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Rigorous scheduling of mesh-structure refined petroleum pipeline networks

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ABSTRACT

Pipeline networks represent the major mode of transportation for crude oil and refined fuels. Recent data suggest that this trend will persist in coming years. A multiproduct pipeline network can be described as a set of interconnected pipelines with several input and receiving terminals. In the most general case, it has a mesh-like configuration with alternative paths between two terminals. Pumping and delivery operations should be scheduled all at once in an integrated fashion. This work introduces a novel MILP continuous-time formulation for the scheduling of mesh pipeline networks. The pipeline operational plan is conceived as a sequence of composite pumping runs each one involving at most a batch injection at every input station. The model solution simultaneously provides the timing of batch inputs at every source, the product sequence and lot sizes at every pipeline, and the flows diverted to terminals. Three examples of growing complexity were successfully solved at low CPU times.

1. Introduction

Pipeline networks represent the major mode of transportation for crude oil and refined fuels. Because they are widely recognized as the most efficient, reliable and safe way of moving liquid fuels to distant destinations, pipelines have largely become the shippers' first choice. Almost all gasoline in the U.S. is transported by pipeline. Tanker trucks usually carry gasoline to local gas stations only the last few miles, after picking it up from a pipeline at a distribution terminal. Recent data confirm that pipelines continue increasing their share of the U.S. petroleum transportation market from 66.7% in 2007 to 71% in 2008. Moreover, this trend will persist in coming years because the liquid pipeline industry is actively investing in new capacity expansions and the upgrade of existing lines to accommodate new supply resources and consumer markets (Association of Oil Pipe Lines, 2011).

A multiproduct pipeline network can be regarded as a set of interconnected pipelines with several entry and exit points, whose operations should be scheduled all at once in an integrated fashion. Each individual pipeline presents a single source at its origin and has one or several terminals over the line. Batches of different products injected at entry points usually travel through several pipelines before reaching their final destinations. An interesting feature of multi-source pipeline networks is the fact that pumping operations are simultaneously performed at several input stations. At the same time, product deliveries from in-transit batches to multiple pipeline terminals can also take place.

In addition, the network structure is rather complex. A source node can be directly connected to multiple receiving terminals through different pipelines or vice versa. Moreover, two or more sequences of pipelines may be connecting a source node with a distant receiving depot, and the flow direction in some pipelines can be reversed. In other words, there may be several alternative paths to move a batch from a particular entry station to the assigned destination, i.e. a mesh-type network structure. A proper route selection for every shipment is a key issue to both avoid congestion of some pipelines and reduce pumping and interface costs. Accounting for the structural and operational issues to be considered, it is quite clear that the short-term scheduling of pipeline networks is a very difficult problem that requires efficient supporting tools to even find good feasible solutions.

1.1. Literature review

Shah, Li, and Ierapetritou (2011) presented an extensive review of available methodologies for addressing scheduling, planning and supply chain management in the petroleum refining industry. Though substantial work in the literature has been devoted to oil refinery operations, the authors pointed out that the research focus is currently shifting to a more integrated approach based on an enterprise-wide viewpoint. Enterprise-wide optimization for the petroleum industry involves the optimization of the whole supply chain, including manufacturing and distribution operations.

In fact, most papers on short-term scheduling of refined products pipelines have addressed rather simple transportation systems

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Nomenclature

(a) Sets

FS	ordered pairs of incompatible products
I	chronologically arranged composite runs ($I^{old} \cup I^{new}$)
I^{new}	new composite runs
I^{old}	old composite runs performed in a previous horizon
J	terminals of the pipeline network
J_l	receiving depots over pipeline l
J_p	distribution terminals demanding product p
P	refined petroleum products
P_l	subset of products transported through pipeline l
PL	set of pipelines in the mesh-structure network
$PL_j^{(IN)}$	subset of pipelines supplying products to depot j
$PL_j^{(OUT)}$	subset of pipelines emerging from terminal j

(b) Parameters

$cb_{p,j}$	unit backorder penalty cost to tardily meet a requirement of product p at depot j
$cf_{p,p',l}$	unit reprocessing cost of interface material involving products p and p' into line l
$cid_{p,j}$	unit inventory carrying cost for product p at terminal j
$cp_{j,l,p}$	unit pumping cost to transport product p through pipeline l to depot j
cmk	unit cost of utilization of the pipeline network
$\text{dem}_{p,j}$	overall demand of product p to be satisfied at depot j before the horizon end
$(d_{\max})_l$	maximum delivery size from a batch in line l to a distribution terminal
$(d_{\min})_l$	minimum delivery size from a batch in line l to a distribution terminal
$h_{\max}$	horizon length
$(\text{id}_{\max})_{p,j}$	maximum allowed inventory level for product p at terminal j
$(\text{id}_{\min})_{p,j}$	minimum allowed inventory level for product p at terminal j
$ido_{p,j}$	initial inventory of product p in tanks of depot j
$\text{if}_{p,p',l}$	volume of interface between batches containing products p and p' into line l
$(l_{\max})_{l,p}$	maximum length of a new batch injection of product p in pipeline l
$(l_{\min})_{l,p}$	minimum length of a new batch injection of product p in pipeline l
pv_l	total volume of pipeline l
$(q_{\max})_{l,p}$	maximum injection size for product p in pipeline l
$(q_{\min})_{l,p}$	minimum injection size for product p in pipeline l
$(vb_{\max})_l$	maximum pumping rate over pipeline l
$(vb_{\min})_l$	minimum pumping rate over pipeline l
$vm_{p,j}$	maximum supply rate of product p to the local market from depot j
$vp_{p,j}$	production rate of product p coming from nearby refineries to terminal j
$\sigma_{j,l}$	volumetric coordinate of depot j from the origin of pipeline l

(c) Continuous variables

$B_{p,j}$	backorder of product p for depot j
C_i/L_i	completion time/length of the composite run i
$D_{(i,l),j}^{(i')}$	volume of batch (i, l) diverted from pipeline l to depot j during the composite run i'
$\text{DMO}_{p,j}^{(i')}$	amount of product p sent to local market j during the time interval $[C_{i'-1}; C_i - L_{i'}]$
$\text{DMP}_{p,j}^{(i')}$	amount of product p sent to local market j during the time interval $[C_{i'} - L_{i'}; C_{i'}]$
$\text{DP}_{(i,l),p,j}^{(i')}$	amount of product p supplied by batch (i, l) to depot j during the composite run i'
$F_{(i,l)}^{(i')}$	upper coordinate of batch (i, l) from the origin of pipeline l at time $C_{i'}$
$\text{ID}_{p,j}^{(i')}$	inventory of product p in depot j at the end of the composite run i'
$L_{(i,l)}$	length of the batch injection (i, l) into pipeline l during the composite run i
$Q_{(i,l)}$	original size of the batch (i, l) injected in pipeline l during run i
$QP_{(i,l),p}$	volume of product p injected in pipeline l during run i
$\text{WIF}_{i,p,p',l}$	interface volume between batch i and its predecessor containing products p' and p in pipeline l
$W_{(i,l)}^{(i')}$	size of batch (i, l) in pipeline l at time $C_{i'}$

(d) Binary variables

$x_{(i,l),j}^{(i')}$	denotes that a portion of batch (i, l) is transferred to depot j during the composite run i'
$y_{(i,l),p}$	denotes that batch (i, l) contains product p
z_i	denotes the existence of the composite run i

with no pipeline branching, i.e. chain-like network structures. They usually deal with single-source, unidirectional pipelines joining an input station to multiple distribution terminals. Source nodes feeding two or more pipelines or branching terminals supplying products to several downstream ducts rarely arise. Furthermore, terminal demands are to be satisfied before the end of the planning horizon, i.e. a common due date for all product requirements is assumed. Different types of approaches, including rigorous optimization models, knowledge-based techniques (Sasikumar, Prakash, Patil, & Ramani, 1997), discrete-event simulation (García-Sánchez, Arreche, & Ortega-Mier, 2008; Mori et al., 2007), and decomposition methods (Hane & Ratliff, 1995; Neves et al., 2007) were proposed. Rigorous approaches generally rely on mixed-integer linear programming (MILP) or mixed-integer non-linear programming (MINLP) models and are usually classified into two classes: discrete and continuous. On one hand, discrete MILP formulations divide both the pipeline volume into a number of single-product packs of equal size, and the planning horizon into time intervals of fixed duration (Magatão, Arruda, & Neves-Jr, 2004; Rejowski & Pinto, 2003, 2004; Zyngier & Kelly, 2009). Because they are based on approximate representations, discrete approaches will not provide feasible schedules unless a fine discretization is used. Rejowski and Pinto (2008) introduced an improved continuous-time MINLP formulation that yields better solutions, but still divides the pipeline volume into single-product packs of fixed size.

On the other hand, a few closely related continuous formulations for the operational planning of multiproduct pipelines connecting a single origin to one or multiple depots have been developed (Cafaro & Cerdá, 2004, 2008a; Relvas, Matos, Barbosa-Póvoa, Fialho, & Pinheiro, 2006, 2007). Continuous representations in both time and volume domains permit to exactly determine the optimal sequence of batch injections, lot sizes, pump rates, start/end times of pumping runs, interface volumes to be reprocessed, and amounts and types of products diverted from the pipeline to distribution terminals during every run. Cafaro and Cerdá (2008b) extended the approach to tackle the operational planning of a similar pipeline system over a monthly rolling horizon, with product deliveries due at the end of each weekly period. As time goes on, the planning horizon moves forward and a new period with further product demands is considered. Consequently, a rescheduling process based on updated demand data is triggered over the new time-horizon instance.

The first continuous formulation for the scheduling of chain-like pipeline networks with multiple origins and destinations was developed by Cafaro and Cerdá (2009). It is a single-level approach that efficiently determines both input and delivery schedules all at once. Given the product requirements and delivery dates at distribution terminals, the proposed MILP model chooses the size, origin and destination for each batch, the product sequence inputted into the pipeline from every source, and the start/end times of pumping and extraction operations. Pumping runs at intermediate locations can either insert a new lot or increase the size of a batch in transit. Therefore, batches travelling in a particular pipeline are no longer arranged by increasing input times. Later, Cafaro and Cerdá (2010) generalized the MILP formulation for the short-term scheduling of chain-like multi-source pipeline networks to allow the execution of simultaneous batch injections at two or more input stations. Results indicate that the execution of simultaneous pumping runs allows a better use of the pipeline transport capacity and a significant decrease in the time needed to meet all terminal demands.

MirHassani and Jahromi (2011) presented a continuous-time MILP formulation for the operational planning of tree-structure single-source pipeline systems. In networks with this topology, pipeline branches emerge from the trunk line to transport smaller volumes of oil derivatives to multiple, nearby market areas. However, the model assumes that at most a single product can be diverted to a delivering line during the execution of a pumping run. Cafaro and Cerdá (2011) introduced an improved formulation for tree-structure pipeline networks that allows to divert lots of different products to a delivering line during the same pumping run. In addition, it makes a rigorous tracking of batch and interface movements along trunk and secondary lines, including the identity of the original batch from which a lot moving through a branch was diverted.

Recent works in the field of multiproduct pipeline scheduling are focused on pipeline networks with more complex structures. García-Sánchez et al. (2008) developed a hybrid methodology that combines tabu search and discrete-event simulation for the scheduling of pipeline systems with branching terminals. Tabu search guides a local search procedure that eventually succeeds in obtaining satisfactory schedules in terms of some relevant criteria like product shortages, pipeline stoppages, and interface costs. At each stage, the searching procedure starts from a given schedule, and randomly explores a neighborhood around the current solution allowing a single type of move. Four different kinds of moves on the batch sequence were considered: batch insertion, batch mass-exchange, batch splitting, and batch merging. Every stage attempts to improve one of the evaluation criteria while keeping the others within reasonable values. In turn, the simulation model provides an accurate and suitable tool for a quick quality assessment of such schedules.

Herrán, de la Cruz and de Andrés (2010) presented a new mathematical approach for the short-term scheduling of a multi-pipeline transportation system with a branching structure. The proposed MILP formulation is based on a discrete problem representation and the problem goal is to minimize the total cost including pumping, start/stop, interface reprocessing, and inventory carrying expenses. A simplified version of the model was derived by assuming the compulsory execution of a pack insertion at every pipeline during each time interval, except for reversible (bidirectional) lines. Such a version can only be used for a scenario of high production and demands, and low pumping costs. A case study involving a pipeline system that transports four products from two sources to three distribution terminals through two intermediate nodes was solved. The branch-like network consists of seven pipelines (including one reversible duct) and every line is roughly divided into three packs of uniform size. Product demands are to be satisfied before the end of the planning horizon featuring a length of 100 h. When using the complete model to solve the case study under different scenarios, several hours of CPU time are required to find the optimal solution.

Lopes, Ciré, de Souza, and Moura (2010) introduced a hybrid framework for the planning and scheduling of mesh-like pipeline networks. In a mesh network structure, a batch has several alternative paths to move from the entry point to the assigned destination. The approach is based on a two-phase decomposition strategy comprising: (i) a heuristic planning phase generating the delivery orders (product and batch size) to be transported between two depots, and (ii) a constraint programming-based scheduling phase sequencing the delivery orders to be pumped into each pipeline or unloaded from a given depot tank. The procedure was applied to find a feasible operational plan for a very large mesh-like pipeline network transporting petroleum derivatives and ethanol.

