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Price-Based Resource Allocation for Self-Backhauled Small Cell Networks

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Abstract

Heterogeneous cellular networks are promising solutions to address the need for the exponentially increasing data traffic demands by ensuring an acceptable level of quality of service. In such networks, base stations with different cell sizes serve the cellular areas (i.e., macro cells along with small cells). The access technologies of such base stations can be different as well. Small cell access points (SAP) are typically connected either directly to the core network through a wired link or to a macro-cell base station through a wireless backhaul link. In this paper, we consider the scenario where the SAP is connected to a macro-cell base station through a wireless backhaul link operating at the same frequency band as the access links from the SAP to its users. We consider amplify and forward (AF) protocol under both full/half duplex transmission modes for the SAP. Under such circumstances, we study the price-based resource allocation where the SAP charges each user equipment (UE) proportional to the amount of the power it allocates for transmission to that UE. A Stackelberg game is employed to model and investigate the joint utility maximization problem of the SAP and UEs. In our game model, the SAP is the leader and the UEs are the followers. We formulate the utility maximization problems for both the leader

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and the followers as optimization problems. We consider two pricing schemes, namely non-uniform and uniform. Moreover, we present a condition which gives a proper criterion for resignation of the UEs from the proposed Stackelberg game when transmitted power of the SAP is limited. We prove that both sub-games are convex optimization problems which ensures their tractability. We also propose a novel algorithm to obtain the optimal prices of Stackelberg equilibrium of the game. Numerical results validate the efficiency of our proposed priced based resource allocation scheme.

Keywords: Heterogeneous cellular networks, full/half duplex self-backhauled small cells, Stackelberg game, amplify and forward, price-based resource allocation.

1. Introduction

Mobile data traffic has been growing dramatically in recent years. An ordinary smartphone is expected to generate approximately 1.3 GB data traffic per month in 2015, which leads to a 11-fold increase in global data traffic in 2015 in comparison to 2013 [1]. To alleviate this data traffic storm, mobile operators have increased the capacity of the radio access and backhaul links not only through development of new technologies but also via operating at higher and wider frequency bands. Despite of all these efforts, the demand is expected to exceed the capacity of the fourth generation cellular networks in near future [1],[2].

The use of small cell access points (SAP) operating at the same frequency band with the same access technology or different frequency bands with different access technologies are attractive solutions to cope with the explosive data traffic demand. Such networks which consist of various types of base stations with various sizes (i.e., macro cells along with small cells) and possibly various access technologies are employed. is called heterogeneous networks. Heterogeneous networks are attractive solutions providing better quality of service (QoS) and support higher number of users. SAPs are connected to the core network

through a wired or wireless link. Wired backhaul links which employ optical transmission technology are costly. Microwave backhaul links require extra frequency bands which is costly as well. One solution to solve this issue is to use a SAP capable of communicating with the macro base station (BS) at the same frequency band that it uses for the users. In the downlink channel, the SAP would receive data from the macro BS while simultaneously transmitting the data to the UEs. In this method, the SAP uses the same channel for access and backhaul. Therefore, there is no need for a separate backhaul channel which decreases the cost and capital expenditure (CAPEX) [3, 4, 5, 6]. This is called a *self-backhauled* SAP.

The enabling technology behind the self-backhauled SAPs is *full duplex* (FD) operation of the SAP transceivers. We shall notice that there is no need to change BS or UEs. Authors in [7] have designed and implemented the first real FD WiFi radio using single antenna for simultaneous transition and reception on the same channel. They have proposed novel analog and digital self-interference cancellation techniques that cancel the self-interference and enables FD mode of operation. uplink, downlink, and backhaul transmissions. Self-backhauling is particularly efficient when coupled with FD relaying. Antenna design, as well as cancellation in radio frequency and digital domains at an FD relay enables reuse of the same resources for backhaul and access hops. The use of radio resources in the self-backhauling and access hops can be coordinated to maximize end-to-end performance [8].

Recently, different aspects of pricing and economics of heterogeneous cellular networks and mobile data offloading are investigated. In [9], authors investigated the virtual resource allocation issues in small cell networks with FD self-backhauls and virtualization. They formulated the virtual resource allocation problem as an optimization problem by maximizing the total utility of mobile virtual network operators (MVNOs). In [10], authors studied the economics of mobile data offloading and proposed a Stackelberg game model for multiple BSs and APs. The authors in [11] studied the economic incentive issue by using an iterative double auction mechanism. In [12], authors proposed an

economic framework and formulated the interactions between the users and the BS by using a Stackelberg game model. In [13], a well described price-based resource allocation for a two tier spectrum sharing femtocell network is considered. A Stackelberg game was formulated to study the joint utility maximization of the macrocell and femtocell and a closed form solution for the Stackelberg Equilibrium of the game is proposed. In [14], a price-based resource allocation for a hybrid spectrum femtocell network is investigated using Stackelberg game. In [15], authors studied the economic incentive issue by using the cooperative game framework, precisely Nash bargaining model. They used a one-to-one bargaining model between the mobile operator and fixed-line operator. In [16], the authors consider a capacity maximizing power allocation based on a Stackelberg game, where the MBS is the leader and the FBSs are assumed as followers. The game is formulated as a mathematical program with equilibrium constraints, and an iterative algorithm has been presented to reach the Stackelberg equilibrium. Most of these works only consider SAPs with wired backhaul which is cost-prohibitive.

In this paper, we investigate a price-based power allocation in an imperfect self-backhauled FD SAP using Stackelberg game. We also consider the half duplex SAP as a benchmark. A Stackelberg game is proposed in which the SAP plays as the leader and the UEs are the followers. The SAP has limited power and shares it between the UEs. Moreover, the SAP uses AF protocol to transmit the information of the UEs. The UEs need to pay based on the portion of the power they use from the SAP. The higher the power SAP uses for transmission to each UE, the higher its data rate as well as the higher its payment to the SAP. Our goal is to jointly maximize the utility of the SAP and UEs. In the first stage of the game, the FD SAP proposes a set of prices to the UEs by maximization of its utility subject to an aggregate power constraint. Then, each UE calculates its optimal downlink transmit power for the given prices. We consider two pricing schemes: *non-uniform pricing or discriminatory pricing* in which the SAP imposes different prices to different UEs, and *uniform pricing or nondiscriminatory pricing* in which the same price is imposed to all

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UEs. In the paper, we will use these words interchangeably. In this paper, both half duplex and full duplex transmission modes are considered for comparison.

