Distributed Adaptive Leader-follower and Leaderless Consensus Control of a Class of Strict-feedback Nonlinear Systems: A Unified Approach

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Abstract

In this paper, distributed adaptive consensus for a class of strict-feedback nonlinear systems under directed topology condition is investigated. Both leader-follower and leaderless cases are considered in a unified framework. To design distributed controller for each subsystem, a local compensatory variable is generated based on the signals collected from its neighbors. Such a technique enables us to solve the leader-follower consensus and leaderless consensus problems in a unified framework. And it further allows us to treat the leaderless consensus as a special case of the leader-follower consensus. For leader-follower consensus, the assumption that the leader trajectory is linearly parameterized with some known functions as required in most recent relevant literatures is successfully relaxed. It is shown that global uniform boundedness of all closed-loop signals and asymptotically output consensus could be achieved for both cases. Simulation results are provided to verify the effectiveness of our schemes.

Key words: Adaptive control; Distributed Consensus Control; Leader-follower Consensus; Leaderless Consensus.

1 Introduction

Consensus of multi-agent systems has become a rapidly emerging topic in various research communities over the past decades due to its wide potential applications. Distributed consensus control normally aims at achieving an agreement for the states or the outputs of network connected subsystems by designing a control protocol for each agent based on only locally available information collected within its neighboring area. This control issue can be further classified into leaderless consensus control (see Ren and Beard (2005) and many other references) and leader-following consensus control, such as Abdessameud (2017); Arcak (2007); Wang et al. (2014); Zhang and Lewis (2012); Hong et al. (2006); Huang et al. (2015, 2017); Yoo (2013); Wang et al. (2016a,b, 2017); Zhang et al. (2015a,b, 2016).

Leaderless consensus means that the outputs of all agents reach a common state in a cooperative manner through distributed controls with no specified leader in the systems. Over the past few years, the leaderless consensus problem has been investigated by many scholars. In Ren (2009), a distributed leaderless consensus algorithms is proposed for networked Euler-Lagrange systems. In Qiu et al. (2016), leaderless quantized consensus for a kind of high-order linear systems is considered. In Yu and Xia (2017), the leaderless consensus problem of first-order nonlinear multi-agent systems with jointly connected topologies is addressed. However, the research on the leaderless consensus control of uncertain strict-feedback nonlinear systems is still unsatisfactory. The main reason is that the unmatched unknown pa-

⋆ This paper was not presented at any IFAC meeting. Corresponding author Wei Wang.

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rameters will be intertwined with the Laplacian matrix, which makes distributed parameter update laws difficult to design. The leaderless consensus of high-order nonlinear systems such as strict-feedback nonlinear systems with unmatched parametric uncertainties and external disturbances still remain unsolved.

On the other hand, the leader-follower consensus control also receives lots of attention. In Abdessameud (2017) the leader-follower synchronization of uncertain networked Euler-Lagrange systems under directed graphs with communication constraints is considered, where the dynamics of the leader is governed by a matrix whose eigenvalues are pure imaginary. In Li et al. (2013a), distributed control of multi-agent systems with general linear dynamics is investigated with the input of the leader's dynamics assumed to be nonzero. However, the control signals designed for the agents are non-smooth. Similarly in Lu et al. (2016), two non-smooth leader-following formation protocols are presented for nonlinear multi-agent systems with directed communication network topologies. In Wang et al. (2014) and Yu and Xia (2012), the reference trajectory is assumed to be linearly-parameterized with the basic time-varying functions known by all agents. Based on this assumption, the distributed adaptive control approaches are proposed by adopting backstepping technique. In Wang et al. (2017), the leader-follower consensus control problem for a group of uncertain Euler-Lagrangian systems is addressed. The controller is smooth but the system model appears in Brunovsky form, i.e., there is no unmatched unknown parameter. Other representative works are reported in Zhang and Lewis (2012); Ferik et al. (2014); Wang et al. (2017); Arcak (2007); Hong et al. (2006); Bai et al. (2009); Das and Lewis (2010); Ren (2007) etc., where linear model or a simple nonlinear model is considered.

The main difficulty of leader-follower consensus is that not all agents have direct access to the trajectory of the leader. To handle this difficulty, the existing results of leader-follower consensus can be generally classified into three categories. i) The behavior of the leader is set by a specific node with similar dynamics to the followers with zero/known inputs, e.g. in Zhang et al. (2015a), Cao et al. (2015) and many reference therein. ii) The desired reference trajectory is assumed to be linearly parameterized with basis function vectors known by all subsystems, e.g. in Bai et al. (2009); Yu and Xia (2012); Wang et al. (2014); Hu and Zheng (2014). iii) The reference is time-varying but non-smooth signum function based distributed control approaches are adopted, which is undesirable due to chattering phenomenon, e.g. in Lu et al. (2016); Dong (2012); Li et al. (2013a); Mei et al. (2011). In Huang et al. (2017), by introducing an nth-order filter and a group of n estimators for counteracting the effects due to totally unknown trajectory information in each agent, a new backstepping based smooth distributed adaptive tracking control protocol is proposed. However, this control scheme needs a considerable amount of communication among agents over the communication channel for updating the estimated parameters, which may be unsatisfactory if the communication channel bandwidth and computation resources are limited.

In this paper, a unified consensus control approach will be proposed to address both leader-follower consensus and leaderless consensus problems for a group of strict-feedback nonlinear multi-agent systems under directed topology condition, where intrinsic mismatched unknown parameters and uncertain non-vanishing disturbances are simultaneously involved. Based on such an approach, the leaderless consensus can be treated as a special case of leader-follower consensus in terms of control design process. For the leader-follower consensus case, the time-varying leader trajectory $y_r(t)$ no longer needs to be linearly parameterized with basis functions. Asymptotical convergence of consensus is achieved, while the control signals are guaranteed to be smooth. The main contributions of this paper are twofold. Firstly, smooth consensus controllers are designed thus undesired chattering phenomenon is avoided. The assumptions on linearly parameterized reference signals are no longer needed. Furthermore, all closed-loop signals are globally uniformly bounded and asymptotically consensus tracking for all agent outputs are achieved, despite of the presence of uncertainties and external disturbances mentioned above. Secondly, local compensatory variables are generated which unifies the leader-follower consensus and leaderless consensus and makes leaderless consensus as a special case of leader-follower consensus in terms of control design process. The compensatory variables are specially generated in such a way so that the distributed controllers and parameter estimators can be designed under the framework of backstepping approach. Finally, simulation results are provided to verify the effectiveness of the proposed control schemes.

The paper is organized as follows. In Section 2, the control problem is formulated and some necessary preliminaries are provided. In Section 3, the consensus control schemes and stability analysis are given. In Section 4, two simulation examples are shown to illustrate the effectiveness of the control schemes and finally the paper is concluded in Section 5.

2 Problem formulation

2.1 System Model

We consider a group of N nonlinear agents which can be modeled as follows.

$$\begin{aligned} \dot{x}_{i,q} &= x_{i,q+1} + \psi_{i,q}(x_{i,1}, \ldots, x_{i,q})\theta_{i,q}, \quad q = 1, \ldots, n-1 \\ \dot{x}_{i,n} &= u_i + \psi_{i,n}(x_i)\theta_{i,n} + d_i(t) \\ y_i &= x_{i,1} \end{aligned} \tag{1}$$

where $i = 1, 2, \ldots, N$, $x_i = [x_{i,1}, \ldots, x_{i,n}]^{\mathrm{T}} \in \Re^{\mathrm{n}}$, $u_i \in \Re$, $y_i \in \Re$, $d_i(t)$ are the state, control input and output of the ith agent and external disturbance respectively. $\theta_{i,q} \in \Re$, $q = 1, ..., n$, is an unknown constant. $\psi_{i,j} : \Re^{\mathrm{j}} \to \Re$ for $j = 1, \ldots, n$ are known smooth nonlinear functions.