On the other hand, Boschetto et al. (2010) developed a hierarchical scheduling methodology for mesh pipeline networks. Primarily based on the work of Neves et al. (2007), this approach additionally introduces an MILP model using the data generated by a set of heuristic modules making most of the discrete decisions. The main goal of the new MILP formulation is to determine the exact times at which to pump products into the pipelines and to deliver products to receiving terminals. Those times should satisfy pipeline operational constraints defined by a discrete-event simulation module, that include pipeline stoppages, movement of batches through branching terminals, use of preferential routes to avoid contamination losses, on-peak demand hours, and change of flow direction in reversible pipelines. The approach was applied to a large real-world pipeline network, where more than 14 oil derivatives and ethanol are transported and distributed between supply and demand nodes.

This paper introduces a novel MILP continuous formulation for the operational planning of mesh-like pipeline networks that allows simultaneous batch injections at multiple input stations. It is the first monolithic approach determining the optimal schedules of pumping and delivery operations all at once, in an integrated fashion. The planning horizon is conceived as a sequence of time slots of variable length, and a single pumping run is at most allowed at the origin of every pipeline during each time interval. The approach has been illustrated by successfully solving three case studies of growing complexity in quite reasonable CPU times.

The rest of the paper is organized as follows. Section 2 presents a detailed description of a typical mesh pipeline network transporting several refined products from multiple sources to intermediate depots and final destinations. The model assumptions are listed in Section 3, whereas Section 4 extensively describes the variables and equations included in the proposed MILP formulation. Illustrative examples are solved and their results discussed in Section 5, while the final conclusions are presented in Section 6.

2. Description of a mesh-structure pipeline network

Fig. 1 shows a typical mesh pipeline network comprising seven unidirectional pipelines (l_1–l_7) through which several petroleum products are conveyed from two primary sources (nodes $N1$ and $N2$) to nine destinations (nodes $N3$–$N11$). The intermediate node $N4$ is a main distribution center where product streams coming from the input stations $N1$ and $N2$ are received and stored in depot tanks.

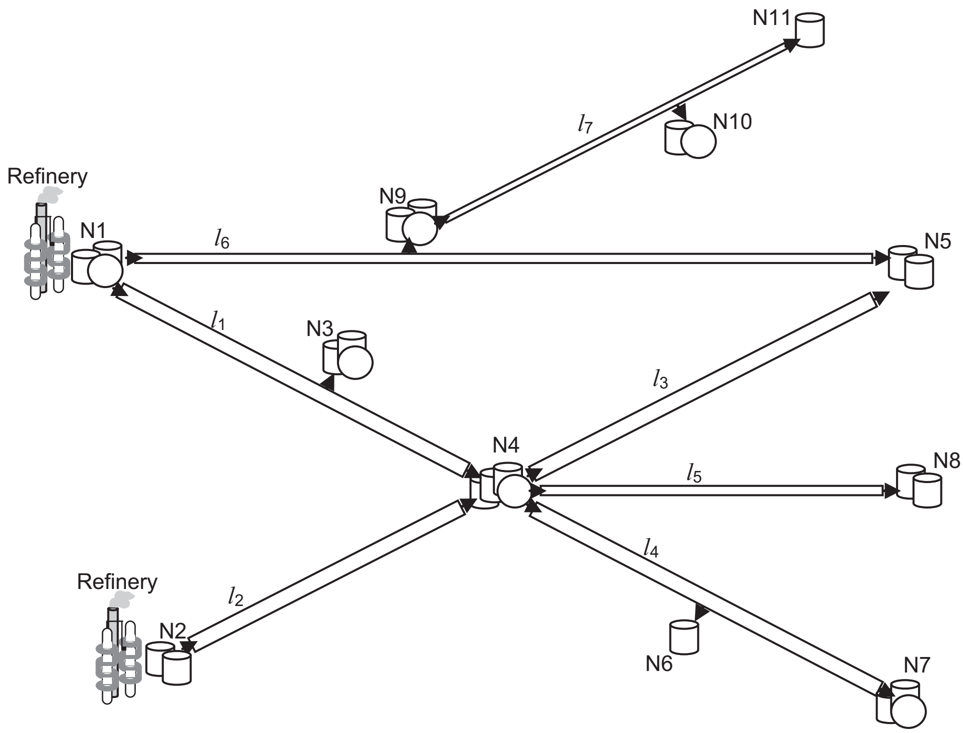

Fig. 1. A multiproduct pipeline network structure.

Simultaneously, some other batches of products taken from the available inventory at $N4$ are pumped into downstream pipelines to meet specific demands at the receiving depots $N5$, $N6$, $N7$ and $N8$. Hence, the intermediate node $N4$ behaves as an input station injecting lots of products into outgoing lines (l_3, l_4, l_5) for delivery to those distant depots, and as a receiving terminal for product flows coming from upstream pipelines (l_1, l_2) to restore inventories at $N4$. Therefore, $N4$ can play a dual role acting as a receiving and a source node at the same time. Dual terminals like $N4$ are usually located at the intersection of two or more pipelines.

In contrast, nodes $N3$, $N6$, and $N10$ located over pipelines l_1, l_4 and l_7, respectively, can be regarded as intermediate off-take points where some of the flow is diverted from the line to the receiving terminal, thus reducing the flow-rate downstream of such points. Therefore, $N3$, $N6$, and $N10$ are "pure" receiving terminals sited midway between the origin and the farthest depot of lines l_1, l_4, and l_7, respectively. Similarly, the outmost depots of pipelines l_3–l_7 (i.e. $N5$, $N7$, $N8$, and $N11$) also behave as "pure" receipt stations. So far, three different types of nodes have been identified: input stations ($N1$ and $N2$), dual terminals ($N4$ and $N9$) and "pure" receiving terminals ($N3$, $N5$, $N6$, $N7$, $N8$, $N10$, and $N11$). However, node $N9$ is a special case of dual node. In fact, depot $N9$ behaves as an off-take point for pipeline l_6 and as an input station for line l_7. It is one of the destinations for product streams coming from line l_6 and the source of line l_7 injecting new batches at its origin to meet demands of depots $N10$ and $N11$. Some dual terminals can be simultaneously supplied by multiple upstream pipelines, thus receiving the same or different products from distinct sources.

The pipeline network shown in Fig. 1 is a set of interconnected, unidirectional pipelines whose operations are scheduled all at once in an integrated fashion. Each individual pipeline presents a single primary or secondary source at its origin and has one or several receiving terminals over the line. For instance, pipeline l_1 conveys products from source $N1$ to a pair of terminals: one of them is a "pure" receiving depot ($N3$) and the other is a dual terminal ($N4$). Accounting for the double function (receiving/injecting) of dual nodes $N4$ and $N9$, the pipeline network of Fig. 1 can be better described by Fig. 2. In the new representation, the hybrid nature of node $N9$ is clearly shown because it appears as an off-take point for pipeline l_6 and as a source node for line l_7.

2.1. Planning the operation of multiple pipelines

As mentioned before, pumping/delivery pipeline operations must be scheduled all at once in an integrated fashion. To this end, the planning horizon is divided into multiple time slots of variable length. In each time slot, at most a single pumping run can be performed at the origin of every pipeline. Such simultaneous pumping operations allocated to the same time slot constitute a so-called "composite" pumping run $i \in I^{new}$.

During a composite run, multiple batch injections (as many as the number of single pipelines in the network) can be simultaneously carried out. The longest batch injection sets up the length of the related composite run. The other pumping operations must be accomplished within the time slot of the associated run. As there is a one-to-one relationship between time slots and composite runs, both terms look equivalent. Therefore, it can be said that the pipeline operational plan comprises a series of composite pumping runs $i \in I^{new}$ of variable length. The elements of the set I^{new} are chronologically ordered which means that composite run i is performed before run i' if $i < i'$. Before developing the network operational plan, the lengths of the composite runs (or time slots) are unknown. As a result, the number of runs to be carried out over a weekly or monthly horizon can only be estimated. To guarantee the discovery of the best pipeline schedule, the cardinality of the set I^{new} (a model parameter) should never be lower than the number of composite runs at the optimal solution. For short-term pipeline scheduling problems typically comprising time horizons ranging from 7 to 10 days, it is likely that at most one batch of every product will be pumped through each pipeline. As a result, a good way to initially estimate the number of time slots needed to achieve the optimal solution is by adopting $|I^{new}| = \max_l \{|P_l|\}$, i.e. the maximum number of products that can be transported by any of the pipelines.

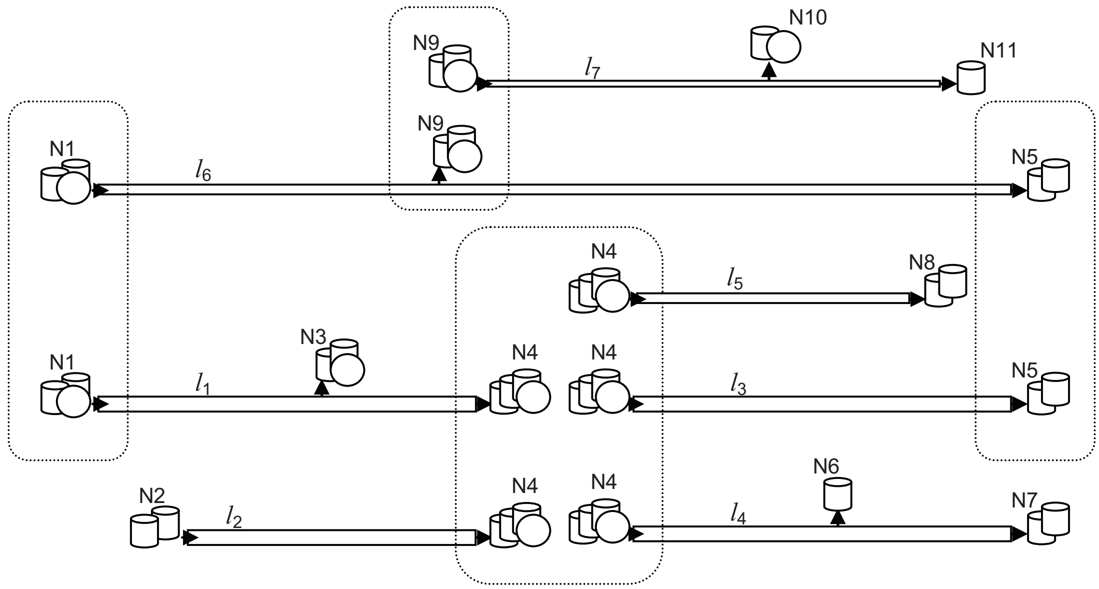

Fig. 2. Representing the network of Fig. 1 as an integrated set of single pipelines.

Each individual batch inputted to the pipeline network during a particular composite run i is identified through both the associated run i and the index $l \in PL$ of the pipeline into which it is inserted, i.e. batch (i, l). PL is the set of single lines that compose the pipeline network. To allow the execution of quality control operations, the amount of product $p \in P$ received by a dual terminal through the composite run i cannot be pumped into downstream pipelines during the same run. In other words, if a dual terminal injects a new lot of product p over run i, such a lot should come from the inventory of p available in the terminal at the start of run i.

3. Model assumptions

The continuous-time mathematical formulation for the scheduling of refined products pipeline networks to be presented in the next section has been developed based on the following assumptions:

(A1) The pipeline network is regarded as an arrangement of interconnected, unidirectional pipelines, each one featuring a single source node at its origin and one or several destination nodes along the line.
(A2) Some individual pipelines can share either the source node or some destination nodes. In Fig. 2, $N4$ is the common destination for pipelines l_1 and l_2, and the common source of lines l_3, l_4 and l_5.
(A3) Batches of the same (or different) product(s), supplied by two or more pipelines can be simultaneously received at a common destination node. Conversely, lots of the same (or different) product(s) can be shipped at the same time from a common origin through several pipelines to multiple destinations.
(A4) A terminal collecting products from several pipelines during a pumping run will have at least one tank connected to each incoming line. Similarly, a depot providing products to several pipelines will have at least one tank feeding each of them.
(A5) Some intermediate terminals can act as dual nodes, receiving material flows from upstream sources and injecting batches of products destined to downstream terminals at the same time (e.g. node $N4$ in Fig. 2).
(A6) The transfer of material between interconnected pipelines cannot be directly made but through intermediate terminals where the incoming product flows are temporarily stored. Some fractions of the batches are shipped by truck to customers, while other volumes can be pumped into outgoing pipelines to meet specific demands at farther depots.
(A7) To allow the execution of quality control operations on the arriving batches, the volume of product pumped into outgoing pipelines at dual nodes must be available in inventory at the starting time of the new injection.
(A8) For a better coordination between incoming and outgoing flows at each distribution terminal of the pipeline network, pumping operations have been combined into groups of pumping runs, i.e. *composite* runs.
(A9) During a composite run, at most a single batch can be pumped into any single pipeline. Therefore, a composite run may include as many batch injections (i.e. individual pumping runs) as the number of single pipelines. Moreover, the duration of an individual batch injection must never exceed the length of the composite run to which it belongs.
(A10) Some free storage capacity is kept at any dual terminal to temporarily compensate the positive difference between simultaneous incoming and outgoing flows of a certain product that may briefly arise throughout a composite run.

4. Mathematical formulation

To develop a mathematical formulation for the operational planning of a complex pipeline network, this work introduces the notion of composite pumping runs $i \in I^{new}$. A composite run stands for a group of pumping/delivery operations taking place all over the pipeline network within some time interval of the planning horizon. The lower/upper limits of such a time slot define the starting/completion times of the composite run whose values are determined by solving the proposed formulation. In this way, the pipeline operational plan can be seen as a sequence of composite runs. Then, the cardinality of the set I^{new} should be large enough to guarantee that the problem solution space encloses the optimal pipeline schedule. The starting/completion times of a composite run are the problem time events dividing the planning horizon into a number of time intervals of variable length. All the pipeline operations associated to a certain composite run should be carried out within the time slot allocated to it. Besides, a batch injected in pipeline $l \in PL$ during run i is called the lot (i, l).