To find the Stackelberg equilibrium, backward induction method is used. The UEs' sub-game is proved to be a convex optimization problem and a closed form solution is proposed. The leader's sub-game is also proved to be a convex optimization problem. We propose an algorithm to find the best response of the leader sub-game. At the end, the proof of optimality of the algorithm is proposed. In addition, in case that the transmitted power of the SAP is limited, some UEs should resign from the game. We present a condition so that based on that, UEs can decide weather resign the game or not. Each UE can check the proposed price with a given threshold, and decide to resign the game or not. To the best of our knowledge, this is the first work which investigates the price-based resource allocation in an imperfect self-backhauled FD SAP using Stackelberg game. Most of works which investigated the pricing and economics of SAPs and heterogeneous networks assumed that the SAPs use wired backhaul. The underlying self-backhauld system eliminates the need for a separate backhaul solution and also separate frequency band (whether licensed or unlicensed), that can effectively decrease the cost and complexity of rolling out a small cell network [6].

The rest of the paper is organized as follows. The system model is described in Section II. We formulate the Stackelberg game problem in Section III. Section IV presents the Stackelberg equilibrium point of the proposed game under both pricing schemes and the numerical results are presented in section V. Finally, section VI concludes the paper.

2. System Model

We consider a two-tier heterogeneous network consisting of one macro BS and N UEs. We assume there is a SAP within the coverage area of the BS. As shown in Fig.1, due to the distance between the BS and the UEs, the BS can route information through SAP to provide better QoS for the UEs. Since
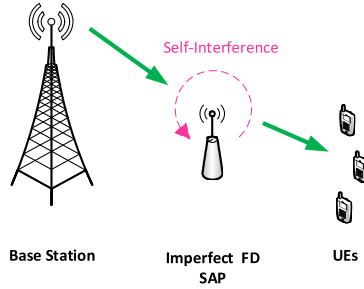
Figure 1: System Model for the FD SAP

the SAP is a selfish node, it charges the UEs for forwarding their information. Each UE adjusts its downlink power (i.e., rate) taking into the account the price charged by SAP. The SAP aims to maximize its revenue under the aggregate power constraint on the downlink power. In this paper, we focus on the downlink transmissions but it is worth pointing out that this scenario can be extended to the uplink direction with slight modifications.

We assume that the SAP uses AF protocol to transmit the information of BS to UEs. For AF protocol, the SAP amplifies its received information and forwards it to the UEs. Let γ_0 and $\gamma_i, \forall i$ denote the channel gain from BS to SAP and from SAP to UE i, respectively. The additive noise is modeled as white Gaussian with zero mean and unit variance. Each slot is called a frame and T_f denotes the frame duration. The data link packets are divided into frames at the physical (PHY) layer. The frame duration is assumed to be less than the fading coherence time. So we assume channel gains will be fixed for the duration of each time slot. Both half and full duplex transmission modes are investigated under AF protocol in the following subsections.

2.1. Half-Duplex Transmission Mode

Under the half duplex transmission mode, the frame time is divided into two portions. The BS transmits data to the SAP during the first portion and then the SAP forwards data to the UEs during the second half of the frame. The achievable rate of the AF protocol under half duplex transmission mode of SAP

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for the UE i is [17, 18]

$$R_i^{HD} = \left(\frac{T_f W_i}{2}\right) \log_2 \left(1 + \frac{4\gamma_0 p_0 \gamma_i p_i}{1 + 2\gamma_0 p_0 + 2\gamma_i p_i}\right), \qquad (1)$$

where W_i is the bandwidth allocated to each UE. To make the comparison fair, in half duplex mode, we assume the BS transmits with power $2p_0$ during the first half of the frame time and it is silent during the second half so the average is p_0. The SAP is silent in the first half and it forwards data with power $2p_i$ during the second half of the frame time on a bandwidth of W_i for UE i.

2.2. Full-Duplex Transmission Mode

Under the full duplex transmission mode, the SAP can transmit and receive simultaneously at the same bandwidth in one frame but since the SAP is an imperfect FD node, it needs to endure the self-interference due to simultaneous transmission and reception of data. It is critical to accurately measure, and suppress the self-interference in FD communication. The self-interference cancellation techniques are classified into to categories: passive and active self-interference suppression methods. Three key passive suppression mechanisms are directional isolation, absorptive shielding, and cross-polarization. In directional isolation, directional antennas is used such that the gain of the transmit antenna is low in the direction of the receive and visa versa. Directional isolation is the simplest passive self-interference cancellation mechanism, and it consists in loss in interference power due to propagation losses caused by separating the transmit and receive antennas at a node [19]. On the other hand, in absorptive shielding, lossy materials is used to attenuate the self-interference, and transmit and receive antennas in orthogonal polarization states is employed for cross-polarization implementation [20].

Active methods include digital and analog cancellation methods. Analog Cancellation mechanism sends a canceling signal through another radio chain and adds it to the signal at the receiving antenna. On the other hand, Digital Cancellation uses the knowledge of the interfering signal to cancel the interfering signal in baseband [19].

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We consider κ as a coefficient which reflects the ability of SAP in suppressing its own self interference under full duplex transmission mode. Self-interference cancellation strategies can depend on the transmit power [17, 21] or can be independent of the transmit power [22]. We assume the self-interference is independent of the transmit power. The value of cancellation coefficient κ depends on a number of factors, such as system bandwidth, antenna displacement error, and transmit signal amplitude difference, etc [18]. We have $0 < \kappa \leq 1$. When κ is one, it means that the SAP is able to completely cancel its own self interference (i.e., perfect full duplex operation). On the other hand, when κ approaches zero, FD SAP is not able to cancel any self-interference. The achievable rate of AF protocol under full duplex transmission mode of SAP for the UE i can be written as [17, 18]

$$R_i^{FD} = (T_f W_i) \log_2 \left(1 + \frac{\kappa \gamma_0 p_0 \gamma_i p_i}{1 + \kappa \gamma_0 p_0 + \gamma_i p_i} \right). \tag{2}$$

where the data of UE i is transmitted by the BS with the power p_0 on bandwidth W_i to SAP and the SAP forwards it to UE i with power p_i using AF protocol during the whole frame time. That is why the pre-log coefficient $\frac{1}{2}$ is omitted for T_f but there is self-interference cancellation coefficient which decreases the achievable rate.

3. Problem Formulation

In this section, at first the price-based power allocation problem is formulated using Stackelberg game. Then, the definition of Stackelberg equilibrium for the proposed game is investigated.