2.2 Information Transmission Among the N agents

Suppose that the communications among the N agents can be represented by a directed graph $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{1, \ldots, N\}$ denotes the set of indexes (or vertices) corresponding to each agent, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges between two distinct agents. An edge $(i, j) \in \mathcal{E}$ indicates that agent j can obtain information from agent i, but not necessarily vice versa (Ren and Cao (2010)). In this case, agent i is called a neighbor of agent j. We denote the set of neighbors for agent i as $\mathcal{N}_i$. The connectivity matrix $A = [a_{ij}] \in \Re^{N \times N}$ is defined such that $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ if $(j, i) \notin \mathcal{E}$. Clearly, the diagonal elements $a_{ii} = 0$. We introduce an in-degree matrix $\triangle$ such that $\triangle = \mathrm{diag}(\triangle_i) \in \Re^{N \times N}$ with $\triangle_i = \sum_{j \in \mathcal{N}_i} a_{ij}$ being the sum of ith row in A. Then, the Laplacian matrix of $\mathcal{G}$ is defined as $\mathcal{L} = \triangle - A$. A direct path from agent i to agent j is a sequence of successive edges in the form $\{(v_i, v_l), (v_l, v_m), ..., (v_k, v_j)\}$. A digraph has a spanning tree, if there is an agent called root, such that there is a directed path from the root to each other agent in the graph. If there exists a directed path between any two distinct nodes in directed graph $\mathcal{G}$, the graph is said to be strongly connected.

We now use $b_i = 1$ to indicate the case that $y_r(t)$ is accessible directly to agent i; otherwise, b_i is set as 0. Throughout this paper, the following notations are used. $\|\cdot\|$ is the Euclidean norm of a vector. Let Q be a matrix, then $\lambda_{\min}(Q)$ denotes the minimum eigenvalue of Q.

The control objectives of this paper are to design distributed adaptive controllers for all the N subsystems (1) under the directed graph condition such that:

1) For the leader-follower case, all subsystem outputs reach a consensus by tracking a common desired trajectory $y_r(t)$ asymptotically, i.e., $\lim_{t \to \infty} y_i(t) - y_r(t) = 0$, $\forall i \in \mathcal{N}$.

2) For the leaderless case, all subsystem outputs reach a consensus, i.e. $\lim_{t \to \infty}(y_i(t) - y_j(t)) = 0$, $\forall i, j \in \mathcal{N}$,

where $\mathcal{N}$ denotes the set of all agents.

Before proceeding to the control design, the following lemmas are introduced, which will play important roles in the control design and stability analysis.

Lemma 1 *Zhang and Lewis (2012). If the directed $\mathcal{G}$ contains a spanning tree, then matrix $(\mathcal{L} + \mathcal{B})$ is nonsingular where $\mathcal{B} = diag\{b_1, \ldots, b_N\}$. Define*

$$\begin{aligned} \bar{q} &= [\bar{q}_1, \ldots, \bar{q}_N]^{\mathrm{T}} = (\mathcal{L} + \mathcal{B})^{-1}[1, \ldots, 1]^{\mathrm{T}} \\ P &= diag\{P_1, \ldots, P_N\} = diag\left\{\frac{1}{\bar{q}_1}, \ldots, \frac{1}{\bar{q}_N}\right\} \\ Q &= P(\mathcal{L} + \mathcal{B}) + (\mathcal{L} + \mathcal{B})^{\mathrm{T}} P, \end{aligned} \tag{2}$$

then $\bar{q}_i > 0$ for $i = 1, \ldots, N$ and Q is positive definite.

Lemma 2 *Ren and Cao (2010). Let $\mathcal{G}$ be a directed graph and L be the associated Laplacian matrix, then L has a single zero eigenvalue and all other eigenvalues have positive real parts if and only if $\mathcal{G}$ contains a directed spanning tree.*

3 Control Design and Stability Analysis

3.1 Leader-Follower Consensus Control

To achieve the leader-follower control objective, some necessary assumptions are imposed.

Assumption 1 *The directed graph $\mathcal{G}$ contains a spanning tree.*

Assumption 2 *There exists an unknown but bounded positive constant F such that $|y_r(t)| \leq F$, $\forall t > 0$. The first nth-order derivatives of $y_r(t)$ are bounded and piecewise continuous.*

Remark 1 *Assumption 2 is a reasonable and mild assumption since in practice, such as a group of mechanical systems, the leader always moves in a certain region. Then there always exists a constant F such that $|y_r(t)| \leq F$, $\forall t > 0$.*

The compensatory variable $z_{i,1}$ is generated for the ith agent

$$\dot{z}_{i,1} = -\sum_{j=1}^{N} a_{ij}(y_i - y_j) - b_i(y_i - y_r) \tag{3}$$

where $i \in \mathcal{V}$. Let $z_1 = [z_{1,1}, z_{2,1}, ..., z_{N,1}]^{\mathrm{T}}$, $y = [y_1, y_2, ..., y_N]^{\mathrm{T}}$ and $\underline{y}_r = [y_r, y_r, ..., y_r]^{\mathrm{T}}$, then

$$\begin{aligned} \dot{z}_1 &= -(\mathcal{L} + \mathcal{B})(y - \underline{y}_r) \\ &= -H(y - \underline{y}_r) \end{aligned} \tag{4}$$

where $H = \mathcal{L} + \mathcal{B}$. The local parameter update law for $\dot{\hat{\mathcal{F}}}_i$ is designed as

$$\dot{\hat{\mathcal{F}}}_i = -\sum_{j=1}^{N} a_{ij}(\hat{\mathcal{F}}_i - \hat{\mathcal{F}}_j) - b_i(\hat{\mathcal{F}}_i - \mathcal{F}) \tag{5}$$

where $i \in \mathcal{V}$, $\mathcal{F}$ is a positive constant which will be assigned later and let $\tilde{\mathcal{F}}_i = \hat{\mathcal{F}}_i - \mathcal{F}$ and $\underline{\tilde{\mathcal{F}}} = [\tilde{\mathcal{F}}_1, \tilde{\mathcal{F}}_2, ..., \tilde{\mathcal{F}}_N]^{\mathrm{T}}$, then

$$\dot{\underline{\tilde{\mathcal{F}}}} = -(\mathcal{L}+\mathcal{B})\underline{\tilde{\mathcal{F}}} \tag{6}$$

