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Research Report

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Comparing classical performance measures for a multi-period, two-echelon supply chain network design problem with sizing decisions

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Abstract

This paper addresses a new problem to design a two-echelon supply chain network over a multi-period horizon. Strategic decisions are subject to a given budget and concern the location of new facilities in the upper and intermediate echelons of the network as well as the installation of storage areas to handle different product families. A finite set of capacity levels for each product family is available at each potential location. Further decisions concern the quantities of products to be shipped through the network. Two mixed-integer linear programming models are proposed that differ in the type of performance measure that is adopted to design the supply chain. Under a cost minimization objective, the network configuration with the least total cost is to be determined. In contrast, under a profit maximization goal the aim is to design the network so as to maximize the difference between total revenue and total cost. In this case, it may not always be attractive to completely satisfy demand requirements. To investigate the implications that the choice of these performance measures have on network design, an extensive computational study is conducted using randomly generated instances that are solved with a general-purpose solver.

Keywords: supply chain network design, facility location, capacity acquisition, profit maximization, cost minimization.

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1 Introduction

Supply chain network design (SCND) is at the core of strategic planning in supply chain management (SCM). Whether to create a new network configuration or to redesign an existing network is one of the major strategic decisions to be made, as the configuration of the network defines the operating basis of the supply chain. According to Harrison [7], up to 80% of the total cost of a product is driven by network design decisions.

SCND is a complex undertaking. It involves determining which facilities to include in the supply chain network (e.g., plants, warehouses), their size and location, and establishing the transportation links among the members of the supply chain as well as setting the flow of materials through them. This paper focuses on a comprehensive problem arising in SCND. It includes various key elements that are of practical relevance but have not yet been considered simultaneously in the literature. Focus is given to the design of a supply chain network as depicted in Figure 1.

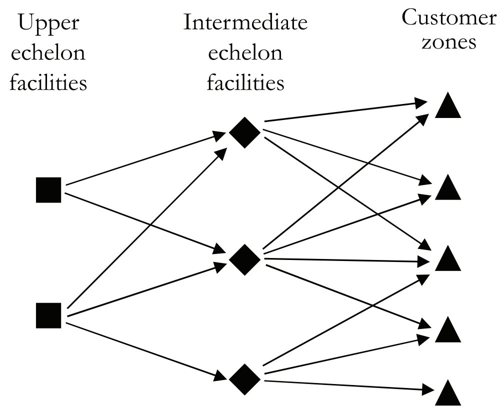


Figure 1: A two-echelon supply chain network.

The network comprises two echelons involving two types of distribution facilities such as central and regional warehouses. Demand for multiple products originates at the lowest layer of the supply chain, namely, at customer zones. Moreover, products are grouped into families, each having specific storage requirements. This is commonly known as *class-based storage*, a policy used nowadays in many warehouses. Compared to other storage strategies, class-based storage reduces floor space utilization, order picking times and costs (see, e.g., Muppani and Adil [15, 16]). For each product family, a given set of storage areas with different sizes are available to be installed at each potential location. Fixed installation costs for new storage areas reflect economies of scale.

Figure 2 shows an example of three possible facility configurations for the storage of three families of refrigerated goods, each demanding specific temperature and moisture conditions. Small and medium-sized storage areas are available for frozen goods. The chilled area (e.g., for dairy products) can occupy a small, medium or large space. For the special chilled area (e.g., for fruit and vegetables) either a medium or a large-sized area can be installed. In total, 12 different configurations are possible.

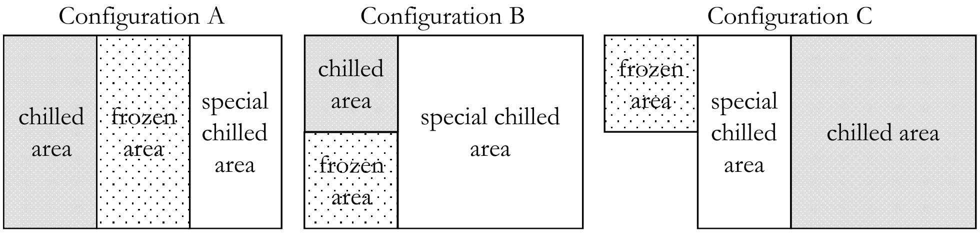


Figure 2: Three possible configurations for a refrigerated storage facility.

Structural decisions to be made over a multi-period planning horizon are as follows: (i) selection of new facilities from a given set of candidate locations to operate at the upstream and intermediate echelons of the network; (ii) facility sizing through the installation of storage areas with given capacities for product families at each open location; (iii) the investment of an available budget for facility location and sizing. Further decisions concern the quantities of products to be shipped from upper level facilities to intermediate level facilities, and from the latter to customer zones (see Figure 1).

Through the consideration of a multi-period horizon, the design of the supply chain network also entails the specification of when and where capacity expansions for given families should be made, and how large they should be. A further distinctive feature of our model is that the cost of operating a facility depends on the capacity utilization rate of the installed storage areas at the facility. This aspect has been neglected in the literature as facility operating costs are usually assumed to be fixed, although this does not apply to many situations.

A recent survey conducted by Melo et al. [14] has shown that cost minimization is the most widely used objective in SCND. This means that the decision-making process aims at identifying the network configuration with the least total cost. In contrast, a profit oriented objective has received much less attention, although most organizations pursue this goal. In this case, the physical distribution structure of the network that maximizes the difference between total revenue and total cost is to be determined. In this paper, we investigate the SCND problem from both perspectives and discuss the implications that the choice of each of these performance measures have on network design. Our comparison is based on the development of appropriate mixed-integer linear programming (MILP) models and on their analysis through an extensive computational study. In recent years, commercial optimization software has become a powerful tool for solving a wide variety of large-scale optimization problems. As a result, many organizations resort to optimization tools for decision support nowadays. Using a well-known general-purpose solver, we will show that the above performance criteria have a strong impact on the quality of the solutions obtained.

The contribution of this paper is twofold. First, it offers a new SCND model that significantly generalizes previous models. This is accomplished through the integration of various strategic features of practical relevance into a single model. Second, it analyzes the SCND problem from two different viewpoints, namely, a cost minimization goal and a profit maximization objective. Our study is - to the best of our knowledge - the first to investigate the impact of these objectives on the configuration of the supply chain network and on the ability of commercial optimization software to solve problem instances within a reasonable time limit.

The remainder of the paper is organized as follows. In the next section, we briefly review the literature dedicated to SCND. We will show that there exist several SCND models that consider some but not all of the characteristics of our problem. In Section 3, we describe two mathematical formulations for our SCND problem, one considering a cost minimization goal and the other following a profit maximization objective. Section 4 proposes various types of valid inequalities to enhance the original formulations in an attempt to strengthen the bounds of the corresponding linear relaxations. Section 5 focuses on an extensive computational study. A new methodology for the random generation of test instances reflecting real-world situations is introduced, followed by the analysis of the results obtained with a general-purpose solver. In particular, our findings provide managerial insights on how the decision-making process is impacted by the choice of the objective function. We conclude with a few general remarks and present future research directions.

2 Related research

Over the last decades, facility location decisions have attracted a great deal of attention from researchers. In recent years, increasing attention has also been paid to the interaction of these decisions with key features of strategic supply chain planning such as supplier selection, production planning, technology acquisition, inventory planning, transportation mode selection, and vehicle routing. The importance of integrating location decisions with other decisions relevant to SCND has been highlighted in a recent survey by Melo et al. [14]. Economic globalization has also prompted the development of more comprehensive facility location models as evidenced by Meixell and Gargeya [13].

In this section, we provide a review of the literature with a focus on the development of models for SCND rather than solution methods. In doing so, references will be limited to relevant deterministic models in the extensive SCND literature that consider at least one of the characteristics captured by our model. In particular, the following features will be covered: multi-echelon, multi-commodity, facility sizing and capacity utilization, multi-period, investment restrictions, and type of objective.

Multi-echelon supply chain networks

A crucial aspect of many realistic location problems concerns the existence of different types of facilities, each one playing a specific role (e.g., production or warehousing), and a natural material flow (that is, a hierarchy) between them. However, in their review of hierarchical location models, Sahin and Süral [20] observe that facility location problems have been mostly studied for networks comprising a single echelon of facilities and a customer layer. These findings are also supported by Melo et al. [14]. In supply chain networks with multiple facility echelons, location decisions often concern more than one type of facilities as in our case. Contributions in this area include Jang et al. [10] and Pirkul and Jayaraman [17, 18]. Both plants and warehouses are to be located to satisfy customers' demands for multiple commodities. An upper limit is specified on the number of facilities that can be open in each echelon. Altiparmak et al. [1] and Jayaraman and Pirkul [11] extend this problem by including supplier selection and production decisions.

Location and capacity acquisition

Facility sizing and location decisions are intertwined. The most common modeling approach is to consider potential locations to be either uncapacitated or capacitated. In the former case, the size of each new facility is an outcome of solving the location problem. However, in SCND, it is more realistic to incorporate the capacity limitations in the facilities or equipment to be established. Mazzola and Neebe [12] combine the location of new facilities with the purchase of capacity in a multi-commodity single-echelon network. Capacity acquisition involves the selection of a capacity level for each new facility from a set of available discrete sizes with fixed installation costs that capture economies of scale. This form of capacity acquisition is not widely spread in strategic supply chain planning, whereas it is often considered in the design of telecommunication networks (see, e.g., Gourdin et al. [6]). For a single product, Amiri [2] adopt the same form of capacity acquisition as Mazzola and Neebe [12] in a two-echelon network, where both plants and warehouses are to be located. Sadjady and Davoudpour [19] extend the model in [2] by considering multiple commodities and the choice of transportation modes. The acquisition of technology is also strongly associated with capacity decisions. Elhedhli and Gzara [5] incorporate this feature into a model for the location of plants and warehouses to meet demands for multiple commodities. In all the previous works [2], [5], [12], and [19] at most one technology type can be installed at an open facility. Recently, Amrani et al. [3] have also modeled capacity acquisition in conjunction with the location of plants and distribution centers (DCs). An interesting feature of their model is that the demand of each customer must be supplied by a single facility, either a DC or a plant.

Multi-period SCND

Facility sizing issues are also handled in the literature in conjunction with a multi-period planning horizon. Although the timing of facility locations and expansions over an extended time horizon is of major importance to decision-makers in SCND, significantly less attention has been given in the literature to this aspect compared to the static case (i.e. a single time period), see Melo et al. [14]. One of the early contributions in this research area is by Shulman [21], who focuses on a single product, single-echelon network design problem with location and capacity acquisition decisions. Multiple capacity levels are available for selection in each time period. In contrast, the models proposed by Antunes and Peeters [4], Hugo and Pistikopoulos [8], and Thanh et al. [22, 23] are more comprehensive. The redesign of an existing network of schools is studied in [4] through the gradual capacity expansion of new locations and the gradual capacity reduction of existing facilities. Moreover, a maximum size is imposed on every new facility, thereby limiting its expansion over the time horizon. In [8], the number of open plants and the number of capacity expansions of given technologies are controlled by pre-defined minimum and maximum values. In the problem studied by Thanh et al. [22, 23], the set of strategic decisions includes facility location and capacity acquisition as well as production, distribution, and inventory planning in a three-echelon network. Location decisions concern plants and warehouses whose capacities may be extended over the time horizon. In addition, supplier selection occurs at the upstream layer of the network. Technology acquisition is also included in a large-scale model developed by Vila et al. [24] which comprises location, production, inventory, and transportation decisions along with international factors such as tariffs and duties. However, location decisions regarding plants and DCs are only made at the beginning of the planning horizon, while all other decisions including the purchase of capacity for a particular technology may change over time.