3.1. Price-Based Power Allocation Problem

We consider a price-based power allocation scheme where each UE pays for the power that SAP allocates for it. The design parameters are the power that SAP allocates to UE i, i.e. $p_i, \forall i$ and the price, i.e. $\mu_i, \forall i$, the UE pays for that. Both the UEs and the SAP would like to maximize their utility. To

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formulate the joint problem of maximizing the utility of the UEs and the SAP, a Stackelberg game model is employed. A Stackelberg game is a strategic game where there is a leader and several followers [23]. The followers compete with each other for a certain commodity. The leader moves first and the followers move subsequently. In our model, the SAP plays the leader's role and the UEs are followers. In the first stage of the game, the SAP imposes a set of prices for each unit of the power allocated to UE i. In the second stage, based on the given prices, each UE optimizes its individual utility U_i over p_i. At each frame duration, the two stages of the game should be done by players. So, the Stackelberg game consists of two sub-games, i.e., leader sub-game and the followers' sub-game. At the SAP side, the aggregate transmit power should not be greater than P_{max} as

$$\sum_{i=1}^{N} p_i \leq P_{max}. \tag{3}$$

The price that UE i pays for the power p_i is a linear function of p_i where μ_i denotes the price per power unit for simplification. The revenue of the SAP can be formulated as a function of powers $\boldsymbol{p} = \{p_1, \ldots, p_N\}$ and their corresponding prices $\boldsymbol{\mu} = \{\mu_1, \ldots, \mu_N\}$ as

$$U_{\mathrm{SAP}}(\boldsymbol{\mu}, \boldsymbol{p}) = \sum_{i=1}^{N} \mu_i p_i(\mu_i), \tag{4}$$

Note that p_i is a function of the proposed μ_i by the SAP in the Stackelberg game formulation. So the Stackelberg game formulation for full duplex and half duplex transmission modes are described in the following subsections.

3.1.1. Problem Formulation for Full Duplex Transmission Mode

Under full duplex transmission mode for SAP, the SAP sub-game problem can be formulated as

Problem 1: (FD SAP Sub-game):

$$\max_{\boldsymbol{\mu} \succeq 0} \quad U_{SAP}(\boldsymbol{\mu}, \boldsymbol{p}) \tag{5}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} p_i \leq P_{max}. \tag{6}$$

The SAP aims to maximize its revenue subject to an aggregate power threshold. The UEs pay for the power allocated to them by the SAP. Under the full duplex transmission mode of the SAP, the utility of UE i can be formulated as

$$U_i^{FD}(p_i, \mu_i) = \lambda_i R_i^{FD} - \mu_i p_i, \forall i, \tag{7}$$

where λ_i is the utility gain per unit transmission rate for UE i. For each UE i the optimization problem can be formulated as follows

Problem 2: (UE i Sub-game for FD SAP):

$$\max_{p_i \geq 0} \quad U_i^{FD}(p_i, \mu_i). \tag{8}$$

Each UE maximizes its utility based on the announced price by SAP by choosing p_i. Problem 1 and Problem 2 together form a Stackelberg game for full duplex transmission mode.

3.1.2. Problem Formulation for Half Duplex Transmission Mode

For the Stackelberg formulation under half duplex transmission, the leader sub-game can be written as

Problem 3: (HD SAP Sub-game):

$$\max_{\boldsymbol{\mu} \succeq 0} \quad U_{SAP}^{HD}(\boldsymbol{\mu}, \boldsymbol{p}) \tag{9}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} p_i \leq P_{max}, \tag{10}$$

and the UE i sub-game can be formulated as

Problem 4: (UE i Sub-game for HD SAP):

$$\max_{p_i \geq 0} \quad U_i^{HD}(p_i, \mu_i), \tag{11}$$

where the individual utility of each UE i under half duplex transmission mode can be written as follows:

$$U_i^{HD}(p_i, \mu_i) = \lambda_i R_i^{HD} - \mu_i p_i, \forall i, \tag{12}$$

Problem 3 and Problem 4 together form a Stackelberg game for half duplex transmission mode.

The goal of the games are to find the Stackelberg Equilibrium (SE) points. The SE of the proposed game is investigated in the following subsection.

3.2. Stackelberg Equilibrium

The Stackelberg game equilibrium is a point where neither the leader nor the followers have incentive to deviate from their strategy unilaterally. In our proposed game under full duplex transmission mode, it can be written as follows:

Definition 1: Stackelberg Equilibrium

Let $\boldsymbol{\mu}^*$ and $\boldsymbol{p}^*$ denote the optimal power prices and optimal powers for UEs, respectively. Then, the point $(\boldsymbol{\mu}^*, \boldsymbol{p}^*)$ is a Stackelberg equilibrium if the following conditions are satisfied [23]:

$$U_{SAP}(\boldsymbol{\mu}^*, \boldsymbol{p}^*) \geq U_{SAP}(\boldsymbol{\mu}, \boldsymbol{p}^*), \tag{13}$$

$$U_i^{FD}(\boldsymbol{p}^*, \boldsymbol{\mu}^*) \geq U_i^{FD}(p_i, \boldsymbol{p}_{-i}^*, \boldsymbol{\mu}^*), \quad \forall p_i \geq 0. \tag{14}$$

where $\boldsymbol{p}_{-i}^*$ is the optimal power vector for all the UEs except UE i. This definition can be used for half duplex transmission mode of SAP as well.

To find the SE point, the sub-game perfect Nash Equilibrium (NE) of the proposed game should be investigated. We use the well-known backward induction method to find the SE of the proposed game which is described in [23]. For the proposed Stackelberg games, the backward induction method can be described as follows: For a given price vector imposed by SAP, the followers' sub-game is solved first. After substituting the best responses of the followers sub-game into the leader sub-game the optimal power price vector is obtained as best response of the SAP.

4. Analysis of the Proposed Stackelberg Game

In this section, using the backward induction method, we aim to find the SE of the proposed Stackelberg games (i.e., the optimal power allocation for the followers' sub-game and the optimal pricing strategy for the leader sub-game). We consider two different pricing schemes for the SAP, namely non-uniform and uniform pricing. In the non-uniform pricing scheme, the SAP charges each UE with different power prices while in uniform pricing scheme, the SAP use the same price for all the UEs. In the following subsections, these two pricing schemes are investigated under both full duplex and half duplex transmission modes.

4.1. Non-Uniform (discriminatory) Power Pricing

In the non-uniform power pricing, the SAP imposes different power prices to different UEs. For a given price vector in full duplex mode, the optimal power vector $\boldsymbol{p}^*$ is given in the following lemma.

Lemma 1. *Given the power price μ_i, Problem 2 has a global optimal solution as*

$$p_i^*(\mu_i) = \left(\frac{1}{2\gamma_i} \sqrt{(\kappa a)^2 + \frac{4\kappa a b_i \gamma_i}{\mu_i}} - \frac{\kappa a + 2}{2\gamma_i} \right)^+ , \quad \forall i, \tag{15}$$

where $a = \gamma_0 p_0$ and $b_i = \lambda_i T_f W_i / \ln 2$.