Define an error variable

$$e_i = y_i - c_0 z_{i,1} - \xi_{i,1} \tag{7}$$

where c_0 is a positive constant, and $\xi_{i,1}$ is a variable to be defined later. Let $e = [e_1, e_2, ..., e_N]^{\mathrm{T}}$, $\xi_1 = [\xi_{1,1}, \xi_{2,1}, ..., \xi_{N,1}]^{\mathrm{T}}$, then

$$\dot{z}_1 = -H(c_0 z_1 + e + \xi_1 - \underline{y}_r) \tag{8}$$

Let $\dot{\xi}_{i,j} = \xi_{i,j+1}$, $j = 1, ..., n$ and $\xi_{i,n+1} = -c_0\chi_i - \sum_{j=0}^{n-1} C_{n-1}^{j}\xi_{n-j} - s(\chi_i)\hat{F}_i$ with $\delta_i = \sum_{j=1}^{N} a_{ij}(\xi_{i,1} - \xi_{j,1}) - b_i(\xi_{i,1} - y_r)$ and $\chi_i = (\frac{\mathrm{d}}{\mathrm{dt}}+1)^{(n-1)}\delta_i$. Consider the first Lyapunov function as

$$V_1 = \frac{1}{2}\chi^{\mathrm{T}}P\chi + \frac{1}{2\gamma_F}\underline{\tilde{\mathcal{F}}}^{\mathrm{T}}P\underline{\tilde{\mathcal{F}}} \tag{9}$$

where γ_F is a positive constant and F being the bound of ξ_r with $\xi_r = \sum_{j=0}^{n-1} C_{n-1}^{j} y_r^{(n-j)}$, $s(x) = \frac{x}{\sqrt{x^2+\eta^2}}$, $\chi = [\chi_1, ..., \chi_N]^{\mathrm{T}}$, $\eta = e^{-2t}$. From (4) and (6), the derivative of V_1 is calculated as

$$\begin{aligned}\dot{V}_1 =& \chi^{\mathrm{T}}P\dot{\chi} + \frac{1}{\gamma_F}\underline{\tilde{\mathcal{F}}}^{\mathrm{T}}P\dot{\underline{\tilde{\mathcal{F}}}}\\ =& z_1^{\mathrm{T}}PH[-e - c_0\chi - diag\{s(\chi_i)\}\underline{\hat{\mathcal{F}}} + 1_N \otimes \xi_r]\\ &+ \frac{1}{\gamma_F}\underline{\tilde{\mathcal{F}}}^{\mathrm{T}}P\dot{\underline{\tilde{\mathcal{F}}}}\\ =& \chi^{\mathrm{T}}PHe - \chi^{\mathrm{T}}P(\Delta - A)diag\{s(\chi_i)\}\cdot 1_N \otimes \mathcal{F}\\ &- \chi^{\mathrm{T}}PBdiag\{s(\chi_i)\}\cdot 1_N \otimes \mathcal{F} - c_0\chi^{\mathrm{T}}PH\chi\\ &- \chi^{\mathrm{T}}PH\cdot 1_N \otimes y_r - \chi^{\mathrm{T}}PHdiag\{s(\chi_i)\}\underline{\tilde{\mathcal{F}}}\\ &- \frac{1}{\gamma_F}\underline{\tilde{\mathcal{F}}}^{\mathrm{T}}PH\underline{\tilde{\mathcal{F}}}\end{aligned} \tag{10}$$

The bounds of the following two terms can be calculated as
1)

$$\begin{aligned}&-\chi^{\mathrm{T}}P(\Delta - A)diag\{s(\chi_i)\}\cdot 1_N \otimes \mathcal{F}\\ =& -\chi^{\mathrm{T}}P\begin{pmatrix}\Delta_1 & -a_{12} & \cdots & -a_{1N}\\ -a_{21} & \Delta_2 & \cdots & -a_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ -a_{N1} & -a_{N2} & \cdots & \Delta_N\end{pmatrix}\begin{bmatrix}s(\chi_1)\mathcal{F}\\ s(\chi_2)\mathcal{F}\\ \vdots\\ s(\chi_N)\mathcal{F}\end{bmatrix}\end{aligned} \tag{11}$$

$$\begin{aligned}=& -\sum_{i=1}^{N}\chi_i P_i\Delta_i s(\chi_i)\mathcal{F} + \sum_{i=1}^{N}\sum_{k=1;k\neq i}^{N}\chi_i P_i\mathcal{F}a_{ik}s(\chi_k)\\ \leq& -\sum_{i=1}^{N}\chi_i P_i\Delta_i s(\chi_i)\mathcal{F} + \sum_{i=1}^{N}\sum_{k=1;k\neq i}^{N}\mathcal{F}P_i a_{ik}|\chi_i|\\ \leq& \sum_{i=1}^{N}\mathcal{F}P_i\Delta_i[|\chi_i| - \chi_i s(\chi_i)]\\ \leq& \sum_{i=1}^{N}\mathcal{F}P_i\Delta_i\eta\end{aligned} \tag{12}$$

2)

$$\begin{aligned}&-\chi^{\mathrm{T}}P\mathcal{B}diag\{s(\chi_i)\}\cdot 1_N \otimes \mathcal{F} - \chi^{\mathrm{T}}PH\cdot 1_N \otimes \xi_r\\ =& -\chi^{\mathrm{T}}P\mathcal{B}diag\{s(\chi_i)\}\cdot 1_N \otimes \mathcal{F} - \chi^{\mathrm{T}}P\mathcal{B}\otimes \xi_r\\ =& -z_1^{\mathrm{T}}[P_1 b_1 s(\chi_1)\mathcal{F}, ..., P_N b_N s(\chi_N)\mathcal{F}]^{\mathrm{T}}\\ &- z_1^{\mathrm{T}}P[b_1\xi_r, b_2\xi_r, ..., b_N\xi_r]^{\mathrm{T}}\\ =& -\sum_{i=1}^{N}\chi_i P_i b_i s(\chi_i)\mathcal{F} - \sum_{i=1}^{N}\chi_i P_i b_i\xi_r\\ \leq& -\sum_{i=1}^{N}\chi_i P_i b_i s(\chi_i)\mathcal{F} + \sum_{i=1}^{N}|\chi_i|P_i b_i\mathcal{F}\\ \leq& \sum_{i=1}^{N}P_i b_i\mathcal{F}(|\chi_i| - \chi_i s(\chi_i))\\ \leq& \sum_{i=1}^{N}P_i b_i\mathcal{F}\eta\end{aligned} \tag{13}$$

Substituting (11) and (13) into (10) yields that

$$\begin{aligned}\dot{V}_1 \leq& \chi^{\mathrm{T}}PHe - \frac{1}{2}c_0\chi^{\mathrm{T}}Q\chi + \sum_{i=1}^{N}(\Delta_i + b_i)P_i\mathcal{F}\eta\\ &- \chi^{\mathrm{T}}PHdiag\{s(\chi_i)\}\underline{\tilde{\mathcal{F}}} - \frac{1}{2\gamma_F}\underline{\tilde{\mathcal{F}}}^{\mathrm{T}}Q\underline{\tilde{\mathcal{F}}}\\ \leq& \parallel \chi \parallel\parallel PH \parallel\parallel e \parallel - \frac{1}{4}c_0\lambda_{min}(Q)\parallel \chi \parallel^2\\ &+ \sum_{i=1}^{N}(\Delta_i + b_i)P_i\mathcal{F}\eta - \frac{\lambda_{min}(Q)}{4\gamma_F}\parallel \underline{\tilde{\mathcal{F}}} \parallel^2\\ &+ (\frac{\parallel PH \parallel^2}{c_0\lambda_{min}(Q)} - \frac{\lambda_{min}(Q)}{4\gamma_F})\parallel \underline{\tilde{\mathcal{F}}} \parallel^2\end{aligned} \tag{14}$$

Choose γ_F such that

$$0 < \gamma_F < \frac{c_0\lambda_{min}(Q)}{\parallel PH \parallel^2}, \tag{15}$$

then we obtain

$$\begin{aligned}\dot{V}_1 \leq & \parallel \chi \parallel\parallel PH \parallel\parallel e \parallel -\frac{1}{4}c_0\lambda_{min}(Q) \parallel \chi \parallel^2 \\ &+ \sum_{i=1}^{N}(\Delta_i + b_i)P_i\mathcal{F}\eta - \frac{\lambda_{min}(Q)}{4\gamma_F} \parallel \underline{\tilde{\mathcal{F}}} \parallel^2\end{aligned} \tag{16}$$

Backstepping technique Krstic et al. (1995) will now be applied to design the adaptive control law for each subsystem.

