## The Rogue Waves with Quintic Nonlinearity and Nonlinear Dispersion effects in Nonlinear Optical Fibers

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We present exact rational solution for a modified nonlinear Schrödinger equation that takes into account quintic nonlinearity and nonlinear dispersion corrections to the cubic nonlinearity, which could be used to describe rogue wave in nonlinear fibers. We find the rogue wave with these higher order effects has identical shape with the well-known one in nonlinear Schrödinger equation. However, the quintic nonlinear term and nonlinear dispersion effect affect the velocity of rogue wave, and the evolution of its phase.

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Introduction—- Rogue wave(RW) phenomena in the ocean, are reported to have disastrous consequence, such as destroy ships, oil platform, etc [1, 2]. To avoid its negative effects, people need to know its character, mechanism, even find the ways to control it. Recently, scientists have done lots of studies on them [3]. It is shown that it possesses many exceptional properties, such as much higher than surrounding waves, abrupt appearance and disappear without any trace, etc. Between the studies, the nonlinear theory have been paid much attention [4–8]. It has been found that the dynamics equations for RW in ocean, Bose-Einstein condensate, plasmas, and nonlinear optical fibers are identical fundamentally. Therefore, the studies on RW in nonlinear fibers would help us to understand the ones in other systems [9]. It is well known that the nonlinear Schrödinger equation (NLSE)

$$i\psi_z + \psi_{tt} + 2|\psi|^2 \psi = 0, \tag{1}$$

which describes the optical pulse propagation in optics fibers when the pulse width is greater than 100 femtosecond [10–12], Where  $\psi=\psi(t,z)$  is the slowly varying amplitude of the pulse envelope, z represents the distance along the direction of propagation and t is the retarded time. It is shown that the rational form solution of the NLSE, could be used to describe RW well [8]. Furthermore, RW has been observed recently in one-mode nonlinear fibers experimentally [10, 11], which would highly stimulate RW studies in a lot of nonlinear systems.

However, the dynamics of these nonlinear systems is significantly more complicated than the one modeled by the simple NLSE. For example, for femtosecond optical pulse, higher-order terms that take into account third-order dispersion, self-steepening and other nonlinear effects have to be added to this equation [13]. Thus, a question arises: do rogue wave solutions exist for these more complicated equations? Considering third-order dispersion and delayed nonlinear response effects, RW in Hirota

equation have been studied in [14, 15]. Distinct from them, we study RW for the integrable Kundu-Eckhaus (KE) equation as following [16–19],

$$i\psi_z + \alpha\psi_{tt} + \gamma|\psi|^2\psi + 4\beta^2|\psi|^4\psi - 4i\beta(|\psi|^2)_t\psi = 0,$$
 (2)

where the subscripts represent the partial derivatives,  $\alpha$  is the group velocity dispersion coefficient,  $\gamma$  is the non-linear parameter responsible for the self-phase modulation,  $\beta^2$  is the quintic nonlinearity coefficient, the last term is a nonlinear term which results from the time-retarded induced Raman process. Eq. (2) has been derived in [16, 17] and possesses some applications in the nonlinear optics [21], quantum field theory [20] and weakly nonlinear dispersive matter waves [22].

In this letter, we present exact rational solution for the KE model through Darboux transformation. It is found that properties of the rational solution are similar to RW's. Therefore, it could be used to describe RW in the model as the previous works [6, 7]. We find that the quintic nonlinear coefficient and nonlinear dispersion effect affect the velocity of rogue wave, and change the evolution of its phase, under the integrable condition, which is the additional requirement on the coefficients to solve it analytically. Interestingly, the rogue wave with these higher order effects has identical shape with the well-known one for NLSE.

Exact rational solution and rogue waves— The Eq.(2) has been solved to get soliton solution on trivial background through Darboux transformation(DT) method in [19]. As done in NLSE, one can get rational solution on nonzero plane wave background. We perform the DT method to derive rational solution from a plane wave seed solution. With  $\alpha=1,\ \gamma=2$ , the corresponding Lax-pair is given in Appendix part. The nontrivial seed solutions, which can be seen as the background for RW, are derived as follows

$$\psi_0 = s \exp\left[ikt + i(4\beta^2 s^4 + 2s^2 - k^2)z\right], \quad (3)$$

where s and k are two arbitrary real constants, and denote the amplitude of background and its wave vector respectively. Because there are trivial transformations connected with different s and k, we can set them s=1 and

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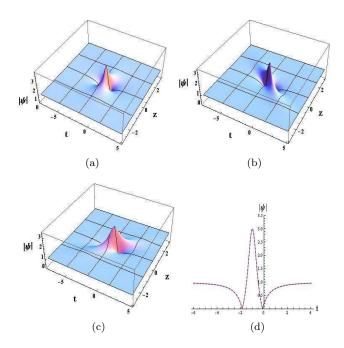


FIG. 1: (color online) The evolution of RW with different nonlinear parameters  $\beta$ . (a)  $\beta=0$ ; (b)  $\beta=0.5$ ; (c)  $\beta=-0.5$ ; and (d) the density distribution of RW with different  $\beta$  at z=0 (the red dashed one with  $\beta=0$ , and the blue one with  $\beta=0.5$ ). It is shown that the density distribution on time are unchanged with  $\beta$ , and the parameter  $\beta$  just affect its velocity.