Facility configuration and capacity utilization

Capacity utilization is also closely related to facility configuration. Typically, once capacity or technology has been acquired for a location, all products handled by the new facility compete for the installed capacity. This aspect is present in the models of Elhedhli and Gzara [5], Jang et al. [10], Mazzola and Neebe [12], Sadjady and Davoudpour [19], Thanh et al. [22, 23], and Vila et al. [24]. In contrast, Hugo and Pistikopoulos [8] consider the installed production capacity to be product-dependent. Amrani et al. [3] also capture this aspect for production facilities. However, at the DC layer the typical view of non-dedicated technology is adopted by these authors. Often in SCND, it is only desired to establish a new facility if it will achieve at least a given minimum throughput. This minimum activity level is usually specified as a percentage of the available installed capacity. This aspect is considered by Amrani et al. [3], Hugo and Pistikopoulos [8], and Thanh et al. [22, 23].

Capacity utilization is an important issue as the model to be presented in the next section takes on a new perspective that differs from the literature presented above. This has to do with the fact that in our case products are grouped into families and the configuration of a new facility entails the acquisition of capacity for one or more families over a time horizon. Dedicated technology offers economies of scale and is motivated by different families having different storage and handling requirements (recall Figure 2). As will be shown in Section 3, this aspect has a strong impact on the formalization of a mathematical model. Moreover, the utilization rate of dedicated equipment is also closely related to facility operating costs. In our case, the total quantity processed by a facility for a given family is subject to variable handling costs that account, for example, for energy consumption. Amrani et al. [3] and Shulman [21] are among the few authors that follow this view. In contrast, most of the literature on SCND regards facility operating costs as being fixed (see, e.g., the above references).

Investment capital

The establishment of new facilities is typically a long-term project involving substantial investment capital (e.g., for facility construction and equipment acquisition). This aspect is rarely captured by SCND models, see Melo et al. [14]. An exception is the model developed by Antunes and Peeters [4] which includes budget limitations for facility location, expansion and reduction over multiple time periods. A tight budget (as is usually the case in location projects) strongly limits the investment options in a given time period. This impacts not only the selection of new facilities but also other strategic decisions (e.g., capacity acquisition).

Performance measures

The primary objective of SCND models has been the identification of the network configuration with the least total cost. Almost 80% of the articles surveyed by Melo et al. [14] fall into this category. The same applies to the majority of the references given in this section. Facility location and logistics costs (e.g., for production and distribution) are among the most frequent cost components. In contrast, profit maximization has received much less attention. Among the references given before, only Vila et al. [24] have adopted an after-tax profit objective, that is, the maximization of the difference between revenues and total costs, and taxes. Observe that under a profit maximization goal, it may not always be attractive to meet all customers' demands. This occurs when servicing certain customers yields additional costs that are higher than the corresponding revenues. Moreover, in some cases, an organization may intentionally lose customers when the costs of maintaining them are prohibitively high. This contrasts with a cost minimization model in which the demand of every customer has to be satisfied. Finally, there are a few contributions that propose multi-objective models. Altiparmak et al. [1] consider three conflicting objectives: minimization of the total supply chain costs, maximization of the total demand that can be satisfied within a pre-specified time limit, and maximization of a balanced distribution of capacity utilization among the open facilities. The multi-objective function of the model by Hugo and Pistikopoulos [8] represents a trade-off between maximizing profit and minimizing ecological impacts (e.g., emissions, waste).

We conclude this section by observing that to the best of our knowledge, the impact of considering a cost minimization objective versus a profit maximization goal has not been investigated so far. As mentioned above, the flexibility provided by a profit maximization perspective to choose those demands to be filled is likely to significantly influence the configuration of the supply chain network, and hence, the interactions across all tiers of the supply chain. This paper is the first to focus on this issue for a comprehensive model capturing various aspects that have not yet been considered simultaneously in a single model.

3 Mathematical models

In this section, we propose two MILP formulations for the problem at hand. The models mainly differ in their objective functions. After introducing the required notation and the decision variables in Section 3.1, we present the constraints that are shared by both formulations in Section 3.2. Finally, Section 3.3 is dedicated to the description of the cost minimization and profit maximization objectives.

We assume that prior to the SCND project, all relevant data (demands, costs, capacity acquisition options, available capital, and other factors) were collected by using, for example, appropriate forecasting methods and company-specific business analyses.

3.1 Notation and definition of decision variables

In our SCND problem, a network comprising two types of facilities (e.g., central and regional DCs) is to be established (recall Figure 1). A finite set of candidate sites for locating new facilities in each echelon is available. Over a multi-period planning horizon, new facilities can be established and their capacity can be gradually extended through the installation of storage areas dedicated to families of products. In particular, we assume that the same type of storage area can be selected in successive time periods for a given family. Table 1 introduces the index sets to be used.

Table 2 summarizes all costs. These are divided into two categories, namely *investment* and *network operating costs*. The first category comprises fixed costs for establishing new facilities ($F^1_{t,i}, F^2_{t,i}$) over the time horizon and fixed costs for installing new storage areas ($G^1_{t,i,\ell,k}, G^2_{t,i,\ell,k}$). The latter reflect economies of scale. Investment options are limited by the budget available in each time period. Network operating costs - the second cost category - include fixed facility maintenance costs ($M^1_{t,i}, M^2_{t,i}$) at new locations as well as variable operating ($O^1_{t,i,\ell,k}, O^2_{t,i,\ell,k}$) and shipment costs ($S^1_{t,i,i',p}, S^2_{t,i,j,p}$). Facility maintenance costs account, for example, for business overhead costs (e.g., staff and security costs) that are incurred from the first time period in which the new facility is established until the end of the time horizon. Operating costs are charged to the capacity utilization rate of the installed storage areas and represent handling

Symbol	Description
T	Set of time periods
J	Set of customer zones
I^1	Set of potential sites for locating new facilities in the *upper* echelon
I^2	Set of potential sites for locating new facilities in the *intermediate* echelon
L	Set of product families
P_ℓ	Set of products that are part of family $\ell \in L$
P	Set of all products; $P = \bigcup\limits_{\ell \in L} P_\ell$
K_ℓ^1	Set of different types of storage areas available for family $\ell \in L$ at a location in the *upper* echelon
K_ℓ^2	Set of different types of storage areas available for family $\ell \in L$ at a location in the *intermediate* echelon

Table 1: Index sets.

expenditures. Finally, shipment costs are incurred by the delivery of the products from upper echelon facilities to intermediate level facilities, and from these to the customer zones.

Table 3 introduces additional input parameters. We assume that the available storage areas are sorted in non-decreasing order of their sizes. This means that for each family $\ell \in L$, the various capacity levels that can be installed at site $i \in I^1$ are ordered such that $Q^1_{i,\ell,1} < Q^1_{i,\ell,2} < \cdots < Q^1_{i,\ell,|K^1_\ell|}$. For intermediate echelon locations a similar assumption is made, that is, $Q^2_{i,\ell,1} < Q^2_{i,\ell,2} < \cdots < Q^2_{i,\ell,|K^2_\ell|}$ for every $t \in T, i \in I^2$, and $\ell \in L$.

Customers' demands are expressed in units of measurement that depend on the product type (e.g., kilogram, liter). In contrast, the sizes of storage areas are expressed in a unit of storage space utilization rather than in a unit of measurement, such as feet or meters. A *slot* is considered to be the smallest unit of assignable storage space. At intermediate echelon facilities, such as regional warehouses, a slot has often the dimension of a standard pallet. Hence, the size of a storage area of type $k \in K^2_\ell$ for family $\ell \in L$ in location $i \in I^2$, i.e. $Q^2_{i,\ell,k}$, is expressed as the total number of available slots. For example, demand for milk is usually conveyed in liters. In a regional warehouse, one-liter milk cartons are stored in standard pallets, each holding, e.g., 200 cartons. Therefore, $\mu^2_p = 0.005$ is the conversion factor.

In upper echelon locations, however, bulk storage is more common since other types of storage facilities are used, such as silos and tanks. In this case, a unit of space utilization may not be a slot. For example, in a dairy factory, milk is typically stored in a large tank with

Symbol	Description
$F^1_{t,i}$	Fixed cost of establishing a new facility in site $i \in I^1$ (*upper* echelon) in period $t \in T$
$F^2_{t,i}$	Fixed cost of establishing a new facility in site $i \in I^2$ (*intermediate* echelon) in period $t \in T$
$G^1_{t,i,\ell,k}$	Fixed cost of installing a new storage area of type $k \in K^1_\ell$ for family $\ell \in L$ at location $i \in I^1$ in period $t \in T$
$G^2_{t,i,\ell,k}$	Fixed cost of installing a new storage area of type $k \in K^2_\ell$ for family $\ell \in L$ at location $i \in I^2$ in period $t \in T$
$M^1_{t,i}$	Fixed maintenance cost incurred by a facility established in location $i \in I^1$ in period $t \in T$
$M^2_{t,i}$	Fixed maintenance cost incurred by a facility established in location $i \in I^2$ in period $t \in T$
$O^1_{t,i,\ell,k}$	Unit operating cost in period $t \in T$ of a product belonging to family $\ell \in L$ that uses a storage area of type $k \in K^1_\ell$ installed at location $i \in I^1$
$O^2_{t,i,\ell,k}$	Unit operating cost in period $t \in T$ of a product belonging to family $\ell \in L$ that uses a storage area of type $k \in K^2_\ell$ installed at location $i \in I^2$
$S^1_{t,i,i',p}$	Unit shipment cost in period $t \in T$ of product $p \in P$ from a facility operating at location $i \in I^1$ to a facility operating at location $i' \in I^2$
$S^2_{t,i,j,p}$	Unit shipment cost in period $t \in T$ of product $p \in P$ from a facility operating at location $i \in I^2$ to customer zone $j \in J$

Table 2: Costs.

capacity for up to, e.g., 1000 hectoliters. The sizes of storage areas in such facilities (given by $Q^1_{i,\ell,k}$) are thus expressed in this unit. Using again the above example, the factor $\mu^1_p = 0.00001$ is necessary to convert liters into 1000 hectoliters. In the facility location literature, it is often assumed that both customers' demands and the capacities of new facilities have the same units even when different products are considered. Amrani et al. [3] are among the few authors who explicitly account for unit conversion in a similar way as in our case.

Strategic decisions on facility location and capacity acquisition are ruled by the binary variables in Table 4, while distribution decisions are described by continuous variables given in Table 5. The latter table also includes the auxiliary variables $w^1_{t,i,\ell,k}$ and $w^2_{t,i,\ell,k}$, which gather the total quantity of products belonging to family ℓ that are handled by a storage area of type k at an upper, resp. intermediate, echelon facility in time period t. Finally, the variables u_t account for the non-invested capital in period t.

