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Multi-level network characterization using regular topologies

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***Abstract*— Multi-level networks have been a good solution in large scale networks scenarios. The implementation of a network into different levels or sub-layers, improves the performance and reduces the investment against plain topologies. This paper tries to characterize important parameters on multi-level networks such as diameter, average distance and gateway location so that to be able to optimize the global network topology with no need for additional path calculations. The study focuses on the lower level of the network formed by subnetworks with regular structures such as Single Ring, Double Ring and Torus Grid. The achieved results will ease and improve the network planning of large scale networks.**

***Keywords*—Multilevel Networks, Subnetwork, Single Ring, Double Ring, Torus Grid, Gateway, Diameter, Average Distance**

I. Introduction

The network interconnection planning has to deal with the main properties: *node degree* (links at the nodes) and *diameter* (maximum path distance for any pair of nodes). The ideal network would have a low node degree which optimizes the economical investment on the network and low diameter which optimizes the performance of the network. The problem is that these two properties are in conflict [1]. Low degree (respectively high degree) networks usually involve a high (respectively low) diameter. The problem becomes critical with large scale networks where the transmission performance or the economical investment will not fulfil the requirements due to the size of the network. At scenarios with large number of nodes implemented as a plain topology, the two options are to build a high degree structure which involves a high investment but short distances, or the opposite, lower investment but longer distances [1].

Multi-level networks have become an option for large scale interconnection network schemes obtaining better properties than the mono-level networks. Hence, the multilevel issue is an interesting and useful topic to focus on. Multilevel networks are already being used, especially for distribution networks (backbones) [2]. Nowadays, this kind of networks usually consist of the highest level formed by a mesh network and the lower levels formed by single rings, e.g wired backbone networks. These rings can only offer two independent paths between any pair of nodes which, in case of failure of one link or node, overloads the network due to the rerouting of some of the traffic in the opposite direction. However, while a link or node is down, if another failure occurs some of the nodes will not be connected. Higher degree topologies can solve this problem due to the use of more independent paths. The first motivation for this study is to analyze the effects of substituting those sub-layer rings with higher degree topologies.

The multilevel connections have been treated from different points of view:

In [3]- [5] the goal was to obtain a solution using algorithms, such as path calculations, to interconnect a set of nodes optimizing the diameter and arcs connecting the nodes. This type of optimization can be done only for small networks due to the exponential growth of the number of paths calculated. In [6] the performance of routing at different link configurations are tested and in many other cases a regular topology is studied to find the way of dividing it in multi-levels such as in [7] and [8]. However, these studies do not treat mixing different types of topologies. This paper will only focus on the lowest level subnetworks and especially the performance of three different topologies: Single Ring, Double Ring and Torus Grid. The reason for choosing exactly these topologies is to study the relation between the degree of the network (2,3 or 4) and their performance. In the future many other topologies can be studied in the same way to obtain a complete library for this topic such as N2R [9], Honeycomb and Chordal Rings [10].

The goal is to find mathematical properties of multi-level networks to be able to define and characterize the performance based on known regular topological information about their subnetworks. The key aspect of this depiction is to perform the characterization of using regular topologies without the need of additional path calculations, which is unavoidable in the case of irregular topologies.

The study of regular topologies in mono-level networks identifies relational patterns on the topology parameters, such as diameters and average distances, as function of the number of nodes and path. Extrapolating these ideas, it is possible to define similar patterns for multilevel networks with the difference that at the multilevel communications the distances calculated are from any node to the closest gateways (node connecting the levels). When pattern are identified, equations representing these patterns can be defined, making the performance of a network more deterministic with these characteristics.

It is assumed that the regular topologies as mono-level networks have been extensively studied and we will only focus on the multilevel interaction of the subnetwork forming the complete structure. It is considered that the internal transmissions of each subnetwork will not affect the multilevel transmissions, at least from the structural point of view. Therefore, the multilevel issue can be discussed independently of the mono-level issues of each of the components [4]. Looking into other traffic patterns is interesting, but not within the scope of this paper. It could be interesting for further research within the field.

The idea is to treat each subnetwork as independent and all the subnetworks are connected to a *Black box* representing the higher levels of the network, see Fig. 1. This idea of dividing the network into independent groups is not new. It has been successfully used to plan a two level network using heuristic algorithms. *The parameters of the subnetworks are independent from the higher level, therefore it can be assumed that the global parameters for the complete multilevel communication are the sum of the parameters of each level* (always considering that they are calculated for a multilevel purpose) [4].

Based on this assumption, in order to optimize the performance of the multilevel network, we should optimize the *fairness* among the different subnetworks. When the best configuration of the gateways is identified, formulas that represent the characteristics of the multi-level communications networks at the lowest hierarchical level are presented. The relative positions among the gateways in each sub-layer are important for the optimization of the subnetworks and the balancing of the traffic. The formulas obtained will help to decide the best topology to implement for each subnetwork, taking in consideration the rest of the subnetworks and trying to achieve similar performances for all of the subnetworks. Having similar performance at all the subnetworks guarantees that there is no waste of resources or specific critical situations which affect the whole network behavior.

The parameters used to analyze the performance of the network are *Diameter* and *Average Distance*. These two parameters have been extensively used to analyze and compare many different network structures in terms of *Structural Quality of Service, SQoS* [11]- [14]. *SQoS* can be defined as the level of performance of a network due to its structure or topology. Different network structures that have similar values of parameters, diameter and average distance, may not necessarily offer similar levels of SQoS, but to some extent they have the potential to [11]- [14]. But to be similar in all the performance aspects, maybe, some other addition managing tasks are required, (e.g traffic balancing). It is necessary to use some general parameters in order to obtain general results about the topologies and algorithms. However specific cases will always be subject to specific constraints, requirements, parameters etc. This study is dealing with the lowest network layer (physical) therefore it is focused only on the *SQoS*.

The structure of the rest of the document is as follows. Section II explains the motivation of this study and possible applications. Section III introduces all the concepts useful for the understanding of the paper such as the topologies, parameters, math and network definitions. Section IV explains the procedure of obtaining the mathematical properties and equations. Section V introduces formulas obtained for the three analyzed topologies. In Section VI the results are compared and deeply interpreted. Finally, Section VII reflects the conclusions extracted from the obtained results and it introduces further work to be developed in Section VIII.

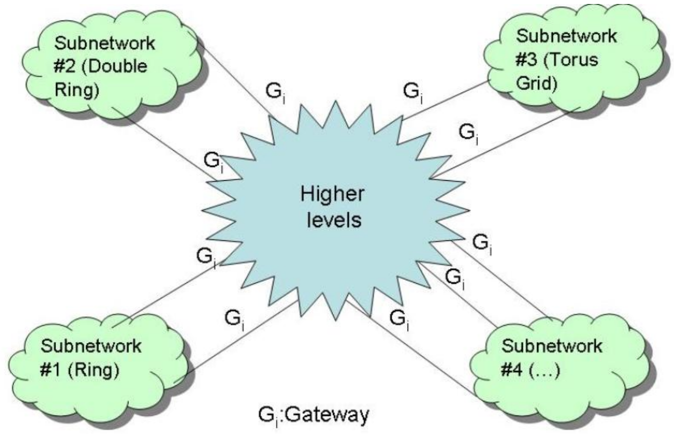


Fig. 1. Network Scheme

II. Motivation and Application

The motivation of this study is to improve and ease the multilevel networks planning and characterizations. For these purposes the first and second paths of the studied topologies are characterized with formulas, but it should be kept in mind that for future studies a third and fourth path could also be characterized. The number of gateways connecting the level will be two or three and possibly more gateways can be characterized in the future. Nowadays, the number of gateways is usually two, which limits the number of independent paths to only two [15]. Studying the possibility of having three gateways will allow in future analysis to consider the possibility of a third path, and similarly in the case of four gateways and fourth path. Adding a third gateway, or even a fourth, can improve the distances at a subnetwork if required by the network planning without increasing the degree of the topology.