Proof: The proof can be found in appendix A. ∎

By setting $p_i^*(\mu_i) \geq 0$ in (15), with some manipulations, one can obtain

$$\mu_i \leq \frac{\kappa a b_i \gamma_i}{\kappa a + 1}, \quad \forall i. \tag{16}$$

Lemma 1 shows that if the power price is above a threshold given in (16), UE i is not willing to pay for the power to the SAP. Therefore, the UE leaves the game if the power price proposed is too high and we have

$$p_i^*(\mu_i) = 0 \text{ if } \mu_i \geq \frac{\kappa a b_i \gamma_i}{(\kappa a + 1)}, \forall i. \tag{17}$$

Substituting equation (15) into Problem 1, we can get

Problem 5:

$$\max_{\boldsymbol{\mu} \succeq 0} \quad \sum_{i=1}^{N} \left(\frac{1}{2\gamma_i} \sqrt{(\kappa a)^2 \mu_i^2 + 4\kappa a b_i \gamma_i \mu_i} - \frac{\kappa a + 2}{2\gamma_i} \mu_i \right)^+ \tag{18}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \left(\frac{1}{2\gamma_i} \sqrt{(\kappa a)^2 + \frac{4\kappa a b_i \gamma_i}{\mu_i}} - \frac{\kappa a + 2}{2\gamma_i} \right)^+ \leq P_{max}. \tag{19}$$

We propose an approach to solve Problem 5 efficiently for large number of UEs. At first, we assume that P_{max} is sufficiently large. Therefore, we remove the constraint from Problem 5 and also we assume the positivity constraint on $p_i, \forall i$ to solve it. We define Problem 6 as

Problem 6:

$$\max_{\boldsymbol{\mu} \succeq 0} \quad \sum_{i=1}^{N} \frac{1}{2\gamma_i} \sqrt{(\kappa a)^2 \mu_i^2 + 4\kappa a b_i \gamma_i \mu_i} - \frac{\kappa a + 2}{2\gamma_i} \mu_i. \tag{20}$$

The objective function of Problem 6 is decomposable. Taking partial derivative with respect to all μ_i and set them to zero, one can obtain the optimal solution of Problem 6 as

$$\tilde{\mu_i}^* = b_i \gamma_i \frac{\kappa a + 2 - 2\sqrt{\kappa a + 1}}{\kappa a \sqrt{\kappa a + 1}}, \quad \forall i. \tag{21}$$

Similarly, the optimal price imposed by SAP, under half duplex transmission mode, can be written as:

$$\tilde{\mu_i}^* = b_i \gamma_i \frac{a + 1 - \sqrt{2a + 1}}{a\sqrt{2a + 1}}, \quad \forall i. \tag{22}$$

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It is observed from (21) and (22) that $\tilde{\mu}_i^* > 0, \forall i$ always holds. As previously discussed, when the power price is higher than a threshold, i.e., $\mu_i \geq \frac{\kappa a b_i \gamma_i}{\kappa a + 1}$, the UE i leaves the game and we have $p_i^*(\mu_i) = 0$. However, it does not happen when there is no limit on the aggregate power which is proved in the following lemma.

Lemma 2. *For the optimal solutions of Problem 6, $\tilde{\mu}_i^*$, no UE leaves the game.*

Proof: UE i is not rejected if

$$\tilde{\mu}_i^* < \frac{\kappa a b_i \gamma_i}{\kappa a + 1}, \quad \forall i. \tag{23}$$

Substituting (21) in (23), we have

$$b_i \gamma_i \frac{\kappa a + 2 - 2\sqrt{\kappa a + 1}}{\kappa a \sqrt{\kappa a + 1}} < \frac{\kappa a b_i \gamma_i}{\kappa a + 1}, \quad \forall i. \tag{24}$$

By some simplification, one can obtain

$$0 < (\kappa a)^2 + 3\kappa a + 3, \quad \forall i. \tag{25}$$

Since $a \geq 0$, it is observed that the inequality (25) always holds for all UEs and no UE is rejected due to high value of $\tilde{\mu}_i^*$. ∎

As a consequence of Lemma 3, we have $p_i^*(\tilde{\mu}_i^*) > 0, \forall i$. We use P to denote

$$P = \sum_{i=1}^{N} p_i^*(\tilde{\mu}_i^*) = \sqrt{\kappa a + 1} \sum_{i=1}^{N} \frac{1}{\gamma_i}. \tag{26}$$

So, for any $P \leq P_{max}$, the optimal solution of Problem 5 is the same as the optimal solution of Problem 6, i.e., $\tilde{\mu}_i^*, \forall i$. To solve Problem 5 for $P > P_{max}$, we use the following approach. We define two functions $f_i(\mu_i)$ and $g_i(\mu_i)$ as

$$f_i(\mu_i) = \frac{1}{2\gamma_i}\sqrt{(\kappa a)^2 \mu_i^2 + 4\kappa a b_i \gamma_i \mu_i} - \frac{\kappa a + 2}{2\gamma_i}\mu_i, \quad \forall i, \tag{27}$$

$$g_i(\mu_i) = \frac{1}{2\gamma_i}\sqrt{(\kappa a)^2 + \frac{4\kappa a b_i \gamma_i}{\mu_i}} - \frac{\kappa a + 2}{2\gamma_i}, \quad \forall i. \tag{28}$$

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To solve Problem 5, a sufficiently small $\epsilon > 0$ is chosen and $I = \frac{P - P_{max}}{\epsilon}$ steps is needed to reach to the optimal solution. The constraint (19) is an active constraint, so we can get

$$\sum_{i=1}^{N} g_i(\mu_i) = P_{max}. \tag{29}$$

It is because $g_i(\mu_i)$ is monotone decreasing for all i and for a $P_{max} < P$, we need to increase the prices in a way that the objective function decreases at least in order to satisfy (29). Besides, by increasing the prices, the objective decreases due to the monotone decreasing property of each $f_i(\mu_i), \forall i$ for $\mu_i > \tilde{\mu}_i^*$.

In the proposed algorithm, we increase μ_i in a way that the constraint (29) is satisfied and the objective function is maximized. The algorithm starts from the optimal prices of Problem 6 , i.e., $\tilde{\mu}_i^*, \forall i$. It starts from P and converts it to P_{max} by subtracting ϵ from P for I times. At step $1 \leq t \leq I$, for a certain P_{max}, let $P' = P - t\epsilon$. To go one step closer to P_{max}, the following problem should be solved

$$\max_{\boldsymbol{\mu}^{t+1} \succeq 0} \quad \sum_{i=1}^{N} f_i(\mu_i^{t+1}) \tag{30}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} g_i(\mu_i^{t+1}) = P' - \epsilon. \tag{31}$$

The proposed algorithm is described in details in Algorithm 1. First, in Step 2, the SAP initializes the prices (i.e., $\mu_i^{(0)} = \tilde{\mu}_i^*, \forall i$). Then, the SAP satisfies constraint (31) by changing each $\mu_i^t, \forall i$ unilaterally and then, it checks UEs condition on prices and if the price is too high for a UE, such UE is rejected. After removing the rejected UE, the problem will be solved again for other UEs (Steps 5 to 9). If the UE is not rejected, SAP computes its corresponding objective function which decreased due to increase in its price in comparison to μ_i^t (Steps 10 and 11). Then the price that decreases the objective function at least is chosen to change (Step 12). The SAP updates the prices (Steps 13 and 14) and checks the termination condition. The algorithm terminates when the P' is equal to P_{max}. When a UE resigns, that UE is omitted from input parameters of Algorithm and the problem is solved again.