Symbol	Description
$d_{t,j,p}$	Demand of customer zone $j \in J$ for product $p \in P$ in period $t \in T$
$Q^1_{i,\ell,k}$	Handling capacity of a storage area of type $k \in K^1_\ell$ installed at location $i \in I^1$ for product family $\ell \in L$
$Q^2_{i,\ell,k}$	Handling capacity of a storage area of type $k \in K^2_\ell$ installed at location $i \in I^2$ for product family $\ell \in L$
$q^1_{i,\ell,k}$	Minimum throughput for the delivery of products of family $\ell \in L$ from a storage area of type $k \in K^1_\ell$ installed at location $i \in I^1$
$q^2_{i,\ell,k}$	Minimum throughput for the delivery of products of family $\ell \in L$ from a storage area of type $k \in K^2_\ell$ installed at location $i \in I^2$
μ^1_p	Unit capacity handling factor of product $p \in P$ in an *upper* echelon location
μ^2_p	Unit capacity handling factor of product $p \in P$ in an *intermediate* echelon location
B_t	Available budget in period $t \in T$
α_t	Unit return factor on capital not invested in period $t \in T \cup \{0\}$ with $\alpha_0 = 0$

Table 3: Further input parameters.

3.2 Network design constraints

The following constraints impose the required conditions for the configuration of the supply chain network.

$$\sum_{t \in T} z^1_{t,i} \leq 1 \qquad i \in I^1 \tag{1}$$

$$\sum_{t \in T} z^2_{t,i} \leq 1 \qquad i \in I^2 \tag{2}$$

$$\sum_{k \in K^1_\ell} y^1_{t,i,\ell,k} \leq \sum_{\tau=1}^{t} z^1_{\tau,i} \qquad t \in T,\ i \in I^1,\ \ell \in L \tag{3}$$

$$\sum_{k \in K^2_\ell} y^2_{t,i,\ell,k} \leq \sum_{\tau=1}^{t} z^2_{\tau,i} \qquad t \in T,\ i \in I^2,\ \ell \in L \tag{4}$$

$$\sum_{p \in P_\ell} \mu^1_p \sum_{i' \in I^2} x^1_{t,i,i',p} = \sum_{k \in K^1_\ell} w^1_{t,i,\ell,k} \qquad t \in T,\ i \in I^1,\ \ell \in L \tag{5}$$

$$\sum_{p \in P_\ell} \mu^2_p \sum_{j \in J} x^2_{t,i,j,p} = \sum_{k \in K^2_\ell} w^2_{t,i,\ell,k} \qquad t \in T,\ i \in I^2,\ \ell \in L \tag{6}$$

Symbol	Description
$z^1_{t,i}$	1 if a new facility is established in an *upper* echelon location $i \in I^1$ in period $t \in T$, 0 otherwise
$z^2_{t,i}$	1 if a new facility is established in an *intermediate* echelon location $i \in I^2$ in period $t \in T$, 0 otherwise
$y^1_{t,i,\ell,k}$	1 if a new storage area of type $k \in K^1_\ell$ is installed in period $t \in T$ at an *upper* echelon location $i \in I^1$ for family $\ell \in L$, 0 otherwise
$y^2_{t,i,\ell,k}$	1 if a new storage area of type $k \in K^2_\ell$ is installed in period $t \in T$ at an *intermediate* echelon location $i \in I^2$ for family $\ell \in L$, 0 otherwise

Table 4: Binary decision variables.

Symbol	Description
$w^1_{t,i,\ell,k}$	Total quantity of family $\ell \in L$ handled in period $t \in T$ at a storage area of type $k \in K^1_\ell$ installed at location $i \in I^1$
$w^2_{t,i,\ell,k}$	Total quantity of family $\ell \in L$ handled in period $t \in T$ at a storage area of type $k \in K^2_\ell$ installed at location $i \in I^2$
$x^1_{t,i,i',p}$	Quantity of product $p \in P$ shipped in period $t \in T$ from location $i \in I^1$ to location $i' \in I^2$
$x^2_{t,i,j,p}$	Quantity of product $p \in P$ shipped in period $t \in T$ from location $i \in I^2$ to customer zone $j \in J$
u_t	Unspent budget in period $t \in T \cup \{0\}$ with $u_0 = 0$

Table 5: Continuous decision variables.

$$q^1_{i,\ell,k} \sum_{\tau=1}^{t} y^1_{\tau,i,\ell,k} \leq w^1_{t,i,\ell,k} \leq Q^1_{i,\ell,k} \sum_{\tau=1}^{t} y^1_{\tau,i,\ell,k} \qquad t \in T,\ i \in I^1,\ \ell \in L,\ k \in K^1_\ell \tag{7}$$

$$q^2_{i,\ell,k} \sum_{\tau=1}^{t} y^2_{\tau,i,\ell,k} \leq w^2_{t,i,\ell,k} \leq Q^2_{i,\ell,k} \sum_{\tau=1}^{t} y^2_{\tau,i,\ell,k} \qquad t \in T,\ i \in I^2,\ \ell \in L,\ k \in K^2_\ell \tag{8}$$

$$\sum_{i' \in I^1} x^1_{t,i',i,p} = \sum_{j \in J} x^2_{t,i,j,p} \qquad t \in T,\ i \in I^2,\ p \in P \tag{9}$$

$$\sum_{i \in I^1} F^1_{t,i}\, z^1_{t,i} + \sum_{i \in I^2} F^2_{t,i}\, z^2_{t,i} + \sum_{i \in I^1} \sum_{\ell \in L} \sum_{k \in K^1_\ell} G^1_{t,i,\ell,k}\, y^1_{t,i,\ell,k}$$
$$+ \sum_{i \in I^2} \sum_{\ell \in L} \sum_{k \in K^2_\ell} G^2_{t,i,\ell,k}\, y^2_{t,i,\ell,k} + u_t = B_t + \alpha_{t-1}\, u_{t-1} \qquad t \in T \tag{10}$$

$$z^1_{t,i} \in \{0,1\} \qquad t \in T,\ i \in I^1 \tag{11}$$

$$z_{t,i}^2 \in \{0,1\} \qquad t \in T,\ i \in I^2 \tag{12}$$

$$y_{t,i,\ell,k}^1 \in \{0,1\} \qquad t \in T,\ i \in I^1,\ \ell \in L,\ k \in K_\ell^1 \tag{13}$$

$$y_{t,i,\ell,k}^2 \in \{0,1\} \qquad t \in T,\ i \in I^2,\ \ell \in L,\ k \in K_\ell^2 \tag{14}$$

$$x_{t,i,i',p}^1 \geq 0 \qquad t \in T,\ i \in I^1,\ i' \in I^2,\ p \in P \tag{15}$$

$$x_{t,i,j,p}^2 \geq 0 \qquad t \in T,\ i \in I^2,\ j \in J,\ p \in P \tag{16}$$

$$u_t \geq 0 \qquad t \in T \tag{17}$$

Constraints (1)-(2) guarantee that at most one new facility can be established in each potential site over the time horizon. Constraints (3) ensure that in each time period at most one type of storage area is installed for a product family at a potential site in the upper echelon, provided that a facility is already operated in that site. Constraints (4) impose similar conditions on intermediate echelon facilities. Constraints (5) and (6) enable the calculation of the total quantity that is actually handled for products of each family in a given facility and time period. Notice that on the left-hand side of equalities (5) and (6) the product units are converted into capacity handling units that are in use in the corresponding echelon. Constraints (7) and (8) state that the total quantity handled by a facility for a given family must be within pre-defined lower and upper limits in each time period. Observe that conditions (1)-(4) together with (7) and (8) are consistent with the definition of the binary variables in Table 4. This implies that new facilities and storage areas operate from the time period they are established until the end of the planning horizon. Constraints (9) are the usual product flow conservation conditions at intermediate echelon facilities over the time horizon. The investment constraints (10) guarantee that the available budget is invested in establishing new facilities as well as installing new storage areas. Observe that the amount of capital not used in a given period earns interest and can later be invested. Finally, conditions (11)-(17) are integrality and non-negativity constraints. We note that it is not necessary to specify that the variables $w_{t,i,\ell,k}^1$ and $w_{t,i,\ell,k}^2$ are non-negative due to inequalities (7) and (8).

Customers' demands do not appear in any of the above constraints. As mentioned at the end of Section 2, demand satisfaction is closely related to the type of objective considered. This aspect is detailed in the next section.

3.3 Cost minimization vs profit maximization

When the decision-making process aims at identifying the supply chain network configuration with the least total cost, then the following objective function needs to be considered:

$$\begin{aligned}
\text{Min} \quad & \sum_{t \in T} \sum_{i \in I^1} M^1_{t,i}\, z^1_{t,i} + \sum_{t \in T} \sum_{i \in I^2} M^2_{t,i}\, z^2_{t,i} \\
& + \sum_{t \in T} \sum_{i \in I^1} \sum_{\ell \in L} \sum_{k \in K^1_\ell} O^1_{t,i,\ell,k}\, w^1_{t,i,\ell,k} + \sum_{t \in T} \sum_{i \in I^2} \sum_{\ell \in L} \sum_{k \in K^2_\ell} O^2_{t,i,\ell,k}\, w^2_{t,i,\ell,k} \\
& + \sum_{t \in T} \sum_{i \in I^1} \sum_{i' \in I^2} \sum_{p \in P} S^1_{t,i,i',p}\, x^1_{t,i,i',p} + \sum_{t \in T} \sum_{i \in I^2} \sum_{j \in J} \sum_{p \in P} S^2_{t,i,j,p}\, x^2_{t,i,j,p} \\
& - u_{|T|}
\end{aligned} \tag{18}$$

The terms in (18) comprise fixed facility maintenance costs as well as variable operating and shipment costs. Observe that a revenue term $u_{|T|}$ is also included in (18), which encourages the minimization of expenditures on establishing new facilities and installing storage areas for product families (see also (10)).

Under a cost minimization objective, it is natural to impose the satisfaction of all customers' demands. This is ensured by the following set of constraints:

$$\sum_{i \in I^2} x^2_{t,i,j,p} = d_{t,j,p} \qquad t \in T,\ j \in J,\ p \in P \tag{19}$$

Due to the above conditions, the total revenue obtained by selling the demanded quantities to the customer zones is a fixed amount as it is known beforehand. Hence, it can be excluded from the objective function (18).

In contrast, if the aim of the SCND project is to determine the network configuration yielding the largest total profit over the time horizon, then it is not required to meet all customers' demands (recall the discussion at the end of Section 2). Therefore, in this case equations (19) are replaced by

$$\sum_{i \in I^2} x^2_{t,i,j,p} \leq d_{t,j,p} \qquad t \in T,\ j \in J,\ p \in P \tag{20}$$

The demand choice flexibility provided by this case allows the identification of those demands that can be satisfied profitably and those that represent a loss for the organization. As a result, the total revenue that can be achieved is an outcome of the SCND project. In this case, the notation introduced in Section 3.1 is extended by defining the per-unit revenue as follows:

$R_{t,j,p}$: Revenue of selling one unit of product $p \in P$ to customer zone $j \in J$ in period $t \in T$

Hence, under a profit maximization objective, the following function is considered:

$$
\begin{aligned}
\text{Max} \quad & \sum_{t\in T}\sum_{i\in I^2}\sum_{j\in J}\sum_{p\in P} R_{t,j,p}\, x^2_{t,i,j,p} + u_{|T|} \\
& - \Bigg(\sum_{t\in T}\sum_{i\in I^1} M^1_{t,i}\, z^1_{t,i} + \sum_{t\in T}\sum_{i\in I^2} M^2_{t,i}\, z^2_{t,i} \\
& + \sum_{t\in T}\sum_{i\in I^1}\sum_{\ell\in L}\sum_{k\in K^1_\ell} O^1_{t,i,\ell,k}\, w^1_{t,i,\ell,k} + \sum_{t\in T}\sum_{i\in I^2}\sum_{\ell\in L}\sum_{k\in K^2_\ell} O^2_{t,i,\ell,k}\, w^2_{t,i,\ell,k} \\
& + \sum_{t\in T}\sum_{i\in I^1}\sum_{i'\in I^2}\sum_{p\in P} S^1_{t,i,i',p}\, x^1_{t,i,i',p} + \sum_{t\in T}\sum_{i\in I^2}\sum_{j\in J}\sum_{p\in P} S^2_{t,i,j,p}\, x^2_{t,i,j,p} \Bigg)
\end{aligned}
\tag{21}
$$

Let P_{cost} denote the SCND model defined by the objective function (18) subject to the constraints (1)–(17) and (19). Moreover, we denote by P_{profit} the SCND model given by the objective function (21) subject to the constraints (1)–(17) and (20).