Furthermore, the application of this study can be extended to two different scenarios:

A. Multilevel network from the scratch

Regions, subnetworks, with a given number of nodes divided into clusters to be interconnected with other nodes from other cluster groups. In the case of planning a network with these characteristics, the best solution is to obtain subnetworks with similar performances, it is what we define as *fairness*. If one of the subnetworks has better characteristics than the rest of the subnetworks, referring to multilevel communications (between two nodes from different groups), it will be a waste of resources since the limited performance of the global network would be lower than the performance of the specific better subnetwork. Therefore, the characteristics of the subnetwork will probably never be totally useful. In the opposite situation, if there is a subnetwork worse than the rest this network would limit the total performance and therefore the limitation on the QoS of possible implemented applications. Considering this assumption the characterization of the parameters of the topologies used at the subnetworks will help to optimize the global network as much as possible.

If the topology of each of the subnetworks is identified, then there are already several studies about the physical implementation of the network [16] - [19]. Furthermore, these results allow the network to be optimized not only in terms of performances but also budget.

B. Analysis and Enhancement of already deployed networks

In the case of networks already on the ground with the same characteristics (formed by regular subnetworks) the studied parameters would help to identify the performances of the network and therefore, to identify existing problems, their location and can offer possible improvements.

These two cases could be analyzed just using tables with the values of the parameters for each topology as a function of the number of nodes. Problems arise when there are several topologies to choose from and/or subnetworks to implement. It would be necessary to have as many tables as topologies and the computational time required would be long. Hence, the formulas will speed up the process for huge scenarios and in a theoretical way they ease the understanding of the behavior (performances) of the topologies.

III. Definitions

This section introduces the important concepts which must be explained for the proper understanding of the paper.

A. Topologies

The three main reasons for analyzing regular topologies are:

a) It is possible to define and document well-known parameters and metrics (e.g. number of independent paths) which ease the network characterization. Besides, based on well-know metrics it is easy to compare different topology designs in a proper way.

b) Based on regular topologies it is possible to define topological routing techniques which allow faster communications and the reduction of routing traffic within the network [20].

c) It is easier to add links to update a network in an organized way in case the improvements of the performances of the network are required. For example, it can be possible to update a Double Ring to a Torus-Grid in a systematic way, which makes the consequences of adding new links easier and predictable. Using irregular topologies this update will require longer and more difficult analysis.

This Section discusses the three topologies used in the study. The explanation only treats the structure and notation, for further information about the topological properties and advantages it is recommend to read the given references for each of the topologies [9] and [21]. Table I exposes the principal characteristics of each of the three topologies.

The number of nodes of the subnetwork is given by *N*, and each of the nodes has a label or id . The id is given by ID_T, where T is the topology of the nodes that will fulfil condition (1). Then, for example, when $id = 3dr$ it means the node 3 in the Double Ring structure.

$$0 \leqslant ID_T < N \tag{1}$$

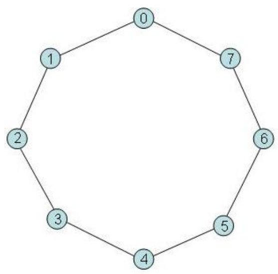
- Single Ring(SR): The number of nodes, *N*, is any positive integer larger than 1. All nodes in a Single Ring network are connected to two other nodes; thus the nodes in the structure are of second degree. The complete structure forms a circle [9]. See Fig. 2(a). The id of each node is called as ID_{SR}. Starting from any node of the network as $ID_{SR} = 0$ the rest of the nodes ID_{SR} will increment by one at each node counterclockwise.
- Double Ring (DR):It consists of two rings denoted inner and outer rings, see Fig. 2(b). These rings each contain the same number of nodes *(n)*; hence the number of nodes, $N = 2 * n$, being $n \geq 2$. The rings are interconnected by links between each corresponding pair of nodes in the inner and outer ring. The DR network is a third node degree network structure. [9]. The notation of this structure is similar to the ring, in this case the outer ring $0 \leqslant ID_{DR,outer} < n$ and the inner ring values $n \leqslant ID_{DR,inner} < N$. In this way each outer ring node $ID_{DR} = X$ is connected with the inner ring node $ID_{DR} = X + n$. For example, the Double Ring in Fig. 2(b) the outer ring nodes will have labels form 0 to 7 and the inner ring from 8 to 15.
- Torus Grid (TG): A torus grid network is obtained from a rectangular grid network by adding links between opposite nodes at the border grid [21], see Fig. 2(c). The result is a fourth degree structure. Depending on the number of nodes, there can be more than one possible grid structure. The rectangular structure is defined as $N = F_1 * F_2$. F_1 is the number of rows of the rectangle and F_2 is the number of columns. F_1, $F_2 \geq 2$. The increment of ID_{TG} is from left to right and form top to bottom. The configuration that has the minimal average distance is considered, this issue is deeply explained at subsection V-C.

	SR	DR	TG
N	n	$2n > 3$	$n_1 * n_2 > 3$
n	> 1	> 1	> 1
Degree	2	3	4

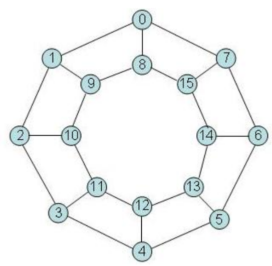
TABLE I
Topologies characteristics

B. Hierarchy

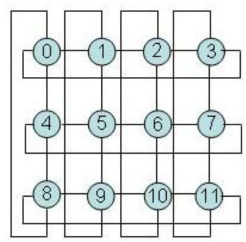
Network hierarchy is defined in [22] as a technique used to build scalable complex systems. Network hierarchy is an abstraction of the parts of a network's topology, routing and signaling mechanisms. Abstraction may be used as a mechanism to build large networks, "divide and conquer" or as a technique for enforcing administrative, topological, or geographic boundaries.



(a) Single Ring, N=8



(b) Double Ring, N=16



(c) Torus Grid, N=12 $F_1 = 3$ $F_2 = 4$

Fig. 2. Single Ring, Double Ring and Torus Grid Topologies

In terms of topological hierarchy it should be mentioned that it is a horizontal oriented hierarchy. It is defined as a large network that is partitioned into multiple smaller, non-overlapping sub-networks [22]. The partitioning criteria can be based on topology, network function, administrative policy, or service domain demarcation. In this paper the partitioning is based on topologies.

C. Notation

Along the document there are several abbreviations and mathematical notations that are summarized and explained as follows:

Math Notation:

- $[x]$ is the integer part of x
- $\{x\}$ is the fractional part of x

Parameter's attributes:

- T: Topology. This attribute can be:
  - SR: Single Ring
  - DR: Double Ring
  - TG: Torus Grid
- N: Number of nodes at each subnetwork
- L: Nodes linked to the higher level.
- P: Path number

Parameters:

- $D_{T,N,L,P}$: Diameter
- $A_{T,N,L,P}$: Average distance

The notation along the document uses the known parameters as subindex and the parameters working as variables between the brackets. For example, $A_{SR,1}(N, L)$ represents the Single Ring's average distance of the first path ($P = 1$) in function of the number of nodes N and the number of gateways L.

Other Variables:

- $E_{T,N,L,P}$: Correction variable
- G_i: Gateway. Going from G_1 to G_L (L is the number of used gateways)

D. Parameters:

The parameters used to characterize the three topologies are:

- Diameter ($D_{T,N,L,P}$): This value corresponds to the maximum number of hops needed to leave the subnetwork to the higher level or vice versa. This value is calculated in function of previous commented attributes T, N, L and P:
  The gateways are the nodes linking levels named as G_j being $0 < j \leq L$, each subnetwork will have L gateways. For logical representation each gateway is linked to another node belonging to the higher level, but in reality, probably these gateway nodes would be located in the same building and physically belonging to both levels. The reason for considering two separate nodes is that logically the process of level changing even at the same node can be considered as one hop just by itself.
  Let $d_i(P)$ be the shortest distance ($P = 1$) or the second shortest distance considering no links nor nodes in common with the first path ($P = 2$) from any node i to the closest gateway G_j. Then the diameter value is given by Equation (2).
  $$D_{T,N,L,P} = max(d_i(P)) \quad (2)$$
- Average distance ($A_{T,N,L,P}$): This value corresponds to the average number of hops needed to leave the subnetwork to the higher level or vice versa in function of the same parameters as the diameter.
  The value of the average distance considering the same rules as the diameter for $P = 1$ and $P = 2$ is given by Formula (3).
  $$A_{T,N,L,P} = \frac{1}{N}\sum_{i=1}^{N} d_i(p) \quad (3)$$

IV. Methodology

This section treats the methodology for obtaining the equations for the different cases. The number of independent or disjoint paths treated in this paper is just two, but in the future a third or fourth path analysis can be included (for the topologies that allow them, the number of independent paths is given by the node degree of the topology). The procedure is defined as the four following steps:

1) Sweeping calculation
2) Patterns identification
3) Formulas definition
4) Verification

The first step is to perform a sweeping to obtain information about the possible optimal values of the studied parameters and the configuration. The sweeping process calculates the distances of the paths from all the nodes to the gateways (nodes linked to the higher level). Due to the use of regular topologies, the distances calculation is relatively simple applying distance formulas defined at [8] and [9]. Based on these formulas an algorithm is implemented to obtain the values of the studied parameters. This procedure must be repeated for all the possible gateway combinations to identify the optimal configuration.