A composite run $i \in I^{\mathrm{new}}$ is characterized through three problem variables: the 0–1 variable z_i denoting the existence of run i, and the continuous variables C_i and L_i representing its completion time and its duration, respectively. If run i does not exist, its length is null and no pumping operation takes place during run i. In turn, five problem variables are associated to every batch (i, l): (1) the allocation variable $y_{(i,l),p}$ indicating the product it contains; (2) its original size, $Q_{(i,l)}$; (3) the duration of the related batch injection, $L_{(i,l)}$; (4) its location in pipeline l, $F_{(i,l)}^{(i')}$; and (5) its current size, $W_{(i,l)}^{(i')}$; with the latter two measures given at the time event $C_{i'}$. On the other hand, product deliveries are described through the 0–1 variable $x_{(i,l),j}^{(i')}$ standing for the existence of a product delivery from batch (i, l) to depot j during run i', and the continuous variable $D_{(i,l),j}^{(i')}$ representing the amount of product diverted from batch (i, l) to depot $j \in J$ during run i'.

4.1. Pumping run related constraints

4.1.1. Product allocation

Every new batch (i, l) that is pumped at the origin of pipeline $l \in PL$ during a new run $i \in I^{\mathrm{new}}$ will contain at most a single refined petroleum product.

$$\sum_{p \in P_l} y_{(i,l),p} \leq 1 \; \forall i \in I^{\mathrm{new}}, \quad l \in PL \tag{1}$$

The 0–1 variable $y_{(i,l),p}$ denotes the existence of the new batch (i, l) containing product $p \in P_l$ in pipeline l whenever $y_{(i,l),p} = 1$. The subset P_l stands for all of the products to be transported through pipeline l over the current planning horizon. Fictitious batches never pumped into pipeline l feature $y_{(i,l),p} = 0$ for all $p \in P_l$.

4.1.2. Sequencing composite pumping runs

The execution of a new composite pumping run $i \in I^{\mathrm{new}}$ involving batch injections at one or more pipelines should start after completing all the pumping operations related the previous composite run $(i - 1)$.

$$C_i - L_i \geq C_{i-1} \quad \forall i \in I^{\mathrm{new}} \tag{2}$$

$$L_i \leq C_i \leq h_{\max} \quad \forall i \in I^{\mathrm{new}} \tag{3}$$

The continuous variable C_i denotes the completion time for run $i \in I^{\mathrm{new}}$, i.e. the time at which all the lot injections related to run $i \in I^{\mathrm{new}}$ have ended. Such lot injections (i, l) into one or several pipelines $l \in PL$ are executed within the time interval $[C_i - L_i;\, C_i]$. The longest batch injection determines the length of the composite run $i \in I^{\mathrm{new}}$, represented by the continuous variable L_i. In turn, $h_{\max}$ is the specified length of the current scheduling horizon, also measured in time units.

4.1.3. Sizing lot injections

Let $Q_{(i,l)}$ denote the size of a new lot (i, l) injected at the origin of pipeline l during the pumping run $i \in I^{\mathrm{new}}$. The continuous variable $Q_{(i,l)}$ will be positive only if the composite run i is really performed and a new lot (i, l) is inserted at the inlet section of pipeline l, i.e. $y_{(i,l),p} = 1$. Therefore,

$$\sum_{p \in P_l} y_{(i,l),p} (q_{\min})_{l,p} \leq Q_{(i,l)} \leq \sum_{p \in P_l} y_{(i,l),p} (q_{\max})_{l,p} \quad \forall i \in I^{\mathrm{new}}, \quad l \in PL \tag{4}$$

where $(q_{\min})_{l,p}$ and $(q_{\max})_{l,p}$ stand for the minimum and maximum permissible sizes for lots of product p pumped into pipeline l.

4.1.4. Batch injection length

Let $L_{(i,l)}$ be the length of the batch injection (i, l) into pipeline l during the composite run i. Hence,

$$(vb_{\min})_l L_{(i,l)} \leq Q_{(i,l)} \leq (vb_{\max})_l L_{(i,l)} \quad \forall i \in I^{\mathrm{new}}, \quad l \in PL \tag{5}$$

The interval $[(vb_{\min})_l;\, (vb_{\max})_l]$ represents the feasible pump rate range for pipeline l. If no batch is injected during run i at pipeline l, then $y_{(i,l),p} = 0$ for any product $p \in P_l$ and, from Eq. (6), $L_{(i,l)} = 0$. Besides, $L_{(i,l)}$ must belong to the length range $[(l_{\min})_{l,p};\, (l_{\max})_{l,p}]$ only if batch (i, l) exists and contains product p.

$$\sum_{p \in P_l} y_{(i,l),p} (l_{\min})_{l,p} \leq L_{(i,l)} \leq \sum_{p \in P_l} y_{(i,l),p} (l_{\max})_{l,p} \quad \forall i \in I^{\mathrm{new}}, \quad l \in PL \tag{6}$$

Therefore, the length of a composite run i will be given by: $L_i = \max_{l \in PL}\{L_{(i,l)}\}$, or:

$$L_i \geq L_{(i,l)} \quad \forall i \in I^{\mathrm{new}}, \quad l \in PL \tag{7}$$

In other words, the longest batch injection (i, l) determines the duration of run i.

If all the batch injections related to the composite run $i \in I^{\mathrm{new}}$ are never performed ($\sum_l \sum_p y_{(i,l),p} = 0$), then run i does not exist and represents a fictitious element of the set I^{new}. In such a case, the related binary variable z_i denoting the existence of the composite run i will be equal to zero. When at least a single batch injection is executed at the inlet of some active pipeline during the time interval $[C_i - L_i;\, C_i]$, run i does exist and the variable z_i turns to one. Both conditions are modeled through constraints (8), where $|PL|$ stands for the number of pipelines in the network.

$$z_i \leq \sum_{l \in PL} \sum_{p \in P_l} y_{(i,l),p} \leq z_i |PL| \quad \forall i \in I^{\mathrm{new}} \tag{8}$$

To avoid multiple equivalent solutions, Eq. (9) reserves the last elements of the set I^{new} for fictitious runs featuring $z_i = 0$.

$$z_i \leq z_{i-1} \quad \forall i \in I^{\mathrm{new}}, \quad i > 1 \tag{9}$$

4.1.5. Interface material between consecutive batches

Through Eqs. (2) and (9), the composite runs $i \in I^{\mathrm{new}}$ have been arranged in the same order as they are performed. Then, an existent run $(i+1)$ is executed right after run i. Batch injections can be accomplished into some, but not all, of the pipelines $l \in PL$ during the execution of a new composite run i. As a result, lot (i', l) may directly succeed lot (i, l), with $i < i'$, if the elements $(i+1, l)$, $(i+2, l)$,. $(i' - 1, l)$ are not inserted into pipeline l during runs $(i+1)$, $(i+2)$,. . ., $(i' - 1)$, and $y_{(i+1,l),p} = y_{(i+2,l),p} = \ldots = y_{(i'-1,l),p} = 0$ for all $p \in P_l$. Consequently, some mixing volume will be generated at the interface of batches (i, l) and (i', l). To rigorously account for interface volumes, the model should be able to identify every pair of non-fictitious batches flowing one after the other through any pipeline. Lot (i', l) will directly chase lot (i, l) in pipeline l if the two following conditions hold:

(a) Batches (i, l) and (i', l) with $i < i'$ are not fictitious and therefore contain some oil refined products $(p, p') \in P_l$, i.e. $y_{(i,l),p} + y_{(i',l),p'} = 2$.
(b) Every potential lot (k, l) that could have been injected during an intermediate run k (with $i < k < i'$) does not exist, and therefore $\sum_{k \in I^{\mathrm{new}} i<k<i'} \sum_{p \in P_l} y_{(k,l),p} = 0$.

Let $if_{p,p',l}$ denote the characteristic size of the interface between consecutive batches containing products p and p' in pipeline l, i.e. a model parameter. Let us also define the variable $\mathrm{WIF}_{i',p,p',l}$ representing the interface volume between a new batch (i', l) and its direct predecessor (i, l) in pipeline l. As stated by constraint (10), the mixing volume $\mathrm{WIF}_{i',p,p',l}$ will never be lower than $if_{p,p',l}$ if the batches (i, l) and (i', l) are consecutively pumped and contain products p and p', respectively. In case batches (i, l) and (i', l) are not adjacent in line l or do not contain products p and p', the RHS of Eq. (10) becomes zero or negative, and the lower bound $if_{p,p',l}$ will not be imposed upon the value of $\mathrm{WIF}_{i',p,p',l}$.

$$\mathrm{WIF}_{i',p,p',l} \geq if_{p,p',l}\Big(y_{(i,l),p} + y_{(i',l),p'} - \sum_{i<k<i'}\sum_{q\in P_l} y_{(k,l),q} - 1\Big) \quad \forall i, i' \in I^{\mathrm{new}},\ i < i',\ l \in PL,\ (p,p') \in P_l \qquad (10)$$

Similarly to previous approaches, the value of the parameter $if_{p,p',l}$ is assumed to be known and independent of the pump rate. All interface volumes are traced by the proposed mathematical model along every pipeline from the source point to the farthest destination, where they are stored in separate tanks.

As the initial linefill is given, the product p_l^o contained in the last batch inserted in pipeline l during the previous horizon is also known. Then, constraint (10) reduces to constraint (10′) for the first batch (i', l) inserted into line l during the current horizon. If lot (i', l) is the first element pumped into line l, then any batch (i, l) with $i \in I^{\mathrm{new}}$ and $i < i'$ does not exist, and the related variables $y_{(i,l),p}$ will be equal to zero for any $p \in P_l$. Otherwise, Eq. (10′) becomes a redundant constraint.

$$\mathrm{WIF}_{i',p,p',l} \geq if_{p,p',l}\Big(y_{(i',l),p'} - \sum_{\substack{k\in I^{\mathrm{new}}\\ k<i'}}\sum_{q\in P_l} y_{(k,l),q}\Big) \quad \forall i' \in I^{\mathrm{new}},\quad l \in PL,\quad p = p_l^o,\quad p' \in P_l \qquad (10')$$

4.1.6. Forbidden product sequences

Due to product contamination, some product sequences are strictly forbidden. If FS represents the set of forbidden product sequences and $(p, p') \in \mathrm{FS}$, then batches containing products p and p' must never be consecutively pumped into any pipeline. Batches (i, l) and (i', l) move one after the other through line l only if they both exist and the intermediate lots (k, l) between them $(i < k < i')$ are all fictitious. To avoid forbidden product sequences, constraints (11) and (11′) have been included in the problem formulation. Eq. (11′) just applies to the first batch pumped into pipeline l, right behind the last old batch containing a known product p_l^o.

$$y_{(i,l),p} + y_{(i',l),p'} - \sum_{i<k<i'}\sum_{q\in P_l} y_{(k,l),q} \leq 1 \quad \forall i, i' \in I^{\mathrm{new}},\quad i < i',\quad l \in PL,\quad (p,p') \in FS \qquad (11)$$

$$y_{(i',l),p'} - \sum_{k\in I^{\mathrm{new}}}\sum_{k<i'}\sum_{q\in P_l} y_{(k,l),q} \leq 0 \quad \forall i' \in I^{\mathrm{new}},\quad l \in PL,\quad (p_l^o, p') \in FS \qquad (11')$$

4.1.7. Amount of product p injected into pipeline l during a new composite run

If the new lot (i, l) pumped into pipeline l during run $i \in I^{\mathrm{new}}$ does not contain product p, the associated volume of p in batch (i, l) given by the variable $QP_{(i,l),p}$ is equal to zero. Otherwise, $QP_{(i,l),p}$ is equal to the initial size of lot (i, l), i.e. $Q_{(i,l)}$. Therefore,

$$QP_{(i,l),p} \leq (q_{\max})_{l,p}\, y_{(i,l),p} \quad \forall i \in I^{new},\quad l \in PL,\quad p \in P_l$$

$$\sum_{p\in P_l} QP_{(i,l),p} = Q_{(i,l)} \quad \forall i \in I^{\mathrm{new}},\quad l \in PL \qquad (12)$$

4.2. Batch-tracking constraints

4.2.1. Location of a batch at the end of a composite pumping run

Let $F_{(i,l)}^{(i')}$ denote the location of the front boundary of batch (i, l) after completing the new pumping run i' $(i' \geq i)$. Such a continuous variable $F_{(i,l)}^{(i')}$ represents the volume between the origin and the interface between batch (i, l) and the preceding lot in pipeline l at the completion time of run i' $(C_{i'})$. Based on the continuity condition, the value of $F_{(i,l)}^{(i')}$ is found by adding the content of lot (i, l), given by $W_{(i,l)}^{(i')}$, to the front coordinate of the succeeding batch $(i+1, l)$ in line l, both at time $C_{i'}$.

$$F_{(i+1,l)}^{(i')} + W_{(i,l)}^{(i')} = F_{(i,l)}^{(i')} \quad \forall i \in I,\quad i' \in I^{new},\quad i' \geq i,\quad l \in PL \qquad (13)$$

4.2.2. Size of a batch at the end of the run during which it is injected

Let $W_{(i,l)}^{(i)}$ be the volume of batch (i, l) in pipeline l at the completion time C_i of the composite run i. If $Q_{(i,l)}$ is the size of batch (i, l) originally pumped into pipeline l, then $[Q_{(i,l)} - W_{(i,l)}^{(i)}]$ is the amount of material transferred from batch (i, l) to depots $j \in J_l$ (located along line l) during run i. Obviously, $Q_{(i,l)} \geq W_{(i,l)}^{(i)}$ and the lower coordinate of batch (i, l) at time C_i is equal to zero.

$$Q_{(i,l)} = W_{(i,l)}^{(i)} + \sum_{j\in J_l} D_{(i,l),j}^{(i)};\quad F_{(i,l)}^{(i)} - W_{(i,l)}^{(i)} = 0 \quad \forall i \in I^{\mathrm{new}},\quad l \in PL \qquad (14)$$

In Eq. (14), $D_{(i,l),j}^{(i)}$ represent the volume of product transferred from batch (i, l) to terminal $j \in J_l$ during run i.

