A Game-based Power Optimization for 5G Femtocell Networks

Azadeh Pourkabirian, Mohammad Hossein Anisi and Fereshteh Kooshki

Abstract*— Spectrum sharing deployment of femtocells brings interferences which dramatically degrade network performance. Hence, interference control is a crucial challenge for femtocell networks. In this paper, we propose a power optimization approach for 5G femtocell networks consisting of macrocell and underlying femtocells to manage the interference. Firstly, we formulate the problem based on a non-cooperative game to analyze the competition among the users to access shared spectrum. We then design a pricing mechanism in the utility function to guarantee quality of service (QoS) requirements of macro users. The mechanism lets the macro users experience lower interference and achieve the minimum required data rate. As a result, QoS requirements of both macro and femto users are fulfilled in a non-cooperative manner. We also design a minimax decision rule to optimize the worst-case performance and find an optimal transmission power for each user. By adjusting the optimal power for each user, the maximum aggregate interference is minimized, and the network throughput is maximized. Finally, we develop an iterative learning- based algorithm to implement the proposed scheme and achieve the game equilibrium. Theoretical analysis and simulation results verifies the effectiveness of the proposed mechanism in terms of throughput maximization, QoS assurance and interference mitigation.

***Keywords*— 5G Femtocell networks, game theory, power optimization, QoS guarantees.**

I. INTRODUCTION

A two-tier 5G femtocell network consisting of femto base stations (FBSs) underlaid with macro base stations (MBSs) that helps to achieve load balancing and capacity enhancement through the network [1], [2]. In this way, Femtocells considerably increase cellular coverage and capacity; particularly, for indoor users. On the other hand, the deployment of FBSs in MBS coverage area and shared use of the spectrum cause the mutual interference between macro user equipment (MUEs) and femto user equipment (FUEs) [3]-[5] which dramatically degrades network performance. Therefore, the interference management is one of the most important challenges in femtocell networks. Resource allocation [6]-[12] and power control strategies [13]-[15] are promising techniques for interference mitigation [16], [17] in wireless femtocell networks.

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There are many power control approaches proposed in the literature for wireless femtocell networks. The authors in [18] proposed a power control and subcarrier allocation scheme considering rate maximization and proportional fairness. Study [19] developed a power allocation scheme for downlink communications in two-tier femtocell network. The authors considered co-channel interference to improve the performance of the network. Authors in [20] introduced a joint resource and power allocation problem in hybrid spectrum access femtocells. They aimed to guarantee users' QoS requirements, while allowing spectrum sharing between MBS and the underlying FBSs. In [21], a power control approach was designed in a downlink heterogeneous network (HetNet). They formulated a worst-case robust power minimization problem considering imperfect channel estimation. A weighted bandwidth–power optimization was proposed in [22] for orthogonal frequency division multiple access (OFDMA)-based femtocell networks. The authors improved the network throughput while mitigating cross-tier interference between MUEs and FUEs. A power minimization scheme with interference allowance was presented for wireless HetNet in [23]. The authors designed an on-off scenario for FUEs activity in which MBSs measure and alleviate the aggregated interference from FUEs for uplink transmission. The authors in [24] developed a rate adaptation power control scheme to mitigate cross-tier interference for macrocell-femtocell networks. The Foschini-Miljanic (FM) algorithm is applied to maximizes their individual utility. The authors claimed that the algorithm achieves the maximum data rate in each tier. Study [25] designed a robust power control method under uncertain channel state information (CSI) for the two-tier femtocell network. The authors used the fuzzy logic system (FLS) to adapt the power allocation to dynamic channel states. FLS-based method estimates the instantaneous channel gain and then adjust the power optimal allocation to guarantee the quality of service of each user. They also introduced a price regulation strategy to reduce the intra-tier interference. The author also developed the successive convex approximation (SCA) method based on logarithmic approximation to transform the original nonconvex problem into a convex form. They then applied the Lagrangian decomposition to solve the optimization problem. Finally, they proposed an iterative algorithm with a fast convergence speed to implement the proposed method. Game theory is a powerful distributed framework that analyzes the strategic decision-making problems in interactive multiuser systems. In [26], a game-based power allocation approach was formulated that maximizes the FUEs' utilities with satisfactory QoS. The authors developed a worst-case scenario in order to take into account the imperfect CSI in the power optimization problem. The authors in [27] formulated a joint price assignment and power control mechanism for interference avoidance using the Stackelberg game. Study [28] presented a hierarchical game-based framework for the power optimization problem. Authors claimed that different service requirements for users are satisfied. A distributed game-based framework was designed in [29] to uplink power control and interference mitigation in OFDMA-based femtocell networks. The authors also took into account different service requirements for wireless users. A robust game-based algorithm was developed in [30] for joint resource allocation and power adjustment in downlink femtocell network. The authors modeled the problem as a Stackelberg game and defined a utility function based on the particle swarm optimization to find the best response for each player of the game. They also used a water-filling algorithm to implement the game and obtain the best response. Authors proposed a non-cooperative game-based setting in [31] for robust power allocation in hierarchical OFDMA femtocell network system. Authors in [32] proposed a robust resource allocation algorithm under bounded channel gain uncertainties which maximizes the sum data rate of microcell users. A game-theory based approach was presented in [33] for femtocell networks under spectrum sharing. The problem is modeled as a convex optimization problem to obtain maximum utility function. In [34], users' sum-rate maximization problem was formulated for HetNet. The problem was converted to convex optimization problem and optimal power allocation scheme was derived. Study [35] proposed a join sub-channel allocation and power optimization for wireless cognitive networks. They presented a coalitional game in partition form considering both co-tier and cross tier interference. The objective was to maximize the uplink data rate while satisfying delay constraint for FUEs. A novel power allocation scheme was developed in [36]. The authors claimed that the method can mitigate cross-tier interference efficiently by assigning sub-channels in a profit-calculating method when the idle sub-channels are unavailable. In particular, the total interference from FUEs to the MUE is kept under an acceptable level. Authors in [37] formulated the joint Multi-service Up-link transmission Power and data Rate Allocation Problem in two-tier femtocell networks (MUPRAP). They defined a two-variable utility function to express resource allocation via the independent variables (i.e., power and rate) based on the relevant tier of the users - either macrocell or femtocell. The problem then was formulated as a non-cooperative game while the theory of supermodular games was utilized to achieve the Nash equilibrium as the solution of the game. The theory of supermodular games help to tackle the inherent challenges stemming from the joint two-variable consideration. Finally, they proposed a distributed and iterative algorithm to show the convergence to the NE point and simultaneously update the optimal values of transmission power and data rate for each user. A power control scheme was proposed for OFDMA-based cellular networks under the channel uncertainty in [38]. The authors presented a hierarchical game to minimize both the inter-tier and intra-tier interferences through the network and obtain the tradeoff between QoS satisfaction and lower power consumption. In [39], authors studied the resource allocation and the channel estimation problem for multiuser 5G systems under imperfect CSI. The problem was formulated by Lagrangian duality to compute an optimal power for wireless users. In [40], the authors proposed a joint power and frequency allocation scheme for Self–Organizing OFDMA Femtocell Networks using Foschini-Miljanic algorithm. A frequency allocation method was presented in [41] for 5G HetNet using master–slave algorithm in which works are assigned to the slave nodes by the master node. The authors defined three areas: inner area, outer area, and most-outer area and frequencies are assigned to femtocells in these areas. However, power optimization faces technical challenges such as providing minimum target Signal-to-Interference-Plus-Noise Ratio (SINR) for macro users due to the lack of coordination between the macro user and femto user.