Finally, we point out that both models, P_{cost} and P_{profit}, apply not only to the situation in which a new supply chain is to be created but also to the re-design of an existing network. In the latter case, facilities that already operate have the corresponding binary variables set to one in the first time period (i.e. $z^1_{1,i} = 1, z^2_{1,i} = 1$). Moreover, the binary variables corresponding to the storage areas already in use in each one of these facilities are also assigned the value one in the first period. In this way, the expansion of an existing supply chain can also be modeled over the planning horizon.

4 Enhancing the mathematical models

To be able to solve large-scale discrete optimization problems within a branch-and-bound or branch-and-cut framework, a MILP formulation is often enhanced by adding valid inequalities. Extensive computational experience suggests that the success of this approach critically depends on the choice of the valid inequalities. The principal difficulty with this approach, however, is that it is not a priori clear which class of valid inequalities is better for particular instances. As P_{cost} and P_{profit} are both large-scale MILP models, we follow this approach in this section through devising several additional inequalities in an attempt to strengthen the bounds of the corresponding linear relaxations. Ideally, this step also leads to a reduction of the computational time required to solve to optimality an instance with a general-purpose solver.

We first introduce inequalities that are valid for both types of models, P_{cost} and P_{profit}. Afterwards, we focus on valid inequalities that are specific to the cost minimization model.

4.1 Valid inequalities common to both models

The following constraints are redundant to the original models but may be relevant to their linear relaxations:

$$x^2_{t,i,j,p} \le d_{t,j,p} \sum_{\tau=1}^{t} z^2_{\tau,i} \qquad t \in T,\ i \in I^2,\ j \in J,\ p \in P$$

This set comprises a huge number of inequalities and thus, strongly affects the size of both models, P_{cost} and P_{profit}. A way of overcoming this drawback is to aggregate them as described by (22) and (23), although this may weaken the impact of the inequalities:

$$\sum_{j \in J} x^2_{t,i,j,p} \le \sum_{j \in J} d_{t,j,p} \sum_{\tau=1}^{t} z^2_{\tau,i} \qquad t \in T,\ i \in I^2,\ p \in P \tag{22}$$

$$\sum_{p \in P} x^2_{t,i,j,p} \le \sum_{p \in P} d_{t,j,p} \sum_{\tau=1}^{t} z^2_{\tau,i} \qquad t \in T,\ i \in I^2,\ j \in J \tag{23}$$

As the number of customer zones $|J|$ is usually much larger than the number of products $|P|$, the size of the models increases considerably when constraints (23) are added instead of (22). However, these inequalities are usually stronger than (22) with respect to the linear relaxation bounds.

4.2 Valid inequalities specific to cost minimization

In addition to the constraints described in the previous section, it is also possible to devise further inequalities that are specific to the case in which customers' demands must be satisfied, that is, to model P_{cost}. In particular, we will show that the minimum number of storage areas required by each family in each time period can be determined. Recall that under a maximization objective it is not possible to derive similar conditions due to the flexibility of not having to meet demands completely.

Let us assume that if a new storage area is installed for family ℓ in a new location, then the one with the largest size is selected. Furthermore, we assume that this choice can be made in every time period. For an upper echelon location $i \in I^1$ and a family $\ell \in L$ this means that only the $|K^1_\ell|$-th storage area (with size $Q^1_{i,\ell,|K^1_\ell|}$) can be selected. Similarly, in an intermediate echelon location $i \in I^2$, the largest storage area has size $Q^2_{i,\ell,|K^2_\ell|}$ for family $\ell \in L$.

For each family $\ell \in L$ we sort the largest storage sizes in non-decreasing order over the set of candidate sites for new facilities. Regarding the upper echelon locations, this entails building the sequence $Q^1_{[1],\ell}, Q^1_{[2],\ell}, \ldots, Q^1_{[|I^1|],\ell}$ such that

$$Q^1_{[1],\ell} \geq Q^1_{[2],\ell} \geq \ldots \geq Q^1_{[|I^1|],\ell}$$

For intermediate echelon facilities, the ordered sequence is as follows:

$$Q^2_{[1],\ell} \geq Q^2_{[2],\ell} \geq \ldots \geq Q^2_{[|I^2|],\ell}$$

A minimum number of storage areas required by each family in each time period can now be determined.

Consider an instance to the problem with 3 locations in the upper echelon. Additionally, suppose that the largest capacity of a storage area for some family ℓ is 100, 80 and 70 at locations 1, 2 and 3, respectively. Assume that the total demand for the products in family ℓ is 200 in period 2. Taking the above capacities into account and also the fact that we can install the same type of storage area in different periods, we can directly state that at least two storage areas must be available for family ℓ in period 2 (possibly one of size 100 installed in period 1 and another installed in period 2). In fact, it is important to recall that the storage areas available in every period $t \in T$ may have been installed in any period $1, \ldots, t$.

Let us denote by $R^1_{t,\ell}$ a lower bound on the total number of storage areas operating in period t for family ℓ, and let $\widetilde{Q}^1_{t,\ell}$ be their total capacity with $t \in T$ and $\ell \in L$. In addition, we define $R^1_{0,\ell} = 0$ and $\widetilde{Q}^1_{0,\ell} = 0$.

Let $\tilde{d}^1_{t,\ell}$ define the total quantity demanded in period t for products in family ℓ, that is,

$$\tilde{d}^1_{t,\ell} = \sum_{p \in P_\ell} \mu^1_p \sum_{j \in J} d_{t,j,p} \qquad t \in T,\ \ell \in L$$

Observe that $\tilde{d}^1_{t,\ell}$ is expressed in the storage units used at upper echelon facilities.

If the inequality $\widetilde{Q}^1_{t-1,\ell} \geq \tilde{d}^1_{t,\ell}$ holds, it means that the total capacity installed until period $t-1$ is also large enough to cover all the demand in period t. Therefore, $R^1_{t,\ell} = R^1_{t-1,\ell}$. Otherwise, it is necessary to expand the available capacity by installing additional storage areas in period t. The capacity thus acquired must cover $u^1_{t,\ell} = \tilde{d}^1_{t,\ell} - \widetilde{Q}^1_{t-1,\ell}$ units of demand. If $u^1_{t,\ell} < Q_{[1],\ell}$ then it suffices to install a single storage area, namely the largest one. In this case, $R^1_{t,\ell} = R^1_{t-1,\ell} + 1$. Otherwise, k new storage areas are required with k being a number greater than or equal to 2 such that the following inequalities hold:

$$\sum_{m=1}^{k-1} Q^1_{[m],\ell} < u^1_{t,\ell} \leq \sum_{m=1}^{k} Q^1_{[m],\ell}$$

It follows that $R^1_{t,\ell} = R^1_{t-1,\ell} + k$ as this is the minimum number of storage areas operating in the upper echelon in period t for family ℓ.

Note that the above reasoning relies on selecting only the largest storage areas in each location. However, in practice, also smaller storage areas can be installed. Hence, $R^1_{t,\ell}$ is in fact a lower bound on the actual number of storage areas that must be available in period t for family ℓ. Consequently, the following inequalities can be added to model P_{cost}:

$$\sum_{\tau=1}^{t} \sum_{i \in I^1} \sum_{k \in K^1_\ell} y^1_{\tau,i,\ell,k} \geq R^1_{t,\ell} \qquad t \in T,\ \ell \in L \tag{24}$$

Regarding the intermediate echelon facilities $i \in I^2$, the minimum number of storage areas that must operate in period $t \in T$ for family $\ell \in L$, that is $R^2_{t,\ell}$, is determined in a similar way. In each period t, we need to compare the available storage capacity with the demand requirements $\tilde{d}^2_{t,\ell}$ of family ℓ. The latter are determined by

$$\tilde{d}^2_{t,\ell} = \sum_{p \in P_\ell} \mu^2_p \sum_{j \in J} d_{t,j,p} \qquad t \in T,\ \ell \in L$$

Hence, if the installed capacity is large enough then $R^2_{t,\ell} = R^2_{t-1,\ell}$. Otherwise, $R^2_{t,\ell} = R^2_{t-1,\ell} + k$ with k denoting the number of storage areas that are additionally installed in period t ($k \geq 1$). The calculation of k follows the same steps as for upstream facilities.

The following inequalities are also valid for model P_{cost}:

$$\sum_{\tau=1}^{t} \sum_{i \in I^2} \sum_{k \in K^2_\ell} y^2_{\tau,i,\ell,k} \geq R^2_{t,\ell} \qquad t \in T,\ \ell \in L \tag{25}$$

Note that we assume that the following necessary conditions for optimality hold for each family $\ell \in L$ in each period $t \in T$:

$$\tilde{d}^1_{t,\ell} \leq t \times \sum_{i=1}^{|I^1|} Q^1_{[i],\ell} \qquad \text{and} \qquad \tilde{d}^2_{t,\ell} \leq t \times \sum_{i=1}^{|I^2|} Q^2_{[i],\ell}$$

In particular, in the first time period, the values of $R^1_{1,\ell}$ and $R^2_{1,\ell}$ are also useful to derive the minimum number of new facilities that must be established in each echelon. For instance, as $R^1_{1,\ell}$ indicates the minimum number of storage areas required by family ℓ and these storage areas can only operate at $R^1_{1,\ell}$ different locations due to constraints (3), it follows that at least $R^1_{1,\ell}$ new facilities must be established in time period 1. Furthermore, the family with the largest number of required storage areas also imposes a lower bound on the number of new upper echelon facilities that must be established in the first period. A similar reasoning also applies to intermediate echelon facilities. Hence, the following inequalities are valid for model P_{cost}:

$$\sum_{i \in I^1} z^1_{1,i} \geq \max_{\ell \in L} \left\{ R^1_{1,\ell} \right\} \tag{26}$$

$$\sum_{i \in I^2} z^2_{1,i} \geq \max_{\ell \in L} \left\{ R^2_{1,\ell} \right\} \tag{27}$$

Finally, we note that the class of inequalities (24)-(27) may be very useful, especially when the right-hand side terms are significantly larger than one. In this case, the total number of possible combinations of new locations and new storage areas decreases.

5 Computational experiments

Although problems P_{cost} and P_{profit} are NP-hard, being generalizations of the multi-period uncapacitated facility location problem (see Jacobsen [9]), nowadays state-of-the-art optimization software can handle very large instances of real-world combinatorial problems. As a result, many organizations resort to general-purpose solvers for decision support. In this section, we assess the performance of a commercial optimization tool, namely CPLEX, on a set of randomly generated instances. A further aim of our empirical study is to analyze the impact of the type of objective function on the design of the supply chain network.

5.1 Characteristics of test instances

As benchmark instances are not available for the problems at hand, we randomly generated 72 instances by combining the values indicated in Table 6.