4.2.3. Size of batch (i, l) at the end of a later composite run i′ (i′ > i)

By definition, $C_{i'}$ is the time at which the injection of all new batches (i', l) into different pipelines $l \in PL$ have been completed. Let us assume that batch (i, l), with $i < i'$, still travels through pipeline l right before starting a later run $i' > i$, i.e. at time $C_{i'-1}$. Then, the volume of batch (i, l) at time $C_{i'}$ will be given by the difference between its size at time $C_{i'-1}$ and the total volume transferred to receiving depots $j \in J_l$ over line l during run i'.

$$W_{(i,l)}^{(i')} = W_{(i,l)}^{(i'-1)} - \sum_{j\in J_l} D_{(i,l),j}^{(i')} \quad \forall i \in I,\quad l \in PL,\quad i' \in I^{\mathrm{new}},\quad i' > i \qquad (15)$$

4.3. Constraints on product deliveries from batches to terminals

4.3.1. Feasibility conditions for diverting material from in-transit batches to depot tanks

Diverting material from batch (i, l) to depot $j \in J_l$ during the new run $i' \in I^{\mathrm{new}}$ $(i' \geq i)$ is feasible only if the interconnection to depot j is accessible from batch (i, l). To fulfill such a feasibility condition, it is required that:

(a) The front edge of batch (i, l) at time $C_{i'}$ decreased by the volume of the interface material $(\mathrm{WIF}_{i,p,p',l})$ should never be lower than the coordinate of terminal j over pipeline l $(\sigma_{j,l})$, except for the farthest depot of the line, where interface material is removed. The feasibility condition for the farthest depot $j = |J_l|$ is achieved when $F_{(i,l)}^{(i')} = \sigma_{|Jl|,l}$.

(b) The back edge of batch (i, l) at time $C_{i'-1}$ must be less than the depot coordinate $\sigma_{j,l}$ by at least a certain volume φ. The value of φ represents the total volume of product diverted from batch (i, l) to terminals $j' \in J_l$ (with $j' \leq j$), while pumping batch (i', l).

Let $x_{(i,l),j}^{(i')}$ be a binary variable denoting that a portion of batch (i, l) is diverted to terminal $j \in J_l$ while performing run i' whenever

$x^{(i')}_{(i,l),j}=1$. Otherwise, $x^{(i')}_{(i,l),j}=0$ and no material will be transferred from batch (i, l) to depot j. Note that $x^{(i')}_{(i,l),j}$ can be driven to zero because of two reasons: (1) batch (i, l) has not still reached or has already overpassed the location of terminal $j \in J_l$, or (2) the model decides not diverting material from batch (i, l) to depot j despite it has an adequate location to do it. Therefore,

$$(d_{\min})_l x^{(i')}_{(i,l),j} \le D^{(i')}_{(i,l),j} \le (d_{\max})_l x^{(i')}_{(i,l),j} \quad \forall i \in I,\, i' \in I^{\text{new}}, \quad i' \ge i, \quad l \in PL, \quad j \in J_l \tag{16}$$

where $(d_{\min})_l$ and $(d_{\max})_l$ stand for the lower and upper bounds on the amount of material that can be transferred from pipeline l to a receiving depot during a composite run. Usually, $(d_{\min})_l$ takes a fairly low value. Constraints (17) and (18) stand for the feasibility conditions (a) and (b) described before. In such restrictions, the parameter pv_l is the total volume of pipeline l.

$$\begin{aligned} &F^{(i')}_{(i,l)} - \sum_{p \in P_l}\sum_{p' \in P_l} \text{WIF}_{i,p',p,l} \ge \sigma_{j,l} x^{(i')}_{(i,l),j} \quad \forall i \in I,\, i' \in I^{\text{new}}, \\ &i' \ge i,\, l \in PL,\, j \in J_l,\, j < |J_l| \\ &F^{(i')}_{(i,l)} \ge \sigma_{j,l} x^{(i')}_{(i,l),j} \quad \forall i \in I,\, l \in PL,\, i' \in I^{\text{new}},\, i' \ge i,\, j = |J_l| \end{aligned} \tag{17}$$

$$F^{(i'-1)}_{(i,l)} - W^{(i'-1)}_{(i,l)} + \sum_{k=1}^{j} D^{(i')}_{(i,l),k} \le \sigma_{j,l} + (pv_l - \sigma_{j,l})(1 - x^{(i')}_{(i,l),j}) \quad \forall i \in I,\, i' \in I^{new} (i' > i),\, l \in PL,\, j \in J_l \tag{18}$$

4.3.2. Bound on the total amount of product diverted from a batch to depot tanks during the execution of a composite run

The total volume transferred from batch (i, l) to depots $j \in J_l$ during run $i' > i$, must never exceed the content of lot (i, l) at time point $C_{i'-1}$.

$$\begin{aligned} &\sum_{\substack{j \in J_l \\ j < |J_l|}} D^{(i')}_{(i,l),j} \le W^{(i'-1)}_{(i,l)} - \sum_{p \in P_l}\sum_{p' \in P_l} \text{WIF}_{i,p',p,l} \\ &\sum_{j \in J_l} D^{(i')}_{(i,l),j} \le W^{(i'-1)}_{(i,l)} \quad \forall i \in I,\, i' \in I^{\text{new}},\, i' > i,\, l \in PL \end{aligned} \tag{19}$$

In line with Eq. (17), constraint (19) states that the interface volume can be removed for reprocessing just at the farthest destination of every pipeline ($j = |J_l|$).

4.3.3. Type and amount of product diverted from a batch while executing a composite run

Let $DP^{(i')}_{(i,l),p,j}$ be the amount of product p contained in batch (i, l) that is diverted to depot $j \in J_l$ during the new composite run $i' \ge i$. The variable $DP^{(i')}_{(i,l),p,j}$ will be equal to zero whenever: (a) batch (i, l) does not convey product p (i.e. $y_{(i,l),p} = 0$), and/or (b) there is no transfer of material from batch (i, l) to terminal j during run i' (i.e. $y_{(i,l),p} = 1$ but $D^{(i')}_{(i,l)j} = 0$). Otherwise, $DP^{(i')}_{(i,l),p,j} = D^{(i')}_{(i,l),j}$. These operational conditions are enforced by constraints (20).

$$\begin{aligned} &DP^{(i')}_{(i,l),p,j} \le (d_{\max})_l y_{(i,l),p} \quad \forall i,\, i' \in I^{\text{new}} (i' \ge i),\, l \in PL,\, p \in P_l,\, j \in J_l \\ &\sum_{p \in P_l} DP^{(i')}_{(i,l),p,j} = D^{(i')}_{(i,l),j} \quad \forall i,\, i' \in I^{\text{new}} (i' \ge i),\, l \in PL,\, j \in J_l \end{aligned} \tag{20}$$

On the other hand, the value of $DP^{(i')}_{(i,l),p,j}$ for an old batch (i, l) with $i \in I^{\text{old}}$ (already in the pipeline network at $t = 0$) is given by:

$$DP^{(i')}_{(i,l),p,j} = D^{(i')}_{(i,l),j} \quad \forall i \in I^{\text{old}},\, i' \in I^{\text{new}},\, l \in PL,\, p = p_{(i,l)},\, j \in J_l \tag{21}$$

where the product $p_{(i,l)}$ contained in an old batch (i, l) is a known problem datum.

4.3.4. Overall balance between incoming and outgoing material flows during a composite run around every pipeline

Because of the liquid incompressibility condition, the overall volume diverted from all batches moving along pipeline l to depots $j \in J_l$ must be equal to the size of the new batch (i', l), i.e. $Q_{(i',l)}$, pumped at the origin of line l while performing the composite run $i' \in I^{\text{new}}$.

$$\sum_{i \in I\, i \le i'} \sum_{j \in J_l} D^{(i')}_{(i,l),j} = Q_{(i',l)} \quad \forall i' \in I^{\text{new}},\; l \in PL \tag{22}$$

4.4. Constraints on product deliveries from terminals to markets

Let us introduce variables $\text{DMO}^{(i')}_{p,j}$ and $\text{DMP}^{(i')}_{p,j}$ to represent the amounts of product p delivered from depot $j \in J_p$ to neighboring markets during the time intervals $[C_{i'-1};\, C_{i'} - L_{i'}]$ and $[C_{i'} - L_{i'};\, C_{i'}]$, respectively. The former stands for the total volume of product p sent from the available inventory at depot j to consumer markets during the idle period between the completion of run ($i' - 1$) and the start of run i'. In turn, $\text{DMP}^{(i')}_{p,j}$ represents the total amount of p delivered to markets during run i' itself. If $vm_{p,j}$ is the maximum delivery rate of product p from terminal j to neighboring markets, then upper bounds on the values of $\text{DMO}^{(i')}_{p,j}$ and $\text{DMP}^{(i')}_{p,j}$ are provided by Eq. (23).

$$\begin{aligned} &\text{DMO}^{(i')}_{p,j} \le (C_{i'} - L_{i'} - C_{i'-1}) vm_{p,j} \\ &\text{DMP}^{(i')}_{p,j} \le L_{i'} vm_{p,j} \quad \forall p \in P,\, j \in J_p,\, i' \in I^{\text{new}} \end{aligned} \tag{23}$$

For the first element of the set I^{new} ($i' = \text{first}(I^{\text{new}})$), variable $C_{i'-1}$ in Eq. (23) has no meaning and is fixed to zero. Moreover, the model parameter $\text{dem}_{p,j}$ stands for the total demand of product p that should be sent from node j to consumer markets before the end of the time horizon. Then, the fulfillment of such product demands is ensured by adding Eq. (24) to the problem formulation.

$$\sum_{i' \in I^{\text{new}}} (\text{DMO}^{(i')}_{p,j} + \text{DMP}^{(i')}_{p,j}) + B_{p,j} = \text{dem}_{p,j} \quad \forall p \in P,\, j \in J_p \tag{24}$$

Eq. (24) includes the nonnegative term $B_{p,j}$ standing for back-orders of product p at node j. Then, potential product shortages are taken into account by the proposed model. The incorporation of variables $B_{p,j}$ in Eq. (24) not only avoids model solution failures because of a depleted inventory of product p at terminal j, but also permits to know when and where such product shortages arise, and which product is run out.

4.5. Inventory management constraints

Given the overall storage capacity available for each product at every depot, it is necessary to guarantee that: (i) enough amount of product p will be available in terminal tanks at the time of injecting new batches of p into outgoing pipelines, and (ii) the maximum inventory level at every depot tank should never be exceeded.

Storage tanks at any node of the pipeline network can be directly filled up with oil derivatives produced by nearby refineries. For simplicity, it is assumed that product flows from refineries to depot tanks are discharged at a fixed rate, with no interruptions. Let us introduce the parameter $vp_{p,j}$ representing the constant feed rate of product p from nearby refineries to depot j all over the planning horizon.

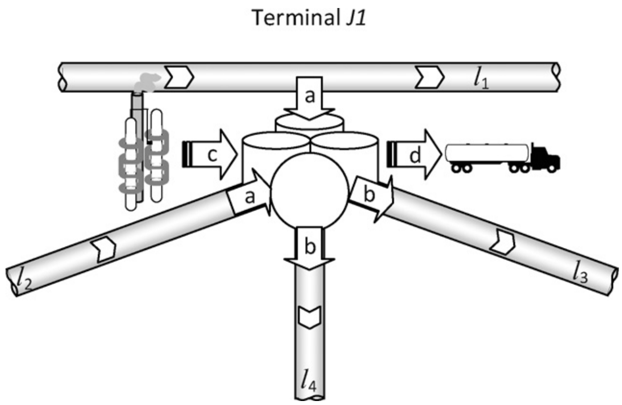

Fig. 3. Flow scenario for a generic terminal in the pipeline network.

4.5.1. Monitoring product inventories at every terminal of the pipeline network

The key issue of the proposed formulation is the coordination among incoming flows from upstream pipelines and/or oil refineries, and outgoing flows to downstream pipelines and/or consumer markets at every terminal of the network. The problem goal is to fulfill specified market demands before the end of the time horizon, while keeping product inventory levels at pipeline terminals within the feasible range. In this way, pipeline stoppages due to tank overloading, or backorders due to product shortages may be avoided. Fig. 3 illustrates the general product flow scenario for a node involving:

a. Incoming flows of refined oil products from upstream pipelines.
b. Outgoing flows of refined oil products to downstream pipelines.
c. Incoming flows of refined petroleum products from nearby refineries.
d. Outgoing flows of oil derivatives to neighboring consumer markets.