To find the optimal position of the gateways at the network, an implemented algorithm performs a sweep of the possible combinations of the nodes selected. The mechanism of the algorithm is to select one of the nodes (it can be any node due to the regularity of the topologies studied) that can be named as G_1. Then starting to sweep for all the possible values of the rest of the gateways the parameters are calculated (diameters and average distances) and the best options are selected as G_2 and G_3 (in case of three links to the higher level). The result obtained gives the relative positions of G_2 and G_3 from G_1, and these relative positions can be expressed in function of the number of nodes *N*, deeply explained in Section V. G_1 can be any of the nodes of the network, therefore there will be *N* optimal configurations for the position of the gateways.

The best configurations change depending on the considered parameters, in this case, and in order, the minimum values of the following criteria:

1) Diameter of the first path ($D_{T,N,L,P=1}$)
2) Diameter of the second path ($D_{T,N,L,P=2}$)
3) Average distance of the first path ($A_{T,N,L,P=1}$)
4) Average distance of the second path ($A_{T,N,L,P=2}$)

The reason for giving priority to the diameter values is due to their role at the time of guaranteeing a certain level of performance for the network. These values can be considered as the worst case possible. Therefore, there is a defined limit for communications between any pair of nodes (in this case from a node to a corresponding gateway).

The rest of the calculation consists in the increase of the number of nodes N which gives as a result a deterministic series for each of the four studied parameters, ($D_{T,N,L,P=1}, D_{T,N,L,P=2}, A_{T,N,L,P=1}, A_{T,N,L,P=2}$). The performance of this calculation results in numeric series in function of the number of nodes and the optimal gateway configuration. Based on these series, patterns on the values of the parameters can be identified and these patterns are used to define mathematical formulas to characterize each of the topologies at Section V. The gateways optimal position can also be defined with formulas as a function of the number of nodes N. These formulas will also be developed at Section V.

The last step in the procedure is to verify that the formulas obtained are correct. The results of the sweep calculation and the formulas are compared and verified that they are identical.

V. Mathematical Analysis

This section treats the equations and parameters found to characterize the subnetworks forming the lowest level of the network. As an introduction, the following paragraphs introduce the properties and formats of the formulas obtained for a better understanding.

Diameter:

Theorem 1: The diameter follows a stair distribution with N. *The difference between $D_{T,N,L,P}$ and $D_{T,N-1,L,P}$ is always 0 or 1 hop, which defines the increment of the stair step on 1 hop in respect to the previous step (constant increment). The same value of $D_{T,N,L,P}$ is related to a set of consecutive values of* N. *The number of consecutive values of* N *is constant in each of the cases (the same for the same values of* T, L *and* P*) given by $\#SET$. O_f, gives the starting value of N for each of the sets with the same diameter. See Equation (4)*

$$D_{T,N,L,P} = \left\lceil \frac{N + O_f}{\#SET} \right\rceil \tag{4}$$

Average Distance:

Theorem 2: The average distance of a subnetwork can be defined as a sum with two terms, an initial condition IC and S, depending on the index of summation. The index range is directly related to the number of nodes. See Equation (5)

$$A_{T,N,L,P} = IC + \sum S \tag{5}$$

Theorem 3: Let U_B be the upper bound and L_B the lower bound of the sum for N *nodes. The equations are defined in such a way that the upper bound of a sum of* N-1 *nodes correspond to $S(U_B - 1)$. Then $S(U_B)$ can be defined as the difference between $A_{T,N,L,P}$ and $A_{T,N-1,L,P}$. See Equations (6)*

$$\begin{aligned} &A_{T,N-1,L,P} = IC + \textstyle\sum_{i=L_B}^{U_B-1} S(i) \\ &A_{T,N,L,P} = IC + \textstyle\sum_{i=L_B}^{U_B} S(i) = IC + A_{T,N-1,L,P} + S(U_B) \\ &A_{T,N,L,P} - A_{T,N-1,L,P} = S(U_B) \end{aligned} \tag{6}$$

Therefore, applying this assumption to any value of *N*, $S(i)$ is the marginal increment of the sum.

Equation (5) in some special cases is not completed. The series has small variations on some specific values of *N*. This variation is due to the variance of the relation between N and L (N/L). The variable $E_{T,N,L,P}$ is introduced to correct this variance and it will have the format illustrated by Equation (7) where *REP* is the cycle of the error which is constant for the same values of *T,L* and *P*; N_{err} is any value of N where the error occurs and E is the value of the variance between the expected value and the real value.

$$E_{T,N,L,P} = \quad E * \left(\left[\tfrac{N+REP-N_{err}}{REP} \right] - \left[\tfrac{N+REP-N_{err}-1}{REP} \right] \right) \tag{7}$$

Analyzing Equation (7), $E_{T,N,L,P} = 0$ for all N values different than N_{err} and at the exact values of N_{err}, $E_{T,N,L,P} = E$, which is required to have a modification on the series. [1]

Gateway position:

This value represents the relative position of the nodes linked to the higher level to obtain the best results possible concerning the two previous values.

The calculation of the relative position is only required from a fixed value of G_1 due to the regularity of the structure and

[1] As commented at Subsection III-C, the symbols [] are the integer value of the operation.

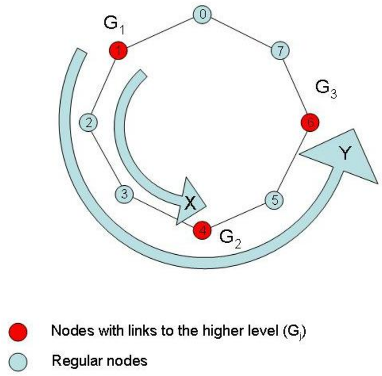


Fig. 3. Gateways Position Example

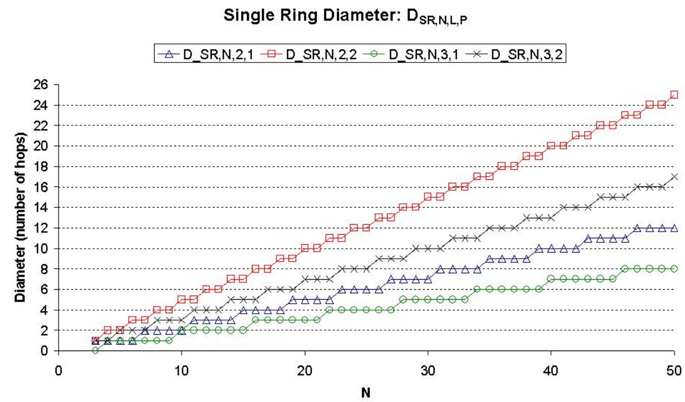


Fig. 4. Single Ring, Diameters for *L=2* and *P=1*; *L=2* and *P=2*; *L=3* and *P=1*; and *L=3* and *P=2* respectively

then the result can be applied to any possible node position of G_1. The notation is defined by Equation (8):

$$\begin{aligned} &G_1 \\ &G_2 = (G_1 + X)mod(N) \\ &G_3 = (G_1 + Y)mod(N) \end{aligned} \tag{8}$$

Being $1 \leq G_1 \leq N$ and X and Y any positive integer, considering $mod(N)$ for any resulting position of G_2 or G_3. In most of the cases there is more than one solution for the relative position of the gateways. To be able to define general equations for all those possibilities variables $V_{T,I}$ is used and depending on the situations its value will change; where I is just the numbering of the variables for the same topology.