The number of products belonging to each family ranges from 3 to 5 commodities, being the exact number randomly selected. Given that either 3 or 5 families are considered, the generated instances comprise at least 9 and at most 25 products. Three capacity levels are assumed for storage areas in both location echelons of the network. For $k \in K^1_\ell$, resp. $k \in K^2_\ell$, $k = 1$ refers to a small storage area, $k = 2$ represents a medium-sized storage area, and $k = 3$ denotes a large storage area. Although test instances are randomly generated, they reflect realistic features of strategic SCND problems to capture a wide range of problem structures (e.g. the fixed installation costs of storage areas reflect economies of scale). Appendix A provides the details of the methodology developed to obtain test instances. Each instance is designed so

Symbol	Description	Values
$\|T\|$	Number of time periods	$3, 4$
$\|J\|$	Number of customer zones	$20, 30, 50$
$\|I^1\|$	Number of potential locations in the *upper* echelon	$3, 5, 7$
$\|I^2\|$	Number of potential locations in the *intermediate* echelon	$10, 15, 20$
$\|L\|$	Number of product families	$3, 5$
$\|P_\ell\|$	Number of products in family $\ell \in L$	$3, 4, 5$
$\|P\|$	Total number of products	$\sum_{\ell \in L} \|P_\ell\|$
$\|K_\ell^1\|$	Number of storage areas for family $\ell \in L$ in a facility of the *upper* echelon	3
$\|K_\ell^2\|$	Number of storage areas for family $\ell \in L$ in a facility of the *intermediate* echelon	3

Table 6: Cardinality of index sets.

that the feasible regions of both P_{cost} and P_{profit} are not empty. This means that the capacity levels of storage areas as well as the budget available in each time period are set in such a way that it is possible to design a supply chain network to satisfy all customers' demands if required (that is, if model P_{cost} is solved). Nonetheless, the investment capital is a scarce resource as it is often the case in practice.

The size of the test instances is summarized in Table 7. The data sets are divided into two classes that differ by the length of their planning horizons. Each class comprises 36 instances. On average, the expansion of the time horizon from 3 to 4 periods results in a 33.5% increase of both the number of variables and constraints. In particular, the set of binary variables is extensive. The set of continuous variables is mainly dominated by the variables ruling the quantities to be shipped through the network. Details about the specific characteristics of each instance are given in Tables 15 and 16 in Appendix B.

	$\|T\| = 3$			$\|T\| = 4$		
	Avg	Min	Max	Avg	Min	Max
# Binary variables	878	540	1296	1170	720	1728
# Continuous variables	29113	10840	60529	39170	14453	83705
# Constraints	4329	2367	6840	5755	3150	9391

Table 7: Size of the test instances.

5.2 Numerical results for the original models

The models were implemented using IBM ILOG Concert Technology 2.9 and solved with IBM ILOG CPLEX 12.1. All experiments were conducted on a PC with an Intel Core i7-2600 3.4 GHz processor and 4 GB RAM. CPLEX was run with a CPU time limit of 8 hours.

Figure 3 displays the quality of the best feasible solution identified by CPLEX for every test instance within the specified time limit. For each model M, with $M = P_{cost}, P_{profit}$, solution quality is measured by the integrality gap as follows:

$$\text{MIP gap (\%)} \quad = \quad \frac{|z^B - z^M|}{z^M} \times 100\% \tag{28}$$

with z^M denoting the objective value of the best feasible solution to model M and z^B representing the best bound. For P_{profit}, z^B corresponds to the best *upper* bound, whereas z^B refers to the best *lower* bound in the case of P_{cost}.

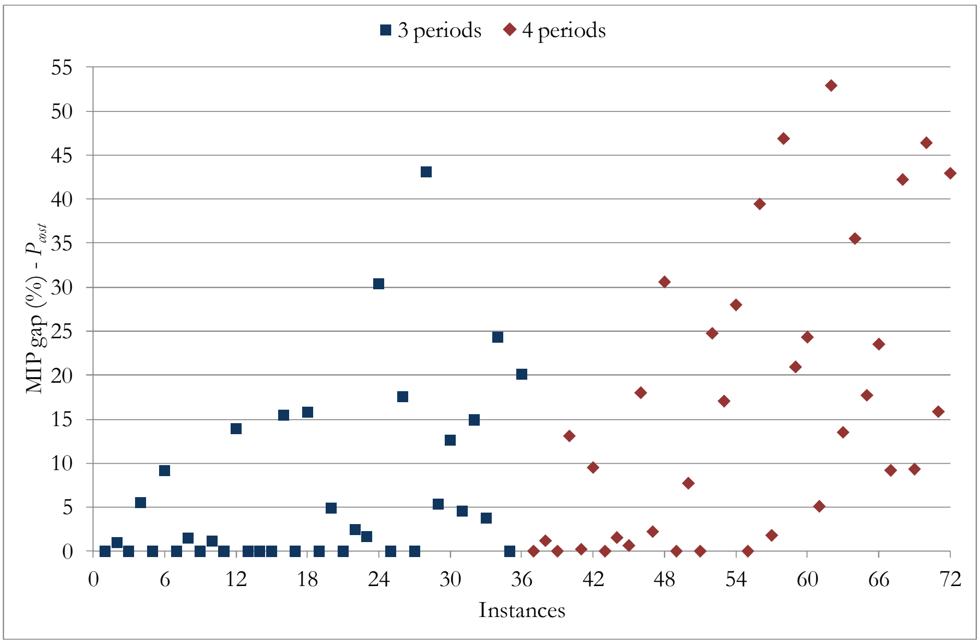
The choice of a performance criterion for SCND has a strong effect on the results obtained as shown in Figure 3. While near-optimal solutions are always identified by CPLEX under a profit maximization goal, the same does not occur when the network configuration is driven by a cost minimization objective. In particular, 40% of the instances exhibit a MIP gap between 10% and 55% in the latter case.

Table 8 gives a summary of the MIP gaps obtained with each type of performance measure. The length of the time horizon and as a result, the size of the instances, have a significant impact on solution quality. The class of problems with 4 time periods has on average a larger MIP gap. This is particularly striking for model P_{cost} whose solution quality deteriorates significantly. Tables 17–20 in Appendix B specify the MIP gap of each instance under the two types of objective.

	$\|T\| = 3$		$\|T\| = 4$	
MIP gap (%)	P_{cost}	P_{profit}	P_{cost}	P_{profit}
Avg	6.92	0.15	16.73	0.26
Min	0.00	0.00	0.00	0.00
Max	43.07	0.67	52.91	0.94

Table 8: Synthesis of the quality of the best solutions identified for the original models.

Regarding the computational time required by CPLEX, Table 9 displays the average, minimum and maximum CPU times (in hours) for the two models P_{cost} and P_{profit}. It can be seen that the CPU time increases sharply with the number of periods. The examination of



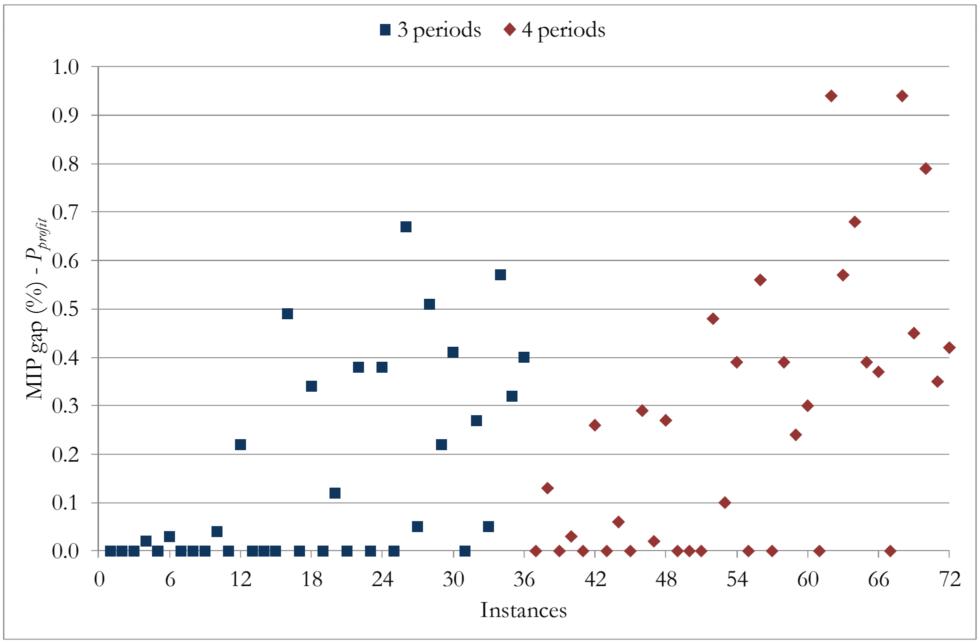


Figure 3: Quality of the best solutions identified for the original models P_{cost} and P_{profit}.

Tables 17–20 (see Appendix B) confirms this observation and also reveals a significant time variability.

The results shown so far indicate that the type of model considered has a strong effect both

CPU (h)	$\lvert T\rvert = 3$		$\lvert T\rvert = 4$	
	P_{cost}	P_{profit}	P_{cost}	P_{profit}
Avg	5.8	5.3	7.2	6.4
Min	0.2	0.1	0.5	0.5
Max	8.0	8.0	8.0	8.0

Table 9: Computational time required by the original models.

on the computational time and on the solution quality. One way to measure the quality of a MILP formulation is to determine the bound provided by its linear relaxation. In general, the tighter the bound provided by the linear relaxation, the higher the quality of the MILP formulation. Hence, in an attempt to understand the large differences in solution quality observed in Figure 3, we also analyzed the LP bound of each instance under both objective functions. Note that the optimal value of P_{cost} is bracketed from below by its LP bound, while the opposite occurs with the LP bound of P_{profit}. Figure 4 displays the relative LP gaps which are calculated as follows:

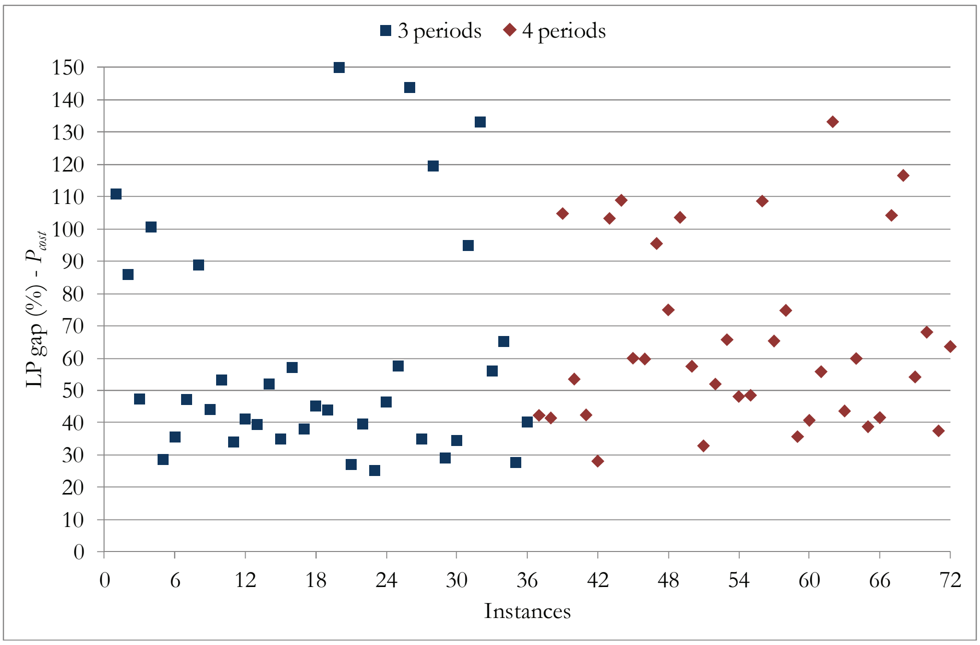
$$\text{LP gap (\%)} \quad = \quad \frac{|z^{LP} - z^{M}|}{z^{M}} \times 100\% \tag{29}$$

with z^M denoting the objective value of the best feasible solution to model M $(M = P_{cost}, P_{profit})$ and z^{LP} representing the corresponding LP bound. The striking differences observed in Figure 4 reveal that the MILP formulation of the profit maximization problem is very tight. Moreover, the quality of these bounds is independent of the instance size. In contrast, the LP bounds of the cost minimization model vary between 25% and 150% and thus, are very poor. Table 10 summarizes these findings (see also Tables 17–20 in Appendix B for details specific to each instance).

