbb{R}^{n_x}$, positive definite matrices $\bar{P}_i \in \mathbb{R}^{n_x}$, $\bar{R} \in \mathbb{R}^{n_x}$, $\bar{F}_{ipq} \in \mathbb{R}^{n_x}$, and a positive definite diagonal matrix $W_{iq} \in \mathbb{R}^m$, for $\forall i,p,q \in \mathscr{N}$, such that the following conditions hold:*

$$\begin{bmatrix} -\bar{P}_i & \Gamma_i \\ * & \Xi_i \end{bmatrix} < 0, \tag{35}$$

$$\begin{bmatrix} \mathscr{U}_{ipq} & \mathscr{V}_{ipq} & \mathscr{W}_{ipq} \\ * & -I & 0 \\ * & * & \mathscr{P} \end{bmatrix} < 0, \tag{36}$$

where

$$\begin{aligned}
\Gamma_i &= \left[\sqrt{\mu_{i11}}\bar{P}_i \ \cdots \ \sqrt{\mu_{ipq}}\bar{P}_i \ \cdots \ \sqrt{\mu_{inn}}\bar{P}_i\right], \\
\Xi_i &= diag\{-\bar{F}_{i11}, \cdots, -\bar{F}_{ipq}, \cdots, -\bar{F}_{inn}\}, \ \mu_{ipq} = \omega_{ip}\sigma_{iq}, \\
\mathscr{U}_{ipq} &= \begin{bmatrix} d\bar{R} + \bar{F}_{ipq} - L^{\mathrm{T}} - L & 0 & 0 & \bar{K}_p^{\mathrm{T}} & 0 \\ * & -\bar{R} & 0 & 0 & 0 \\ * & * & -\bar{\gamma}I & 0 & 0 \\ * & * & * & -W_{iq} & 0 \\ * & * & * & * & -W_{iq} \end{bmatrix}, \\
\mathscr{V}_{ipq} &= \left[C_iL + D_{1i}\bar{K}_p \ C_{di}L \ D_{2i} \ 0 \ D_{1i}\Lambda_q W_{iq}\right]^{\mathrm{T}}, \\
\mathscr{W}_{ipq} &= \left[\sqrt{\pi_{i1}}Z_{ipq}^{\mathrm{T}} \ \sqrt{\pi_{i2}}Z_{ipq}^{\mathrm{T}} \ \cdots \ \sqrt{\pi_{in}}Z_{ipq}^{\mathrm{T}}\right], \\
Z_{ipq} &= \left[A_iL + B_{1i}\bar{K}_p \ A_{di}L \ B_{2i} \ 0 \ B_{1i}\Lambda_q W_{iq}\right], \\
\mathscr{P} &= diag\{-\bar{P}_1, -\bar{P}_2, \cdots, -\bar{P}_n\},
\end{aligned}$$

where these variables, all the system matrices in $\mathbb{S}$ and transition probability matrix Θ, conditional probability matrices Ω, Σ, quantization parameters Λ_q, boundaries of time delay d_1 and d_2 are all pre-given. Moreover, if the LMIs (35) and (36) are feasible, then the controller gain can be parameterized as

$$K_p = \bar{K}_p L^{-1}. \tag{37}$$

Proof. To begin with, we denote

$$\begin{gathered} \bar{P}_i = P_i^{-1}, \ \bar{F}_{ipq} = F_{ipq}^{-1}, \ \bar{\gamma} = \gamma^2, \\ \bar{R} = L^{\mathrm{T}}RL, \ \bar{K}_p = K_pL, \end{gathered} \tag{38}$$

where L is a slack matrix, and it is invertible, which is guaranteed by (36). By performing a congruence transformation to (35) using $diag\{P_i, I, \cdots, I\}$, the following matrix inequality holds

$$\begin{bmatrix} -P_i & \bar{\Gamma}_i \\ * & \Xi_i \end{bmatrix} < 0, \tag{39}$$

where $\bar{\Gamma}_i = \left[\sqrt{\mu_{i11}}I \ \cdots \ \sqrt{\mu_{ipq}}I \ \cdots \ \sqrt{\mu_{inn}}I\right]$. By using Schur Complement, it can be seen that (39) is equivalent to (15).