Algorithm 1: Proposed Algorithm under Non-Uniform Pricing

Input: N, ϵ, $\tilde{\mu_i}^*, \forall i$, P and P_{max}

Output: The optimal solutions $\mu_i^*, \forall i$

1 $t \leftarrow 0$, $P' \leftarrow P$

2 Initialize $\mu_i{}^{(0)} = \tilde{\mu_i}^*, \forall i$

3 **while** $P_{max} \leq P' - \epsilon$ **do**

4   **for** $j \leftarrow 1$ **to** N **do**

5   Calculate $\mu_j'^{(t+1)} = \dfrac{4\kappa a b_j \gamma_j}{\left(2\gamma_j \left(P' - \epsilon - \sum_{i=1, i \neq j}^{N} g_i(\mu_i^{(t)})\right) + \kappa a + 2\right)^2 - (\kappa a)^2}$

6   **if** $\mu_j'^{(t+1)} > \frac{\kappa a b_j \gamma_j}{\kappa a + 1}$ **then**

7 $\mu_j^* \leftarrow \frac{\kappa a b_j \gamma_j}{\kappa a + 1}$

8 $p_j^* \leftarrow 0$

9   Remove UE j and run Alg. 1 by $N - 1$ UEs.

10   **else**

11   Calculate $f_j(\mu_j'^{(t+1)}), \forall j$

12 $k = \text{argmin}_j \;\; f_j(\mu_j^{(t)}) - f_j(\mu_j'^{(t+1)})$

13 $\mu_k^{(t+1)} \leftarrow \mu_k'^{(t+1)}$

14 $\mu_i^{(t+1)} \leftarrow \mu_i^{(t)}, \forall i, i \neq k$

15 $P' \leftarrow P' - \epsilon$

16 $t \leftarrow t + 1$

17 $\mu_i^* \leftarrow \mu_i^{(t)}, \forall i$

Lemma 3. *The Algorithm 1 converges to the optimal solution of Problem 5.*

Proof: The proof can be found in appendix C. ∎

Corollary 1. *The SE of the proposed Stackelberg for non-uniform pricing scheme is $(\boldsymbol{\mu}^*, \boldsymbol{p}^*)$, where $\boldsymbol{\mu}^*$ can be obtained by Algorithm 1 and $\boldsymbol{p}^*$ is given by (15).*

4.2. Half Duplex versus Full Duplex transmission mode Under Non-Uniform Power Pricing

In this section, we compare the full duplex and half duplex transmission modes for non-uniform pricing. For this goal, since there is a closed-form solution for optimal power prices for Problem 6, the revenue of the SAP under full duplex and half duplex transmission mode are compared for $P \geq P_{max}$. Substituting (21) in objective of (20), the revenue of the SAP under full duplex transmission mode can be written as

$$U_{SAP}^{FD}(\tilde{\boldsymbol{\mu}}^*) = \frac{\kappa a + 2 - 2\sqrt{\kappa a + 1}}{\kappa a} \sum_{i=1}^{N} b_i \qquad (32)$$

It can be observed that the revenue of the SAP is independent on channel gains for $P \geq P_{max}$ because the SAP compensates the effect of channel gains due to the relaxation of the power constraint.

Similarly, the revenue of the SAP under half duplex transmission mode can be written as

$$U_{SAP}^{HD}(\tilde{\boldsymbol{\mu}}^*) = \frac{a + 1 - \sqrt{2a + 1}}{2a} \sum_{i=1}^{N} b_i \qquad (33)$$

Comparing the revenue of the SAP under both full duplex and half duplex transmission mode, we can find a self-interference cancellation coefficient threshold κ_{th} which can be used to determine whether to use full duplex or not. The κ_{th} can be found by setting the revenue of the SAP equal under both half duplex and full duplex transmission modes, as follows:

$$U_{SAP}^{FD}(\tilde{\boldsymbol{\mu}}^*) = U_{SAP}^{HD}(\tilde{\boldsymbol{\mu}}^*) \qquad (34)$$

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By some manipulations, the self-interference cancellation coefficient threshold can be written as:

$$\kappa_{th} = \frac{8(a + 1 - \sqrt{2a + 1})}{(\sqrt{2a + 1} - 1 + a)^2} \tag{35}$$

Considering sufficiently large values of P_{max}, for $\kappa \geq \kappa_{th}$, the revenue of the SAP under full duplex transmission mode is higher than that of half duplex transmission mode while for $\kappa < \kappa_{th}$, the revenue of the SAP under half duplex transmission mode is higher than that of full duplex transmission mode.

4.3. Uniform (nondiscriminatory) Power Pricing

In uniform power pricing, the SAP imposes the same power price to all UEs, i.e., $\mu_i = \mu, \forall i$. For a given uniform power price μ, under full duplex transmission mode, the optimal power allocation for UEs can be obtained by setting $\mu_i = \mu, \forall i$ in (15) as follows:

$$p_i^*(\mu) = \left(\frac{1}{2\gamma_i} \sqrt{(\kappa a)^2 + \frac{4\kappa a b_i \gamma_i}{\mu}} - \frac{\kappa a + 2}{2\gamma_i} \right)^+, \quad \forall i, \tag{36}$$

Setting the $p_i^*(\mu) \geq 0, \forall i$, the following condition should hold in order to no UEs resigns the game:

$$\mu \leq \frac{\kappa a b_i \gamma_i}{\kappa a + 1}, \quad \forall i. \tag{37}$$

Based on this, by decreasing the P_{max}, let j denotes the first UE that resigns the game. The first UE that resigns the game can be obtained by

$$j = \text{argmin}_{i=1,2,\dots,N} \; \frac{\kappa a b_i \gamma_i}{\kappa a + 1}. \tag{38}$$

Substituting (36) in Problem 2 with $\mu_i = \mu, \forall i$, the optimization problem at SAP side can be written as

Problem 7:

$$\max_{\mu \geq 0} \sum_{i=1}^{N} \left(\frac{1}{2\gamma_i} \sqrt{(\kappa a)^2 \mu^2 + 4\kappa a b_i \gamma_i \mu} - \frac{\kappa a + 2}{2\gamma_i} \mu \right)^+ \tag{39a}$$

$$\text{s.t.} \sum_{i=1}^{N} \left(\frac{1}{2\gamma_i} \sqrt{(\kappa a)^2 + \frac{4\kappa a b_i \gamma_i}{\mu}} - \frac{\kappa a + 2}{2\gamma_i} \right)^+ \leq P_{max}. \tag{39b}$$