•*Step 1:* Taking the derivative of e_i yields

$$\dot{e}_i = x_{i,2} + \psi_{i,1}(x_{i,1})\theta_{i,1} - c_0\dot{z}_{i,1} - \xi_{i,2} \tag{17}$$

Let $\alpha_{i,1}$ be the virtual control of $x_{i,2}$ and let $z_{i,2} = x_{i,2} - \alpha_{i,1}$. Then $\alpha_{i,1}$ is designed as

$$\alpha_{i,1} = -\hat{c}_{i,1}e_i - \psi_{i,1}(x_{i,1})\hat{\theta}_{i,1} + c_0\dot{z}_{i,1} + \xi_{i,2} \tag{18}$$

where $\hat{c}_{i,1}$ is the estimate of $c_{i,1}$, which is an unknown positive constant to be given later since the graph information is unknown to each agent and $\tilde{c}_{i,1} = \hat{c}_{i,1} - c_{i,1}$. $\hat{\theta}_{i,1}$ is the estimate of $\theta_{i,1}$ with $\tilde{\theta}_{i,1} = \hat{\theta}_{i,1} - \theta_{i,1}$. Consider the following Lyapunov function

$$V_2 = V_1 + \frac{1}{2}e^{\mathrm{T}}e + \frac{1}{2\gamma_{\theta,i}}\sum_{i=1}^{N}\tilde{\theta}_{i,1}^2 + \frac{1}{2\gamma_{c,i}}\sum_{i=1}^{N}\tilde{c}_{i,1}^2 \tag{19}$$

Then the derivative of V_2 is

$$\begin{aligned}\dot{V}_2 \leq & \parallel \chi \parallel\parallel PH \parallel\parallel e \parallel -\frac{1}{4}c_0\lambda_{min}(Q) \parallel \chi \parallel^2 \\ &+ \sum_{i=1}^{N}(\Delta_i + b_i)P_i\mathcal{F}\eta - \frac{\lambda_{min}(Q)}{4\gamma_F} \parallel \underline{\tilde{\mathcal{F}}} \parallel^2 \\ &- c_{i,1}e^{\mathrm{T}}e - \frac{1}{\gamma_{c,i}}\sum_{i=1}^{N}\tilde{c}_{i,1}\Big(\gamma_{c,i}e_i^2 - \dot{\hat{c}}_{i,1}\Big) \\ &- \frac{1}{\gamma_{\theta,i}}\sum_{i=1}^{N}\tilde{\theta}_{i,1}\Big(\gamma_{\theta,i}e_i\psi_{i,1}(x_{i,1}) - \dot{\hat{\theta}}_{i,1}\Big) + \sum_{i=1}^{N}e_iz_{i,2}\end{aligned} \tag{20}$$

The parameter estimators for $\dot{\hat{c}}_{i,1}$ and $\dot{\hat{\theta}}_i$ are designed as

$$\begin{aligned}\dot{\hat{c}}_{i,1} &= \gamma_{c,i}e_i^2 \\ \dot{\hat{\theta}}_{i,1} &= \gamma_{\theta,i}e_i\psi_{i,1}(x_{i,1})\end{aligned} \tag{21}$$

Furthermore, if

$$c_{i,1} \geq \frac{4 \parallel PH \parallel^2}{c_0\lambda_{min}(Q)} \tag{22}$$

then

$$\parallel \chi \parallel\parallel PH \parallel\parallel e \parallel -\frac{1}{8}c_0\lambda_{min}(Q) \parallel z_1 \parallel^2 - \frac{c_{i,1}}{2}e^{\mathrm{T}}e \leq 0 \tag{23}$$

Thus with $c_{i,1}$ being a positive satisfying (22), we have

$$\begin{aligned}\dot{V}_2 \leq & -\frac{1}{8}c_0\lambda_{min}(Q) \parallel \chi \parallel^2 - \frac{c_{i,1}}{2}e^{\mathrm{T}}e \\ &+ \sum_{i=1}^{N}(\Delta_i + b_i)P_i\mathcal{F}\eta - \frac{\lambda_{min}(Q)}{4\gamma_F} \parallel \underline{\tilde{\mathcal{F}}} \parallel^2 + \sum_{i=1}^{N}e_iz_{i,2}\end{aligned} \tag{24}$$

•*Step k* $(2 \leq k \leq n-1)$: Taking the time-derivative of $z_{i,k}$ yields

$$\begin{aligned}\dot{z}_{i,k} &= x_{i,k+1} + \theta_{i,k}\psi_{i,k}(\bar{x}_{i,k}) - \dot{\alpha}_{i,k-1} \\ &= \alpha_{i,k} + z_{i,k+1} - \frac{\partial\alpha_{i,k-1}}{\partial\hat{c}_{i,1}}\dot{\hat{c}}_{i,1} - \frac{\partial\alpha_{i,k-1}}{\partial\hat{\theta}_{i,1}}\dot{\hat{\theta}}_{i,1} \\ &\quad - \sum_{m=2}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial\hat{\Theta}_{i,m}}\dot{\hat{\Theta}}_{i,m} - \sum_{m=1}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial\eta^{(m-1)}}\eta^{(m-1)} \\ &\quad - \Theta_{i,k}^{\mathrm{T}}\Psi_{i,k}(\bar{x}_{1,k}, ..., \bar{x}_{N,k})\end{aligned} \tag{25}$$

where $\bar{x}_{i,k} = [x_{i,1}, ..., x_{i,k}]^{\mathrm{T}}$, $\alpha_{i,k}$ is the virtual control of $x_{i,k+1}$, $z_{i,k+1} = x_{i,k+1} - \alpha_{i,k}$ and

$$\Theta_{i,k}=[-\theta_{i,k}, \underset{m=1,...,k-1}{vec}(\theta_{i,m})^{\mathrm{T}}, \underset{j\in\mathcal{N}_i, m=1,...,k-1}{vec}(\theta_{j,m})^{\mathrm{T}}]^{\mathrm{T}}$$

and

$$\begin{aligned}\Psi_{i,k} = [\psi_{i,k}(\bar{x}_{i,k}), & \underset{m=1,...,k-1}{vec}(\frac{\partial\alpha_{i,k-1}}{\partial x_{i,m}}\psi_{i,m}(\bar{x}_{i,m}))^{\mathrm{T}}, \\ &\underset{j\in\mathcal{N}_i, m=1,...,k-1}{vec}(\frac{\partial\alpha_{i,k-1}}{\partial x_{j,m}}\psi_{j,m}(\bar{x}_{j,m}))^{\mathrm{T}}]^{\mathrm{T}}\end{aligned} \tag{26}$$

The virtual control $\alpha_{i,k}$ is designed as

$$\begin{aligned}\alpha_{i,k} = & -z_{i,k-1} - c_{i,k}z_{i,k} + \frac{\partial\alpha_{i,k-1}}{\partial\hat{c}_{i,1}}\dot{\hat{c}}_{i,1} + \frac{\partial\alpha_{i,k-1}}{\partial\hat{\theta}_{i,1}}\dot{\hat{\theta}}_{i,1} \\ &+ \sum_{m=1}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial\eta^{(m-1)}}\eta^{(m-1)} + \xi_{i,k} + \hat{\Theta}_{i,k}^{\mathrm{T}}\Psi_{i,k}(\bar{x}_{1,k}, ..., \bar{x}_{N,k}) \\ &+ \sum_{m=2}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial\hat{\Theta}_{i,m}}\dot{\hat{\Theta}}_{i,m}\end{aligned} \tag{27}$$

where $c_{i,k}$ is a positive constant, $\hat{\Theta}_{i,k}$ is the estimate of $\Theta_{i,k}$ and $\tilde{\Theta}_{i,k} = \hat{\Theta}_{i,k} - \Theta_{i,k}$. Define a new Lyapunov function

$$V_{k+1} = V_k + \frac{1}{2}\sum_{i=1}^{N}z_{i,k}^2 + \frac{1}{2}\sum_{i=1}^{N}\tilde{\Theta}_{i,k}^{\mathrm{T}}\tilde{\Theta}_{i,k} \tag{28}$$