LP gap (%)	$\lvert T\rvert = 3$		$\lvert T\rvert = 4$	
	P_{cost}	P_{profit}	P_{cost}	P_{profit}
Avg	59.80	1.18	65.70	1.18
Min	25.20	0.58	28.11	0.61
Max	149.94	2.16	133.18	1.90

Table 10: Synthesis of the quality of the linear relaxation gap of the original models.

In view of these results, a further strengthening of the original MILP formulation of model P_{cost} is required. In the next section, we evaluate the effect that the valid inequalities proposed in Section 4 have on the linear relaxation bound.



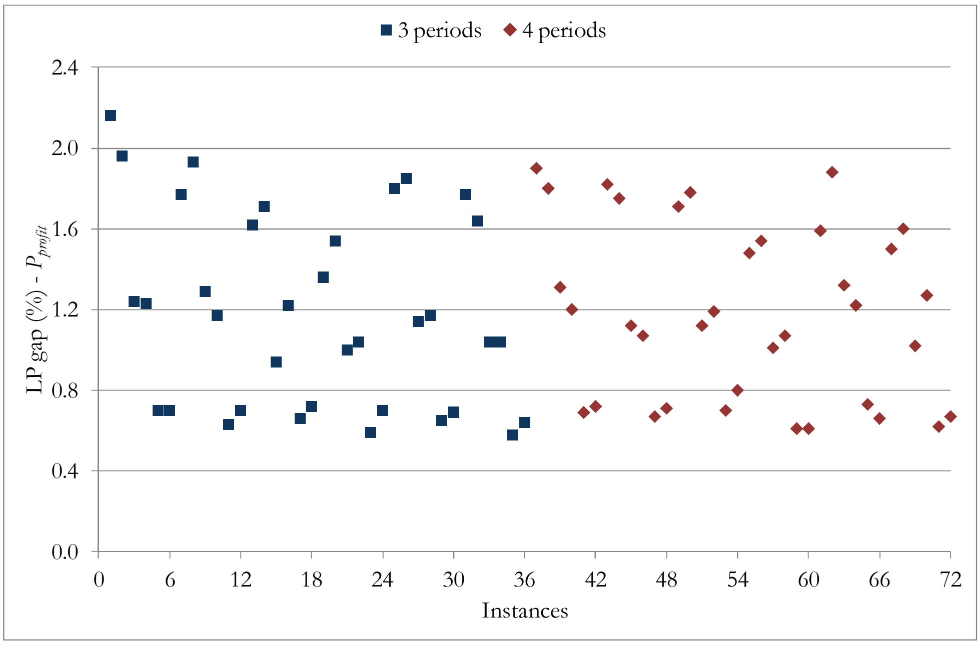


Figure 4: Quality of the LP gaps of the original models P_{cost} and P_{profit}.

5.3 Impact of valid inequalities on the cost minimization model

The degree to which the original model P_{cost} is expanded depends on the number of valid inequalities that are added. The first column in Table 11 indicates two different ways of strengthening the original formulation P_{cost} that were considered. The column under "Size expansion (%)" gives the relative increase of the number of constraints resulting from combining different inequalities. While constraints (24)–(27) are computationally inexpensive (in total $2 \cdot |T| \cdot |L| + 2$ inequalities), the same does not apply to constraints (22). For each class of test instances, the percentage LP gap closed is given. This value is calculated as $100 \times (z^{CUT} - z^{LP})/(\overline{z} - z^{LP})$, where z^{CUT} denotes the value of the linear relaxation of P_{cost} enhanced with valid inequalities and z^{LP} represents the linear relaxation value of the original model. In addition, $\overline{z}$ denotes the objective value of the best feasible solution available.

Additional inequalities	Size expansion (%)	$\|T\| = 3$ Avg	Min	Max	$\|T\| = 4$ Avg	Min	Max
(24)+(25)+(26)+(27)	0.6	26.38	2.65	48.80	21.58	4.73	42.25
(24)+(25)+(26)+(27)+(22)	17.6	26.40	2.68	48.80	21.61	4.76	42.33

Table 11: Cost minimization model with additional inequalities - LP gap closed (%).

From Table 11, it is evident that the most significant enhancement is due to adding the valid inequalities (24)–(27). Although the linear relaxation is further tightened with constraints (22), the improvement is unimportant. In Tables 12 and 13, the original model is compared with its best enhancement with respect to the best feasible solutions identified by CPLEX and the required computational time.

Although the linear relaxation is strengthened, it does not significantly impact neither the solution quality (see Table 12) nor the CPU time (see Table 13). For example, on average CPLEX solves an instance with 4 time periods within 6.9 hours and the best feasible solution has an integrality gap of 12.85%. As these results are rather disappointing, we further investigated why a cost minimization problem is considerably more difficult to solve than its profit maximization counterpart.

Model	$\|T\| = 3$ Avg	Min	Max	$\|T\| = 4$ Avg	Min	Max
P_{cost}	6.92	0.00	43.07	16.73	0.00	52.91
P_{cost}+(24)+(25)+(26)+(27)	5.20	0.00	25.48	12.85	0.00	54.70

Table 12: Cost minimization: Final MIP gap (%) using the original model and its best enhancement.

	$\|T\| = 3$			$\|T\| = 4$		
Model	Avg	Min	Max	Avg	Min	Max
P_{cost}	5.8	0.2	8.0	7.2	0.5	8.0
P_{cost}+(24)+(25)+(26)+(27)	5.3	0.1	8.0	6.9	0.3	8.0

Table 13: Cost minimization: CPU time (hours) required by the original model and by its best enhancement.

To this end, for each test instance we compared the best feasible solutions to P_{cost} and P_{profit}. Recall that each instance is generated in such a way that the feasible solution space of P_{cost} is not empty and thus, the same holds with respect to the feasible region of P_{profit}. However, the solutions to the two problems differ considerably. Although the available budget permits to design a network that satisfies all customers' demands in each time period, this only occurs in two instances (out of 72) under a profit maximization goal. In the remaining 70 instances, the percentage demand met in each time period ranges from 96.04% to 99.98% (see Tables 19 and 20 in Appendix B). This indicates that the costs incurred by installing further storage areas and shipping the products so as to cover all demand requirements are higher than the corresponding revenues. This fact has an important managerial implication as it enables the identification of those demands that represent a loss for the organization. Furthermore, the solutions to P_{profit} include storage areas with large capacity utilization, a characteristic that is relevant in practice.

The linear relaxation of the cost minimization model tries to mimic this situation through the assignment of fractional values to some of the variables ruling the installation of new storage areas (i.e. $y^1_{t,i,\ell,k}$ and $y^2_{t,i,\ell,k}$). The fractional values are selected in such a way that only the required capacity is installed (see constraints (7) and (8)) to cover any remaining demand that otherwise could not be satisfied with the capacity available in the network. As a result, the linear relaxation bound is very weak. In contrast, due to constraints (19), to obtain a feasible solution the setup of a few storage areas is enforced, even if they are operated at their minimum capacity levels. Therefore, the relative difference between the objective value of a feasible solution and the LP bound tends to be rather large.

6 Conclusions

In this paper, we introduced a new SCND problem that generalizes several other problems studied in the literature. The aim of the new problem is to design a two-echelon supply chain network over a multi-period horizon. This entails locating new facilities in the upper and intermediate echelons of the network and installing storage areas to handle different product families. A finite set of capacity levels for each product family is available at each potential location. Decisions concerning the quantities of products to be shipped through the network are also to be made. A further novel aspect captured by our SCND problem is the investment of an available budget for facility location and sizing in each time period. Moreover, variable facility operating costs are considered which depend on the capacity utilization rate of the storage areas installed at each new location. Application scenarios for our problem include the design of a new network and/or the gradual expansion of (existing/new) facilities over a time horizon.

Our extensive computational study focused on the implications that the choice of different performance measures have on network design. Under a profit maximization objective, it may not always be attractive to completely satisfy demand requirements. In this case, the results obtained by solving randomly generated instances with CPLEX indicate that our MILP formulation is very good due to the tight upper bound provided by its linear relaxation. Furthermore, CPLEX was able to identify feasible solutions with integrality gaps below 1% to all test instances within the pre-specified time limit. In practice, as data estimates often contain errors, it may not be meaningful to solve a problem instance to optimality. Hence, the near-optimal solutions found by CPLEX provide a good basis for decision-making. In addition, the positive results obtained under a profit maximization goal encourage the further development of even more comprehensive models for SCND. For example, the current model could be extended by considering direct shipments from upper echelon facilities to customer zones.

In contrast, the study of the SCND problem under a cost minimization objective provided significantly different insights on the effect of this performance measure on solution quality and computational time. Our numerical experiments suggest that the enforcement of demand satisfaction yields a much more difficult problem. The linear relaxation bound of the MILP formulation proved to be rather weak in most of the test instances. Our attempt to enhance the original formulation by adding several valid inequalities did not produce on average significant improvements. Moreover, the integrality gaps of the best feasible solutions identified by CPLEX vary widely (up to 53%). In particular, solution quality seems to deteriorate as the number of time periods increases. These results indicate that further research is required to improve the polyhedral description of the feasible solution set under a cost minimization objective. Hence, future research will explore the application of decomposition techniques in an attempt to develop a promising solution approach.

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Appendix A: Data generation

The methodology for the random generation of test instances is described in detail below. Recall that Table 6 presented in Section 5.1 gives the cardinality of the index sets.

The next table describes the generation of various parameters. In what follows, we denote by $U[a,b]$ the random generation of numbers in the interval $[a,b]$ according to a uniform distribution.

Symbol	Description	Values
μ_p^1	Unit capacity handling factor of product $p \in P$	
	in an *upper* echelon location	$U[0.0001, 0.001]$
μ_p^2	Unit capacity handling factor of product $p \in P$	
	in an *intermediate* echelon location	$U[0.01, 0.05]$
β_t^d	Parameter for demand generation, $t \in T \setminus \{1\}$	$U[1.05, 1.10]$
$d_{1,j,p}$	Demand of customer zone $j \in J$ for product $p \in P$	
	in period 1	$U[20, 100]$
$d_{t,j,p}$	Demand of customer zone $j \in J$ for product $p \in P$	
	in period $t \in T \setminus \{1\}$	$\beta_t^d \cdot d_{t-1,j,p}$
α_t	Unit return factor on capital not invested in	
	period $t \in T \cup \{0\}$	$\alpha_0 = 0$, otherwise $U[1.01, 1.03]$

Table 14: Selected parameters.

As indicated in Table 14, the demand requirements increase between 5% and 10% between two consecutive time periods.

To obtain the minimum throughput and the capacity level of each storage area in a given location, the following procedure is employed.