According to the proposed representation of the pipeline network, incoming flows can arrive from any upstream pipeline l converging to terminal $J1$; that is, from any $l \in PL_{J1}^{(\mathrm{IN})}$. The set $PL_{J1}^{(\mathrm{IN})}$ comprises all the upstream pipelines supplying refined products to node $J1$. In the example depicted in Fig. 3 $(l_1, l_2) \in PL_{J1}^{(\mathrm{IN})}$. Hence, incoming flows of product p may be received at depot $J1$ from any lot (i, l_1) or (i, l_2) conveying product p through pipelines l_1 or l_2, during a later run $i' \geq i$. Such product deliveries are represented by the variables $\mathrm{DP}_{(i,l1),p,J1}^{(i')}$ and $\mathrm{DP}_{(i,l2),p,J1}^{(i')}$, respectively. In turn, outgoing flows can be shipped from node $J1$ through any downstream pipeline l originating at depot $J1$, i.e. to any $l \in PL_{J1}^{\mathrm{OUT}}$. Therefore, batches of product p may be pumped from node $J1$ to pipelines l_3 and l_4 during a new run $i' \in I^{\mathrm{new}}$ whenever variables $\mathrm{QP}_{(i',l3),p}$ and $\mathrm{QP}_{(i',l4),p}$ take non-zero values. In the most general case, it may also occur that a flow of product p sent from nearby refineries is discharged into storage tanks of terminal $J1$ at a fixed rate of $vp_{p,J1}$ units per hour. Finally, the amount of product p delivered from node $J1$ to neighboring markets since the completion of run $(i' - 1)$ to the end of the next run i' is given by the sum of $\mathrm{DMO}_{p,J1}^{(i')}$ and $\mathrm{DMP}_{p,J1}^{(i')}$.

Because the time domain is handled in a continuous manner, the aggregate inventory level of product p in node j, given by the variable $\mathrm{ID}_{p,j}^{(i')}$, is monitored at every time event $t = C_{i'}$ for all $i' \in I^{\mathrm{new}}$. The value of $ID_{p,j}^{(i')}$ is computed by simultaneously adding and/or subtracting the following terms to the available stock of p in terminal j at time $t = C_{i'-1}$:

(a) Add the total amount of product p provided by batches (i, l) containing p coming from upstream pipelines $l \in PL_j^{(\mathrm{IN})}$, with the set $PL_j^{(\mathrm{IN})}$ comprising all pipelines supplying terminal j. This term is null for depots j featuring $PL_j^{(\mathrm{IN})} = \varnothing$, i.e. pure source terminals.
(b) Subtract the overall volume of p injected into downstream pipelines $l \in PL_j^{(\mathrm{OUT})}$ at terminal j. This term just arises for terminals acting as input stations.
(c) Add the product flow coming from neighboring refineries to tanks of terminal j. This flow contribution should only be considered for pure sources or dual nodes supplied by nearby refineries.
(d) Deduct product deliveries dispatched from terminal j to local markets, generally by truck.

All these terms are included in Eq. (25).

$$\mathrm{ID}_{p,j}^{(i')} = \mathrm{ID}_{p,j}^{(i'-1)} + \sum_{l \in PL_j^{(IN)}} \sum_{i \in I\, i \leq i'} DP_{(i,l),p,j}^{(i')} - \sum_{l \in PL_j^{(\mathrm{OUT})}} QP_{(i',l),p} + vp_{p,j}(C_{i'} - C_{i'-1}) - (DMO_{p,j}^{(i')} + DMP_{p,j}^{(i')}) \quad \forall p \in P,\ j \in J_p,\ i' \in I^{\mathrm{new}} \tag{25}$$

For the first pumping run of the current time horizon, the variable $ID_{p,j}^{(i'-1)}$ in Eq. (25) is equal to the initial stock of product p in tanks of node j given by the parameter $\mathrm{ido}_{p,j}$. Besides, variable $ID_{p,j}^{(i')}$ should remain within the feasible range given by the specified minimum and maximum inventory levels.

$$(\mathrm{id}_{\min})_{p,j} \leq ID_{p,j}^{(i')} \leq (\mathrm{id}_{\max})_{p,j} \quad \forall p \in P,\ j \in J_p,\ i' \in I^{\mathrm{new}} \tag{26}$$

In general, $(\mathrm{id}_{\min})_{p,j}$ is a safety stock level preventing from product shortages when demand fluctuates. In turn, $(\mathrm{id}_{\max})_{p,j}$ is an upper bound on the aggregate stock of product p that can be stored into tanks of node j. This parameter normally keeps some tank capacity free for compensating temporary unbalances between incoming and outgoing flows during the execution of a composite run.

4.5.2. Product availability for new product injections

By assumption (A7), the total volume of product p shipped from the source node j through downstream pipelines $l \in PL_j^{(\mathrm{OUT})}$ during the new composite run $i' \in I^{\mathrm{new}}$ must be available in depot tanks at the start of the run i', i.e. at time $(C_{i'} - L_{i'})$. Such a condition is enforced by Eq. (27).

$$\sum_{l \in PL_j^{(\mathrm{OUT})}} QP_{(i',l)p} \leq \mathrm{ID}_{p,j}^{(i'-1)} + vp_{p,j}(C_{i'} - L_{i'} - C_{i'-1}) - \mathrm{DMO}_{p,j}^{(i')} \quad \forall p \in P, j \in J_p, i' \in I^{\mathrm{new}} \tag{27}$$

This rather conservative constraint is usually enforced at intermediate terminals where direct transfers of products from incoming to outgoing pipelines (called "tightlinings") are forbidden. In such terminals, product batches that arrive from upstream pipelines are temporarily stored in dedicated tanks to accomplish quality control operations, before sending some portions of them through downstream pipelines at later pumping runs.

Fig. 4 illustrates the coordination among incoming and outgoing flows at the dual node $J2$. Based on quality assessment conditions, product batches that arrive at terminal $J2$ are sent to tanks and controlled before moving on to downstream destinations. The first line in Fig. 4 depicts batches of products P_1, P_2, and P_3 moving along

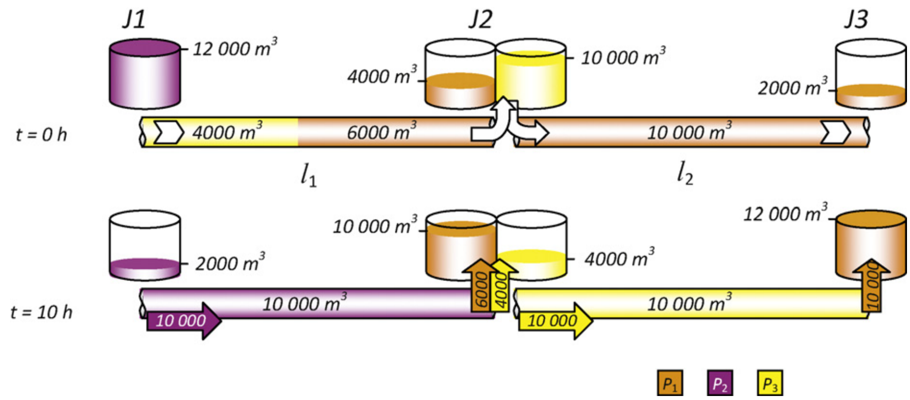

Fig. 4. Coordinating incoming and outgoing flows at a dual terminal.

pipelines l_1 and l_2, as well as the inventory levels in terminal tanks at time $t = 0$ h. Initially, batches (i_1, l_1) and (i_2, l_1) with 6000 and 4000 m^3 of products P_1 and P_3, respectively, travel along pipeline l_1, meanwhile line l_2 just contains a single lot (i_2, l_2) of product P_1 with a volume of 10,000 m^3.

During the new composite run i_3 starting at time $t = 0$ h and ending at $t = 10$ h, two batches are simultaneously pumped into both pipelines l_1 and l_2 at a fixed rate of 1000 m^3/h. They are the batch (i_3, l_1), with 10,000 m^3 of product P_2, and the batch (i_3, l_2), containing 10,000 m^3 of product P_3 (see the last line of Fig. 4). Let us analyze the evolution of the inventory level of product P_3 at terminal $J2$. At time $t = 0$ h, there are 10,000 m^3 of P_3. After the injection of product P_3 through batch (i_3, l_2), the stock of P_3 at $J2$ would run out. However, the lot (i_2, l_1) containing P_3 is discharged into tanks of $J2$ during run i_3, thus rising the stock of P_3 up to 4000 m^3. Despite the incoming volume of product P_3 at terminal $J2$, the size of the injection (i_3, l_2) cannot be further enlarged because the flow of P_3 arriving from pipeline l_1 should be stored and controlled before shipping to destination $J3$. Fig. 5 illustrates the variation of the aggregate inventory levels of products P_1 and P_3 at terminal $J2$ during the new composite run i_3 described in Fig. 4.

In the first 6 h, the stock of P_1 at depot $J2$ shows an increase from 4000 m^3 to 10,000 m^3 caused by the arrival of batch (i_1, l_1). Meanwhile, a volume of P_3 is removed from tanks of terminal $J2$ to inject the batch (i_3, l_2) into the outgoing line l_2. At $t = 6$ h, the available inventory of P_3 has dropped from 10,000 to 4000 m^3. In the last 4 h, the inventories of P_1 and P_3 show no variation because of two reasons: (a) no batch of P_1 is neither leaving nor arriving at node $J2$; and (b) the volume of P_3 in batch (i_2, l_1) that is fully transferred to $J2$ from $t = 6$ h to $t = 10$ h is exactly equal to the amount of P_3 pumped from $J2$ into line l_2 during the same time interval, i.e. 4000 m^3.

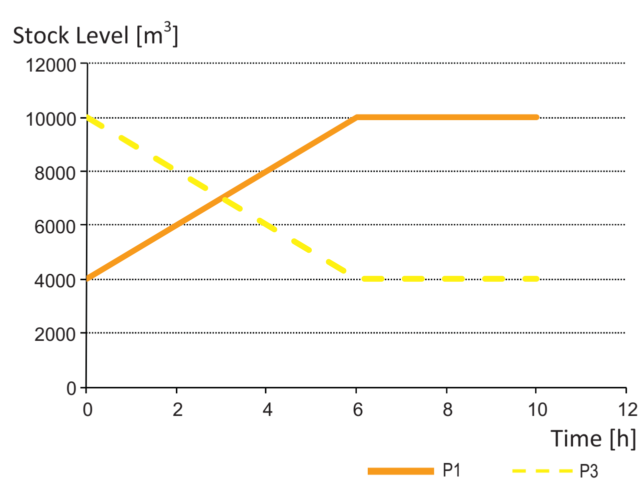

Fig. 5. Stocks of products P_1 and P_3 at terminal $J2$ for the illustrating example.

4.6. The objective function

One of the most important terms in the objective function is the pumping cost. The energy consumed for transporting a unit volume of product p to depot j depends on the pipeline l through which the product will be delivered. Revisiting the pipeline network depicted in Fig. 1, there are two alternative routes for supplying refined products to node $N5$: (i) directly from refinery $N1$ through pipeline l_6, or (ii) from the intermediate node $N4$ through pipeline l_3. Hence, moving a single unit of product p from refinery $N1$ to node $N5$ may cost $cp_{N5,l6,p}$ if it is directly sent through line l_6, or $(cp_{N4,l1,p} + cp_{N5,l3,p})$ if the unit of product p is first moved from $N1$ to the intermediate depot $N4$ through line l_1, and then re-pumped from $N4$ to $N5$ through line l_3. Though it is more likely that direct supplies from refinery are less expensive than indirect pathways, pumping costs will also depend on the pipeline dimensions and the network topography. For instance, pipeline l_6 can be shorter than the sum of the lengths of pipelines l_1 and l_3, but it may have a much smaller diameter, thus increasing the pressure drop along the pipeline for a given pump rate.

The first term in Eq. (28) accounts for the total cost of pumping batches along every pipeline of the network during the current horizon. It is obtained by summing all the volumes delivered from batches (i, l) containing product p to depot $j \in J_l$ during the execution of composite runs $i' \in I^{new}$ $(i' \geq i)$. The delivered amount $DP^{(i')}_{(i,l),p,j}$ is then multiplied by the cost of moving a unit volume of p to depot j through line l $(cp_{j,l,p})$. If the overall volume of product p delivered to depot j during the current horizon is not enough to fulfill market requirements, an unsatisfied demand represented by the variable $B_{p,j}$ will arise (see Eq. (24)). As stated by the second term of the objective function, a relatively expensive backorder cost $(cb_{p,j})$ will be paid for failing to supply a unit amount of product p to the consumer market of terminal j before the end of the time horizon.

$$
\begin{aligned}
\min z = & \sum_{l \in PL} \sum_{p \in P_l} \sum_{j \in J_l} \left(cp_{j,l,p} \sum_{i \in I} \sum_{i' \in I^{new}\, i' \geq i} \mathrm{DP}^{(i')}_{(i,l),p,j} \right) \\
& + \sum_{p \in P} \sum_{j \in J_p} cb_{p,j} B_{p,j} + \sum_{l \in PL} \sum_{p \in P} \sum_{\substack{p' \in P \\ p' \neq p}} \sum_{\substack{i \in I^{new} \\ i > 1}} cf_{p',p,l} \mathrm{WIF}_{i,p,p',l} \\
& + \sum_{p \in P} \sum_{n \in J_p} \mathrm{cid}_{p,j} \left(\frac{1}{|I^{new}|} \sum_{i' \in I^{new}} \mathrm{ID}^{(i')}_{p,j} \right) + cmk\, MK \qquad (28)
\end{aligned}
$$

Table 1
Product inventories and demands at pipeline depots for examples 1–3.