On the other hand, due to the fact that

$$(\bar{F}_{ipq} - L)^{\mathrm{T}}\bar{F}_{ipq}^{-1}(\bar{F}_{ipq} - L) \geq 0, \tag{40}$$

namely,

$$-L^{\mathrm{T}}\bar{F}_{ipq}^{-1}L \leq \bar{F}_{ipq} - L^{\mathrm{T}} - L, \tag{41}$$

then (36) implies

$$\begin{bmatrix} \bar{\mathscr{U}}_{ipq} & \mathscr{V}_{ipq} & \mathscr{W}_{ipq} \\ * & -I & 0 \\ * & * & \mathscr{P} \end{bmatrix} < 0 \tag{42}$$

holds, where

$$\bar{\mathscr{U}}_{ipq} = \begin{bmatrix} d\bar{R} - L^{\mathrm{T}}\bar{F}_{ipq}^{-1}L & 0 & 0 & \bar{K}_p^{\mathrm{T}} & 0 \\ * & -\bar{R} & 0 & 0 & 0 \\ * & * & -\bar{\gamma}I & 0 & 0 \\ * & * & * & -W_{iq} & 0 \\ * & * & * & * & -W_{iq} \end{bmatrix}.$$

Denote $\mathscr{L} = diag\{(L^{\mathrm{T}})^{-1}, (L^{\mathrm{T}})^{-1}, I, I, I, I, I, \cdots, I\}$, pre- and post-multiply (42) by $\mathscr{L}$ and $\mathscr{L}^{\mathrm{T}}$ respectively, we have

$$\begin{bmatrix} X_{ipq} & Y_{ipq}^{\dagger\mathrm{T}} & \mathscr{Y}_{ipq} \\ * & -I & 0 \\ * & * & \mathscr{P} \end{bmatrix} < 0, \tag{43}$$

where

$$X_{ipq} = \begin{bmatrix} dR - F_{ipq} & 0 & 0 & K_p^{\mathrm{T}} & 0 \\ * & -R & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -W_{iq} & 0 \\ * & * & * & * & -W_{iq} \end{bmatrix},$$

$$Y_{ipq}^{\dagger} = \begin{bmatrix} C_i + D_{1i}K_p & C_{di} & D_{2i} & 0 & D_{1i}\Lambda_q W_{iq} \end{bmatrix},$$

$$\mathscr{Y}_{ipq} = \begin{bmatrix} \sqrt{\pi_{i1}}Y_{ipq}^{*\mathrm{T}} & \sqrt{\pi_{i2}}Y_{ipq}^{*\mathrm{T}} & \cdots & \sqrt{\pi_{in}}Y_{ipq}^{*\mathrm{T}} \end{bmatrix},$$

$$Y_{ipq}^{*} = \begin{bmatrix} A_i + B_{1i}K_p & A_{di} & B_{2i} & 0 & B_{1i}\Lambda_q W_{iq} \end{bmatrix}.$$

By applying Schur Complement to (43), we obtain (16), and the proof is completed. □

Remark 5 *The parameter γ denotes the H_∞ performance index of the resulting closed-loop system $\mathbb{S}_{cl}$, the smaller γ is, the better the performance is. By means of LMI toolbox in Matlab, the optimized γ, denoted as γ^*, can be obtained by minimizing $\bar{\gamma}$ subject to* (35) *and* (36)*, then $\gamma^* = \sqrt{\bar{\gamma}_{min}}$.*

4 Numerical Example

In this section, our design method will be supported by a numerical simulation, where a 2-mode MJS of the form $\mathbb{S}$ is considered, and corresponding parameters are as follows:

$$A_1 = \begin{bmatrix} 1.45 & 1 \\ 0.1 & 0.6 \end{bmatrix},\ A_{d1} = \begin{bmatrix} 0.1 & -0.2 \\ 0.1 & 0.15 \end{bmatrix}, B_{11} = \begin{bmatrix} 1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.1 & 0.6 \\ 0.8 & -1.1 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.1 & -0.2 \\ 0 & 0.1 \end{bmatrix}, B_{12} = \begin{bmatrix} 0.5 & 0.8 \\ 0.2 & 0.1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1.6 & 0.2 \end{bmatrix}, C_{d1} = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, D_{11} = \begin{bmatrix} 0.1 & 1 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 1 & 1.5 \end{bmatrix},\ C_{d2} = \begin{bmatrix} 1 & 0.5 \end{bmatrix}, D_{12} = \begin{bmatrix} 0.9 & 0.5 \end{bmatrix},$$

$$B_{21} = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}, B_{22} = \begin{bmatrix} 0.1 \\ 0.02 \end{bmatrix}, D_{21} = 0.2, D_{22} = 0.4.$$