In this work, we propose a game-based power optimization approach to guarantee both MUEs and FUEs requirements. In the presented approach, wireless users compete with each other to raise the transmission rate and satisfy their QoS requirements. The main contributions of the proposed work are summarized as follows:

- We propose a power control approach using game theory for 5G femtocell networks. Game theory helps model the strategic interactions among wireless users and find an optimal transmission power for each user. In the proposed game model, the transmission power levels are the strategies. Each user as a player chooses a power level and then receives payoff in terms of received SINR. Users compete with each other to obtain higher payoff and fulfill their QoS requirement.
- Different from prior studies in the literature, we define a pricing strategy in utility function formulation to restrain both co-tier and cross-tier interference. MUEs charge an interference-price to interfering FUEs which discourages the FUEs for transmitting at high power. This helps MUEs to be better protected from FUEs interferences. Thus, MUEs' QoS requirements will be guaranteed since MUEs experience lower interference and their target SINR are satisfied.
- We design a minimax strategy that optimizes the worst-case performance. The proposed strategy reflects the worst operating point in the game with the maximum interference level. We then find an optimal transmission power for each user to maximize the users' utilities under the worst-case interference scenario.
- We then theoretically analyze the existence and uniqueness of an equilibrium point as the solution of game. Theoretical analyses prove the convergence of the game to the equilibrium point and achieve the global solution of the problem.
- Finally, we develop an iterative learning-based algorithm to implement the power optimization game. Interference reduction is obtained through the optimal power allocation for all the users in several rounds of the algorithm. Simulation results demonstrate that the algorithm can effectively suppress both co-tier and cross-tier interferences, maximizes all users' utilities and meets target SINR for all MUEs.

The rest of the paper is organized as follows: Section II describes the system model and assumptions of the problem. Section III provides a game-theoretic formulation of the problem followed by theoretical analyses for existence and characterization of the game equilibrium (GE). A distributed Q-learning based algorithm is developed for the proposed model in Section V. Section VI gives the simulation results and performance evaluation of the proposed approach. Finally, the conclusions are drawn in Section VII.

II. SYSTEM DESCRIPTION

We consider a 5G femtocell network consists of a MBS and $K-1$ FBSs serving F active FUEs coexists with M active MUEs. We focus on the uplink transmission of the two-tier network system. We assume the MUEs and FUEs are distributed according to the homogeneous Poisson point process (PPP), with intensity λ_{Mu} and λ_{Fu} respectively. For simplicity, we assume that both macrocell and femtocell coverage area are circular. Thus the number of users is $N_u = Poisson(\lambda_u \pi R^2)$ in the circular covered area of radius R, ($\lambda_u \in \{\lambda_{Mu}, \lambda_{Fu}\}$, $u \in \{MUE, FUE\}$) and $N_u = |FUEs| \cup |MUEs|$. We consider that the network bandwidth W is divided into N_s sub-channels and sub-channels all are shared by the all users. In fact, a sub-channel/time resource is the smallest resource unit assignable to a user. We model the sub-channels distribution among users as a Poisson process. Thus, the probability of n_{sc} sub-channels are allocated to k users for a given time t can be expressed as follows:

$$P(n_{sc}, kt) = Pr(k) = e^{-kt} \frac{(kt)^{n_{sc}}}{(n_{sc})!}, \quad n_{sc} = 1,2,\dots,N_s \qquad (1)$$

In a two-tier 5G femtocell network, wireless users are constantly looking for a BS that fulfills their QoS requirements. Users choose a BS that provides them the maximum SINR to be able to satisfy their QoS requirements. Each wireless user chooses a BS with probability p and observes the received SINR from this BS. The user then changes its BS if the received SINR is lower than SINR target. The probability that user i will choose the BS j can be calculated as [6]:

$$p_i^j = \left(1 + \frac{\sum_{k=1,k\neq j}^{K} \lambda_K P^k |g_{i,k}|^2}{\lambda_j P^j |g_{i,j}|^2}\right)^{-1} \qquad (2)$$

Where p_i^j is the probability that user i chooses BS j and λ_K represents the intensity of BSs in the k^{th} tier. In a spectrum sharing scenario, FUEs and MUEs simultaneously access to the same channel. As a result, it might be raised two types of interference in the network. One is created by MUE to adjacent FUEs, and the other is caused by nearby FUEs to the MUE. Therefore, the transmission power of UEs need to be reduced to control interference among them. On the other hand, wireless users require a certain transmission rate to satisfy their QoS requirements. As a result, for an interference-limited scenario, an effective power optimization method is essential to limit the transmission power in order to the minimum required transmission rate of users are fulfilled. Let us define the following two requirements for MUEs and FUEs:

1) The maximum transmission rate for FUEs

Each FUE tends to achieve the maximum transmission rate. The transmission rate of a FUE is significantly improved by increasing the FUE's SINR. The SINR of FUE i^{th} on sub-channel sc can be calculated by:

$$SINR_{i,sc} = \begin{cases} \frac{P_{i,sc}|g_{i,sc}|^2}{\sigma_n^2 + \sum_{j=1,j\neq i}^{F} P_{j,sc}|g_{j,sc}|^2} & if\ I_{sc} = 0 \\ \frac{P_{i,sc}|g_{i,sc}|^2}{\sigma_n^2 + \varrho_I + \sum_{j=1,j\neq i}^{F} P_{j,sc}|g_{j,sc}|^2} & if\ I_{sc} = 1 \end{cases} \qquad (3)$$