Fig. 3 illustrates an example of these values and their meaning.

The rest of the Section is divided into Subsections V-A, V-B and V-C which analyze the cases of Single Ring, Double Ring and Torus Grid respectively. Section IX completes the topologies analysis by presenting several tables with all the formulas developed. Some of these formulas are used to represent more general equations when it is possible. These tables, presented at Section IX, are very useful for following the mathematical approach of each of the topologies and the comparison between them. The presentation of the tables in an independent Subsection at the end of this Section is for esthetic purposes.

Along the rest of the Section graphical representations of the equations obtained are given. The graphs are generated by the formulas and the results perfectly match the values obtained at the sweeping calculation. These graphs help to illustrate the patterns followed in the different cases and topologies used to define the formulas.

A. Single Ring (SR)

The Single Ring analysis gives simple equations that are very useful to identify each term explained at the introduction of this Section. Based on the Single Ring equations at Table V, general equations for the diameter, Equation (9), and average distance, Equation (10) can be formulated for all the possible configurations.

Theorem 4: The diameter for the Single Ring topology follows a stair distribution with N. The number of consecutive values of N with the same diameter, ($\#SET$), is given by $2L/P$ and O_f corresponds to the term $L-P$. See Equation (9).

PROOF:

$$D_{SR}(N,L,P) = \left\lceil \frac{(N+L-P)*P}{2L} \right\rceil \tag{9}$$

Fig. 4 illustrates the graphical representation of Equation (9), for all the possible values of L and P up to 50 nodes in the network. It can be easy to identify the pattern described by the variable $\#SET$ of each of the cases and the increment of 1 hop in respect of the previous value of diameter (in the case that there is a change on this value).

The values from Table VI for the Single Ring help to define a general equation for the average distances. The average distance ($A_{SR}(N,L,P)$)is related with $D_{SR,1}(N,L)$ and it corresponds to Equation (10):

Theorem 5: The average distance for the Single Ring is always in function of the diameter of the first path, regardless the value of L, N and P. See Equation (19).

$$A_{SR}(N,L,P) = \frac{(2*P-1)}{N} * \sum_{i=3}^{N} \underbrace{\left\lceil \frac{i+L-1}{2L} \right\rceil}_{D_{SR,1}(i,L)} + E_{SR}(N,L,P) \tag{10}$$

The term $E_{SR}(N,L,P)$ corrects the variation at some values of N and its value can be found at Table VIII.

Theorem 6: When $E_{SR}(N,L,P)=0$, the average distance for the second path is 3 times the one for the first path. See Equation (11).

$$A_{SR}(N,L,2) = 3 * A_{SR}(N,L,1) + E_{SR}(N,L,2) \tag{11}$$

Fig. 5 illustrates the average distance of the different cases ($L = 2,3$ and $P = 1,2$) for the Single Ring. As expected the average distances for *L=3* are shorter than *L=2*, of around

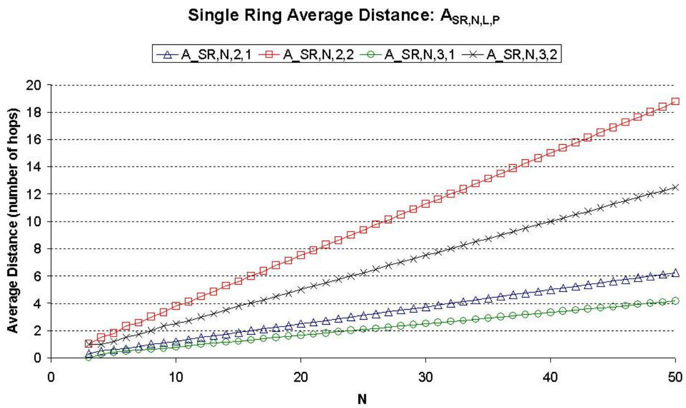


Fig. 5. Single Ring, Average Distance for *L=2* and *P=1*; *L=2* and *P=2*; *L=3* and *P=1*; and *L=3* and *P=2* respectively

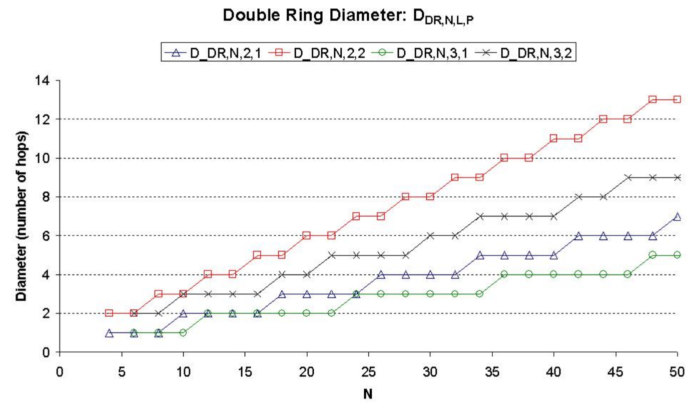


Fig. 6. Double Ring, Diameters for *L=2* and *P=1*; *L=2* and *P=2*; *L=3* and *P=1*; and *L=3* and *P=2* respectively

33%. It can be clearly identified how the increment of the average distances is linearly proportional to N.

The term $E_{SR}(N, L, P)$ of Equations (10) and (11) can be summarized by Equation (12).

$$E_{SR}(N, L, P) = \begin{cases} \frac{4-L+2*(N(mod\ 2))}{N} & \text{if condition (13)} \\ 0 & \text{rest} \end{cases} \tag{12}$$

$$\begin{aligned} &\left\{\tfrac{N+X}{L}\right\} = 0 \ \ \& \ \ \tfrac{N+X}{L}(mod\ \ 2) = 1 \ \ \& \ \ P = 2 \\ &being - (L-2) \leq X \leq (L-2) \end{aligned} \tag{13}$$

The gateways positions, presented at Tables IX and X, represent the relative position of the nodes linked to the higher level in the subnetwork. For most of the values of *N* there are more than one solution for these relative positions. To identify these solutions and the corresponding *N* for each one, *N* can be defined as a function of a variable $k \in \mathbb{N}$

The values of $V_{SR,1}$, $V_{SR,2}$ and $V_{SR,3}$ presented at Tables IX and X are exposed at Table XII. These terms are used to define general equations for the gateways positions. The values of $V_{SR,1}$ and $V_{SR,3}$ are independent but $V_{SR,2}$ depends on $V_{SR,1}$.

The extracted conclusion from these results is that there is a direct relation between the relative position of the gateways and the number of optimal configurations with the value of *L*. Making the proper substitutions and combinations in the equations the number of different optimal solutions for the relative position of the gateways can be identified, (the total number of optimal configuration is *N* times these values since G_1 can be any node of the Single Ring). See Table XIV.

B. Double Ring (DR)

The study of the Double Ring gives some simple equations, shown at V, to be able to define a general equation for the diameter values. See Equation (14).