- For $i \in I^1$, $\ell \in L$, and $k \in K_\ell^1$, the capacity levels $Q_{i\ell k}^1$ are set according to

$$\begin{aligned} Q^1_{i,\ell,|K_\ell^1|} &= \frac{1}{|I^1|} U[4,6] \sum_{p \in P_\ell} \left(\mu_p^1 \sum_{j \in J} d_{|T|,j,p} \right) \\ Q^1_{i,\ell,k} &= 0.7\, Q^1_{i,\ell,k+1}, \quad k = 1, \ldots, |K_\ell^1| - 1 \end{aligned}$$

  The above formulas ensure that the expected maximum capacity available for each family, over all facilities in the upper echelon, is equal to five times the total demand for products belonging to this family.

- A similar procedure is applied to each location in the intermediate echelon. The capacity levels $Q^2_{i,\ell,k}, \ell \in L, k \in K^2_\ell$ are determined according to

$$
\begin{aligned}
Q^2_{i,\ell,|K^2_\ell|} &= \frac{1}{|I^2|} U[1,3] \sum_{p \in P_\ell} \left(\mu^2_p \sum_{j \in J} d_{|T|,j,p} \right) \\
Q^2_{i,\ell,k} &= 0.7\, Q^2_{i,\ell,k+1}, \quad k = 1, \ldots, |K^2_\ell| - 1
\end{aligned}
$$

  In this case, the expected maximum operating capacity for a family in the intermediate level is twice the overall requirement of that family.

- Regarding the minimum throughput levels, they are specified as a percentage of the available capacity:

$$
\begin{aligned}
q^1_{i,\ell,k} &= 0.4\, Q^1_{i,\ell,k}, \quad i \in I^1,\, \ell \in L,\, k \in K^1_\ell \\
q^2_{i,\ell,k} &= 0.4\, Q^2_{i,\ell,k}, \quad i \in I^2,\, \ell \in L,\, k \in K^2_\ell
\end{aligned}
$$

The generation of cost data relies on the parameters $\beta^c_t \in U[1.02, 1.05]$ for $t \in T \setminus \{1\}$. For each $t > 1, \beta^c_t$ indicates that a given cost factor increases between 2% and 5% compared to the previous time period (details are given below).

- The fixed installation costs of storage areas are generated in order to reflect economies of scale. In the first time period, they are set according to

$$
\begin{aligned}
G^1_{1,i,\ell,k} &= 100 \cdot \sqrt{Q^1_{i,\ell,k}/\overline{\mu}^1}, \quad i \in I^1,\, \ell \in L,\, k \in K^1_\ell \\
G^2_{1,i,\ell,k} &= 100 \cdot \sqrt{Q^2_{i,\ell,k}/\overline{\mu}^2}, \quad i \in I^2,\, \ell \in L,\, k \in K^2_\ell
\end{aligned}
$$

  with $\overline{\mu}^1$ and $\overline{\mu}^2$ denoting the averages of the corresponding capacity handling factors, that is,

$$
\overline{\mu}^1 = \frac{\sum_{p \in P} \mu^1_p}{|P|}, \qquad \overline{\mu}^2 = \frac{\sum_{p \in P} \mu^2_p}{|P|}
$$

  In the remaining time periods, these costs are calculated as follows:

$$
\begin{aligned}
G^1_{t,i,\ell,k} &= \beta^c_t \cdot G^1_{t-1,i,\ell,k}, \quad i \in I^1, \ell \in L,\, k \in K^1_\ell,\, t = 2, \ldots, |T| \\
G^2_{t,i,\ell,k} &= \beta^c_t \cdot G^2_{t-1,i,\ell,k}, \quad i \in I^2, \ell \in L,\, k \in K^2_\ell,\, t = 2, \ldots, |T|
\end{aligned}
$$

- The fixed costs of establishing new facilities are set according to

$$
\begin{aligned}
F^1_{t,i} &= \sum_{\ell \in L} G^1_{t,i,\ell,|K^1_\ell|}, \quad t \in T,\, i \in I^1 \\
F^2_{t,i} &= \sum_{\ell \in L} G^2_{t,i,\ell,|K^2_\ell|}, \quad t \in T,\, i \in I^2
\end{aligned}
$$

Observe that the cost of installing the largest storage area for each family sets the fixed setup cost at a potential location.

- The unitary operating costs are generated in order to capture economies of scale. In the first time period they are generated as follows:

$$
\begin{aligned}
O^1_{1,i,\ell,k} &= \frac{1000}{\sqrt{Q^1_{i,\ell,k}/\overline{\mu}^1}}, \quad i \in I^1,\, \ell \in L,\, k \in K^1_\ell \\
O^2_{1,i,\ell,k} &= \frac{1000}{\sqrt{Q^2_{i,\ell,k}/\overline{\mu}^2}}, \quad i \in I^2,\, \ell \in L,\, k \in K^2_\ell
\end{aligned}
$$

In the remaining time periods, we consider:

$$
\begin{aligned}
O^1_{t,i,\ell,k} &= \beta^c_t \cdot O^1_{t-1,i,\ell,k}, \quad i \in I^1,\, \ell \in L,\, k \in K^1_\ell,\ t = 2, \ldots, |T| \\
O^2_{t,i,\ell,k} &= \beta^c_t \cdot O^2_{t-1,i,\ell,k}, \quad i \in I^2,\, \ell \in L,\, k \in K^2_\ell,\ t = 2, \ldots, |T|
\end{aligned}
$$

- In the first period, the unitary shipment costs $S^1_{1,i,i',p}$ are drawn from a uniform $[1, 5]$ distribution for every $i \in I^1$, $i' \in I^2$, and $p \in P$. In the remaining time periods we set

$$
S^1_{t,i,i',p} = \beta^c_t \cdot S^1_{t-1,i,i',p}, \quad t = 2, \ldots, |T|
$$

- The unitary costs of shipping products from intermediate level facilities to customer zones are generated in a similar way. For every $i \in I^2$, $j \in J$, and $p \in P$ we consider:

$$
\begin{aligned}
S^2_{1,i,j,p} &= U[5, 10] \\
S^2_{t,i,j,p} &= \beta^c_t \cdot S^2_{t-1,i,j,p}, \quad t = 2, \ldots, |T|
\end{aligned}
$$

- For the generation of the fixed facility maintenance costs, first an estimation of the maintenance cost in each time period is calculated as follows:

$$\begin{aligned}
m^1_{1,i} &= \frac{F^1_{1,i}}{|L|}, && i \in I^1 \\
m^1_{t,i} &= \beta^c_t \cdot m^1_{t-1,i}, && i \in I^1,\, t = 2, \ldots, |T| \\
m^2_{1,i} &= \frac{F^2_{1,i}}{|L|}, && i \in I^2 \\
m^2_{t,i} &= \beta^c_t \cdot m^2_{t-1,i}, && i \in I^2,\, t = 2, \ldots, |T|
\end{aligned}$$

  Next, the maintenance costs per period are summed up:

$$\begin{aligned}
M^1_{t,i} &= \sum_{\tau=t}^{|T|} m^1_{\tau,i}, \quad i \in I^1,\, t \in T \\
M^2_{t,i} &= \sum_{\tau=t}^{|T|} m^2_{\tau,i}, \quad i \in I^2,\, t \in T
\end{aligned}$$

Regarding the capital available in each time period for opening new facilities and installing storage areas, we set $B_t = B_1$ for $t = 2, \ldots, |T|$. In the first period, B_1 is drawn from a uniform distribution on the range $[2.2\Gamma, 3.5\Gamma]$ with

$$\Gamma = \max_{i \in I^1} F^1_{1,i} + \max_{i \in I^2} F^1_{2,i} + \sum_{\ell \in L} \max_{i \in I^1} G^1_{1,i,\ell,|K^1_\ell|} + \sum_{\ell \in L} \max_{i \in I^2} G^2_{1,i,\ell,|K^2_\ell|}$$

Our numerical tests confirm that the available budget is relatively tight, in particular in the first period.

Finally, to obtain the revenues for selling the products to the customer zones, we combine the average costs incurred to operate the network in the following way:

$$\begin{aligned}
R^2_{t,j,p} = \; & \frac{1}{TD_t} \cdot \left[\frac{\sum\limits_{i \in I^1} M^1_{t,i}}{|I^1|} + \frac{\sum\limits_{i \in I^2} M^2_{t,i}}{|I^2|} \right] + \overline{\mu}^1 \cdot \frac{\sum\limits_{i \in I^1} \sum\limits_{\ell \in L} \sum\limits_{k \in K^1_\ell} O^1_{t,i,\ell,k}}{|I^1| \cdot \sum\limits_{\ell \in L} K^1_\ell} + \\
& \overline{\mu}^2 \cdot \frac{\sum\limits_{i \in I^2} \sum\limits_{\ell \in L} \sum\limits_{k \in K^2_\ell} O^2_{t,i,\ell,k}}{|I^2| \cdot \sum\limits_{\ell \in L} K^2_\ell} + \frac{\sum\limits_{i \in I^1} \sum\limits_{i' \in I^2} S^1_{t,i,i',p}}{|I^1| \cdot |I^2|} + \frac{\sum\limits_{i \in I^2} S^2_{t,i,j,p}}{|I^2|}, \qquad t \in T,\, j \in J,\, p \in P
\end{aligned}$$

with TD_t denoting the total quantity demanded in period $t \in T$, that is, $TD_t = \sum_{j \in J} \sum_{p \in p} d_{t,j,p}$. Preliminary tests showed that the choices described above lead to meaningful instances to the problems.

Appendix B: Complementary results

Table 15 describes the main characteristics of the 36 test instances with a planning horizon comprising 3 periods. Similar information for the second class of 36 instances with 4 time periods is provided in Table 16. The first column of these two tables identifies each instance. The following 5 columns give the number of potential facilities in the upstream and intermediate echelons of the network ($|I^1|$, $|I^2|$), the number of customer zones ($|J|$), the number of product families ($|L|$), and the total number of products ($|P|$). As mentioned in Section 5.1, the latter value is the outcome of randomly choosing at least 3 and at most 5 products in each family. The columns under "# cont. var.", "# bin. var.", and "# const." give the number of continuous variables, binary variables, and constraints, respectively.

Table 17, resp. 18, displays the results obtained with CPLEX for model P_{cost} and the class of instances with 3, resp. 4, time periods. The first column identifies each instance. The CPU time required by CPLEX (in hours) is given in the second column. Recall that CPLEX is run with a time limit of 8 hours. The MIP gap reported by CPLEX upon termination is displayed in the third column (see the definition in (28)). The fourth column ("# nodes") indicates the total number of nodes explored in the branch-and-cut tree. Finally, the last column gives the linear relaxation lower bound as defined in (29). Similar information is included in Tables 19 and 20 for the profit maximization model. In this case, the average percentage demand satisfied by the best feasible solution is also presented ("Avg demand met (%)"). Moreover, the LP gap refers in this case to the linear relaxation upper bound.

A closer examination of Tables 17–20 reveals that the time limit of 8 hours is reached by 70.8% (51) of the instances under a cost minimization objective. Mostly instances with $|T| = 4$ belong to this group. Under a profit maximization goal, 58.3% (42) of the instances required 8 hours of CPU. Furthermore, it can be seen that CPLEX identifies the optimal solution of 29 (40.3%) instances with P_{profit}. In contrast, optimality seems to be more difficult to be achieved with model P_{cost} as it is only obtained in 20 (27.8%) instances.