Where I_{sc} states the macro user's activity over the sub-channel sc at time t, $I_{sc} = 1$ if the sub-channel is under the influence of macro user's activity; otherwise $I_{sc} = 0$, $P_{i,sc}$ is the transmission power of user i over sub-channel sc, $g_{i,sc}$ identifies channel gain on sub-channel sc, and σ_n^2 is the variance of white noise, $\sum_{j=1,j\neq i}^{F} P_{j,sc}|g_{j,sc}|^2$ denotes the total interference caused by all interfering FUEs and $\varrho_I = P_{MUE,sc}|g_{MUE,sc}|^2$ represents the interference caused by MUE transmission over sub-channel sc. In other words, we can rewrite (3) as bellow:

$$\gamma_{i,sc} = E(SINR_{i,sc}) = q\left(\frac{P_{i,sc}|g_{i,sc}|^2}{\sigma_n^2 + \varrho_I + \sum_{j=1,j\neq i}^{F} P_{j,sc}|g_{j,sc}|^2}\right) + (1 - q)\left(\frac{P_{i,sc}|g_{i,sc}|^2}{\sigma_n^2 + \sum_{j=1,j\neq i}^{F} P_{j,sc}|g_{j,sc}|^2}\right) \qquad (4)$$

Where q identifies the probability that the sub-channel sc at time t is busy. Therefore, the SINR of FUE i^{th} can be written as:

$$\gamma_i = \sum_{sc=1}^{N_s} x_{sc,t}\gamma_{i,sc} \qquad (5)$$

Where $x_{sc,t}$ indicates if the sub-channel sc at time t is busy or not. $x_{sc,t} = 1$ when the sub-channel sc is allocated to a MUE at time slot t; otherwise $x_{sc,t} = 0$. Accordingly, the maximization problem is formulated as follows:

$$\max_{P_{i,sc}} \sum_{sc=1}^{N_s} x_{sc,t}\gamma_{i,sc}$$

$$s.t.\ C_1: \quad x_{sc,t} \in \{0,1\}, \quad \forall sc, t$$

$$C_2: \quad 0 \le P_i \le P^{max} \qquad (6)$$

Where P^{max} denotes the maximum transmission power constraint for the user i. The interference from an interfering MUE to a typical FUE can be expressed as:

$$\mu_I = |g_{sc}|^2 \int_0^{r_I} E\{\varrho_I(f)\}\, df \qquad (7)$$

Where $r_I = |f_i - f_j|$ is the frequency reuse distance and $E\{\varrho_I(f)\}$ denotes expected value of MUE interference after L-Fast-Fourier-transform (FFT) processing that can be expressed as [42]:

$$E\{\varrho_I(f)\} = \left(\frac{1}{2\pi L}\right) \int_{-\pi}^{\pi} \phi_{sc}^{MUE}\ (e^{-j2\pi ft}) \left(\frac{\sin(2\pi f - \varphi)L/_2}{\sin(2\pi f - \varphi)/_2}\right)^2 d\varphi \qquad (8)$$

Where $\phi_{sc}^{MUE}(e^{-j2\pi ft})$ the interference power spectral density (PSD) at frequency f.

2) The minimum target SINR of MUE

To MUE's QoS guarantees, the aggregate interference from the all interfering FUEs should not exceed a predefined threshold τ. Thus, we have

$$\sum_{i=1}^{F} P_{i,sc}|g_{i,sc}|^2 < \tau \qquad (9)$$

In addition, we can calculate the interference power spectral density for FUEs at frequency f as:

$$\phi_{sc}^{FUE}(f) = P_f\, T \left(\frac{\sin \pi f T}{\pi f T}\right)^2 \quad (10)$$

Where P_f represents the total transmission power at frequency f in duration T. Thus, the aggregate interference from FUEs to MUE j on sub-channel sc at frequency f can calculated as follows:

$$S_I^{FUE}(f) = |g_{sc}|^2\, P_f T \int_0^{d_{ij}} \left(\frac{\sin \pi f T}{\pi f T}\right)^2\, df \quad (11)$$

where $d_{ij} = |f_i - f_j|$ represents the spectral distance between subcarrier i and center frequency f_j of MUE j.

Hence, the optimization problem in (6) can be formulated as follows:

$$\max_{P_{i,sc}} \sum_{i=1}^{N_u} \sum_{sc=1}^{N_s} x_{sc,t}\gamma_{i,sc}$$

$$s.t.\quad C_1: \quad \sum_{i=1}^{F} P_{i,sc}|g_{i,sc}|^2 < \tau$$

$$C_2: \quad x_{sc,t} \in \{0,1\}, \quad \forall sc, t$$

$$C_3: \quad 0 \le P_i \le P^{max} \quad (12)$$

III. OPTIMAL POWER BASED ON GAME THEORETIC APPROACH

We formulate the strategic power optimization problem using game theory in order to model the users' competition over shared spectrum. In the proposed game, wireless users who adjust a transmission power level to maximize their own utilities are referred to as the players. The utility of a player is evaluated in terms of the transmission rate of the player. We also assume that the set of power levels which a player can select are its strategy set. In a spectrum sharing scenario, we consider the interference levels as the operating points of the game. Now, we define our proposed game model formally as $G(\mathcal{P}, \mathcal{S}, U)$ where $\mathcal{P} = P_1 \times P_2 \times \dots \times P_{N_u}$ denotes the strategy set in which $P_i = \{P_i^1, P_i^2, \dots, P_i^n\}$ is referred to as the set of pure strategies of user i, $\mathcal{S}$ identifies the operating points, and $U: \mathcal{P} \times \mathcal{S} \to \mathbb{R}$ defines the utility function which is minimized over $\mathcal{S}$ and maximized over $\mathcal{P}$. More precisely, the utility function of player i is expressed as below:

$$U_i(P_i, P_{-i}) = R_i(P_i, P_{-i}) - C_i(P_i, P_{-i}) \quad (13)$$

Where $R_i(P_i, P_{-i})$ identifies the revenue of player i in the form of the achieved transmission rate that is given as:

$$R_i(P_i, P_{-i}) = B_i \log_2\left(1 + \frac{P_{i,sc}|g_{i,sc}|^2}{\sigma_n^2 + \sum_{j=1, j\ne i}^{F} P_{j,sc}|g_{j,sc}|^2}\right) \quad (14)$$

And $C_i(P_i, P_{-i})$ is the cost function which denotes the interference-price paid by interfering FUEs. On the other hand, FUEs pay more price when they increase their transmission power. The cost function can be written as follows:

$$C_i(P_i, P_{-i}) = c_i S_I \quad (15)$$

Where c_i denotes the interference-price multiplier and S_I is the interference power from FUE i to the MUE that is calculated similar to (11).