The parameter estimator for $\hat{\Theta}_{i,k}$ is designed as

$$\dot{\hat{\Theta}}_{i,k} = -z_{i,k}\Psi_{i,k}(\bar{x}_{1,k},...,\bar{x}_{N,k}) \tag{29}$$

Then

$$\dot{V}_{k+1} \leq -\frac{1}{8}c_0\lambda_{min}(Q) \parallel \chi \parallel^2 - \frac{c_{i,1}}{2}e^{\mathrm{T}}e - \sum_{m=2}^{k} c_{i,m}z_{i,m}^2 + \sum_{i=1}^{N}(\Delta_i + b_i)P_i\mathcal{F}\eta - \frac{\lambda_{min}(Q)}{4\gamma_F} \parallel \underline{\tilde{\mathcal{F}}} \parallel^2 + \sum_{i=1}^{N} z_{i,k}z_{i,k+1} \tag{30}$$

•*Step n*: Taking the time-derivative of $z_{i,n-1}$ yields

$$\begin{aligned}\dot{z}_{i,n-1} &= u_i + \theta_{i,n}\psi_{i,n}(x_i) - \dot{\alpha}_{i,n-1}\\ &= u_i - \frac{\partial\alpha_{i,n-1}}{\partial\hat{c}_{i,1}}\dot{\hat{c}}_{i,1} - \frac{\partial\alpha_{i,n-1}}{\partial\hat{\theta}_{i,1}}\dot{\hat{\theta}}_{i,1} - \sum_{m=2}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial\hat{\Theta}_{i,m}}\dot{\hat{\Theta}}_{i,m}\\ &\quad - \sum_{m=1}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial\eta^{(m-1)}}\eta^{(m-1)} - \Theta_{i,n}^{\mathrm{T}}\Psi_{i,n}(\bar{x}_{1,n},...,\bar{x}_{N,n})\end{aligned} \tag{31}$$

where

$$\Theta_{i,n} = [-\theta_{i,n}, \underset{m=1,...,n-1}{vec}(\theta_{i,m})^{\mathrm{T}}, \underset{j\in\mathcal{N}_i,m=1,...,n-1}{vec}(\theta_{j,m})^{\mathrm{T}}]^{\mathrm{T}}$$

and

$$\begin{aligned}\Psi_{i,n} = [\psi_{i,n}(x_i), &\underset{m=1,...,n-1}{vec}(\frac{\partial\alpha_{i,n-1}}{\partial x_{i,m}}\psi_{i,m}(\bar{x}_{i,m}))^{\mathrm{T}},\\ &\underset{j\in\mathcal{N}_i,m=1,...,k-1}{vec}(\frac{\partial\alpha_{i,n-1}}{\partial x_{j,m}}\psi_{j,m}(\bar{x}_{j,m}))^{\mathrm{T}}]^{\mathrm{T}}\end{aligned} \tag{32}$$

The control input u_i is designed as

$$\begin{aligned}u_i = &-z_{i,n-1} - c_{i,n}z_{i,n} + \frac{\partial\alpha_{i,n-1}}{\partial\hat{c}_{i,1}}\dot{\hat{c}}_{i,1} + \frac{\partial\alpha_{i,n-1}}{\partial\hat{\theta}_{i,1}}\dot{\hat{\theta}}_{i,1}\\ &+ \sum_{m=1}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial\eta^{(m-1)}}\eta^{(m-1)} + \xi_{i,n} + \hat{\Theta}_{i,n}^{\mathrm{T}}\Psi_{i,n}(\bar{x}_{1,n},...,\bar{x}_{N,n})\\ &+ \sum_{m=2}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial\hat{\Theta}_{i,m}}\dot{\hat{\Theta}}_{i,m} - s(z_{i,n})\hat{D}_i\end{aligned} \tag{33}$$

where $c_{i,n}$ is a positive constant, $\hat{\Theta}_{i,n}$ and $\hat{D}_i$ are the estimates of $\Theta_{i,n}$ and D_i and $\tilde{\Theta}_{i,n} = \hat{\Theta}_{i,n} - \Theta_{i,n}$, $\tilde{D}_i = \hat{D}_i - D_i$, with D_i being the bound of $d_i(t)$. Define a new Lyapunov function

$$V_{n+1} = V_n + \frac{1}{2}\sum_{i=1}^{N} z_{i,n}^2 + \frac{1}{2}\sum_{i=1}^{N}\tilde{\Theta}_{i,n}^{\mathrm{T}}\tilde{\Theta}_{i,n} + \frac{1}{2}\tilde{D}_i^2 \tag{34}$$

The parameter estimator for $\hat{\Theta}_{i,k}$ and $\hat{\vartheta}_i$ are designed as

$$\begin{aligned}\dot{\hat{\Theta}}_{i,n} &= -z_{i,n}\Psi_{i,n}(\bar{x}_{1,n},...,\bar{x}_{N,n})\\ \dot{\hat{D}}_i &= -z_{i,n}s(z_{i,n})\end{aligned} \tag{35}$$

Then

$$\dot{V}_{n+1} \leq -\frac{1}{8}c_0\lambda_{min}(Q) \parallel \chi \parallel^2 - \frac{c_{i,1}}{2}e^{\mathrm{T}}e - \sum_{m=2}^{n} c_{i,m}z_{i,m}^2 - \frac{\lambda_{min}(Q)}{4\gamma_F} \parallel \underline{\tilde{\mathcal{F}}} \parallel^2 + \Omega e^{-2t} \tag{36}$$

where $\Omega = \sum_{i=1}^{N}(\Delta_i + b_i)P_i\mathcal{F} + D_i$ is a positive constant.

3.2 Stability Analysis of Leader-Follower Case

The main result of leader-follower consensus control is formally stated in the following theorem.

Theorem 1 *Consider the closed-loop system consisting of N uncertain nonlinear agents (1) satisfying Assumptions 1-2, the smooth controllers (33) and the parameter estimators (21), (29) and (35). All the signals in the closed-loop system are globally uniformly bounded and asymptotic consensus tracking of all the agents' outputs to $y_r(t)$ is achieved, i.e.* $\lim_{t\to\infty} y_i - y_r(t) = 0$.