Prod.	Stock level [10^2 m^3]	Logistic nodes in the network						
		N1	N2	N3	N4	N5	N6	N7
P1	Min	100	–	50	50	50	–	50
	Max	1000	–	500	500	500	–	500
	Initial	1000	–	50	250	50	–	50
	Demand [10^2m^3]	–	–	150	–	800	–	200
P2	Min	100	100	50	100	100	–	100
	Max	1000	1000	500	1000	1000	–	1000
	Initial	1000	1000	50	700	100	–	100
	Demand [10^2m^3]	–	–	150	–	900	–	400
P3	Min	20	–	10	10	–	–	10
	Max	200	–	100	200	–	–	100
	Initial	200	–	30	100	–	–	10
	Demand [10^2m^3]	–	–	10	–	–	–	250
P4	Min	–	40	–	20	–	20	–
	Max	–	400	–	300	–	200	–
	Initial	–	200	–	100	–	20	–
	Demand [10^2m^3]	–	–	–	–	–	200	–

In turn, the degrading/reprocessing cost of the interface material between consecutive batches depends on the product sequence inputted at the origin of every pipeline of the network. If $\mathrm{WIF}_{i,p,p',l}$ stands for the total volume of interface $p-p'$ generated into pipeline l due to the injection of batch i, and $cf_{p,p',l}$ is the unit cost of degrading and/or reprocessing interfaces of type *p-p'* in pipeline l, then the third term of the objective function will tend to minimize the cost of product transitions. A simple way to reach that goal is to increase the size of the batch injections as much as possible. However, it arises a trade-off between interface and inventory carrying costs because the larger the batches, the higher the average inventory level at pipeline terminals. In Eq. (28), the fourth term accounts for an estimation of the inventory carrying costs at every terminal, based on the average inventory level of each product in depot tanks. Parameter $\mathrm{cid}_{p,j}$ represents the cost of holding a unit volume of p in tanks of node j throughout the time horizon. The average inventory level of product p in every node is approximated by summing product stocks at the end time of every batch injection i', that is $\mathrm{ID}_{p,j}^{(i')}$, and dividing the result by the number of composite pumping runs ($|I^{\mathrm{new}}|$). If no elements of the set I^{new} stand for fictitious runs at the optimum, the term included in the objective function achieves a good estimation of the inventory carrying costs. Otherwise, a better estimation of the total inventory carrying cost is obtained by just accounting for the number of non-fictitious runs.

Finally, the last term in the objective function (28) tends to reduce the problem makespan (MK), i.e. the time needed to fulfill all terminal requirements. By considering this term, unnecessary pipeline stoppages and their negative effects are avoided. During idle time periods, the mixing process is intensified and interface costs significantly rise. In addition, when operations resume, the pumping cost also steps up to restart the flow in the pipeline system. The unit cost *cmk* is paid for every hour of utilization of the pipeline network. The value of *MK* is obtained by imposing $MK \geq C_i$ for all $i \in I^{\mathrm{new}}$.

5. Results and discussion

Short-term operational schedules for three pipeline networks of increasing complexity have been developed by using the proposed mathematical formulation given by Eqs. (1)–(28). The level of difficulty, while proceeding from the first to the third case study, has been raised by including more pipelines and/or logistic nodes in the transportation network. Example 1 is concerned with the operational planning of a multi-product pipeline network comprising four trunk lines and seven logistic nodes (two refineries, an intermediate depot, and four receiving terminals). Example 2 deals with an extended pipeline network that incorporates an additional line directly supplying a major receiving terminal from one of the refineries. The major goal of example 2 is to demonstrate that the use of an auxiliary pipeline as a direct route between a primary source and the receipt location may lead to substantial cost savings. Finally, example 3 considers the complete pipeline network including an additional delivering line to convey refined oil products from the intermediate depot to a new pipeline terminal instead of doing the transportation service by truck.

The three examples assume a 10-day planning horizon. For each case study, the problem goal is to find the optimal pipeline schedule that exactly meets all products requirements from each terminal before the time horizon end at minimum total cost. Product demands at distribution terminals are given in Table 1. This table also includes the minimum, maximum, and initial stock levels (in 10^2 m^3) at every depot. Note that in all but one case (*P*3 at *N*3), terminal demands largely exceed product inventories initially available in storage tanks. Product backorders are not allowed. Moreover, it is not expected to receive further production flows at the primary source nodes *N*1 and *N*2 from nearby refineries during the next ten days. Therefore, terminal demands should be satisfied by using the initial inventories available at *N*1 and *N*2. Shipments received at pipeline terminals are subsequently dispatched by truck to fulfill promised customer orders at a maximum delivery rate of 700 m^3/h for any product and destination. For operational reasons, the duration of a batch injection at any input station should neither be shorter than 1 h (to avoid high changeover and interface costs) nor longer than 60 h (to reduce inventory carrying costs and have a broader variety of products into the pipeline for a fast response to new terminal demands). Besides, the cost per unit time of utilization of the pipeline network (*cmk*) is set to \$1000 per hour.

On the other hand, the interface reprocessing costs for every pair of products sequentially pumped into any pipeline are given in Table 2. Forbidden product sequences are denoted with a letter "X". Moreover, Table 2 shows the inventory carrying cost for holding a single unit of any product during the following ten days at every terminal. Finally, Table 3 presents the pumping cost to be paid for supplying a unit of product p to node j through pipeline l, in \$/m^3. For instance, every unit of product *P*4 supplied from node *N*2 to node *N6* will cost \$0.93 + \$0.56 = \$1.49, because it should first be moved to node *N4* through pipeline l_2 (\$0.93) and then re-pumped and directed to node *N6* through pipeline l_4 (\$0.56). At the end of the time horizon, the intermediate node *N4* should have a final stock of at least 25,000; 50,000; 5000; and 10,000 m^3 of products

Table 2
Interface and inventory carrying costs for examples 1–3.

Interface costs [10^2 $]					Inventory costs [$/m^3]						
	P1	P2	P3	P4	N1	N2	N3	N4	N5	N6	N7
P1		184	340	235	0.19	–	0.24	0.19	0.24	–	0.24
P2	184		250	413	0.16	0.16	0.20	0.16	0.20	–	0.20
P3	340	250		X	0.24	–	0.32	0.24	–	–	0.32
P4	235	413	X		–	0.24	–	0.24	–	0.32	–

Table 3
Unit pumping cost for delivering product p to node j through line l ($cp_{j,l,p}$).

Line	Origin	Product	Pumping Costs [$/m^3] Destination				
			N3	N4	N5	N6	N7
l_1	N1	P1	0.70	1.05	–	–	–
		P2	0.72	1.08	–	–	–
		P3	0.96	1.44	–	–	–
l_2	N2	P2	–	0.90	–	–	–
		P4	–	0.93	–	–	–
l_3	N4	P1	–	–	0.88	–	–
		P2	–	–	0.90	–	–
l_4	N4	P1	–	–	–	–	1.05
		P2	–	–	–	–	1.08
		P3	–	–	–	–	1.44
		P4	–	–	–	0.56	–

*P*1; *P*2; *P*3; and *P*4, respectively. This condition is imposed so that the intermediate node *N*4 have enough starting inventories to meet product demands from distribution terminals *N*5, *N*6, and *N*7 in the next planning horizon. For the three examples, the proposed MILP model was solved to optimality on an Intel Xeon CPU (2.67 GHz) using GAMS/GUROBI 3.0 with 6 parallel threads (in deterministic mode) as the mixed-integer solver (Brooke, Kendrick, Meeraus, & Raman, 2006). An optimality gap of 10^{-9} has been adopted as the stopping criterion.

5.1. Example 1

The mesh-structure pipeline network studied in example 1 involves four mainlines (l_1, l_2, l_3, l_4) distributing four oil refined products (*P*1, *P*2, *P*3, *P*4) to four destination nodes (see Fig. 6). Pipelines l_1 and l_2 connect a pair of oil refineries (*N*1, *N*2) to a major distribution center (*N*4), whereas lines l_3 and l_4 both depart from *N*4 and reach the "pure" receiving terminals *N*5 and *N*7, respectively. Because they process different types of crude oils, refinery *N*1 only supplies products *P*1, *P*2, and *P*3, while refinery *N*2 is a primary source of products *P*2 and *P*4. Every mainline can move products at a flow rate ranging from 800 to 1000 m^3/h. Pipeline l_1 has a total volume of 60,000 m^3 (20 in. diameter, 300 km long) and two destination nodes (*N*3 and *N*4). The intermediate node *N*3 is located 200 km far from the origin of line l_1 ($\sigma_{N3,l1}$ = 40,000 m^3), while *N*4 arises at the farthest extreme of l_1. Batches coming from source *N*1 can be stripped into tanks of terminal *N*3 and/or directly sent to depot *N*4.

On the other hand, pipeline l_2 departing from node *N*2 has a capacity of 50,000 m^3 (20 in. diameter, 250 km long) and directly supplies products *P*2 and *P*4 to the transfer node *N*4. This line has no intermediate destination. Downstream of terminal *N*4 is the line l_3 with a capacity of 50,000 m^3 to convey products *P*1 and *P*2 from *N*4 to the receipt terminal *N*5. Similarly, pipeline l_4 with a capacity of 60,000 m^3 also departs from node *N*4 and transports all the products to terminals *N*6 and *N*7. Node *N*6 is located midway over the line l_4 ($\sigma_{N6,l4}$ = 30,000 m^3) and is the only depot in the network requesting product *P*4. In contrast, the other destination *N*7 demands products *P*1, *P*2, and *P*3.

The initial state of the pipeline network for example 1 is shown in both Fig. 6 and the first row of Fig. 7 (t = 0 h). To solve this example, the proposed formulation initially assumes a maximum number of composite runs $|I^{new}|$ equal to the number of products transported by the pipeline network ($|P|$ = 4). The resulting MILP model was solved in 6.38 CPU s and the solution found presents a total cost of $987,950 and a makespan equal to 183 h. Moreover, four composite runs are executed, i.e. the maximum number of runs available. Therefore, a better operational schedule may be obtained by raising $|I^{new}|$ to five. By doing so, an improved schedule (depicted in Fig. 7) is found in 98.75 CPU s.

For $|I^{new}|$ = 5, the total distribution cost decreases to $982,010, and the time needed to fulfill all terminal demands drops from 183 h to 179 h. Interface and pumping costs remain the same, but the inventory carrying cost and the network utilization cost both decrease. A total of eight new interfaces between lots of different products distributed among the four pipelines (especially in lines l_1 and l_4) are generated. Two of them involve the pair *P2*–*P4*, i.e. the most costly interface. Model sizes, CPU times, optimal total costs, and other relevant data are reported in Table 4. From this table, it follows that a further increase of $|I^{new}|$ has no impact on the makespan and produces minor savings on inventory costs by splitting some batch injections in two or more composite runs.

The best solution with $|I^{new}|$ = 5 comprises the execution of five batch injections into pipelines l_1, l_3, and l_4, and only three into pipeline l_2, i.e. a total of 18 single pumping runs. During the first 60 h, pipelines l_1 and l_3 operate at full capacity (1000 m^3/h), to inject 60,000 m^3 of product *P*2 from nodes *N*1 and *N*4, respectively. Through the injection of *P2* in line l_1, depot *N3* receives products $P1_{10,000}$ and $P2_{15,000}$ (with the subscripts indicating the receiving volume, in m^3) while products $P3_{10,000}$ and $P1_{25,000}$ are discharged into tanks of the transfer node *N*4. In turn, the pumping of 60,000 m^3 of product *P*2 into line l_3 pushes some amounts of products $P1_{35,000}$ and $P2_{25,000}$ to terminal *N*5. During the same composite run, node *N*2 injects product $P2_{56,000}$ in line l_2, causing the discharge of products $P4_{20,000}$ and $P2_{36,000}$ into tanks of node *N*4. At the same time interval, node *N*4 pumps an additional volume of product $P3_{10,000}$ through pipeline l_4, displacing some quantity of product $P2_{10,000}$ to node *N*7.

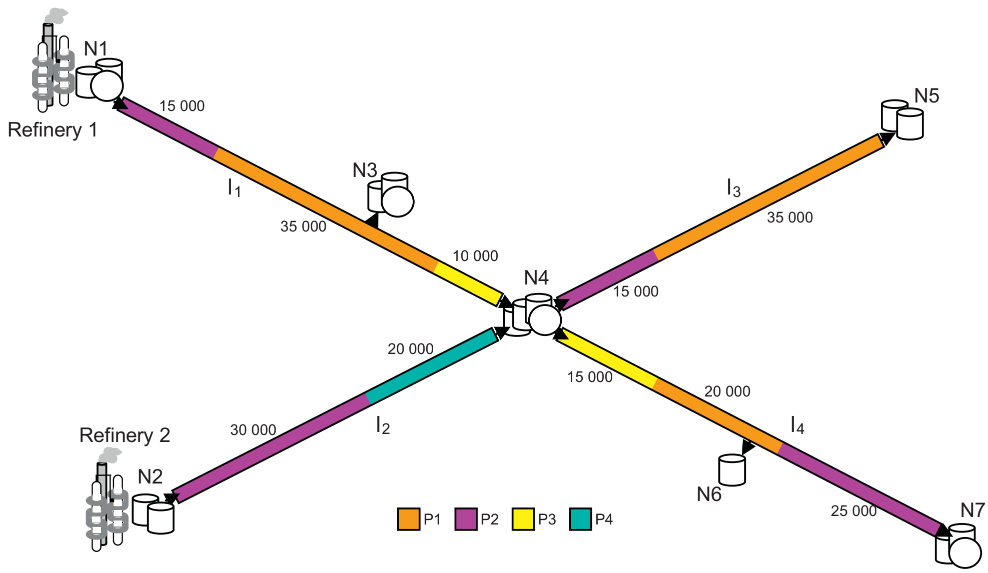

Fig. 6. Initial state of the multiproduct pipeline network of example 1.