k=0 without losing generality. With spectral parameter  $\lambda=\beta+i$  and the nontrivial seed solutions, we can derive the rational solution as following, through making the matrix U be Jordan forms,

$$\psi_1 = \left[ -A_1^2(t,z) + \frac{i16z+4}{K(t,z)} A_1^2(t,z) \right] \exp{[i4\beta^2 z + i2z]},\!\! (4)$$

where

$$K(t,z) = (2t + 8\beta z)^2 + 4(2t + 8\beta z) + 16z^2 + 5,$$
  

$$A_1(t,z) = \exp\left[4i\beta \frac{2t + 8\beta z + 2}{(2t + 8\beta z + 2)^2 + 16z^2 + 1}\right].$$

Based on the rational solution, we can study how quintic nonlinear and nonlinear dispersion term affect on RW in nonlinear fibers through varying the value of  $\beta$ . When  $\beta=0$ , namely, the higher order effects are neglected, the solution will become the well-known one for NLSE, shown in Fig.1(a). When  $\beta>0$ , the RW will have negative velocity on the retarded time, such as Fig.1(b). When  $\beta<0$  it will have positive velocity, such as Fig.1(c). The velocity of RW increases with the value of  $|\beta|$ . Compare the density distribution of the localized waves, it is found that they have identical shape. As an example, we show their intensity distribution at z=0 in Fig.1(d). It is pointed that they emerge on different retarded time with the same shape at  $z\neq 0$ . This indicates that the higher-order effects affect RW's velocity on retarded time.

Furthermore, this character can be verified through calculating  $|\psi_1|^2$  expression.

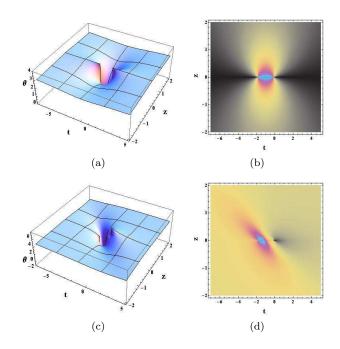


FIG. 2: (color online) The evolution of RW's phase with different nonlinear parameters  $\beta$ . (a)  $\beta=0$ , and (b) is the density plot of (a). (c)  $\beta=0.5$ , and (d) is the density plot of (c). It is seen that the symmetric character of RW's is violated with nonzero  $\beta$ .

Furthermore, we could plot the phase evolution of RW with different  $\beta$ . To show the evolution more clear, we ignore the phase of the background,  $(4\beta^2+2)z$ , which just increase with propagation distance z. Then, the RW's phase can be given as

$$\theta = 8\beta \frac{2t + 8\beta z + 2}{(2t + 8\beta z + 2)^2 + 16z^2 + 1} + Arccos\left[\frac{M_1}{\sqrt{M_1^2 + M_2^2}}\right],$$

$$M_1 = -1 + \frac{4}{K(t, z)},$$

$$M_2 = \frac{16z}{K(t, z)}.$$
(5)

The phase evolution of RW with  $\beta=0$  is shown in Fig.2(a) and (b). When  $\beta\neq 0$ , the phase evolution is shown in Fig.2(c) and (d). It is seen that the symmetric evolution of RW in NLSE is broken by the quintic nonlinear coefficient and nonlinear dispersion effect. Moreover, we find that the phase distribution are opposite for  $\beta$  and  $-\beta$  ( $\beta\neq 0$ ).

Discussion and Conclusion—— In summary, we present exact rational solution for the KE model through Darboux transformation method. This indicates that RW can exist with proper higher-order effects. Based on the analytical solution, it is convenient to study dynamics

of RW with the cubic and quintic nonlinear terms and nonlinear dispersion effects. It is found that RW with these higher-order effects has identical shape with the one of NLSE. The quintic nonlinear terms and nonlinear dispersion effects just affect the velocity of RW on the retarded time. Moreover, they could violate the symmetric evolution of the standard NLSE RW's phase. The phase distribution with  $-\beta$  is opposite to the one for  $\beta$  ( $\beta \neq 0$ ). We believe this character would help us to understand properties of RW in many related nonlinear systems with the higher-order effects. However, we just investigate this character theoretically, the underlying reason is still unknown. It is known that there are always some requirements on the nonlinear coefficients to solve it analytically. Maybe these certain requirements make related nonlinear effects balance with each other. This brings that RW with higher-order term has identical shape with standard NLSE one, which just considering second-order dispersion term and Kerr nonlinear effect.

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## Appendix

The Lax pair of Eq.(2) with  $\alpha = 1$ ,  $\gamma = 2$  could be given as

$$\partial_t \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = U \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \tag{6}$$

$$\partial_z \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = V \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \tag{7}$$

where

$$U = \left( \begin{array}{cc} -i\lambda + i\beta |\psi|^2 & \psi \\ -\bar{\psi} & i\lambda - i\beta |\psi|^2 \end{array} \right),$$

$$V = \begin{pmatrix} -2i\lambda^2 + a_1 & 2\lambda\psi + b_1 \\ -2\lambda\bar{\psi} + c_1 & 2i\lambda^2 - a_1 \end{pmatrix}.$$

and

$$a_{1} = -\beta(\psi_{t}\bar{\psi} - \psi\bar{\psi}_{t}) + 4i\beta^{2}|\psi|^{4} + i|\psi|^{2},$$
  

$$b_{1} = i\psi_{t} + 2\beta|\psi|^{2}\psi,$$
  

$$c_{1} = i\bar{\psi}_{t} - 2\beta|\psi|^{2}\bar{\psi}.$$

Between the above expressions, the over bar denotes complex conjugate, and  $\lambda$  is spectral parameter.

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