Algorithm 2: Proposed Algorithm under Uniform Pricing

Input: N, ϵ, $\tilde{\mu}^*$, and P_{max}

Output: The optimal solution μ^*

1 $t \leftarrow 0$

2 Initialize $\mu^{(0)} = \tilde{\mu}^*$

3 while $P_{max} \leq P'$ **do**

4 $\mu^{(t+1)} \leftarrow \mu^{(t)} + \epsilon$

5 $j = \text{argmin}_{i=1,2,...,N} \; \frac{\kappa a b_i \gamma_i}{\kappa a + 1}$

6   **if** $\mu^{(t+1)} > \frac{\kappa a b_j \gamma_j}{\kappa a + 1}$ **then**

7 $\mu_j^* \leftarrow \frac{\kappa a b_j \gamma_j}{\kappa a + 1}$

8 $p_j^* \leftarrow 0$

9   Remove UE j and run Alg. 1 by $N-1$ UEs.

10   **else**

11   Calculate $P' = \sum_{i=1}^{N} g_i(\mu^{(t+1)})$

12 $t \leftarrow t + 1$

13 $\mu^* \leftarrow \mu^{(t)}$

The characteristics of this Problem is the same as Problem 5. So this problem can be solved by the same approach. Assuming that (38) holds, i.e., no UE leaves the game, Problem 7 is a convex optimization problem. To solve Problem 7 with no constraint, as the objective function of Problem 7 is not decomposable, finding a closed form expression for unconstrained problem is not trivial. So, this problem can be solved using gradient descent method or another similar methods. After solving Problem 7 without power constraint, the optimal price solution and sum power corresponding to this price are denoted by $\tilde{\mu}^*$ and $P = \sum_{i=1}^{N} p_i(\tilde{\mu}^*)$, respectively.

So, for any $P \leq P_{max}$, the optimal solution of Problem 7 is the same as the optimal solution Problem 7 without power constraint, i.e., $\tilde{\mu}^*$. To solve Problem 7 for $P > P_{max}$, we proposed an approach as Algorithm 2.

Corollary 2. *The SE of the proposed Stackelberg for uniform power pricing*

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scheme is $(\mu^*, \boldsymbol{p}^*)$, where μ^* can be obtained by Algorithm 2 and $\boldsymbol{p}^*$ is given by (36).

5. Numerical Results

In this section, numerical results are presented to evaluate the performance of the proposed price-based power allocation algorithm. We assume that there are three UEs, i.e., $N = 3$. In our setup, the coverage area of BS is a circle with 500 m diameter in which the SAP is randomly located, and SAP UEs are randomly distributed inside the coverage of it. The coverage area of the SAP is considered as 50 m. Furthermore, for channel gains, we assume $\gamma_i = \Psi_i d_i^{-\beta}$ where $\beta = 2$ is the path-loss exponent, Ψ_i is exponential random variable (i.e., representing the Rayleigh fading) with mean one, and d_i is the distance between UE i to SAP. For the channel gain between BS and SAP similar channel model is considered. The other parameters are $p_0 = 10$W, $W_i = 1$ MHz, $\lambda_i = 1, \forall i$, and $T_f = 1$. We assume $\epsilon = 10^{-3}$. Figs. 2 and 3 show the revenue of SAP and the sum-utility of the UEs versus the maximum power constraint at SAP for different pricing schemes, under full duplex transmission mode. The self-interference cancellation coefficient is fixed at $\kappa = 0.5$. For similar P_{max}, in Fig. 2, the revenue of the SAP for non-uniform pricing is greater than that of the uniform pricing. However, in Fig. 3, the sum-utility of UEs for uniform pricing is larger than that of the non-uniform pricing because in uniform pricing the SAP can not impose different prices proper to the channel gains of the UEs. It is observed that for small values of P_{max}, the revenue of the SAP is equal for both pricing schemes and also this is true for sum-utility of UEs. It is because for small P_{max}, there is just one UE in the game and uniform pricing and non-uniform pricing results in similar solutions. For sufficiently large values of P_{max}, there is a trade-off in SAP to sell at optimal price. Therefore, due to the concavity of both the revenue of SAP and sum utility of UEs, both Figs. converge to a certain value, i.e., increasing the prices does not increase the objective function anymore.
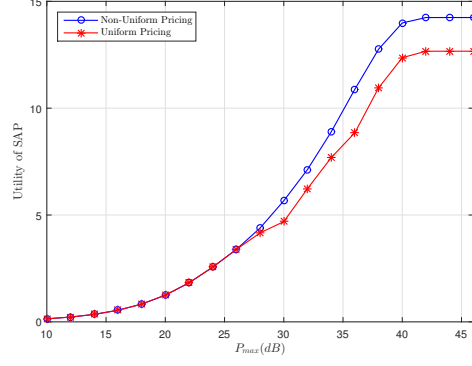
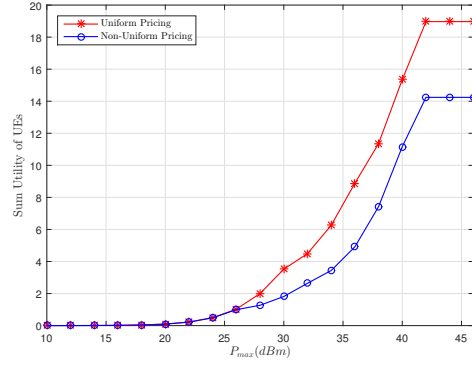Figure 2: Utility of the SAP versus P_{max} under uniform and non-uniform pricing schemes.

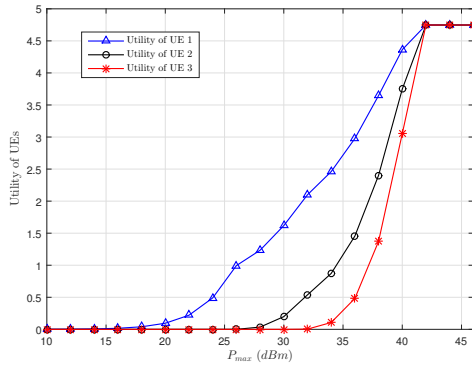Figure 3: Sum-utility of the UEs versus P_{max} under uniform and non-uniform pricing schemes.

Figure 4: Utility of UEs versus P_{max} for non-uniform pricing, under full duplex transmission mode.
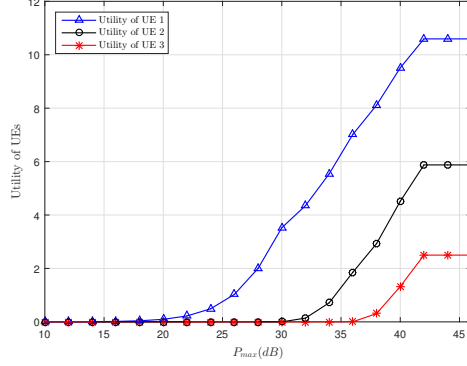
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Figure 5: Utility of UEs versus P_{max} for uniform pricing, under full duplex transmission mode.