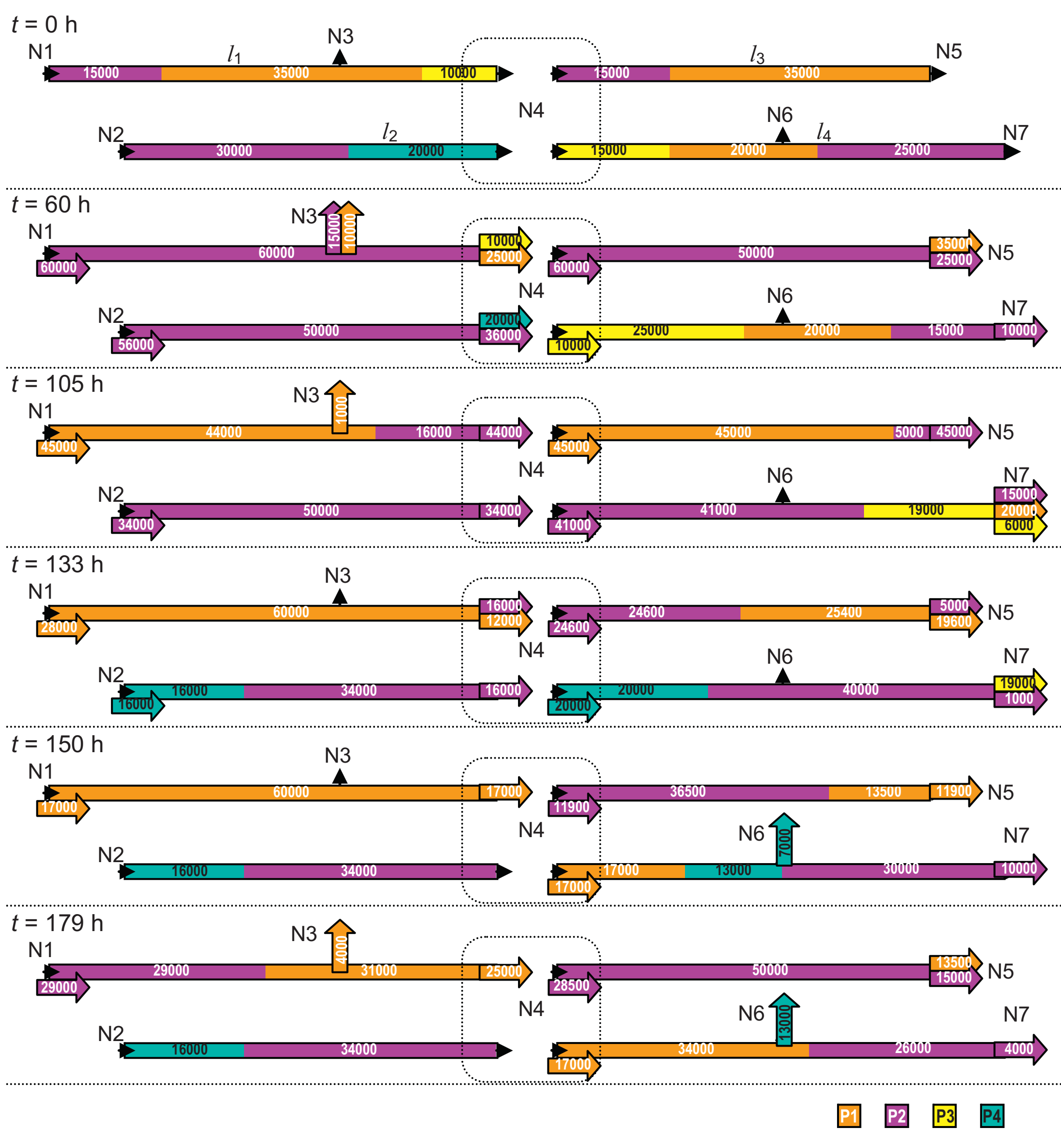

Fig. 7. Optimal operational schedule with $|I^{new}| = 5$ for example 1.

Table 4
Model sizes, computational requirements and results for examples 1–3.

Ex.	$\lvert I^{new} \rvert$	Eqs.	Cont. variables	Binary variables	CPU time (s)	Iter. (10^6)	Optimal solution (10^2 \$)	Interface cost (10^2 \$)	Pumping cost (10^2 \$)	Inventory cost (10^2 \$)	Make-span (h)
1	4	2068	1071	196	6.32	0.25	9879.5	2047.0	5422.5	580.0	183
	5	2736	1412	260	98.7	5.99	9820.1	2047.0	5422.5	560.6	179
	6	3464	1785	330	1602.6	83.31	9789.7	2047.0	5422.5	530.2	179
	7[a]	4252	2190	406	28,161.9	1111.4	9789.7	2047.0	5422.5	530.2	179
2	5	3160	1591	390	59.3	3.26	8588.4	1516.0	5020.5	551.9	150
	6[a]	4003	2015	393	654.4	30.13	8588.4	1516.0	5020.5	551.9	150
3	5	3606	1779	360	174.5	6.42	9646.0	2342.0	5265.0	539.0	150
	6[a]	4566	2252	456	2435.9	110.91	9646.0	2342.0	5265.0	539.0	150

[a] One of the new pumping runs is fictitious at the optimum.

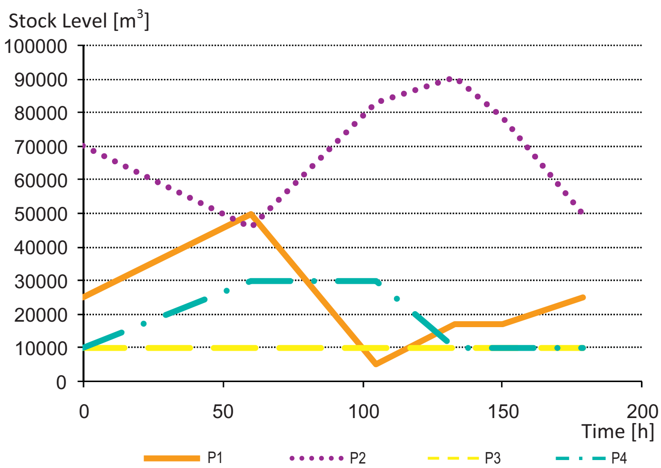

Fig. 8. Inventory profiles at node N4 for example 1.

One of the critical points in the pipeline network is the transfer node *N*4. The coordination among incoming and outgoing flows at node *N*4 is a very complex task effectively managed by the proposed formulation. Fig. 8 illustrates the product inventory profiles at node *N*4 over the planning horizon. After accomplishing the first composite run, the stocks of products *P*1 and *P*4 in tanks of node *N*4 are increased by 25,000 and 20,000 m^3, respectively, and the inventory of product *P*2 is reduced by 24,000 m^3 (60,000 m^3 out – 36,000 m^3 in). Because the same amount of *P*3 is received from l_1 and simultaneously injected into line l_4 during the first run, the inventory level of *P*3 at node *N*4 shows no change.

During the second run going from time t = 60 h to t = 105 h, pipelines l_1 and l_3 continue working at the maximum pump rate, but this time nodes *N*1 and *N*4 both start the injection of product $P1_{45,000}$. Besides, refinery *N*2 keeps sending $P2_{34,000}$ through line l_2, while the intermediate node *N*4 pumps the same product $P2_{41,000}$ into pipeline l_4 forcing the discharge of products $P2_{15,000}$, $P1_{20,000}$, and $P3_{6000}$ into tanks of node *N*7. Despite a batch of $P4_{20,000}$ received from line l_2 is available in tanks of *N*4 at the start time of the second run (t = 60 h), it cannot be immediately pumped through line l_4 because the transition *P*3–*P*4 is forbidden. Therefore, the inventory level of *P*4 remains above its initial value for a while. From Fig. 8, it is also observed that the inventory level of *P*2 at *N*4 experiences a significant increase of 37,000 m^3 caused by the new arrivals from lines l_1 and l_2 (i.e. a total volume of 78,000 m^3), much larger than the amount of *P*2 shipped through l_4 (41,000 m^3). On the contrary, *P*1-stock at *N*4 is depleted by pumping a new batch of 45,000 m^3 into line l_3. In fact, at time t = 105 h, the stock of product *P1* reaches its minimum admissible level (5000 m^3). Nonetheless, tanks of *P*1 will be replenished during the following runs by new arrivals coming from refinery *N1* through line l_1.

Through the five composite runs, refinery *N1* pumps three new lots of products $P2_{60,000}$, $P1_{90,000}$ and $P2_{29,000}$ into pipeline l_1, and works at the maximum flow rate all over the time horizon. The

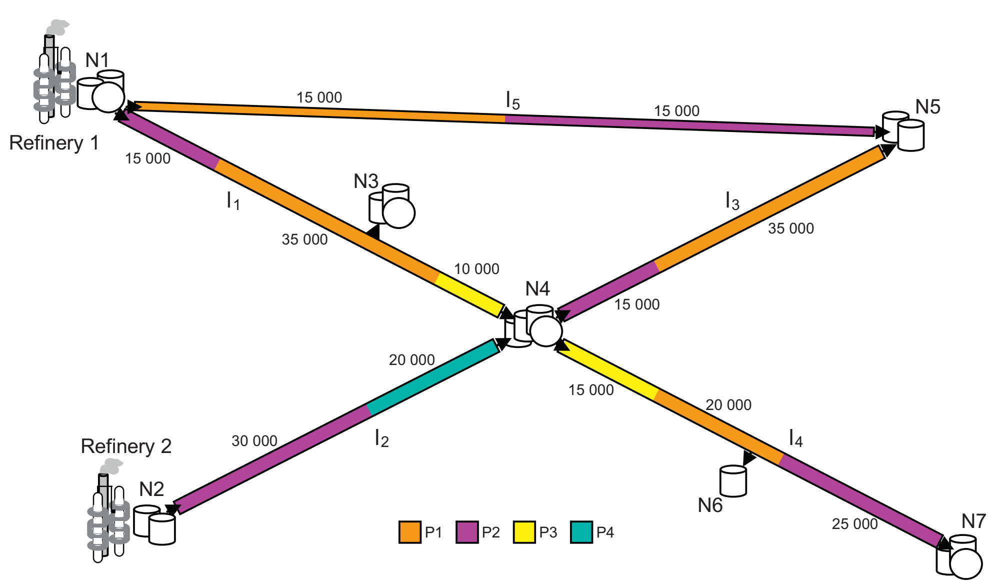

Fig. 9. Initial state of the multiproduct pipeline network of example 2.

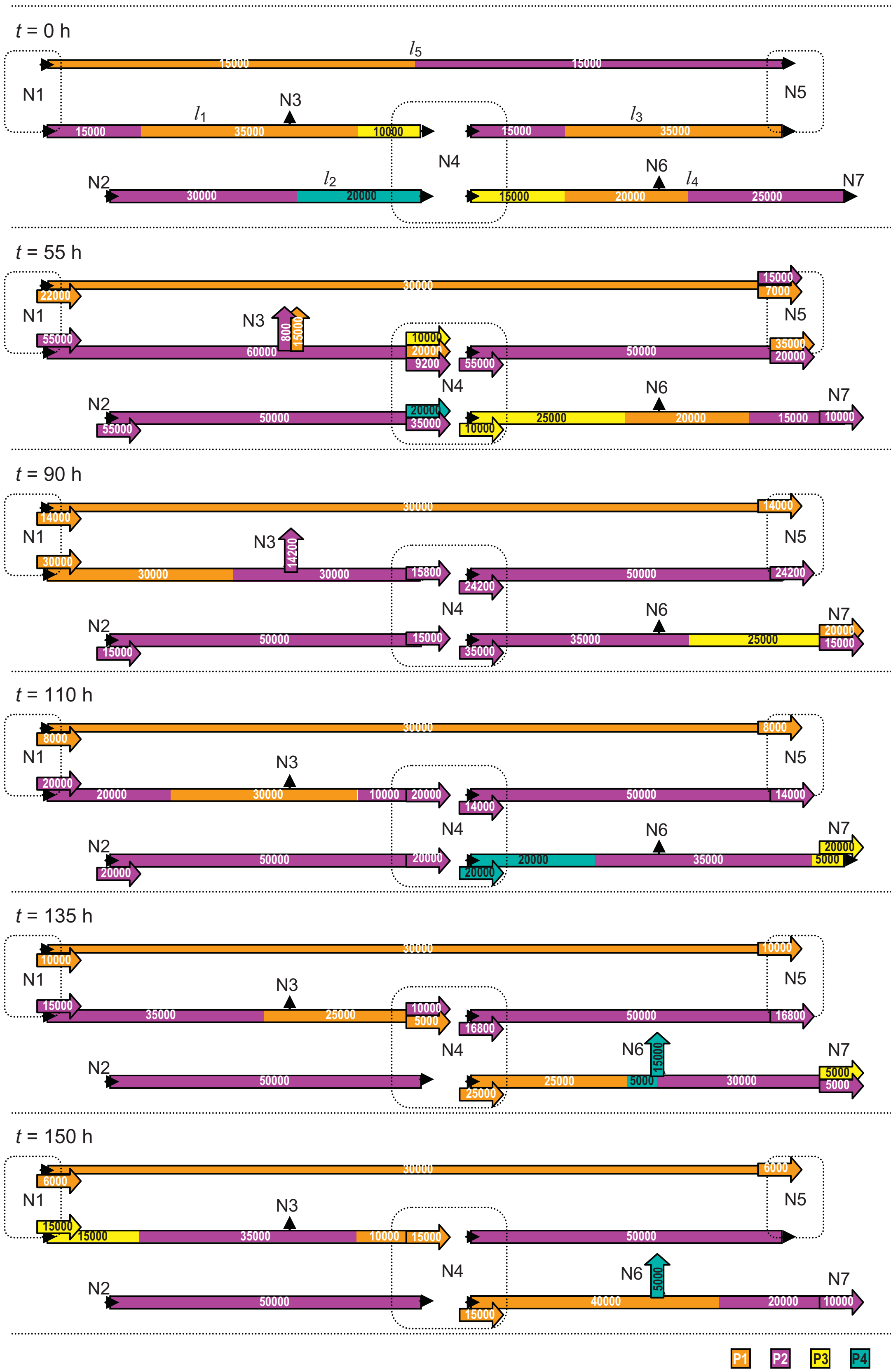

Fig. 10. Optimal pump schedule for the pipeline network of example 2.