Proof: Taking integration of both side of (36), it has

$$\begin{aligned}&V_{n+1}(\infty) + \frac{1}{8}c_0\lambda_{min}(Q)\int_0^\infty \parallel \chi \parallel^2 d\tau + \frac{c_{i,1}}{2}\int_0^\infty e^{\mathrm{T}}ed\tau\\ &+ \int_0^\infty \sum_{m=2}^{n} c_{i,m}z_{i,m}^2 d\tau + \frac{\lambda_{min}(Q)}{4\gamma_F}\int_0^\infty \parallel \underline{\tilde{\mathcal{F}}} \parallel^2 d\tau\\ &\leq V_{n+1}(0) + \frac{\Omega}{\beta}\end{aligned} \tag{37}$$

which means all signals in V_{n+1} are bounded, thus u_i is also bounded. Furthermore, $\int_0^\infty \parallel \chi \parallel^2 d\tau$, $\int_0^\infty e^{\mathrm{T}}ed\tau$, $\int_0^\infty \sum_{m=2}^{n} z_{i,m}^2 d\tau$ and $\int_0^\infty \parallel \underline{\tilde{\mathcal{F}}} \parallel^2 d\tau$ are bounded, and it is easy to check that their respective first-order derivatives are bounded, thus from Barbalat's Lemma, it has

$$\begin{aligned}&\lim_{t\to\infty}\chi_i = 0, \quad \lim_{t\to\infty} e_i = 0,\\ &\lim_{t\to\infty}\tilde{\mathcal{F}}_i = 0, i = 1,...,N, j = 1,...,n.\end{aligned} \tag{38}$$

Therefore it could be obtained that $\lim_{t\to\infty}\varepsilon_i = 0$ where $\varepsilon_i = \xi_{i,1} - y_r$. Consider the following Lyapunov function

$$V_z = \frac{1}{2}z_1^{\mathrm{T}}Pz_1 \tag{39}$$

whose time-derivative is

$$\dot{V}_z \leq -\frac{1}{4}c_0\lambda_{min}(Q)z_1^{\mathrm{T}}z_1 + \iota(e^{\mathrm{T}}e + \varepsilon^{\mathrm{T}}\varepsilon) \tag{40}$$

where $\iota = 2\|PH\|$ and $\varepsilon = [\varepsilon_1, ..., \varepsilon_N]^{\mathrm{T}}$. Therefore z_1 is ISS with respect to e_i and ε_i and it is easy to check that $\lim_{t\to\infty} z_{i,1} = 0$. With the fact that $\ddot{z}_{i,1}$ is also bounded, from Barbalat's Lemma, it has

$$\lim_{t\to\infty} \dot{z}_{i,1} = 0 \tag{41}$$

which means $\lim_{t\to\infty}(\mathcal{L}+\mathcal{B})(x_1 - \underline{y}_r) = \mathbf{0}$. Since $\mathcal{L}+\mathcal{B}$ is nonsigular, thus

$$\lim_{t\to\infty} y_i(t) - y_r(t) = 0. \tag{42}$$

This ends the proof of Theorem 1. □

Remark 2 *The main difficulty of the leader-follower consensus control for strict feedback nonlinear multi-agent systems is that not all the agents have direct access to the leader $y_r(t)$. Also the unmatched uncertainties and the Laplacian matrix will be intertwined together, which makes the problem more complicated. The key technique to solve these problems is introducing compensatory variables $z_{i,1}$ and e_i in (3) and (7), which makes the unmatched uncertainties $\psi_{i,q}(x_{i,1}, \ldots, x_{i,q})\theta_{i,q}$ be easily handled by the virtual controller (18) and (27) by adopting backstepping technique.*

Remark 3 *The controller (33) and parameter estimators (21), (29) and (35) are all designed in such a way that the derivative of the Lyapunov function (34) is made to satisfy (36).*

3.3 Leaderless consensus control

To achieve the leaderless consensus control objective, a necessary assumption is imposed.

Assumption 3 *The directed graph $\mathcal{G}$ is strongly connected.*

Similar to the leader-follower case, the following compensatory variable $z_{i,1}$ is generated for ith agent

$$\dot{z}_{i,1} = -\sum_{j=1}^{N} a_{ij}(y_i - y_j) \tag{43}$$

where $i \in \mathcal{V}$, $z_{i,1}$ is a local variable depends on the output of the ith agent and its neighbors with $z_{i,1}(0) = y_i(0)$.

Define

$$e_i = y_i - z_{i,1}, \tag{55}$$

then the time-derivative of e_i is given as

$$\dot{e}_i = x_{i,2} + \theta_{i,1}\psi_{i,1}(x_{i,1}) + \sum_{j=1}^{N} a_{ij}(x_{i,1} - x_{j,1}) \tag{56}$$

Table 1: The design of distributed adaptive controllers.

Introducing error variables:

$$z_{i,k+1} = x_{i,k+1} - \alpha_{i,k}, \quad k = 1, \ldots, n-1$$
$$\Theta_{i,k} = [\theta_{i,k}, \underset{m=1,\ldots,k-1}{vec}(\theta_{i,m})^{\mathrm{T}}, \underset{j\in\mathcal{N}_i, m=1,\ldots,k-1}{vec}(\theta_{j,m})^{\mathrm{T}}]^{\mathrm{T}}$$
$$\Psi_{i,k} = [\psi_{i,k}(\bar{x}_{i,k}), \underset{m=1,\ldots,k-1}{vec}(\tfrac{\partial \alpha_{i,k-1}}{\partial x_{i,m}}\psi_{i,m}(\bar{x}_{i,m}))^{\mathrm{T}}, \underset{j\in\mathcal{N}_i, m=1,\ldots,k-1}{vec}(\psi_{j,m}(\bar{x}_{j,m}))^{\mathrm{T}}]^{\mathrm{T}}$$

Control Laws:

$$\alpha_{i,1} = -k_i e_i - \hat{\theta}_{i,1}\psi_{i,1} - \sum_{j=1}^{N} a_{ij}(x_{i,1} - x_{j,1}) \tag{44}$$
$$\alpha_{i,2} = -e_i - k_i z_{i,2} - f_{i,2} - \hat{\Theta}_{i,2}\Psi_{i,2} \tag{45}$$
$$\alpha_{i,k} = -k_i z_{i,k} - z_{i,k-1} - f_{i,k} - \hat{\Theta}_{i,k}\Psi_{i,k} \tag{46}$$
$$u_i = -k_i z_{i,n} - z_{i,n-1} - f_{i,n} - \hat{\Theta}_{i,n}\Psi_{i,n} - s(z_{i,n})\hat{D}_i \tag{47}$$

Parameter Update Laws:

$$\dot{\hat{\theta}}_{i,1} = e_i\psi_{i,1}(x_{i,1}) \tag{48}$$
$$\dot{\hat{D}}_i = z_{i,n}s(z_{i,n}) \tag{49}$$
$$\dot{\hat{\Theta}}_{i,k} = z_{i,k}\Psi_{i,k}(\bar{x}_{1,k}, ..., \bar{x}_{N,k}) \tag{50}$$
$$\dot{\hat{\Theta}}_{i,n} = z_{i,k}\Psi_{i,n}(\bar{x}_{1,n}, ..., \bar{x}_{N,n}) \tag{51}$$

Lyapunov functions:

$$\bar{V}_{i,k} = V_{i,k-1} + \frac{1}{2}z_{i,k}^2 \tag{52}$$
$$V_{i,k} = \bar{V}_{i,k} + \frac{1}{2}\tilde{\Theta}_{i,k}^{\mathrm{T}}\tilde{\Theta}_{i,k}, k = 1, ..., n-1 \tag{53}$$
$$V_{i,n} = \bar{V}_{i,n-1} + \frac{1}{2}\tilde{\Theta}_{i,n}^{\mathrm{T}}\tilde{\Theta}_{i,n} + \frac{1}{2}\tilde{D}_i^2, \tag{54}$$

Define

$$V_{i,n} = V_{i,n-1} + \frac{1}{2}\tilde{\Theta}_{i,k}^{\mathrm{T}}\tilde{\Theta}_{i,k} + \frac{1}{2}z_{i,n}^2 + \frac{1}{2}\tilde{D}_i^2 \tag{57}$$

we get

$$\dot{V}_{i,n} \le -k_i e_i^2 - \sum_{j=2}^{n} k_i z_{i,j}^2 + D_i e^{-2t} \tag{58}$$

3.4 Stability Analysis of Leaderless Case

The main results of our distributed adaptive leaderless consensus control of multiple high-order nonlinear systems can be formally stated in the following theorems.