Theorem 7: The diameter for the Double Ring topology follows a stair distribution with N. The number of consecutive values of N with the same diameter, ($\#SET$), is given by $4L/P$. See Equation (14).

$$D_{DR}(N, L, P) = \left\lceil \frac{(N - O_{fDR}) * P}{4L} \right\rceil + 1 + E_{DRDiam} \tag{14}$$

The term $E_{DR(Diam)}$ is an error on the series for the value of the diameter $D_{DR,3,2}(N)$. The error in this case comes from the chosen positions of the gateways. As explained at Section IV the first criterion to decide the optimal position of the gateways is to optimize the first path diameter ($D_{DR,3,1}(N)$). In this case the optimization costs an increment of the value of the second path diameter ($D_{DR,3,2}(N)$). If the criterion to be to optimize is ($D_{DR,3,2}(N)$) then the error would be at the $D_{DR,3,1}(N)$ equation. In this case $E_{DR(Diam)}$ corresponds to formula (15).

$$E_{DR(Diam)} = \begin{cases} 1 & \text{if } \left\{\frac{N+2}{6}\right\} = 0, \ P = 2 \text{ and } L = 3 \\ 0 & \text{rest} \end{cases} \tag{15}$$

The value of O_f of Equation (14) is presented at Condition (16). Of is 2 when $L = 2$ and $P = 1$. Fig. 6 illustrates the graphical representation of Equation (14), the representation is under the same conditions as the Single Ring case. It can also be easily identified the pattern described by $\#SET$ and the increment of the diameters.

$$O_{fDR} = \begin{cases} 2 & \text{if } P = 1 \text{ and } L = 2 \\ 0 & \text{rest} \end{cases} \tag{16}$$

Unfortunately for the average distances ($A_{DR}(N, L, P)$) it was not possible to find a general equation based on the results presented at Table VI. Therefore, each case of L and P has its own equation. In any case, looking at each equation individually, the terms S and IC can be easily identified. Fig. 7 illustrates the results for the average distances. It has to be

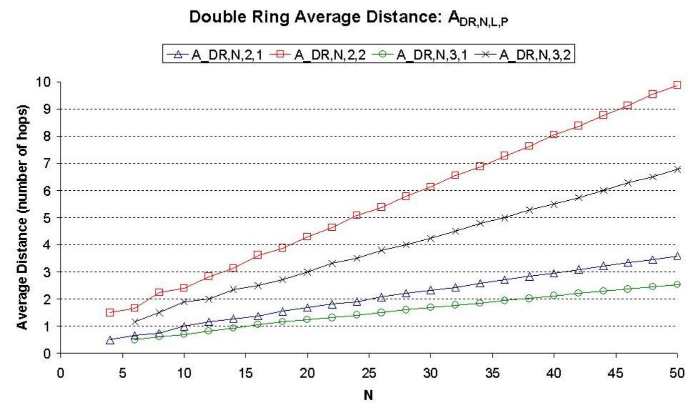


Fig. 7. Double Ring, Average Distance for *L=2* and *P=1*; *L=2* and *P=2*; *L=3* and *P=1*; and *L=3* and *P=2* respectively

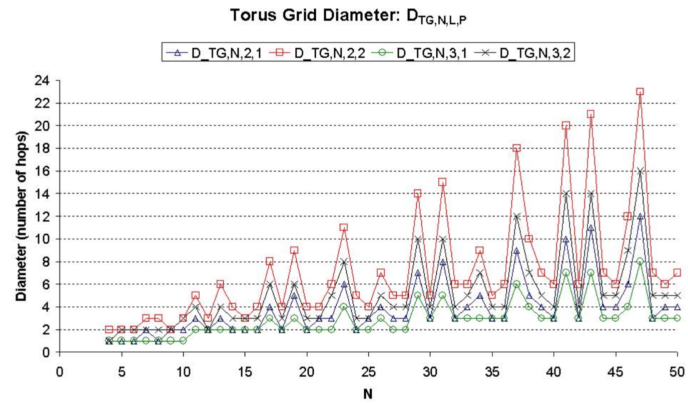


Fig. 8. Torus Grid, Diameters for *L=2* and *P=1*; *L=2* and *P=2*; *L=3* and *P=1*; and *L=3* and *P=2* respectively

noted that for a Double Ring there is no possibility to have odd values of *N*.

Tables IX and XI, show the optimal relative position of the gateways in a Double Ring topology. The value of *N* is defined as a function of a variable $k \in \mathbb{N}$, as in the Single Ring case, to identify the different solutions. Table XII exposes the values of the variable used at Table IX, $V_{DR,1}$, $V_{DR,2}$ and $V_{DR,3}$, in this case as a function of $V_{DR,2}$ since $V_{DR,3}$ depends on it.

In the case of $L = 3$ the location of the gateways equations becomes complex. Therefore, the gateways possibilities are split in *m* options where $1 \leq m \leq Number\ of\ possibilities$ and they are treated separately from the positions when $L = 2$.

The division is basically following the scheme represented at Table XI, in function of m and the values $V_{DR,4}, V_{DR,5}, V_{DR,6}\ and\ V_{DR,7}$ (Table XIII) to represent the different solutions for the same configurations.

Making the proper substitutions on the equations on Table XI with the values of Tables XIII and XII the number of possible optimal solutions for the relative position of the nodes is presented at Table XIV.

C. Torus-Grid (TG)

For this topology depending on the number of nodes there can be more than one configuration. As explained at subsection III-A the structure can be defined as $F_1 \mathrm{X} F_2$, F_1 being the number of rows and F_2 the number of columns. F_1 and F_2 must be factors of *N*, $F_1 * F_2 = N$. Therefore, when *N* is a number that can be divided into more than 2 different factors, different than 1 or *N* (that are not primes), there is more than one possibility; (the truth is that when there is only 2 factors there are two possibilities $F_1 \mathrm{X} F_2$ or $F_2 \mathrm{X} F_1$; but it will be assumed to be the same topology for the rest of the study).

Theorem 8: Assuming that $F_1 \leq \sqrt{N}$ and, therefore $F_2 \geq \sqrt{N}$ the best configuration in terms of diameter and average distance from any pair of nodes is the one which F_1 is the possible factor of N *closest to $\sqrt{N}$.*

In any case the following formulas are valid for all possible configurations and not only the optimal ones. At Table V the diameter equations are presented. The term E_{TGDiam} at Table V is the same as the Double Ring case (E_{DRDiam}). A Torus-Grid with $F_1 = 2$, in this study, behaves exactly the same way as a Double Ring. The structure of the topology is different since the degree of the Double Ring is 3 and of the Torus-Grid is 4, but it does not affect the parameters and cases studied. Therefore, E_{TGDiam} will be different than 0 only when $F_1 = 2$ and condition (15) is fulfilled.

The value of the offset O_{fTG} which appears at one term of Table V is presented by Formula (17).

$$O_{fTG} = \begin{cases} 4 * \left\{\frac{N+1}{2}\right\} + 10 * \left\{\frac{N}{2}\right\} & P = 1 \text{ and } L = 3 \\ 0 & \text{rest} \end{cases} \tag{17}$$

Fig. 8 illustrates the representation of the diameters values for the Torus Grid. For each value of N the best configuration is represented (best F_1 possible). The shape of the graph is different than the previous representations of the diameter of the Single Ring or Double Ring which have values are incrementing in a smooth way. The reason for this fluctuation in the graph of the Torus Grid is caused by the values of F_1 and F_2. For some values of N, the best $F1$ possible is $F_1 << \sqrt{N}$, for example, for N being prime, the only possibility is $F_1 = 1$. Small values of F_1 give longer distances than the values of $F_1 \approx \sqrt{N}$, hence, the best values of *N* for being implemented as a Torus Grid are the ones with a possible $F_1 \approx \sqrt{N}$.

The average distance formulas obtained are complex but dividing them in different parts they are much simpler to understand. The studied parameters on the Torus Grid depend on the mentioned factors F_1 and F_2. For the different values of F_1 and F_2 the pattern on the series of the results changes. Therefore, it is necessary to unify all the possibilities in one global equation.

So far, the average distance was represented as sum, and at some occasions an initial condition was added to the sum. In this case the average distance $A_{TG}(N, L, P)$ needs a third term considered as another initial condition. Of course, the two values of IC_1 and IC_2 could be treated as one variable, but it is much easier to use two to analyze the progression of the series. See Equation (18). The value of each term is represented at Table VII

$$A_{TG}(N,L,P) = IC_1 + IC_2 + \sum S \tag{18}$$

IC_1 is the initial condition of the series of the average distance, depending on F_1. All the configurations with the same F_1 have the same IC_1. The term IC_2, depending on F_1 and F_2, is the initial condition of S. The term S can be defined as usual, see *Theorem 2*, but with the difference that this increment is not constant, it depends on F_2.

IC_1/N corresponds exactly to the average distance when $F_1 = F_2$ and $(IC_1 + IC_2)/N$ correspond to the average distance when $F_1 + 1 = F_2$ (reason why at Table VII the sum starts at a value of $F_1 + 1$). These terms are very helpful at the time of understanding the behavior, but in the case of $L = 3$ it was not possible to find any of these values due to the high asymmetry of the structure. The reason of this asymmetry is the combination of L, an odd number, with the topology with even degree (the degree of the Simple Ring is an even number as well but due to the simplicity of the structure it was possible to find a solution). Only for some value of F_1 there is a predictable series.