In Figs. 4 and 5, the utility of UEs are presented for non-uniform and uniform pricing schemes respectively, under full duplex transmission mode. For sufficiently large values of P_{max}, under non-uniform pricing, it is observed that for the same P_{max}, all the UEs achieve similar utility and the utility of the UEs are independent of their channel gains from SAP. We notice that other parameters such as the utility gain and the allocated bandwidth are identical for all UEs. It is because the SAP can provide the power to each UE as much as it needs due to lack of constraint on P_{max}. In uniform pricing, the UE with better channel condition gains more utility compared to the non-uniform pricing while for the UE with poor channel condition, a higher price is proposed and its utility is less than that of non-uniform pricing. It can be mentioned that although the sum-utility of the UEs for uniform pricing is larger than that of uniform pricing, the non-uniform pricing is more fair for UEs. For $P_{max} < P$, the utility of the UEs depends on their channel gains and the UE with higher channel gain achieves higher utility for both pricing schemes.

In Fig. 6, as P_{max} increases, under both pricing schemes, the power price for all UEs decreases. It means that when the supply goes up, the price comes down as it is expected. Moreover, for sufficiently large values of P_{max}, as the demand is unlimited, the SAP is not able to increase its revenue by increasing the price.
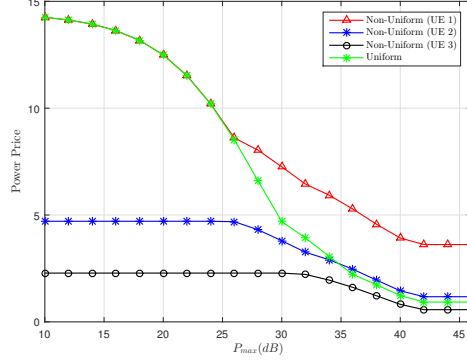
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Figure 6: Power prices versus P_{max} under both uniform and non-uniform pricing schemes.
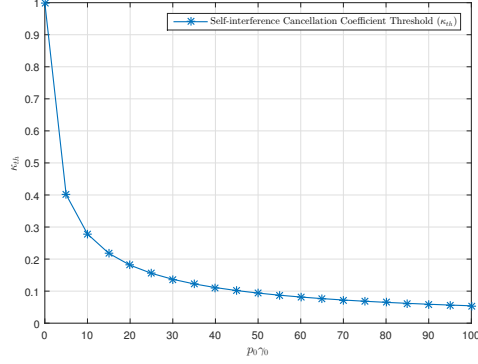

Figure 7: Self-interference cancellation coefficient threshold (κ_{th}) versus $p_0\gamma_0$.

Also, in non-uniform pricing, the power price for the UE with better channel gain is higher due to the better QoS that it experiences. It can be observed that for small values of P_{max}, the power price for the both pricing schemes are similar. It is because for small values of P_{max}, there is only one UE in the game and both of the problems are similar. Moreover, we observe that the UE with the lowest channel gain is the first one that resigns the game. For uniform pricing, the SAP increases the price when P_{max} decreases and based on (38), the UE with lowest channel gain resigns the game first. The resigned UE receives no power and the SAP sets the price as $\mu_j = \frac{\kappa a b_j \gamma_j}{\kappa a + 1}$ for resigned UE j.

In Fig. 7, the self-interference cancellation coefficient threshold κ_{th} decreases as $\gamma_0 p_0$ increases. Intuitively, when the BS uses higher power for trans-
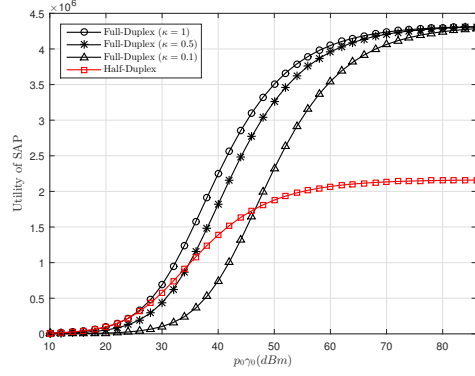
Figure 8: Utility of SAP versus $p_0\gamma_0$ under both full/half duplex transmission modes without power constraint at SAP side.
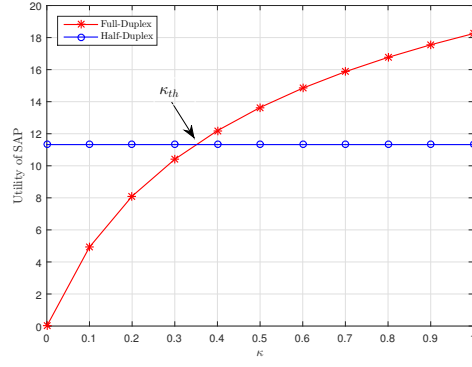

Figure 9: Utility of SAP versus κ under both full/half duplex transmission modes.

mission to the SAP, the simultaneous transmission of the SAP, which causes self-interference, has lower impact on the received signal. Therefore, a lower self-interference cancellation coefficient threshold is needed to achieve the same revenue of half duplex transmission mode for SAP.

In Fig 8, the revenue of he SAP for sufficiently large value of P_{max} is presented for both transmission schemes versus $p_0\gamma_0$. It is observed that for very large values of $p_0\gamma_0$, the revenue of the SAP is independent of the self-interference cancellation coefficient and the revenue of SAP under full duplex transmission mode is two times higher than that of the half duplex mode. Intuitively, when the BS transmits to the SAP at the highest possible power, the

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SAP does not hear its transmission to UEs.

In Fig. 9, the revenue of the SAP versus the self-interference cancellation coefficient is presented. It is observed that when $\kappa_{th} < \kappa$, the SAP revenue under full duplex transmission mode is larger than that under the half duplex mode.

6. Conclusion

In this paper, we introduced a price-based power allocation scheme for UEs using a self-backhauled SAP. We consider both full duplex and half duplex transmission modes for the SAP.. We formulated the price based resource allocation in such a system by using Stackelberg game. We considered both non-uniform and uniform pricing schemes in the game. It is shown that both of followers' and leader's problems are convex optimization. We solved the former and a closed form solution proposed, while for the latter we proposed a novel algorithm to find the optimal power prices. At the end, the optimality of the proposed algorithm is proved. The results of this paper are useful to practically design self-backhauled SAP for the downlink transmissions in heterogeneous networks.