In strategic power optimization scenario, wireless users tend to maximize its transmission power in order to achieve the highest transmission rate. On the other hand, the higher transmission power brings more cross-tier interference over sub-channel. To guarantee the QoS of MUE, the aggregate interference from the FUEs should not be larger than a threshold. Therefore, the transmission power of FUEs should be optimized. We define a worst operating point S_I^{worst} which reflects the maximum aggregate interference from FUEs on a sub-channel. We define S_I^{worst} as:

$$S_I^{worst}(P_j) \in \arg \min_{S_I < \tau} U_i(P_i, P_{-i}; S_I) \triangleq W(S_I, P_j), \; j \in UEs \text{ and } j \ne i \quad (16)$$

The objective is to find an optimal transmission power for each user under S_I^{worst} in order to optimize the worst-case performance as follows:

$$P_i^{opt} \in \arg \max_{0 \le P_i \le P^{max}} U_i(P_i, P_{-i}; S_I) \quad (17)$$

Theorem 1. if (P_i^{opt}, S_I^{worst}) is a saddle point of the game G then P_i^{opt} is an optimal power for the game.

Proof. See Appendix A. ■

Theorem 2. There are two critical values S_I^{Least} and S_I^{worst} for S_I on each sub-channel as follows:

$$S_I^{Least} = \frac{\sqrt{P^{max}}}{\sqrt{-c_i}}|g_i| - (\sigma_n^2 + \varrho_I)$$

And

$$S_I^{worst} = -(\sigma_n^2 + \varrho_I) \quad (18)$$

And there exists a unique GE for the power optimization game if $S_I^{Least} \le S_I \le S_I^{worst}$; otherwise, no GE exists.

Proof. See Appendix B. ■

IV. ITERATIVE LEARNING-BASED POWER OPTIMIZATION ALGORITHM

In this section, we develop a Q-learning based algorithm to achieve the equilibrium of the proposed power optimization game. In the proposed Q-learning based algorithm, the transmission power levels P_i are the actions in state s_i. The received SINR is defined as a reward for each action. The algorithm stores a Q-table consisting Q-values (i.e., Q_i) that reflects the received rewards for actions (i.e., P_i). First, each user randomly adjusts a transmission power level. The user then observes the reward of the chosen action. Users continuously look for an action with the largest reward. Therefore, the user changes its transmission power level if received reward is lower than the minimum required SINR for QoS guarantees as:

$$P_i = arg \max_P \left(\sum_{1 \le j \le N} Q_j(s_j, P)\right) \quad (19)$$

The user then updates the value of Q_i according to the following:

$$Q_i^{new}(s_i(k), P_i) = (1-\rho)Q_i^{old}(s_i(k), P_i) + \rho[U_i(k) + \beta \max Q_i(s_i(k+1), P)] \quad (20)$$

Where $s_i(k)$ identifies the state of the user i in iteration k, $0 \le \rho \le 1$ denotes learning rate, $0 \le \beta \le 1$ is discount factor and $\max Q_i(s_i(k+1), P)$ is an estimate of optimal future value.

The main goal of the algorithm is to maximize the received rewards of all users by taking a series of actions. Users independently learn from their own past information to adjust its best transmission power level and refine the power adaption strategy. Thus, the algorithm reduces the communication overhead. The algorithm is repeated over time and the optimal transmission power is obtained for each user. The pseudo-code of the algorithm is presented in Algorithm 1.