Instance	$\lvert I^1\rvert$	$\lvert I^2\rvert$	$\lvert J\rvert$	$\lvert L\rvert$	$\lvert P\rvert$	# cont. var.	# bin. var.	# const.
1	3	15	20	3	10	10840	540	2367
2	3	15	20	5	17	18409	864	3966
3	3	15	30	3	12	18310	540	2937
4	3	15	30	5	18	27544	864	4611
5	3	15	50	3	12	29110	540	3657
6	3	15	50	5	17	41359	864	5496
7	3	20	20	3	10	14425	690	2882
8	3	20	20	5	17	24499	1104	4826
9	3	20	30	3	9	18445	690	3032
10	3	20	30	5	18	36679	1104	5486
11	3	20	50	3	12	38785	690	4202
12	3	20	50	5	17	55099	1104	6356
13	5	15	20	3	11	12919	600	2618
14	5	15	20	5	17	20029	960	4208
15	5	15	30	3	9	14719	600	2678
16	5	15	30	5	19	30829	960	4988
17	5	15	50	3	11	27769	600	3608
18	5	15	50	5	16	40504	960	5543
19	5	20	20	3	10	15679	750	3028
20	5	20	20	5	18	28129	1200	5188
21	5	20	30	3	11	23779	750	3478
22	5	20	30	5	17	36829	1200	5578
23	5	20	50	3	12	40279	750	4348
24	5	20	50	5	18	60529	1200	6808
25	7	15	20	3	12	15178	660	2869
26	7	15	20	5	16	20434	1056	4345
27	7	15	30	3	11	18913	660	3094
28	7	15	30	5	16	27634	1056	4825
29	7	15	50	3	12	31378	660	3949
30	7	15	50	5	18	47164	1056	6175
31	7	20	20	3	10	16933	810	3174
32	7	20	20	5	17	28759	1296	5310
33	7	20	30	3	10	22933	810	3474
34	7	20	30	5	17	38959	1296	5820
35	7	20	50	3	10	34933	810	4074
36	7	20	50	5	17	59359	1296	6840

Table 15: Characteristics of the test instances - $|T| = 3$.

Instance	$\|I^1\|$	$\|I^2\|$	$\|J\|$	$\|L\|$	$\|P\|$	# cont. var.	# bin. var.	# const.
37	3	15	20	3	10	14453	720	3150
38	3	15	20	5	18	25925	1152	5422
39	3	15	30	3	10	20453	720	3550
40	3	15	30	5	17	34745	1152	5962
41	3	15	50	3	10	32453	720	4350
42	3	15	50	5	18	58325	1152	7582
43	3	20	20	3	10	19233	920	3835
44	3	20	20	5	17	32665	1472	6427
45	3	20	30	3	10	27233	920	4235
46	3	20	30	5	19	51545	1472	7507
47	3	20	50	3	10	43233	920	5035
48	3	20	50	5	19	81945	1472	9027
49	5	15	20	3	10	15725	800	3344
50	5	15	20	5	17	26705	1280	5604
51	5	15	30	3	10	21725	800	3744
52	5	15	30	5	16	34805	1280	6104
53	5	15	50	3	10	33725	800	4544
54	5	15	50	5	17	57305	1280	7644
55	5	20	20	3	10	20905	1000	4029
56	5	20	20	5	17	35505	1600	6749
57	5	20	30	3	12	34505	1000	4829
58	5	20	30	5	19	54705	1600	7829
59	5	20	50	3	11	49305	1000	5509
60	5	20	50	5	17	76305	1600	8789
61	7	15	20	3	11	18617	880	3678
62	7	15	20	5	18	30485	1408	6066
63	7	15	30	3	11	25217	880	4118
64	7	15	30	5	18	41285	1408	6786
65	7	15	50	3	11	38417	880	4998
66	7	15	50	5	19	66305	1408	8486
67	7	20	20	3	10	22577	1080	4223
68	7	20	20	5	18	40505	1728	7231
69	7	20	30	3	12	36497	1080	5023
70	7	20	30	5	17	51945	1728	7751
71	7	20	50	3	11	51137	1080	5703
72	7	20	50	5	18	83705	1728	9391

Table 16: Characteristics of the test instances - $|T| = 4$.

Instance	CPU (h)	MIP gap (%)	# nodes	LP gap (%)
1	0.3	0.00	3557	110.77
2	8.0	1.00	50297	85.84
3	1.8	0.00	5977	47.28
4	8.0	5.51	8626	100.57
5	2.7	0.00	4687	28.69
6	8.0	9.17	1990	35.65
7	0.2	0.00	2634	47.16
8	8.0	1.46	24065	88.87
9	1.5	0.00	11116	44.05
10	8.0	1.15	12123	53.17
11	6.9	0.00	13965	34.06
12	8.0	13.92	1338	41.17
13	0.5	0.00	4713	39.46
14	1.3	0.00	17003	52.06
15	0.6	0.00	5221	34.93
16	8.0	15.48	6004	57.08
17	3.8	0.00	4519	38.09
18	8.0	15.79	3719	45.17
19	0.7	0.00	10912	43.88
20	8.0	4.93	20741	149.94
21	2.1	0.00	6733	27.10
22	8.0	2.47	6815	39.54
23	8.0	1.68	4060	25.20
24	8.0	30.37	1191	46.42
25	3.6	0.00	21017	57.60
26	8.0	17.57	18861	143.79
27	6.5	0.00	21194	34.99
28	8.0	43.07	9680	119.55
29	8.0	5.34	6429	28.98
30	8.0	12.60	1326	34.57
31	8.0	4.58	43068	94.90
32	8.0	14.94	11391	133.21
33	8.0	3.79	25220	56.03
34	8.0	24.31	4220	65.17
35	6.8	0.00	7840	27.68
36	8.0	20.14	981	40.26

Table 17: Cost minimization - Detailed results for $|T| = 3$.

Instance	CPU (h)	MIP gap (%)	# nodes	LP gap (%)
37	0.5	0.00	3652	42.28
38	8.0	1.19	15145	41.48
39	1.8	0.00	13469	104.78
40	8.0	13.10	4857	53.52
41	8.0	0.23	20036	42.42
42	8.0	9.50	1352	28.10
43	4.4	0.00	47138	103.27
44	8.0	1.55	28712	108.89
45	8.0	0.65	25454	59.99
46	8.0	18.01	3111	59.73
47	8.0	2.23	9688	95.49
48	8.0	30.60	1106	74.96
49	1.2	0.00	7904	103.58
50	8.0	7.72	8047	57.47
51	4.9	0.00	24364	32.84
52	8.0	24.75	4317	51.97
53	8.0	17.05	2829	65.71
54	8.0	27.99	1178	48.11
55	6.8	0.00	71257	48.52
56	8.0	39.46	3291	108.63
57	8.0	1.80	9733	65.30
58	8.0	46.89	1368	74.79
59	8.0	20.94	1801	35.69
60	8.0	24.32	691	40.76
61	8.0	5.10	11025	55.80
62	8.0	52.91	8750	133.18
63	8.0	13.51	10930	43.64
64	8.0	35.52	2933	59.89
65	8.0	17.72	3391	38.77
66	8.0	23.53	890	41.59
67	8.0	9.18	20040	104.19
68	8.0	42.23	4005	116.56
69	8.0	9.32	6569	54.17
70	8.0	46.40	1331	68.04
71	8.0	15.86	2048	37.50
72	8.0	42.94	697	63.59

Table 18: Cost minimization - Detailed results for $|T| = 4$.

Instance	CPU (h)	MIP gap (%)	# nodes	Avg demand met (%)	LP gap (%)
1	0.4	0.00	6524	97.81	2.16
2	2.9	0.00	16129	97.20	1.96
3	1.0	0.00	7579	98.71	1.24
4	8.0	0.02	12744	99.22	1.23
5	3.1	0.00	6647	99.76	0.70
6	8.0	0.03	6555	99.45	0.70
7	0.2	0.00	1840	97.13	1.77
8	2.7	0.00	6836	97.92	1.93
9	0.4	0.00	2586	98.88	1.29
10	8.0	0.04	5769	98.55	1.17
11	4.1	0.00	6458	99.79	0.63
12	8.0	0.22	1351	99.79	0.70
13	0.5	0.00	3104	97.72	1.62
14	3.3	0.00	14357	97.20	1.71
15	0.2	0.00	3754	99.80	0.94
16	8.0	0.49	3371	99.43	1.22
17	3.0	0.00	4904	99.86	0.66
18	8.0	0.34	2902	99.80	0.72
19	0.1	0.00	1048	98.06	1.36
20	8.0	0.12	4306	97.92	1.54
21	1.5	0.00	2811	99.27	1.00
22	8.0	0.38	1894	98.39	1.04
23	6.5	0.00	4156	98.94	0.59
24	8.0	0.38	710	99.29	0.70
25	4.4	0.00	16361	98.69	1.80
26	8.0	0.67	10937	99.17	1.85
27	8.0	0.05	17967	98.18	1.14
28	8.0	0.51	5216	99.44	1.17
29	8.0	0.22	3908	99.29	0.65
30	8.0	0.41	991	100.00	0.69
31	5.2	0.00	13331	96.04	1.77
32	8.0	0.27	6493	97.76	1.64
33	8.0	0.05	9898	98.82	1.04
34	8.0	0.57	2393	99.36	1.04
35	8.0	0.32	3268	99.77	0.58
36	8.0	0.40	1018	99.91	0.64

Table 19: Profit maximization - Detailed results for $|T| = 3$.

Instance	CPU (h)	MIP gap (%)	# nodes	Avg demand met (%)	LP gap (%)
37	0.73	0.00	4836	98.45	1.90
38	8.00	0.13	10330	99.21	1.80
39	1.83	0.00	6946	99.48	1.31
40	8.00	0.03	6529	99.71	1.20
41	2.75	0.00	6392	99.57	0.69
42	8.00	0.26	1267	100.00	0.72
43	0.47	0.00	2997	98.38	1.82
44	8.00	0.06	10863	98.39	1.75
45	2.91	0.00	6144	98.71	1.12
46	8.00	0.29	2144	99.67	1.07
47	8.00	0.02	6361	99.98	0.67
48	8.00	0.27	802	99.78	0.71
49	1.87	0.00	9765	96.49	1.71
50	7.20	0.00	16894	99.43	1.78
51	2.38	0.00	14826	99.43	1.12
52	8.00	0.48	1915	99.49	1.19
53	8.00	0.10	4196	99.12	0.70
54	8.00	0.39	849	99.67	0.80
55	0.76	0.00	2972	98.50	1.48
56	8.00	0.56	2279	97.66	1.54
57	7.59	0.00	17858	99.46	1.01
58	8.00	0.39	981	99.09	1.07
59	8.00	0.24	1735	99.87	0.61
60	8.00	0.30	678	99.14	0.61
61	5.23	0.00	10284	97.44	1.59
62	8.00	0.94	4960	99.02	1.88
63	8.00	0.57	6080	99.68	1.32
64	8.00	0.68	1279	99.31	1.22
65	8.00	0.39	2706	99.63	0.73
66	8.00	0.37	540	99.69	0.66
67	2.93	0.00	5089	97.56	1.50
68	8.00	0.94	2740	97.55	1.60
69	8.00	0.45	3855	99.03	1.02
70	8.00	0.79	1335	97.86	1.27
71	8.00	0.35	1373	99.53	0.62
72	8.00	0.42	672	99.67	0.67

Table 20: Profit maximization - Detailed results for $|T| = 4$.

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Keywords: supply chain network design, facility location, capacity acquisition, profit maximization, cost minimization

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