The values of $E_{TG,2,1}(i)$ in Table VI are presented by Formula (19) and the values of $E_{TG,2,2}(j)$ are given by Formula (20). In these cases the errors are inside the sum, therefore, if the errors are not considered, errors propagates with the increment of F_2. Due to the complexity of the formulas three variables (e_1, e_2 and e_3 are defined at Equation (21) to ease the presentation of $E_{TG,2,2}(j)$

$$E_{TG,2,1}(i) = \begin{cases} \left[\frac{F_1}{2}\right] & \text{if } \left\{\frac{i+[(F_1)/2]-1}{4}\right\} \geq 0.5 \\ 0 & \text{rest} \end{cases} \tag{19}$$

$$E_{TG,2,2}(j) = \begin{cases} 3 * F_1 & \text{if } \left\{\frac{i+e_1}{4}\right\} = 0 \ \& \left\{\frac{F_1}{4}\right\} = 0 \\ 2 * F_1 & \text{if } \left\{\frac{i-e_1}{4}\right\} = 0 \ \& \left\{\frac{F_1}{2}\right\} = 0 \\ e_2 + 2 * F_1 + \left[\frac{F_1}{2}\right] & \text{if } \left\{\frac{i+1}{4}\right\} = 0 \ \& \left\{\frac{F_1+1}{2}\right\} = 0 \\ e_3 + 2 * F_1 + \left[\frac{F_1}{2}\right] & \text{if } \left\{\frac{i-1}{4}\right\} = 0 \ \& \left\{\frac{F_1+1}{2}\right\} = 0 \\ 0 & \text{rest} \end{cases} \tag{20}$$

$$\begin{array}{l} e_1 = 1 + \left\{\frac{F_1+2}{4}\right\} * 4 \\ e_2 = \begin{cases} 1 & \text{if } \left\{\frac{F_1+1}{4}\right\} = 0 \\ 0 & \text{rest} \end{cases} \\ e_3 = \begin{cases} 0 & \text{if } \left\{\frac{F_1+1}{4}\right\} = 0 \\ 1 & \text{rest} \end{cases} \end{array} \tag{21}$$

Fig. 9 illustrates the average distances for the Torus Grid case. The values when $L = 3$ are obtained from the sweeping calculation since there was no possible formula found. The value of the average distance is strongly conditioned by the value of F_1 and F_2. In the cases where $F_1 = 1$ (*N* prime number), the values of the averages distances are the same as the single Ring. In the cases where $F_1 = 2$ (*N/2* prime number) these values correspond to the same values of the Double Ring. The same characteristics as at the diameter study that the values of *N* with a possible $F_1 \approx \sqrt{N}$ are more likely to be implemented as a Torus Grid than the ones with lower F_1.

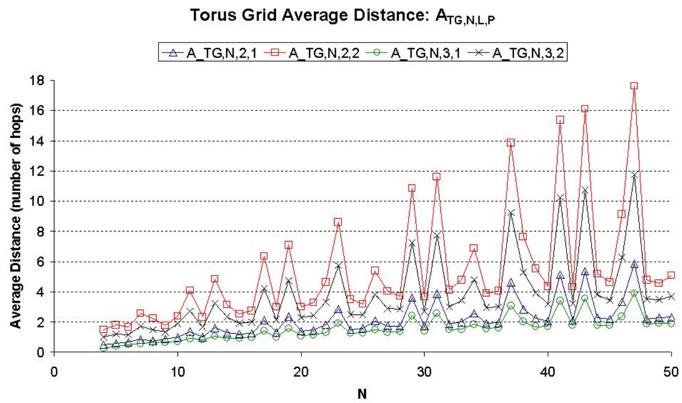


Fig. 9. Torus Grid, Average Distance for *L=2* and *P=1*; *L=2* and *P=2*; *L=3* and *P=1*; and *L=3* and *P=2* respectively

For the gateway position when $L = 2$, it is easier to split the cases in two different tables, when F_1 is an odd or even number (Table IX). The values of $V_{TG,1}$, $V_{TG,2}$, $V_{TG,3}$ and $V_{TG,4}$ are represented at Table XII. In this case as a function of $V_{TG,3}$ since $V_{TG,2}$ and $V_{TG,4}$ depend on it. As in the previous topologies, making the proper substitutions the resulting number of the possible solutions for the relative position of the gateways is presented in Table XIV.

The gateways position when $L = 3$ is very complex due to the huge number of possibilities. For example, when $F_1 = 3$ at some values of *N* there are 18 different solutions. All the possibilities will not be commented, only a few clues are given to know where the gateways are optimally placed. The method is first to select three of the rows that are further away from each other. Then, repeat the same operation with the columns. At this moment, the potential gateways are identified as the crossing points of the rows chosen with the columns chosen. The last step is to select the crossing points that are the furthest away from each other as possible. Equation 22 gives one of the possible solutions for the position of the gateways among all the optimal ones.

$$\begin{array}{l} G_1 \\ G_2 = G1 + \left[\frac{F_1}{3}\right] * F_2 + \left[\frac{2*F_2}{3}\right] \\ G_3 = G1 + \left[\frac{2*F_1}{3}\right] * F_2 + \left[\frac{F_2}{3}\right] \end{array} \tag{22}$$

Basically, what the terms represent are the rows (F_1 terms) and columns selected (F_2 terms). Then the rows are combined with the columns in a way that the column of G_2 is the furthest away from the column of G_1. It is easier to understand this idea with an example, Fig. 10 relates the terms calculated with the scheme of the network. It has to be kept in mind that this is only one of the solutions for the relative position of the gateways. In the case of the example $N = 16$ there are 26 other different solutions to obtain the optimal studied parameters.

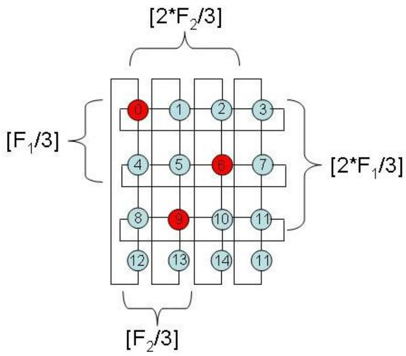


Fig. 10. Torus Grid, Example Of Gateways Position. N=14 $F_1 = 4$ $F_2 = 4$

VI. Case of Study

This Section treats the analysis of the results obtained at Section V. The best way to study the validity and application of all the data and formulas obtained is to test them at a real scenario.

There is a region with 100 fictive cities that has to be interconnected. The possibility of connecting them and forming a monolayer structure is discarded due to the high number of nodes and also the maximum distance among the nodes that can be tolerated. The option is to design a two levels networks. The cities are distributed in a way that the region can be divided in four subregions with 10, 20 ,30 and 40 cities. The goal is to find the best topology for each region but considering the fact that they should have similar properties. For reliability reasons two independent or disjoint paths must be provided between any pair of nodes of the complete network. It is assumed that the higher level (which is out of the scope of this study) can handle the requirements of two independent paths for any pair of nodes.

There are two criteria for choosing each subnetwork topology:

- Lowest degree possible but always considering equal performance characteristics for each of the subnetworks
- Lowest L possible for the topologies selected. Assuming that the topologies selected with $L = 2$ fulfils the requirements. If another gateway is added at the lower level, even thought at the lower level there are only benefits by adding the gateway, at the higher level a new gateway is also included and this extra node will make the distances longer at the higher level. There are as many nodes (gateways) in the higher level as total gateways of the subnetworks.

Table II represents the four studied parameters for the three topologies in relation with the number of nodes at each subnetwork. These data presented helps to explain the procedure of planning.

The procedure should start with the most restrictive subregion which is the one with more nodes. The goal is to optimize this subnetwork and then make the other subnetworks fit the performance of the subnetwork with 40 nodes. The better possibility for this subnetwork is Torus-Grid when $L = 3$. The criteria followed to choose the other topologies is the following:

- Similar diameter of the first path
- Similar diameter of the second path
- Similar average distance of the first path
- Similar average distance of the second path

Therefore, the solution is based on not obtaining diameters higher than the restricting subnetwork for the rest of regions. Table III illustrates the solution found after the proper comparisons. This scenario is relatively small and it can be analyzed with no additional implemented algorithms to find the best combinations, but for similar larger scenarios, many subdivisions and more topologies to choose from this algorithm would be very helpful and easily automatized to obtain the best solution.