Algorithm 1. Iterative power optimization algorithm

```
Initialize P_{i,s}(k) , g_s and starting state s_i(k)
Set k = 1
Repeat
  if rand ≤ ε then
     Randomly adjust 0 ≤ P_i(k) ≤ P^max
  else
     Choose P_i^*(k) = argmax Q_i(s_i(k), P)
  end if
  if Σ_{i=1}^{F} P_{i,sc}|g_{i,sc}|^2 > τ go to step 3
  observe U_i(k) and next state s_i(k)
  update Q_i(s_i(k), P_i) = (1 − α)Q_i(s_i(k), P_i) + α[U_i(k) +
β max Q_i(s_i(k + 1), P)]
  s_i(k) = s_i(k + 1)
  k = k + 1
Until all users' power is adjusted
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V. Performance Evaluation

In the following section, we discuss numerical results to verify the effectiveness of the proposed power optimization approach. We consider the uplink communication of an OFDMA-based femtocell network composed of one MBS, five FBSs and 40 users. The proportion of MUEs is 25% of the total user population. All users are assumed randomly distributed over a 1300×1300 m^2 area and results are averaged over 200 independent runs. According to the distance to BSs, we set the transmission power of MUEs in the range of 10dBm to 35dBm. We compare the network performance of our approach with the results of two existing methods: a fuzzy-based power allocation algorithm [25] and a game-based uplink power control method [37]. The simulation was carried out in MATLAB and the simulation setting is listed in Table I.

TABLE I
SIMULATION PARAMETERS

Parameters	Values
Number of FBS	5
MBS radius	700m
FBS radius	8m
Number of FUEs	30
Number of MUEs	10
Bandwidth	12MHz
τ	20W
Target SINR of MUE	12dB
σ_n^2	0.01
P^{max}	3W
ϱ_I	-14dB
ρ	0.3
β	0.5
θ	3.7

Fig. 1 depicts the FUEs' utilities under different transmission powers. We varied the transmission power of the FUEs from 2dBm to 16dBm. It is observable that, the higher transmission power of FUEs leads to increase FUE's utilities. In the other words, the utilities of FUEs increase with increasing of their transmission power. However, the FUEs' utilities gradually reach a steady level because of the interference-price paid by FUEs. More precisely, MUEs impose a price to FUEs in order to reduce the interference from the interferer FUEs. Increasing the interference level from FUEs results in an increase in interference-price. Consequently, the utilities decrease as the interference-price increases. Thus, FUEs have to decrease their power to decrease the interference-price which leads to decreasing the FUEs' utilities.

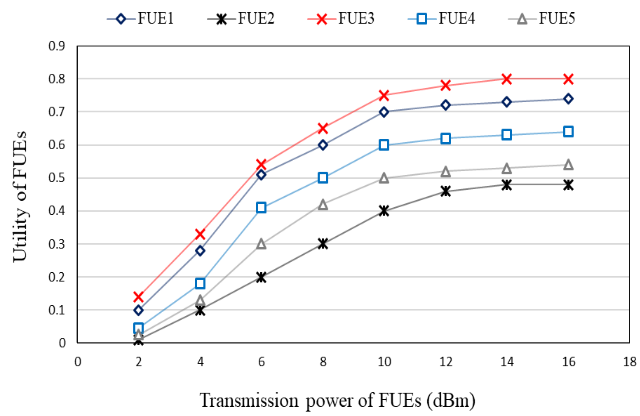

Fig. 1. FUEs' utilities under different transmission power.

Fig. 2 illustrates the relationship between FUEs' transmission power, average utility, and interference-price for the existing approaches. We perform the experiments for 40 users under two different values of the interference-price multiplier $c_i = 0.2$ and $c_i = 0.4$ while keeping the network deployment and the number of FBSs and MBS are the same. The MUEs proportion is considered 20% of the total users. We change FUEs' transmission power from 10dBm to 20dBm. As expected, in all approaches, average utility of users will increase as the transmission power of the FUEs increase. However, the increasing FUEs' transmission power gives rise to interference among FUEs and MUEs results in reducing users' utilities. On the other hand, in high transmission power, MUE charges the interference-price to a larger value to discourages the FUEs for transmitting at high power in order to reduce the cross-tier interference. Thus, high transmission power results in an increase in the interference-price that leads to decrease users' revenues slowly and decline the average utilities for all approaches. The figure also demonstrates a smaller decrease in average utilities of all methods for smaller values of c_i. Finally, average utilities reach a stable level and does not change much for higher transmission power of FUEs. That is because wireless users gradually learn their optimal transmission powers which maximize their utilities. Moreover, there is a significant decrease gap in FUE's utility among the MUPRAP algorithm, FLS scheme and the proposed approach which means our proposed algorithm outperforms than these methods. The gap gradually grows as the transmission power increases.

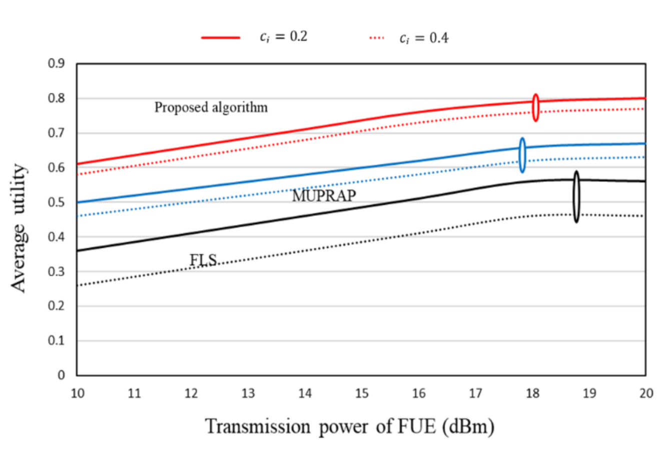

Fig. 2. Average utility of FUEs for different transmission power of FUEs.

Fig. 3 demonstrates the performance improvement of the proposed algorithm in terms of interference mitigation compared to two other schemes. We run the experiments with 8 active FUEs coexists with a MUE. It is shown that our approach significantly suppresses both co-tier and cross-tier interference. This is expected since there is an interference-price in our algorithm. The interference-price is increased with an increase in the UEs' transmission power; this leads to a marked decrease in the utility of users. Thus, users adjust their transmission power to a proper value that not only reduces interference but also improves the user's utility. In fact, using the learning-based algorithm, users learn the best value of their transmission power which help mitigate interference.

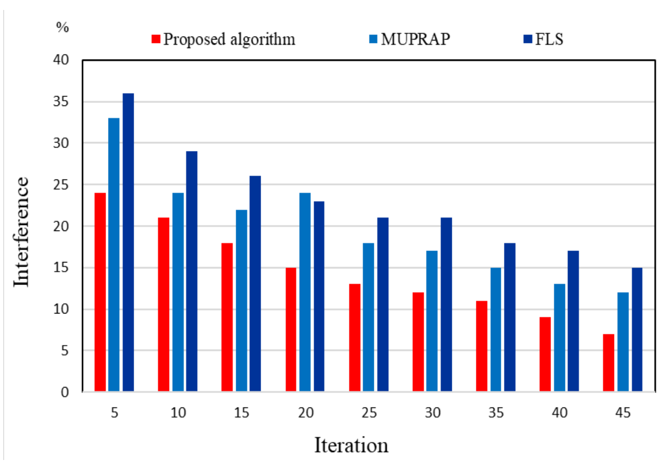

Fig. 3. The interference comparison.