Theorem 2 *Consider the closed-loop system consisting of N uncertain high-order nonlinear sub-systems (1), the distributed controller (47) and the parameter estimators (48)-(51). If Assumption 3 is satisfied, then all the signals in the closed-loop system are globally uniformly bounded. Furthermore, the output of each sub-system will reach consensus asymptotically, i.e., $\lim_{t\to\infty}(y_i - y_j) = 0$ for $i, j \in \mathcal{N}$.*

Proof: Define the Lyapunov function for the overall system as

$$V_n = \sum_{j=1}^{n} V_{i,j} \tag{59}$$

then the derivative of V_n is

$$\dot{V}_n \le -\sum_{i=1}^{N} k_i e_i^2 - \sum_{i=1}^{N}\sum_{j=2}^{n} k_i z_{i,j}^2 + \sum_{i=1}^{N} D_i e^{-2t}. \tag{60}$$

From the definition of V_n in (59), it can be established that e_i, $z_{i,j}$ for $i = 1, ..., N$, $j = 2, \ldots, n$, $\hat{\Theta}_{i,j}$ are bounded for all sub-systems. Then we know $\alpha_{i,j}$ for $i = 1, \ldots, N$ are also bounded. From (47), it concludes that the control signal u_i is also bounded. Thus the boundedness of all signals in the closed-loop system is guaranteed.

From (60), we know $\int_0^\infty \sum_{i=1}^N \sum_{j=1}^n k_i z_{i,j}^2 d\tau$ and $\int_0^\infty \sum_{i=1}^N e_i^2 d\tau$ are bounded. It is easy to check that the time-derivative of $\sum_{i=1}^N e_i^2$ and $\sum_{i=1}^N \sum_{j=2}^n k_i z_{i,j}^2$ are also bounded, then by applying Barbalat's Lemma, it further follows that $\lim_{t\to\infty} e_i(t) = 0$ and $\lim_{t\to\infty} z_{i,j}(t) = 0$ for $i = 1, \ldots, N$ and $j = 2, \ldots, n$.

Now define $x_1 = [x_{1,1}, ..., x_{N,1}]^{\mathrm{T}}$, $z_1 = [z_{1,1}, ..., z_{N,1}]^{\mathrm{T}}$, $e = [e_1, ..., e_N]^{\mathrm{T}}$, then from (43), it has

$$\dot{z}_1 = -\mathcal{L}(z_1 + e) \tag{61}$$

Since $\mathcal{G}$ is strongly connected, then it has a directed spanning, thus $\mathcal{L}$ has a zero eigenvalue, and other eigenvalues of $\mathcal{L}$ lie in the open right half plane. Moreover, the eigenvector associated with the zero eigenvalue of $\mathcal{L}$ is 1_N Ren and Cao (2010). Obviously $\mathcal{L} = PJP^{-1}$ where $J = diag(0, \nu)$ is the Jordan canonical form of $\mathcal{L}$ and P is a positive definite matrix. Furthermore, the columns of P are the right eigenvectors of $\mathcal{L}$.

Defining $\varepsilon = P^{-1} z_1$, then

$$\dot{\varepsilon} = -J\varepsilon - JP^{-1}e \tag{62}$$

Since the first row of J is a zero vector, then obviously $\dot{\varepsilon}_1 = 0$ where ε_1 is the first entry of ε.

It is obvious that $\varepsilon_1(t) = \varepsilon_1(0)$. Let $\hat{\varepsilon} = [\varepsilon_2, ..., \varepsilon_N]^{\mathrm{T}}$, $H = diag(\nu)$, then

$$\dot{\hat{\varepsilon}} = -H\hat{\varepsilon} - \Pi e \tag{63}$$

Since $H > 0$ and $\|e\|$ is bounded, thus $\|\varepsilon\|$ is also bounded. Consider a Lyapunov function $V_\varepsilon = \hat{\varepsilon}^{\mathrm{T}} Q \hat{\varepsilon}$ where Q is the solution of

$$H^{\mathrm{T}} Q + QH = -2I. \tag{64}$$

Then

$$\dot{V}_\varepsilon \le -\hat{\varepsilon}^{\mathrm{T}}\hat{\varepsilon} + 2\|Q\Pi\|\|e\|^2 \tag{65}$$

From (60), we know $\int_0^\infty \|e\|^2 d\tau$ is bounded, thus

$$\int_0^\infty \hat{\varepsilon}^{\mathrm{T}}\hat{\varepsilon} d\tau \le -V_\varepsilon(t) + V_\varepsilon(0) + 2\|Q\Pi\| \int_0^\infty \|e\|^2 d\tau \tag{66}$$

which means $\int_0^t \hat{\varepsilon}^{\mathrm{T}}\hat{\varepsilon} d\tau$ is also bounded. Thus from Barbalat's Lemma, $\lim_{t\to\infty} \|\hat{\varepsilon}\| = 0$. Since the first column of P is 1_N, thus we have

$$\lim_{t\to\infty} x_1 = 1_N p^{\mathrm{T}} x_1(0) \tag{67}$$

where p is the first row of P^{-1}. From (67) the output of each sub-system will reach consensus asymptotically, thus we have

$$\lim_{t\to\infty}(y_i - y_j) = 0, \forall i, j \in \mathcal{N}. \tag{68}$$

This ends the proof of Theorem 2. □

Remark 4 *From the definitions of compensatory variables (3) and (43), the only difference is that (43) does not contain term $b_i(x_{i,1} - y_r)$, since there is no leader in the leaderless consensus and all b_i are equal to 0. In this sense, the leader-follower consensus control and leaderless consensus control can be solved in a unified way.*

Remark 5 *Again, similar comments to Remarks 2 can be made here. Particularly, to overcome the main difficulty of this problem that unmatched uncertainties and the Laplacian matrix will be intertwined together when using existing techniques, we introduce the compensatory variables $z_{i,1}$ so that the unmatched uncertainties $\psi_{i,q}(x_{i,1}, \ldots, x_{i,q})\theta_{i,q}$ could be handled by the virtual controllers (44)-(46) and (47). This enables us to solve the leaderless consensus control problem for strict feedback nonlinear multi-agent systems.*

Remark 6 *The main difficulty in leader-follower and leaderless consensus control of strict-feedback nonlinear* *systems lies in handling the unmatched parametric uncertainties. Take the leaderless consensus control as an example. Traditionally in leaderless consensus control, the following error variable is defined $z_i = \sum_{j=1}^{N} a_{ij}(x_{i,1} - x_{j,1})$. Put z_i into a vector, it is $z = \mathcal{L}x$. If one takes the time-derivative of z, it would be obtained that $\dot{z} = \mathcal{L}\Big(x_2 + \theta_1^T diag\{f_{i,1}(x_{i,1})\}\Big)$. In this case the handling of unknown parameters θ_1 will be intertwined with the Laplacian matrix $\mathcal{L}$, which will bring difficulty in parameter estimator design. The same problem also exists in leader-follower case. To solve this problem, $\dot{z}_i = \sum_{j=1}^{N} a_{ij}(x_{i,1} - x_{j,1})$ and new error variables e_i are defined. These then enable distributed control input u_i and the parameter estimators to be designed to make $\lim_{t\to\infty} e_i = 0$.*