The solution obtained for this scenario has different problems such as number of gateways which is the maximum. The main problem is the number of nodes linked to the higher level, 12, the maximum possible, and it is the number of nodes that the higher level would have. The other two problems are that the solution is not very accurate, there are some differences among the values for the same parameter at the four subnetworks, see Table III. Two of the four topologies are Torus-Grid which implies a high investment due to their degree four. This problem is mainly due to the limited topology

N	**10**	**20**	**30**	**40**
$D_{SR,2,1}$	2	5	7	10
$D_{SR,3,1}$	2	3	5	7
$D_{DR,2,1}$	2	3	4	5
$D_{DR,3,1}$	1	2	3	4
$D_{TG,2,1}$	2	2	3	3
$D_{TG,3,1}$	1	2	3	3
$D_{SR,2,2}$	5	10	15	20
$D_{SR,3,2}$	3	7	10	13
$D_{DR,2,2}$	3	6	8	11
$D_{DR,3,2}$	3	4	6	7
$D_{TG,2,2}$	3	4	11	6
$D_{TG,3,2}$	3	3	7	4
$A_{SR,2,1}$	1.2	2.5	3.73	5
$A_{SR,3,1}$	0.8	1.65	2.5	3.32
$A_{DR,2,1}$	1	1.7	2.3	2.9
$A_{DR,3,1}$	0.7	1.25	1.7	2.1
$A_{TG,2,1}$	1	1.4	1.7	2.05
$A_{TG,3,1}$	0.7	1.1	1.4	1.7
$A_{SR,2,2}$	3.8	7.5	11.26	15
$A_{SR,3,2}$	2.5	5	7.5	10
$A_{DR,2,2}$	2.4	4.3	6.1	8.05
$A_{DR,3,2}$	1.9	3	4.2	5.5
$A_{TG,2,2}$	2.4	3	3.6	4.35
$A_{TG,3,2}$	1.9	2.3	2.7	3.2

TABLE II
Scenario Parameters

N	$Topology$	D_1	D_2	A_1	A_2
40	TG L=3	3	4	1.7	3.2
30	TG L=3	3	4	1.4	2.7
20	DR L=3	2	4	1.25	3
10	SR L=3	2	3	0.8	2.5

TABLE III
Solution 1

N	$Topology$	D_1	D_2	A_1	A_2
40	TG L=2	3	6	2.05	4.35
30	DR L=3	3	6	2.3	4.2
20	DR L=2	3	6	1.7	4.3
10	SR L=2	2	5	1.2	3.8

TABLE IV
SOLUTION 2

options (only 6) (SR *L=2*, SR *L=3*, DR *L=2*, DR *L=3*, TG *L=2* and TG *L=3*), with more topologies available, such as N2R, Chordal Rings or Honey Comb and $L > 3$, the result could be more precise.

Since the best performance criteria did not return appropriate results for this scenario, a better optimization must be achieved. The next example will treat the same scenario trying to find the balance among performance, number of gateways and budget. The solution is based on the previous one, trying not to increase the best parameters found too much .

Table IV illustrates the solution proposed. The performance has decreased but the number of gateways has been reduced which can compensate it. This solution consist on only one Torus-Grid which will also reduce the investment to build the network.

This example presents one of the applications of the study which has been analyzed. The multilevel network planning can be improved and, due to the unnecessary additional paths calculation, to easily find the balance among the subnetworks. It should be kept in mind that the methods help the planning, but the human factor should always be included.

VII. CONCLUSION

The multilevel study has returned some interesting conclusions. Before the parameters characterization the procedure to plan the lowest level of a multilevel network was to test some potential options, calculate all the paths for all the possible communications and then decide the best option.

The studies of regular topologies as monolayer networks give some symmetries that can be useful for a multilevel scenario but always considering that the relations between the number of nodes and gateways will add complexity at some cases. The degree of the structure is directly related with the complexity of the study. At the Single Ring analysis general equations were formulated and expressed in simple terms, but in the case of the Torus Grid topology analysis it was not possible to find an equation for some parameters (*L=3* and *P=1,2*).

The subnetwork of a multilevel network can be characterized using regular topologies. The use of these regular topologies allow to define parameters such as *average distance* and *diameter* as equations in function of the number of nodes *N* of the subnetwork, the links *L*, the path *P* and the *topology* considered. The values of these parameters follow well defined patterns and, therefore, they are deterministic in the way that the exact value of the diameter and the average distance can be estimated with no path calculation at all. The properties of the potential network structures are obtained just using the given equations. These presented equations are useful to optimize the lowest level of the network in such way that each subnetwork has the same performance as the rest of the subnetworks. This balance on the performance allows the optimization of the resources of the complete network and, hence, to take advantage of the network properties such as short diameters and short average distances and therefore short delays. The comparison of these options does not request a long procedure, therefore, the planning can be focused on other important aspects such as the fibre civilian construction and implementation.

However this study is not yet completed and to be able to plan for a multilevel network there must be analysis of the higher levels of the complete network. At the conclusion of this study, a complete large scale network can be planned and optimized in a shorter time and more efficiently than the techniques used nowadays.

VIII. FURTHER WORK

The main topic which must be considered as a future work is the study of higher levels of multilevel networks. In the same way they will be formed by regular topologies and it is expected to follow similar patterns which will allow the definition of general equations. Furthermore, future analysis can include other regular topologies as Honey Comb, N2R and Chordal Rings among others to be able to obtain accurate results for a more balanced performance of the complete network.

ACKNOWLEDGMENT

The authors would like to thank Thibault Julien Renier, Rasmus Hjorth Nielsen and Muhammed Tahir Riaz for their help on improving this work.

IX. APPENDIX:TABLES OF THE RESULTS

SR	**L=2**	**L=3**
P=1	$\left\lfloor \frac{N+1}{4} \right\rfloor$	$\left\lfloor \frac{N+2}{6} \right\rfloor$
P=2	$\left\lfloor \frac{N}{2} \right\rfloor$	$\left\lfloor \frac{N+1}{3} \right\rfloor$

DR	**L=2**	**L=3**
P=1	$\left\lfloor \frac{N-2}{8} \right\rfloor + 1$	$\left\lfloor \frac{N}{12} \right\rfloor + 1$
P=2	$\left\lfloor \frac{N}{4} \right\rfloor + 1$	$\left\lfloor \frac{N}{6} \right\rfloor + 1 + E_{DRDiam}$

TG	**L=2**	**L=3**
P=1	$\left\lfloor \frac{F_2+1+[(F_1+1)/2]*2}{4} \right\rfloor$	$\left\lfloor \frac{F_2-F_1+O_f}{6} \right\rfloor + \left\lfloor \frac{F_1}{2} \right\rfloor$
P=2	$F_2 - \left\lfloor \frac{F_2+1}{2} \right\rfloor + \left\lfloor \frac{F_1}{2} \right\rfloor$	$\left\lfloor \frac{F_2+F_1-2}{3} \right\rfloor + 1 + E_{TGDiam}$