7. Appendices

7.1. Proof of Lemma 1

For concavity of the objective function of Problem 2, the following condition should hold for $p_i \geq 0$:

$$\frac{\partial^2 U_i^{FD}(p_i, \mu_i)}{\partial p_i^2} \leq 0, \forall i \tag{40}$$

Calculating the second derivative of the objective function of Problem 2 and combining with the condition in (40) with some manipulations, one can obtain:

$$(1 + \gamma_i p_i)^2 \leq (1 + \kappa a + \gamma_i p_i)^2, \forall i \tag{41}$$

It is obvious that the inequality (41) always holds for $p_i \geq 0$, so the objective function of Problem 2 is concave. Since the constraint $p_i \geq 0$ is affine, Problem

2 is a convex optimization problem. This guarantees the existence of the global optimal solution [24]. To find the optimal solution, we take the first order derivative of $U_i^{FD}(p_i, \mu_i)$ with respect to p_i. Setting this derivative equal to zero, we obtain

$$\gamma_i^2 p_i^2 + (2 + \kappa a)\gamma_i p_i + 1 + \kappa a - \frac{\kappa a b_i \gamma_i}{\mu_i} = 0, \quad \forall i. \tag{42}$$

Equation (42) is a quadratic equation respect to p_i and the optimal solutions are the roots of this quadratic equation. The solutions for p_i can be written as

$$p_i^*(\mu_i) = \pm \frac{1}{2\gamma_i} \sqrt{(\kappa a)^2 + \frac{4\kappa a b_i \gamma_i}{\mu_i}} - \frac{\kappa a + 2}{2\gamma_i}, \quad \forall i. \tag{43}$$

One solution for p_i^* is always negative. Therefore, it is rejected due to $p_i \geq 0$ constraint. So, the optimal solution of Problem 2 can be written as equation (15). $\blacksquare$

7.2. Proof of Lemma 5

The algorithm will be converged to a solution after I steps. We need to prove that this solution is the optimal solution. It is enough to prove the convergence of the algorithm to the optimal point for only one step. The proof for all steps is identical. At the step $1 \leq t \leq I$ for a special P_{max}, let $P' = P - t\epsilon$.

At step $t + 1$, we only change each of $\mu_i^t, \forall i$ unilaterally to satisfy constraint (31). Using the first order Taylor approximation for sufficiently small ϵ, which leads to a small $\epsilon_i, \forall i$, i.e., $g(\mu_i^t + \epsilon_i) \approx g(\mu_i^t) + \epsilon_i g'(\mu_i^t)$, we can get

$$\epsilon_1 g_1'(\mu_1^t) = \epsilon_2 g_2'(\mu_2^t) = ... = \epsilon_N g_N'(\mu_N^t) = -\epsilon. \tag{44}$$

Now, we change all $\mu_i^t, \forall i$ to satisfy constraint (30) as follows

$$g_1(\mu_1^t + \delta_1) + g_2(\mu_2^t + \delta_2) + ... + g_N(\mu_N^t + \delta_N) = P' - \epsilon. \tag{45}$$

Then, combining (44) and (45) and using first order Taylor approximation, we can get

$$\delta_1 g_1'(\mu_1^t) + \delta_2 g_2'(\mu_2^t) + ... + \delta_N g_N'(\mu_N^t) = \epsilon_1 g_1'(\mu_1^t). \tag{46}$$

Considering the monotone decreasing property of $g_i(\mu_i), \forall i$, we can get $\epsilon_i > \delta_i, \forall i$. By some manipulation and using first order Taylor approximation for sufficiently small ϵ which leads to a small $\epsilon_i, \forall i$, i.e., $f(\mu_i^t + \epsilon_i) \approx f(\mu_i^t) + \epsilon_i f'(\mu_i^t)$, the objective function changes for each μ_i^t unilaterally can be written as

$$\Delta f_i(\mu_i^t) = -\epsilon_i f_i'(\mu_i^t), \quad \forall i. \tag{47}$$

Note that $g_i(\mu_i), \forall i$ is a monotone decreasing function (i.e., $g_i'(\mu_i) < 0, \forall i$). Moreover, $f_i'(\mu_i) \leq 0, \forall i$ for $\mu_i \geq \tilde{\mu}_i^*$ which $\mu_i^t, \forall i$ satisfies $\mu_i^t \geq \tilde{\mu}_i^*$ due to monotone decreasing property of $g_i(\mu_i), \forall i$. Without loss of generality, we assume that $\Delta f_1(\mu_1^t) \leq \min_{i=2,...,N} \Delta f_i(\mu_i^t)$. Combining it with (44) and (47), we can get

$$\frac{f_1'(\mu_1^t)}{g_1'(\mu_1^t)} \leq \min_{i=2,...,N} \frac{f_i'(\mu_i^t)}{g_i'(\mu_i^t)}. \tag{48}$$

The corresponding change in objective function for changing all prices to satisfy (30) can be written as

$$\Delta f(\boldsymbol{\mu^t}) = \sum_{i=1}^{N} -\delta_i f_i'(\mu_i^t). \tag{49}$$

Our aim is to prove

$$\Delta f_1(\mu_1^t) \leq \Delta f(\boldsymbol{\mu^t}). \tag{50}$$

Combining (46), (47), (49), and (50), using the fact that $\epsilon_i > \delta_i, \forall i$, we can get

$$\frac{f_1'(\mu_1^t)}{g_1'(\mu_1^t)} \leq \frac{\sum_{i=2}^{N} -\delta_i f_i'(\mu_i^t)}{\sum_{i=2}^{N} -\delta_i g_i'(\mu_i^t)}. \tag{51}$$

Finally we need to prove (51). Without loss of generality, we sort all $\frac{-\delta_i f_i'(\mu_i^t)}{-\delta_i g_i'(\mu_i^t)}, i = 2, ..., N$ in an ascending order as

$$\frac{-\delta_2 f_2'(\mu_2^t)}{-\delta_2 g_2'(\mu_2^t)} \leq \frac{-\delta_3 f_3'(\mu_3^t)}{-\delta_3 g_3'(\mu_3^t)} \leq ... \leq \frac{-\delta_N f_N'(\mu_N^t)}{-\delta_N g_N'(\mu_N^t)}. \tag{52}$$

We know the inequality $\frac{a}{b} \leq \frac{a+c}{b+d} \leq \frac{c}{d}$ for $a, b, c, d > 0$. Using this inequality for $N - 1$ times for the inequalities in (52) and using (48) we can get

$$\frac{f_1'(\mu_1^t)}{g_1'(\mu_1^t)} \leq \frac{f_2'(\mu_2^t)}{g_2'(\mu_2^t)} \leq \frac{\sum_{i=2}^{N} -\delta_i f_i'(\mu_i^t)}{\sum_{i=2}^{N} -\delta_i g_i'(\mu_i^t)}. \tag{53}$$

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So the change in $f_1(\mu_1^t)$ unilaterally is optimal solution and the optimal solution for step $t+1$ can be written as $\mu_i^{t+1} = \mu_i^t, \forall i = 2,...,N$ and $\mu_1^{t+1} = \mu_1^t + \epsilon_1$. ∎

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