In Fig. 4, we investigate MUEs' QoS guarantees under different number of users in the proposed approach. We consider an interference scenario in which there exist three MUEs coexist with number of FUEs. We vary the number of FUEs from 5 to 40. The minimum required SINR is set to 12 dB in order to fulfill QoS requirements for each MUE. As it can be seen, the number of users has significant impact on the MUEs' QoS guarantees. Clearly, the SINR of MUEs reduces as the number of inferring FUEs increases. In fact, as the number of nearby FUEs grows the total interference from FUEs to MUEs increases and the MUEs experience heavier interference. Therefore, the average received SINR reduces for MUEs. However, the figure confirms that the proposed method fulfills QoS requirements for all MUEs so that the actual SINRs are greater than the minimum required SINR of MUEs.

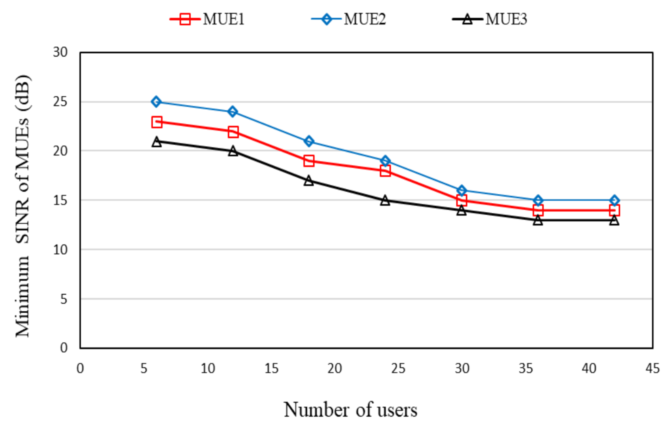

Fig. 4. QoS guarantee for MUEs under different number of users.

We also study the outage probability of the three existing methods under different number of users and fixed BSs density (i.e., with one MBS and five FBSs) in Fig. 5. The outage probability, p_{out} is defined as the probability that the received power of a typical user who is located at a distance r from its serving BS falls below P_{min} as follows [44]:

$$p_{out}(P_{min}, r) = p(P_i^r(r) < P_{min}) = 1 - Q\left(\frac{P_{min} - (P_i^t + 10\log_{10} K - 10\theta \log_{10}(r/R))}{\sigma_{\psi_{dB}}}\right) \quad (21)$$

Where P_{min} is the minimum required transmission power for user i, P_i^r denotes the receive power of user i, P_i^t states the transmission power of user i, K states a constant based on the average channel attenuation, θ is the path loss exponent, R identifies a reference distance and $\sigma_{\psi_{dB}}$ denotes standard deviation of ψ_{dB} in which ψ_{dB} is a random variable with a log-normal distribution that denotes the ratio of transmit-to-receive power.

The experiments were conducted for a given distance of 600m away from BSs under the target outage probability $p_{out,\tau} = 0.36$ and repeated 80 times. We vary the number of users from 10 to 40. The findings show that the outage probability of MUEs are more than the FUEs' outage probability in all existing approach. That is mainly because a MUE may be located at a long distance of the MBS (i.e., edge-user) and not be able to receive the same power level compared to cell-center users. On the other hand, cross-tier interference is much more significant than co-tier interference. The figure also shows that as the number of users increase the outage probability becomes worse. The reason is that more users cause excessive interference to neighboring users that will reduce transmission power of them and leads to performance becomes unacceptable. However, although, in all existing approach, the outage probability is smaller than the target value, the proposed method provides better results. Thus, it can be concluded that the proposed approach supports minimum power requirement for all users even though at high user densities.

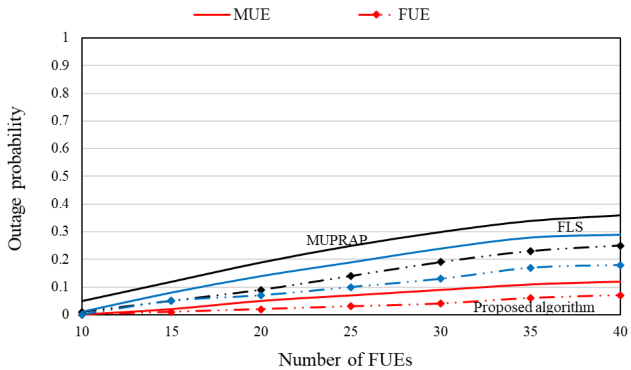

Fig. 5. Outage probability at different number of users for $\theta = 3.7$, $k = -31dB$, $R = 1m$, $P_{min} = 8dB$, $\sigma_{\psi_{dB}} = 3.6dB$.

Fig. 6 and Fig. 7 depict the convergence of the proposed approach in terms of the transmission power and the users' throughput.

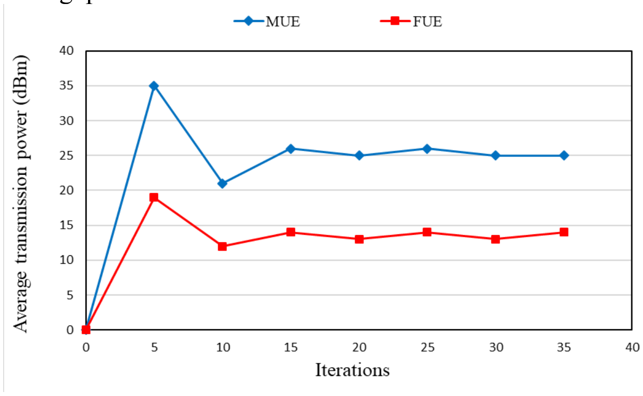

Fig. 6. Average transmission power for MUEs and FUEs at different iterations.

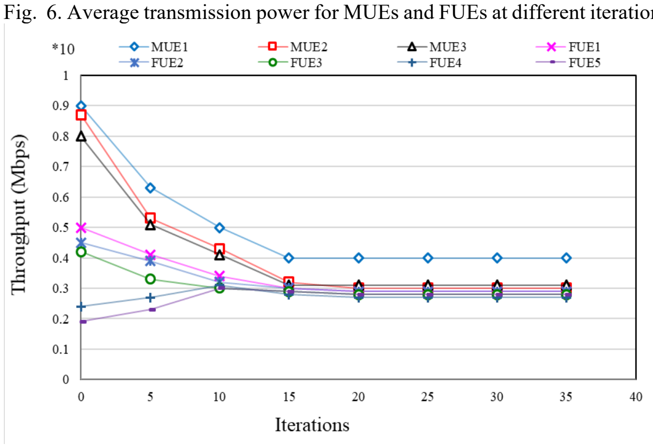

Fig. 7. Convergence of the proposed algorithm.

The results demonstrate that the algorithm converges to the equilibrium point very fast in approximately 15 iterations. We observe that the transmission power of FUEs slightly increases in first iterations. However, FUEs decrease their transmission power as they are penalized for transmitting at high power. Eventually, the transmission power reaches to the stable solution. Similarly, the average throughput of users is felled for some first iterations. Eventually, it remains almost the same after fifteenth iteration. We also present the required number of iterations for convergence to the equilibrium point for the existing methods under different number of users in Table II.

TABLE II
THE NUMBER OF ITERATIONS FOR CONVERGENCE TO THE EQUILIBRIUM

	Average throughput (Mbps)	MUE optimal SINR	MUE target SINR	Iterations
The proposed algorithm	3.8163	14.5237	13	15
MUPRAP	3.2845	13.1369	12	17
FLS	3.5	10.912	10	5

In Fig. 8, we explore the performance of our approach in terms of the spectral efficiency when the network scales up. The spectral efficiency is described as the amount of information that can be transmitted over a given bandwidth and is measured in bps/Hz [45]. We investigate the spectral efficiency for different number of users up to 60 with 5 FBSs and one MBS. The transmission power of FBSs is set to 18 dBW and it is set to 35dBW for MBS. As it can be seen from the figure, spectral efficiencies are relatively close to each other for all existing methods when the number of users is small. However, with the growing in the number of users, the proposed approach achieves higher spectral efficiency compared to the other two methods. The reason is that the transmission power adaption policy degrades both co-tier and cross-tier interference among wireless users that leads to increase data rate and enhance spectral efficiency.

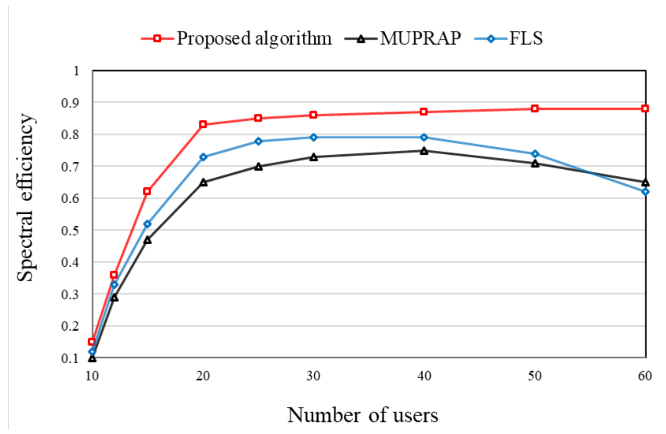

Fig. 8. Spectral efficiency under different number of users.