The transition probability matrix Θ and conditional probability matrices Ω, Σ are chosen as

$$\Theta = \begin{bmatrix} 0.9 & 0.1 \\ 0.36 & 0.64 \end{bmatrix}, \Omega = \begin{bmatrix} 0.6 & 0.4 \\ 0.8 & 0.2 \end{bmatrix}, \Sigma = \begin{bmatrix} 0.75 & 0.25 \\ 0.9 & 0.1 \end{bmatrix},$$

respectively. The time delay takes value 1 or 2 randomly, which implies $d = 2$. The error bounds of the logarithmic quantizer are

$$\Lambda_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ \Lambda_2 = \begin{bmatrix} 0.18 & 0 \\ 0 & 0.18 \end{bmatrix}.$$

Then by Theorem 2, one can readily obtain the optimized H_∞ performance $\gamma^* = 1.6312$ and the following control gain

$$K_1 = \begin{bmatrix} -1.3865 & -0.2973 \\ 1.7847 & -2.9033 \end{bmatrix},\ K_2 = \begin{bmatrix} -1.3926 & -0.2950 \\ 1.7004 & -2.8702 \end{bmatrix}.$$

Based on the parameters mentioned above, we further perform some simulations to show the closed-loop stability, taking initial condition $x(k_0) = [0.28\ 0.17]^{\mathrm{T}}, k_0 = -2, -1, 0,\ \theta_0 = 2$ and disturbance input $w(k) = 0.9^k sin(k)$. It is not difficult to find out that the open-loop system is unstable. Then, a Monte Carlo simulation is carried out with the proposed quantized controller. The simulation results are illustrated in Fig.1, which are state, output and control input trajectories of 1000 repeated simulations (in grey) and their mean values (in full blue line). It can be observed that, when the controller in Theorem 2 is applied, the state and output gradually tend to the equilibrium point, i.e., the closed-loop system becomes stable. Thus the effectiveness of the quantized controller in Theorem 2 is self-evident. Furthermore, to show the H_∞ performance, we repeat the Monte Carlo simulation under zero initial condition and then figure out the ratio

$$\gamma_{y/w} \triangleq \sqrt{\frac{\lim_{k\to\infty}\sum_k E\{\parallel y(k) \parallel^2\}}{\lim_{k\to\infty}\sum_k \parallel w(k) \parallel^2}}$$

which is displayed in Fig.2. Compared with the upper bound $\gamma^* = 1.6312$ calculated by Theorem 2, the ratio mentioned above is smaller, which shows the proposed controller design method is correct though the upper bound γ^* is conservative to some extent. Additionally, we simulate the case of mode-independent controller by letting $\lambda_k \in \{1\}$ and $\Omega = [1\ 1]^{\mathrm{T}}$ according to Remark 2, a worse H_∞ performance $\gamma^* = 1.6458$ is obtained, though this conventional controller is also able to stabilize the system.

In our work, the asynchronization phenomenon is described by hidden Markov models, the key point of this model lies in the conditional probability matrix which can reflect the asynchronization level. Next we will analyze the effect of hidden Markov feature on the H_∞ performance of closed-loop system by simulations with varying conditional probability matrices Ω and Σ. For the considered 2-mode MJS, Ω and Σ can be written as follows:

$$\Omega = \begin{bmatrix} \omega_{11} & 1-\omega_{11} \\ \omega_{21} & 1-\omega_{21} \end{bmatrix}, \; \Sigma = \begin{bmatrix} 1-\sigma_{12} & \sigma_{12} \\ 1-\sigma_{22} & \sigma_{22} \end{bmatrix}.$$

Before proceeding, we make some reasonable conjectures first:

(1) The H_∞ performance γ^* will keep invariant if the columns of Ω are exchanged, because the controller is designable, namely, we can determine the controller gains and can also allocate mode to every controller gain according to certain guidelines. Thus, the system can achieve the same performance by exchanging K_1 and K_2.
(2) The performance of the closed-loop system will improve with the increasing of synchronization level between the controlled system and controller.
(3) The closed-loop system has the worst H_∞ performance when $\omega_{11} = \omega_{21}$. In this case, the rows of Ω are identical, which leads to just one cluster case or mode-independent case as specified in Remark 2 and [22, 23], then the controlled system fails to provide useful mode information for controller design.
(4) The closed-loop system will have better H_∞ performance with smaller σ_{12} and σ_{22}, as smaller σ_{12} and σ_{22} mean that the quantizer is more likely to run in mode 1. By comparing Λ_1 and Λ_2, we can find that the quantizer will produce smaller quantization errors when running in mode 1 according to (9).