Remark 7 *Similar to other adaptive control, the parameter estimate errors are not required to be convergent, which means the asymptotic regulation of the consensus errors is the control objective in control design. However, since part of the agents have direct access to the reference, through distributed estimation it is shown in (38) that the parameter estimation error $\tilde{\mathcal{F}}_i$ will converge to the origin.*

4 Simulation

Now we use an example to illustrate our proposed control scheme and verify the established results. Consider a group of 4 nonlinear sub-systems modeled as

$$\begin{aligned}
\dot{x}_{i,1} &= x_{i,2} + \psi_{i,1}(x_{i,1})\theta_{i,1} \\
\dot{x}_{i,2} &= x_{i,3} + \psi_{i,2}(x_{i,1}, x_{i,2})\theta_{i,2} \\
\dot{x}_{i,3} &= u_i + \psi_{i,3}(x_{i,1}, x_{i,2}, x_{i,3})\theta_{i,3} + d_i(t) \qquad (69)
\end{aligned}$$

where $\psi_{i,1} = \sin(x_{i,1})$, $\psi_{i,2} = \tanh(x_{i,1})\sin^2(x_{i,2})$, $\psi_{i,3} = x_{i,1}x_{i,2}\cos^2(x_{i,3})$, $\theta_{1,1} = \theta_{2,1} = \theta_{3,1} = \theta_{4,1} = 3$, $\theta_{1,2} = \theta_{2,2} = \theta_{3,2} = \theta_{4,2} = 0.5$, $\theta_{1,3} = \theta_{2,3} = \theta_{3,3} = \theta_{4,3} = 1$, $d_i(t) = 0.8\sin(t)$ denotes the external disturbance. Firstly we consider the leader-follower case, where the topology is given in Fig. 1. The initial values of states are set as $x_{1,1}(0) = 1$, $x_{2,1}(0) = 2$, $x_{3,1}(0) = -1$ and $x_{4,1}(0) = -2$. Besides, the design parameters are chosen as $c_0 = 2$. y_r is given as $y_r(t) = \sin(0.5t)$. The outputs of all the sub-systems are shown in Fig. 2. It can be seen that asymptotical consensus is achieved. Control signals u_i, $i = 1, ..., 4$ are respectively shown in Fig.3.

Now we consider the leaderless case, where the topology is given in Fig. 4. The initial values of states are set as $x_{1,1}(0) = 1$, $x_{2,1}(0) = -1$, $x_{3,1}(0) = 3$ and $x_{4,1}(0) = -3$. The outputs of all the sub-systems and the control input u_i, $i = 1, ..., 4$ are shown in Fig.5 and Fig.6 respectively. These simulation results show the effectiveness of the proposed scheme. Since the leaderless consensus of (1)

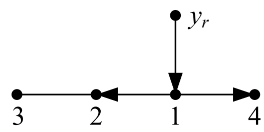

Fig. 1. Topology for leader-follower consensus control of a group of 4 nonlinear sub-systems.

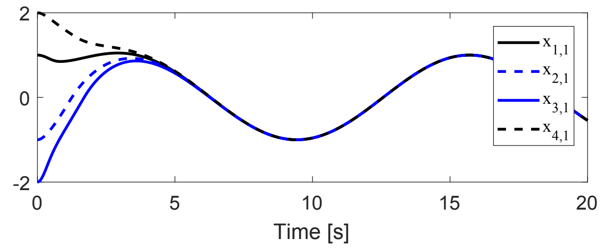

Fig. 2. $x_{i,1}$ of 4 nonlinear sub-systems for leader-follower case.

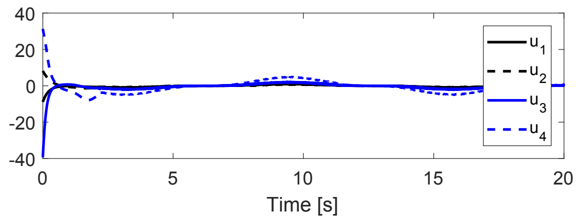

Fig. 3. u_i of 4 nonlinear sub-systems for leader-follower case.

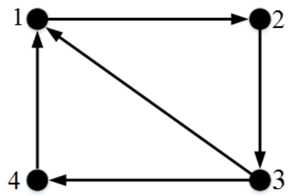

Fig. 4. Topology for leaderless consensus control of a group of 4 nonlinear sub-systems.

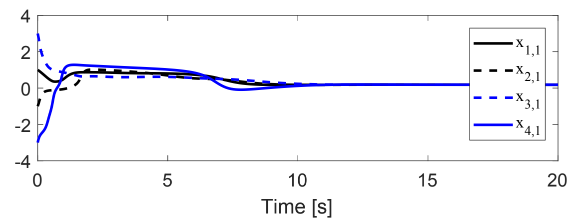

Fig. 5. $x_{i,1}$ of 4 nonlinear sub-systems for leaderless case.

still remains unsolved, there is no comparison for the simulations of leaderless consensus control.

To make comparisons, outputs $x_{i,1}$ and torques u_i of the four agents using the scheme in Huang et al. (2017) are respectively illustrated in Fig.7 and Fig.8. It is shown that both control schemes could achieve leader-follower

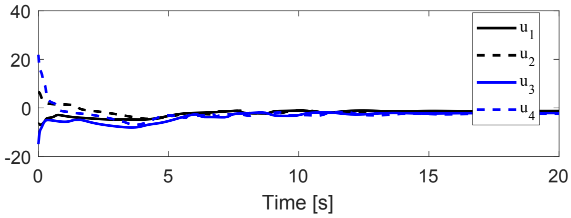

Fig. 6. u_i of 4 nonlinear sub-systems for leaderless case.

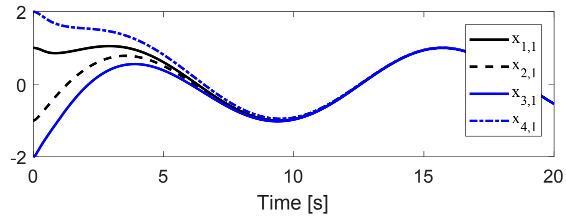

Fig. 7. $x_{i,1}$ of 4 nonlinear sub-systems for leader-follower case with Huang et al. (2017).

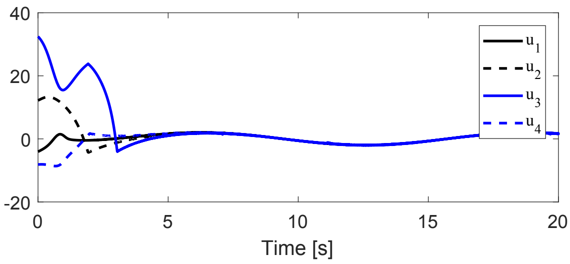

Fig. 8. u_i of 4 nonlinear sub-systems for leader-follower case with Huang et al. (2017).

consensus control. However, the leaderless consensus control is simultaneously achieved only with the scheme proposed in this paper.

5 Conclusion

In this paper, distributed adaptive leader-follower and leaderless consensus control of a class of strict-feedback nonlinear systems under directed topology subjected to mismatched unknown parameters and uncertain external disturbances are investigated. A novel local variable is generated which makes that two consensus problems to be addressed in a unified framework. For leader-follower consensus control, the assumption that the leader is linearly parameterized with known time-varying functions is relaxed. It is shown that global uniform boundedness of all closed-loop signals and asymptotically output consensus can be achieved for both cases. Simulation results are provided to verify the effectiveness of our scheme. The possible future work includes the consensus control of strict-feedback systems with intermittent communication, packet dropouts or under cyber-attacks.

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