Fig. 9 shows the communication overhead of the proposed method for comparison methods. We run the experiments for different number of users ranging from 10 to 90. The results prove that increasing the number of wireless users increases the overall signaling overhead. When the number of users is 20, the number of exchanged signaling messages is 1300 in the proposed method whereas this amount reaches 3800 messages when the number of users exceeds 90. However, the proposed approach significantly reduces the communication overhead by around 22% compared to the schemes under study. That is mainly because the power adaption decisions are made by each user in a distributed manner without any interaction with a central controller. Thus, the number of signaling exchanged messages is reduced across the network that leads to enhance in the available bandwidth for data transmission. Moreover, signaling overhead reduction not only increases the network capacity, but also reduces power consumption that leads to energy efficiency.

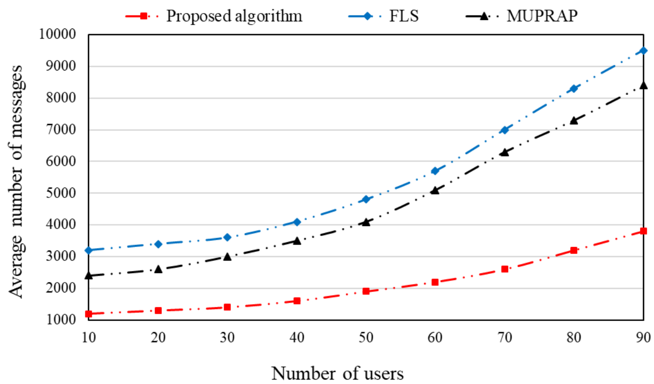

Fig. 9. The average number of messages exchanged in the network.

VI. Conclusion

In this paper, a power optimization approach was proposed for 5G femtocell networks. We formulated the problem as a non-cooperative game to model the competitive behavior of rational users to access shared spectrum. The QoS constraint in terms of minimum required SINR were taken into account for all MUEs. Unlike other studies, we designed a pricing strategy in utility function to suppress cross-tier interference from FUEs to MUEs. We also take into consideration the maximum interference level as a worst-case scenario to find an optimal transmission power for each user and optimize the worst-case performance. We then derived theoretically a saddle point for the game under a given worst operating point. Finally, we designed a learning-based algorithm which solved the problem in a distributed manner. The proposed algorithm helps users make decisions based on their own knowledge toward maximizing utilities. Extensive simulation results demonstrate the performance improvement of our approach in terms of the interference mitigation, QoS satisfaction and throughput improvement.

Our future study focuses on accurate estimation of CSI to adapt the transmissions with the true channel conditions. Some methods in the literature have taken into account the imperfect CSI in the power optimization problem. Nevertheless, they estimate CSI using the known training sequence which leads to huge amount of CSI feedback and weak synchronization between transmitter and receiver. Developing a robust estimation approach can reduce the channel estimation error and optimize the power allocation for wireless users. Also, investigating user mobility and examining different type of service models are among the future plans of this work.

Appendix A

Proof Of Theorem 1

First, we assume that $\left(P_i^{opt}, S_l^{worst}\right)$ is a saddle point to the game, then we show that P_f^{opt} is an optimal power for the game. let consider a game $\mathrm{G}'(\mathcal{P}', \mathcal{S}', U')$ so that

$\mathcal{P} \subset \mathcal{P}'$; $\mathcal{S}' \subset \mathcal{S}$

$$\sup_{P_i \in \mathcal{P}} U\left(P_i, S_j\right) = \sup_{P_i \in \acute{\mathcal{P}}} U\left(P_i, S_j\right), \forall S_j \in \mathcal{S}'$$
$$\inf_{S_j \in \mathcal{S}} U^{opt}\left(S_j\right) = \inf_{S_j \in \acute{\mathcal{S}}} U^{opt}\left(S_j\right) \tag{A.1}$$

We define a least favorable operating point S_j^{Least} as duality of the worst operating point for the game. If the worst operating point does not exist for the game, a least favorable operating point can be replaced. The least favorable operating point can be written as bellow:

$$S_j^{Least} \in \arg \min_{S_j \in \mathcal{S}} U^{opt}\left(S_j\right) \tag{A.2}$$

Now, using (A.1), we can obtain the following:

$$\forall \left(P_i^{Least}, S_j^{Least}\right) \in \mathcal{P}' \times \mathcal{S}',$$
$$\inf_{S_j \in \acute{\mathcal{S}}} U\left(P_i^{Least}, S_j\right) \leq \inf_{S_j \in \mathcal{S}} U\left(P_i^{Least}, S_j\right) \leq$$
$$\sup_{P_i \in \mathcal{P}} \inf_{S_j \in \mathcal{S}} U\left(P_i, S_j\right) \leq \inf_{S_j \in \mathcal{S}} \sup_{P_i \in \mathcal{P}} U\left(P_i, S_j\right) =$$
$$\inf_{S_j \in \acute{\mathcal{S}}} \sup_{P_i \in \acute{\mathcal{P}}} U\left(P_i, S_j\right) \leq \sup_{P_i \in \acute{\mathcal{P}}} U\left(P_i, S_j^{Least}\right) \tag{A.3}$$

It is obvious, $(P_i^{Least} = P_i^{opt})$ is an optimal power for the game G', when $\left(P_i^{Least}, S_j^{Least}\right)$ is a saddle point solution to the game. That means no other transmission power level gives better behavior at S_j^{Least}. Thus, the worst-case performance of the game is obtained at S_j^{Least}.

Since both sides of the equation are equal, we replace the inequalities in (A.3) into the following equalities. Therefore, we have:

$$\inf_{S_j \in \mathcal{S}} U\left(P_i^{Least}, S_j\right) = \sup_{P_i \in \mathcal{P}} \inf_{S_j \in \mathcal{S}} U\left(P_i, S_j\right) \tag{A.4}$$

This completes the second part of the proof. Now, we show that $\left(P_i^{Least}, S_j^{Least}\right)$ is a saddle point solution to the game G. Therefore, the following can be derived mathematically:

$$U\left(P_i, S_j^{Least}\right) \leq U\left(P_i^{Least}, S_j^{Least}\right) \leq U\left(P_i^{Least}, S_j\right) \tag{A.5}$$

We can also transform (A.5) as:

$$U^{opt}\left(S_j^{Least}\right) = U(P_i^{Least}, S_j^{Least}) \tag{A.6}$$

The above equality express that P_i^{Least} is the optimal transmission power under S_j^{Least}, i.e., $P_i^{Least} = P_i^{opt}\left(S_j^{Least}\right)$. We need to prove that S_j^{Least} is the worst operating point for P_i^{Least}, $(S_j^{Least} = S^{worst}(P_i^{Least}))$ if and only if it is the least favorable operating point for the game. We can show $U^{opt}(.)$ is convex in $\mathcal{S}$ as the following:

$$S_j^a = (1-\alpha) S_j^b + \alpha S_j^c, \ \forall \ 0 \leq \alpha \leq 1 \ \ and \ \ S_j^b, S_j^c \in \mathcal{S} \tag{A.7}$$

We also know that:

$$\sup_{P_i \in \mathcal{P}} U\left(P_i, S_j^a\right) \leq \sup_{P_i \in \mathcal{P}} \left\{(1-\alpha) U\left(P_i, S_j^b\right) + \alpha U\left(P_i, S_j^c\right)\right\} \leq$$
$$(1-\alpha) \sup_{P_i \in \mathcal{P}} U\left(P_i, S_j^b\right) + \alpha \sup_{P_i \in \mathcal{P}} U\left(P_i, S_j^c\right) \tag{A.8}$$

Thus, we obtain the right inequality according to the convexity assumption $U(P_i, .)$ on $\mathcal{S}$ for every $P_i \in \mathcal{P}$. Using [43], the following can be powerful:

$$f(0) \leq f(\alpha), \forall \alpha \in [0,1] \ \Leftrightarrow 0 \leq \lim_{\alpha \to 0} \frac{1}{\alpha}[f(\alpha) - f(0)] < +\infty \tag{A.9}$$

where $f: [0,1] \rightarrow \mathbb{R}$ is a convex function. Therefore, we showed that S_j^{Least} is a least favorable operating point for the game $\mathcal{G}$, based on the assumption that $\mathcal{S}$ is considered convex and if

$$\forall S_j \in \mathcal{S}, L\left(S_j, S_j^{Least}\right) \triangleq \lim_{\alpha \to 0} \frac{1}{\alpha}\left[U^{opt}\left(S_j^{Least} + \alpha\left(S_j - S_j^{Least}\right)\right) - U^{opt}\left(S_j^{Least}\right)\right] \geq 0 \quad \text{(A.10)}$$

On the other hand, S_j^{Least} can be defined as the worst operating point for P_i^{Least}, if and only if

$$W\left(S_j, P_i^{Least}, S_j^{Least}\right) \triangleq \lim_{\alpha \to 0} \frac{1}{\alpha}\left[U\left(P_i^{Least}, S_j^{Least} + \alpha\left(S_j - S_j^{Least}\right)\right) - U\left(P_i^{Least}, S_j^{Least}\right)\right] \geq 0 \quad \text{(A.11)}$$

According to the following:

$$S_j^{\alpha} = (1-\alpha)S_j^{Least} + \alpha S_j \quad \text{(A.12)}$$

We can write as:

$$U^{opt}\left(S_j^{\alpha}\right) - U^{opt}\left(S_j^{Least}\right) = U^{opt}\left(S_j^{\alpha}\right) - U\left(P_i^{Least}, S_j^{\alpha}\right) + U\left(P_i^{Least}, S_j^{\alpha}\right) - U\left(P_i^{Least}, S_j^{Least}\right) \quad \text{(A.13)}$$

Taking two opposite limits $\lim_{\alpha \to 0} \frac{1}{\alpha}[.]$ of the above equation results in $L\left(S_j, S_j^{Least}\right) = W\left(S_j, P_i^{Least}, S_j^{Least}\right), \forall S_j \in \mathcal{S}$. Thus, it can be concluded that S_j^{Least} is the worst operating point for P_i^{Least} if and only if it is the least favorable operating point for the game G. It was proved that $\left(P_i^{Least}, S_j^{Least}\right)$ is a saddle point solution to the game and this completes the proof. ■

APPENDIX B

PROOF OF THEOREM 2

We know that S_I denotes the aggregate interference power from interfering FUEs to the user i (i.e., MUE, FUE). Thus, we can write:

$$S_I = \sum_{j=1, j\neq i}^{F} P_{j,sc}\left|g_{j,sc}\right|^2 \quad \text{(B.1)}$$

It is obvious that U_i is differentiable at each point in its domain as follows:

$$\frac{\partial U_i}{\partial S_I} = \frac{\partial}{\partial S_I}\left(\frac{P_{i,sc}\left|g_{i,sc}\right|^2}{\sigma_n^2 + \varrho_I + S_I}\right) - c_i \quad \text{(B.2)}$$

For the sake of simplicity, we use P_i instead of $P_{i,sc}$ and $|g|^2$ for $\left|g_{i,sc}\right|^2, \forall i$, in the rest of this paper. We consider S_I^{Least} and S_I^{worst} as critical points of U_i. As a result, the critical values of S_I can be calculated as follows:

$$\frac{\partial U}{\partial S_I} = 0 \quad \text{(B.3)}$$

Therefore,

$$\frac{\partial U}{\partial S_I} = \frac{-P_i|g_i|^2}{\left(\sigma_n^2 + \varrho_I + S_I\right)^2} - c_i = 0 \quad \text{(B.4)}$$

And

$$S_I = \frac{\sqrt{P_i}}{\sqrt{-c_i}}|g_i| - \left(\sigma_n^2 + \varrho_I\right) \quad \text{(B.5)}$$

Where $c_i < 0$ in order to describe the interference-price which FUE has to pay.

According to $0 \leq P_i \leq P^{max}$, U_i has a minimum value when $P_i = P^{max}$ as the below:

$$S_I^{least} \triangleq \min\left(\frac{\sqrt{P_i}}{\sqrt{-c_i}}|g_i| - \left(\sigma_n^2 + \varrho_I\right)\right) = \frac{\sqrt{P^{max}}}{\sqrt{-c_i}}|g_i| - \left(\sigma_n^2 + \varrho_I\right) \quad \text{(B.6)}$$

We can also obtain a maximum value of U_i when $P_i = 0$ as follows:

$$S_I^{worst} \triangleq \max\left(\frac{\sqrt{P_i}}{\sqrt{-c_i}}|g_i| - \left(\sigma_n^2 + \varrho_I\right)\right) = -\left(\sigma_n^2 + \varrho_I\right) \quad \text{(B.7)}$$

This proves the existence of two critical values S_I^{Least} and S_I^{worst} for S_I. Now, we show that a GE exists for the game only when $S_I^{Least} \leq S_I \leq S_I^{worst}$.

By taking a first-order derivative of P_i, we conclude that U_i is continuous in all P_i. On the other hand, since a second-order derivative of P_i is less than zero, we obtain that the function is concave as follows:

$$\frac{\partial^2 U}{\partial P_i^2} < 0 \quad \text{(B.8)}$$

Let y_i be an indicator that represent whether player i participates in the game or not. If player i select its best strategy (i.e., adjust optimal power) $y_i = 1$; otherwise $y_i = 0$. Thus, we define a nonnegative weighted sum of utility functions as:

$$f(P, y) = \sum_{i=1}^{N_u} y_i U_i(P_i, P_{-i}, S_I) \quad \text{(B.9)}$$

It is obviously that

$$\frac{\partial^2 f(P, y)}{\partial P_i^2} < 0 \quad \text{(B.10)}$$

From (B.3), (B.4) and (B.8), we can conclude that the optimal point of the utility function is unique. On the other hand, we know the optimal point of the function is the solution to the problem where each player selects its optimal power to the strategies of the other players i.e., GE. It can be concluded that there exists a GE for the game and it is unique. So, the theorem holds. ■

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