Next, we perform some simulations to validate the conjectures mentioned above. Letting Σ keep constant and Ω vary, we solve LMIs (35) and (36), and the corresponding γ^* are recorded in Fig.3. The figure is centered with $\omega_{11} = 0.5$ and $\omega_{21} = 0.5$, and γ^* distributes like a parabola at the direction $\omega_{11} + \omega_{21} = 1$, which verifies conjecture 1) and 2) respectively. We can observe the worst H_∞ performance γ^* at the position $\omega_{11} = \omega_{21}$ which illustrates conjecture 3) is right. Note that the worst H_∞ performance $\gamma^* = 1.6458$ equals to that in the case when a mode-independent controller is used, which implies that a mode-independent controller would suffice when $\omega_{11} = \omega_{21}$, otherwise, a mode-dependent controller is preferable. Similarly, by changing Σ, we get Fig.4, which convincingly demonstrates conjecture 4) is correct.

Note that the works [25, 27], especially [22], have provided a detailed discussion on the relationship between H_2 performance and the detector's probability matrix. Though our discussion mentioned above is similar with that in [22], there are some differences, mainly in the following two aspects: Firstly, the work in [22] provided a similar figure as Fig.3 to show the influence of detection accuracy on H_2 performance, and stated that this figure is symmetric with respect to the line $\omega_{11} = \omega_{21}$ ($\rho_2 = 1 - \rho_1$ in [22]), but did not tell the reason therein. However, it is not the case in our work. We find that Fig.3 is centered with $\omega_{11} = 0.5$ and $\omega_{21} = 0.5$. And the reason has been analyzed in conjecture 1). Secondly, in our work, the influence of the synchronization level between the controlled system and quantizer is presented in Fig.4 and further discussed in conjecture 4).

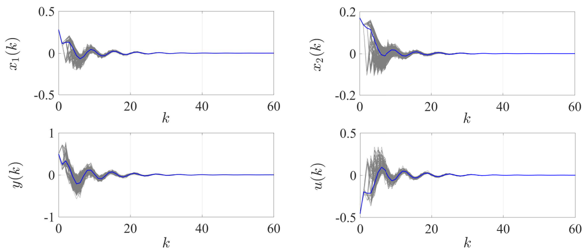

Fig. 1. A Monte Carlo simulation of 1000 repetitions: system state, output and control input

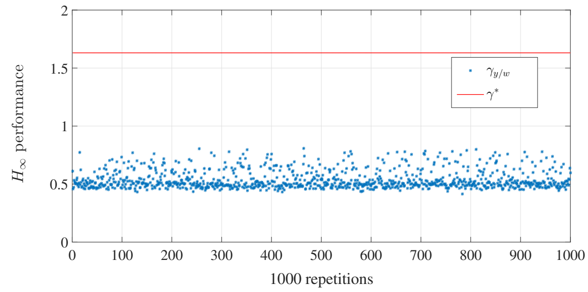

Fig. 2. Comparisons between γ^* and the ratio $\gamma_{y/w}$

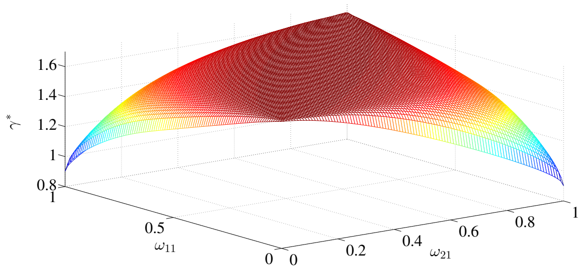

Fig. 3. H_∞ performance with varying Ω

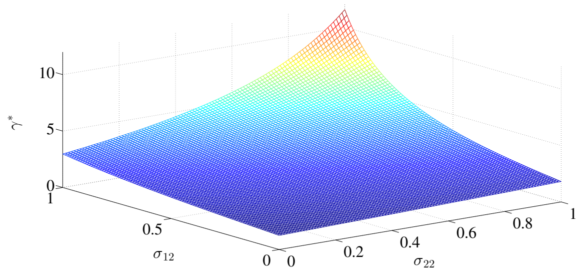

Fig. 4. H_∞ performance with varying Σ

5 Conclusion

The study on asynchronous control has been carried out for time-delay MJSs in the present paper. In the closed-loop system, the control input is the quantized state feedback through a logarithmic quantizer. Moreover, the asynchronizations between the controlled system and controller/quantizer are taken into consideration, and described by hidden Markov models. Under this framework, a sufficient condition has been derived such that the resulting closed-loop system is not only stochastically mean square stable but also has a prescribed H_∞ noise attenuation performance. Then the nonlinearity of the matrix inequalities in the sufficient condition is further handled such that the control gain can be easily figured out with the help of Matlab. Finally, the correctness and effectiveness of the design method is demonstrated by a numerical example.

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