TABLE V
$D(T, N, L, P)$

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SR	**L=2**	**L=3**
P=1	$\frac{1}{N} * \sum_{i=3}^{N} \left[\frac{i+1}{4}\right]$	$\frac{1}{N} * \sum_{i=3}^{N} \left[\frac{i+2}{6}\right]$
P=2	$\frac{1}{N} * \sum_{i=3}^{N} 3 * \left[\frac{i+1}{4}\right] + E_{SR,2,2}(N)$	$\frac{1}{N} * \sum_{i=3}^{N} 3 * \left[\frac{i+2}{6}\right] + E_{SR,3,2}(N)$
DR	**L=2**	**L=3**
P=1	$\frac{1}{N} * \sum_{i=2}^{N/2} 2 * (\left[\frac{i-1}{4}\right] + 1)$	$\frac{1}{N} * (1 + \sum_{i=3}^{N/2} (\left[\frac{i-2}{6}\right] + \left[\frac{i}{6}\right] + 2))$
P=2	$\frac{1}{N} * (2 + \sum_{i=2}^{N/2} (6 * \left[\frac{i+3}{4}\right] - 2) + E_{DR,2,2}(N))$	$\frac{1}{N} * (\sum_{i=1}^{N/2} (2 * \left[\frac{i}{2}\right] + 1) + E_{DR,3,2}(N))$
TG	**L=2**	**L=3**
P=1	$\frac{1}{N} * (IC_{1,TG,2,1} + IC_{2,TG,2,1} + \sum_{i=F_1+1}^{F_2-1} S_{TG,2,1}(i))$	Fig. 9
P=2	$\frac{1}{N} * (IC_{1,TG,2,2} + IC_{2,TG,2,2} + \sum_{i=F_1}^{F_2-1} S_{TG,2,2}(i))$	Fig. 9

TABLE VI

$A(T, N, L, P)$

$IC_{1,TG}(N, L, P)$	**L=2**
P=1	$\sum_{i=1}^{F_1-1} \sum_{j=1}^{i} 2 * i$
P=2	$\sum_{i=1}^{F_1-1} \sum_{j=1}^{i} (6 + 12 * \left[\frac{j}{2}\right]) * \left\{\frac{j}{2}\right\} * 2 + \left\{\frac{j+1}{2}\right\} * 4 * j$
$IC_{2,TG}(N, L, P)$	**L=2**
P=1	if $F_1 = F_2$ $IC_{2,TG} = 0$ else $\sum_{j=1}^{F_1-1} \left[\frac{i+1}{2}\right] * 2$
P=2	$\sum_{i=0}^{F_1-1} (i+1) * \left\{\frac{i}{2}\right\} * 2 + (i * 3 + 2) * \left\{\frac{i+1}{2}\right\} * 2$
$S_{TG}(i, L, P)$	**L=2**
P=1	$(\left[\frac{F_1-2+i-\{N/2\}*2}{4}\right] + 1) * F_1 + E_{TG,2,1}(i)$
P=2	$IC_{2,TG,2,2}(N) + \sum_{j=F1}^{i-1} F_1 * 2 - E_{TG,2,2}(j)$

TABLE VII

TORUS GRID, IC_1, IC_2 AND S WHEN $L = 2$

SR	**L=2**	**L=3**
P=1	0	0
P=2	$2 * (\left[\frac{N-2}{4}\right] - \left[\frac{N-3}{4}\right])/N$	$(3 * (\left[\frac{N-3}{6}\right] - \left[\frac{N-4}{6}\right]) + (\left[\frac{N-2}{6}\right] - \left[\frac{N-3}{6}\right]) + (\left[\frac{N+2}{6}\right] - \left[\frac{N+1}{6}\right]))/N$
DR	**L=2**	**L=3**
P=1	0	0
P=2	$4 * (\left[\frac{N}{8}\right] - \left[\frac{N-1}{8}\right])/N$	$\frac{2}{N} * (\left[\frac{N+2}{12}\right] - \left[\frac{N+1}{12}\right]) + \frac{2}{N} * (\left[\frac{N-2}{12}\right] - \left[\frac{N-3}{12}\right])$
TG	**L=2**	**L=3**
P=1	Eq. (19)	0
P=2	Eq. (20)	0

TABLE VIII

$E(T, N, L, P)$

	$m = 1$	$m = 2$	$m = 3$
G_2	$G_1 + \left[\frac{N}{6}\right] + V_{DR,4}$	$G_1 + \left[\frac{N}{3}\right] + V_{DR,5}$	$G_1 + \left[\frac{2N}{3}\right] + V_{DR,6}$
G_3	$G_1 + G_2(m=1) + G_2(m=3)$	$G_1 + 2 * G_2(m=2) + V_{DR,7}$	$G_1 + G_2(m=1) + G_2(m=3)$

TABLE XI

DOUBLE RING, GATEWAYS POSITION WHEN L=3

SR	G_2
$N = 4 * k$	$G_1 + \frac{N}{2} + V_{SR,1}$
$N = 2 * k + 1$	$G_1 + \left[\frac{N+V_{SR,2}}{2}\right]$
$N = 2 * (2 * k + 1)$	$G_1 + \frac{N}{2}$
DR	G_2
$N = 8 * k$	$G_1 + \frac{3*N}{4}$
$N = 2 * (2 * k + 1)$	$G_1 + \left[\frac{N}{4}\right] + V_{DR,1}$
$N = 4 * (2 * k + 1)$	$G_1 + \frac{V_{DR,2}*N}{4} + V_{DR,3}$
TG $F_1 = 2k\acute{}+1$	G_2
$F_2 = 2 * k + 1$	$G_1 + \frac{N+V_{TG,1}}{2} + V_{TG,2}$
$F_2 = 2 * k$	$G_1 + \frac{N}{2} + V_{TG,2}$
TG $F_1 = 2k\acute{}$	G_2
$F_2 = F_1 + 4 * k$	$G_1 + \frac{N+V_{TG,3}}{2} + V_{TG,4}$
$F_2 = 2 * k + 1$	$G_1 + \frac{N}{2} + \left[\frac{F_2}{2}\right] + V_{TG,1}$
$F_2 = F_1 + 2 * (2 * k + 1)$	$G_1 + \frac{N+F_2}{2}$

TABLE IX

GATEWAYS RELATIVE POSITION WHEN L=2

N	G_2	G_3
$6 * k$	$G_1 + \frac{N}{3} + V_{SR,1}$	$G_1 + \frac{2N}{3} + V_{SR,2}$
$3 * k + 1$ **&** $3 * k + 2$	$G_1 + \left[\frac{N+V_{SR,3}}{3}\right]$	$G_1 + \left[\frac{2N+V_{SR,3}}{3}\right]$
$3 * (2 * k + 1)$	$G_1 + \frac{N}{3}$	$G_1 + \frac{2N}{3}$

TABLE X

SINGLE RING, GATEWAYS POSITION WHEN L=3

SR	$V_{SR,1} = -1$	$V_{SR,1} = 0$	$V_{SR,1} = 1$
$V_{SR,2}$	-1,0	-1,0,1	0,1
$V_{SR,3}$	$(0, L - 1)$	$(0, L - 1)$	$(0, L - 1)$

DR	$V_{DR,2} = 1$	$V_{DR,2} = 3$
$V_{DR,3}$	0	-1,0,1
$V_{DR,1}$	$0, 1$	$0, 1$

TG	$V_{TG,3} = -F_2$	$V_{TG,3} = F_2$
$V_{TG,1}$	0,1	0,1
$V_{TG,2}$	$0, F_2$	$0, F_2$
$V_{TG,4}$	0	$-1, 0, 1, F2$

TABLE XII

V VALUES

N	$V_{DR,4}$	$V_{DR,5}$	$V_{DR,6}$	$V_{DR,7}$
$6k$	0	0	0	0
$2 * (6 * k + 1)$	1	0	1	1
$2 * (6 * k + 2)$	1	0	0,1	0,1
$2 * (6 * k + 4)$	0	1	0,1	-1,0
$2 * (6 * k + 5)$	0	1	1	1

TABLE XIII

$V_{DR,4}, V_{DR,5}, V_{DR,6}$ AND $V_{DR,7}$ VALUES

SR	
N	**Number of possibilities**
$2 * L * k$	L*(L+1)+1
$L * (2 * k + 1)$	1
Rest	L

DR $(1 \leq l_1 \leq L - 1)$	
N	**Number of possibilities**
$4 * L * k$	2*(L-2)+1
$4 * (L * k + l_1)$	2*L
Rest	L

TG (L=2 & $F_1 = 2k\acute{}+1$	
F_2	**Number of possibilities**
$2 * k + 1$	L
$2 * k$	2*L

TG (L=2 & $F_1 = 2k\acute{}$	
F_2	**Number of possibilities**
$F_1 + 4 * k)$	2*L+1
$F_1 + 2 * (2 * k + 1)$	1
Rest	L

TABLE XIV

SR,DR AND TG NUMBER OF OPTIMAL CONFIGURATIONS

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