Table 5
Pumping costs for the new pipeline l_5 (N1–N5) at example 2.

Line	Origin	Product	Pumping costs [\$/m^3] Destination				
			N3	N4	N5	N6	N7
l_5	N1	P1	–	–	1.22	–	–
		P2	–	–	1.25	–	–

Table 6
Product inventory and demand at the new node N8 (example 3).

Prod.	Stock Level [10^2m^3]	Logistic node N8
P4	Min	10
	Max	100
	Initial	10
	Demand [10^2 m^3]	160

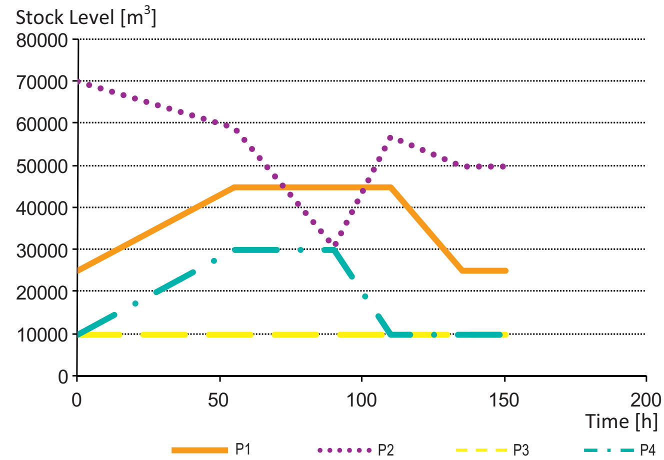

Fig. 11. Inventory levels at node *N4* in example 2.

injection of product $P1_{90,000}$ into line l_1 is distributed among three composite runs. As a result, the stock of *P*1 at terminal *N*1 is lowered to the minimum allowed level at the horizon end. In turn, refinery *N*2 dispatches all the available quantities of products $P2_{90,000}$ and $P4_{16,000}$ during the first three runs. Then, the operation of pipeline l_2 stops earlier at time t = 133 h. When the planned operations are completed, the final stocks of products *P*1, *P*2, and *P*4 at depot *N*4 stay at the specified final levels, while product *P*3 remains 5000 m^3 over the safety stock level required. The usage of pipelines l_1, l_2, l_3 and l_4, defined as the percent ratio between the total volume pumped and the transport capacity (available working time × maximum pump rate) amounts to 100%, 59.2%, 95.0%, and 58.7%, respectively.

5.1.1. *Increasing the number of composite pumping runs* $|I^{new}|$

Like all slot-based continuous time approaches, the total number of composite pumping runs needed to find the optimal pipeline schedule is not known beforehand. Hence, a typical procedure is to increase the cardinality of the set I^{new} by one until no better schedule is discovered. By rising $|I^{new}|$ to six, a new pipeline network schedule is obtained in 1602.6 CPU s. The new best solution is rather similar to the one encountered for $|I^{new}|$ = 5. As shown in Table 4, the makespan is not shortened. By partitioning the last batches pumped into pipelines l_1, l_3, and l_4 into two consecutive composite runs, the inventory level of *P*2 at *N*4 can be slightly reduced, thus producing some savings in inventory carrying costs. In a further step, the set of new pumping runs is raised to seven. After 28,162 CPU s, the model demonstrates that the solution quality can no longer be improved, because one of the proposed composite runs is never executed, i.e. it is a fictitious run.

5.2. *Example 2*

From the results found for example 1, it follows that the most congested route in the network (i.e. the "critical path") comprises the pipelines l_1 and l_3, connecting refinery *N*1 to depot *N*5 through the transfer terminal *N*4. In fact, the usage of pipelines l_1 and l_3 in example 1 is 100% and 95% of their maximum transport capacities. This is so because node *N5* features the highest demands for products *P*1 and *P*2, and refinery *N*1 is the only source of product *P*1 (see Table 1).

One possibility for debottlenecking the critical route is to make use of an additional pipeline, represented by line l_5 in Fig. 9, directly connecting refinery *N1* to depot *N5*. This new pipeline has a capacity of 30,000 m^3 (12 in. diameter, 411 km long) and is initially filled with lots of products *P*1 and *P*2. Because of its lower diameter and

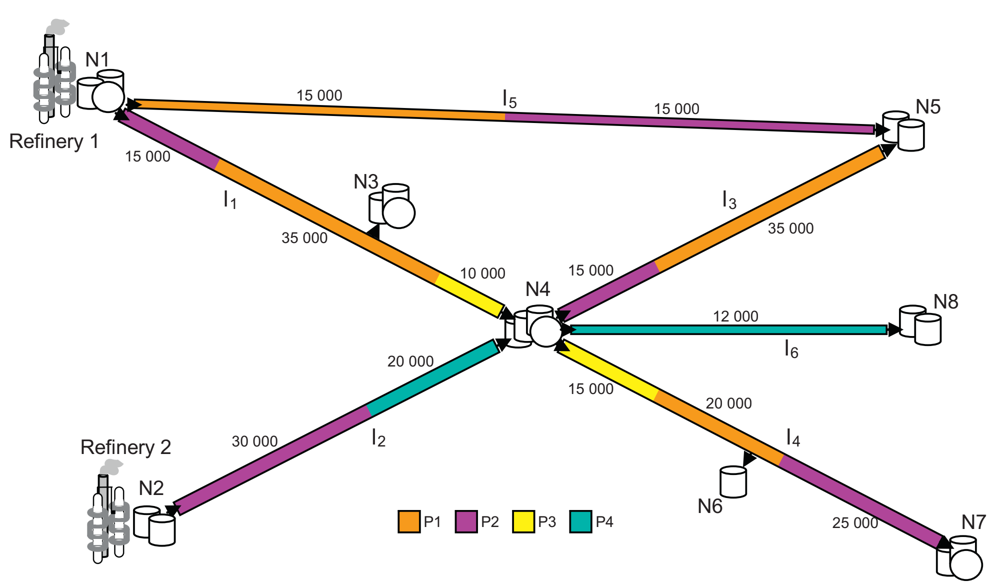

Fig. 12. Initial state of the pipeline network studied at example 3.

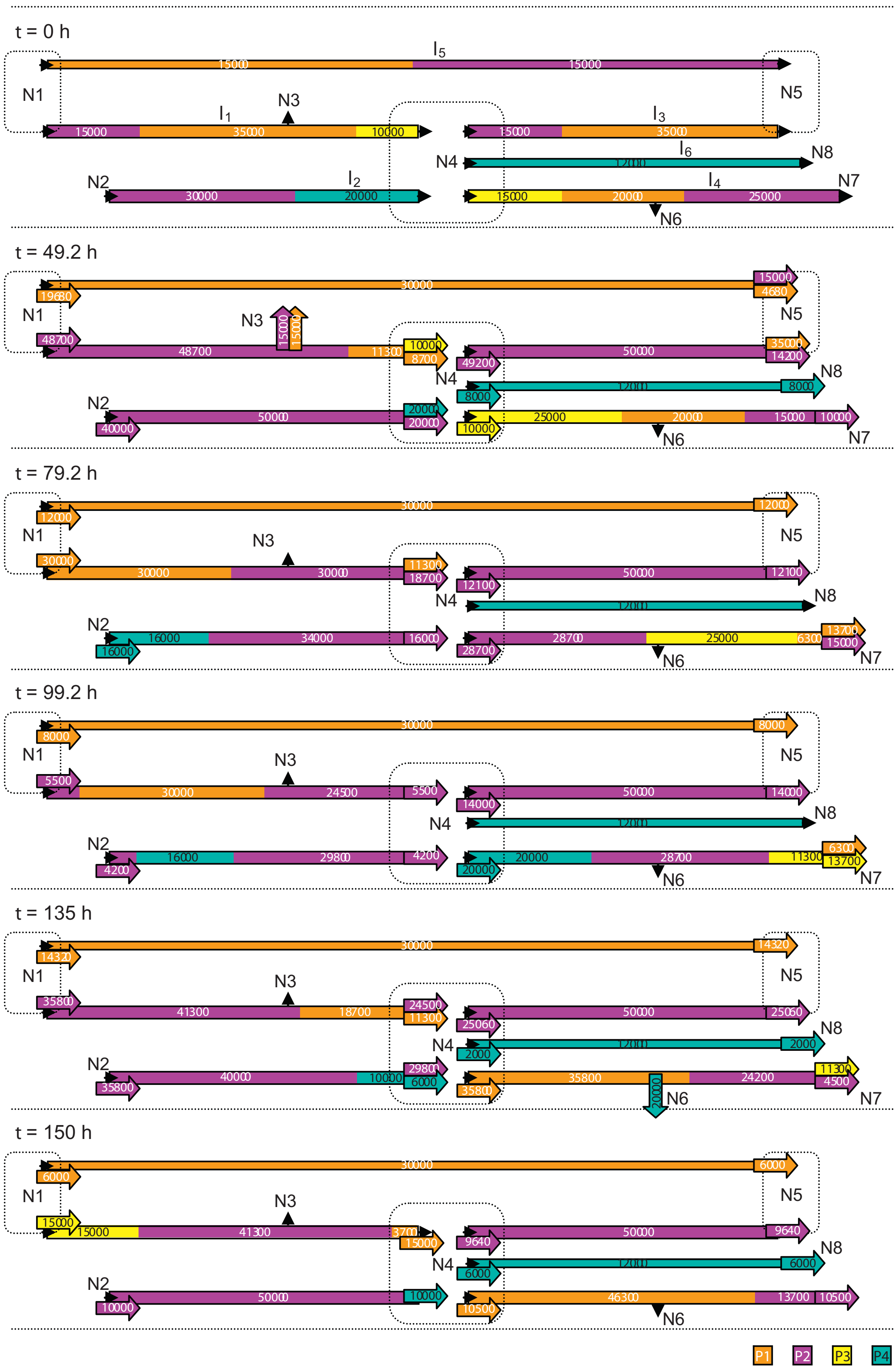

Fig. 13. Optimal pumping and delivery schedule for example 3.

the available pump stations, it will be operated at a much slower flow rate than the trunk lines, ranging from 200 to 400 m^3/h.

The goal of example 2 is to evaluate the convenience of using the alternative route (pipeline l_5) to supply the requested amounts of products to node *N*5 in a lesser lead-time and at a lower operating cost. To this end, a larger MILP formulation is to be solved. Table 5 shows the pumping costs for moving products *P*1 and *P*2 to node *N*5 through pipeline l_5, in \$/m^3. It can be observed that the unit cost for pumping product *P*1 from *N*1 to *N*5 may be reduced from \$1.05 + \$0.88 = \$1.93 (through lines $l_1 + l_3$) to \$1.22 (directly through line l_5), while the unit pumping cost of product P2 decreases from \$1.08 + \$0.90 = \$1.98 to merely \$1.25.

The cardinality of the set I^{new} was initially set to five. The optimal operational plan for example 2 found in 59.34 CPU s comprises the execution of five composite runs involving a total of 22 batch injections (see Table 4 and Fig. 10). As expected, the incorporation of the auxiliary pipeline l_5 in the network shortens the makespan from 179 h (7.5 days) to 150 h (6.2 days). Moreover, the total operating cost diminishes from \$982,010 to \$858,840 (i.e. a 12.54% reduction) if, in both cases, five composite runs are executed. Most of the improvement in the total cost comes from substantial savings in pumping, interface and utilization costs. In the new network configuration, pipeline l_5 is exclusively used for pumping product *P1*, thus avoiding interface losses in that line. Besides, *P*2 is the only product injected into pipelines l_2 and l_3. Consequently, no interfaces are generated in the three lines (l_2, l_3, and l_5). Though the set of batches to be pumped is mostly the same, the pipelines through which they are shipped differ from the ones used in example 1.

The additional pipeline l_5 is continuously operated at the maximum rate (400 m^3/h) to deliver 15,000 m^3 of product *P*2 and 45,000 m^3 of product *P*1 to node *N*5. The remaining demand of *P1* at node *N5*, i.e. (80,000–45,000) m^3 = 35,000 m^3, is covered by shipments initially moving through line l_1. Similarly to example 1, lines l_1 and l_4 are active throughout the five composite runs, but their average percentage usage decreases to 90% and 70%, respectively. Moreover, line l_2 is operated during the first 110 h, and l_3 remains active over the first four runs.

Fig. 11 shows the variation of product inventories with time at the transfer node *N*4 for the optimal solution of example 2. During the first run, stock levels of *P*1 and *P*4 both increase by 20,000 m^3 because *N*4 receives batches containing a similar volume of such products from lines l_1 and l_2, respectively. At the horizon end, those inventories return to their initial values because of subsequent shipments of products *P*4 and *P*1 from node *N*4 through line l_4. By comparing Figs. 8 and 11, it becomes quite clear that the usage of the auxiliary pipeline l_5 considerably simplifies the operation of the intermediate depot *N*4. In fact, the average inventory level of *P*2 substantially decreases and the stock of *P*1 fluctuates within a smaller range.

In short, the use of pipeline l_5 connecting *N*1–*N*5 brings relevant benefits to the operation of the pipeline network:

1. The makespan is shortened by more than one day (16.2% reduction with regards to example 1), by alleviating the operation of the busiest pipelines l_1 and l_3.
2. The usage of the pipelines is more balanced: 90%, 60%, 73.3%, 70%, and 100% for pipelines l_1, l_2, l_3, l_4, and l_5, respectively.
3. The number of interfaces is reduced because some pipelines transport a lesser number of products (26% saving in interface costs).
4. Pumping costs are lowered by 7.4% by making use of a more direct route.
5. The total operating cost is 12.5% less expensive than the previous example.

5.3. Example 3

This example deals with a still more complex pipeline network structure. At the optimal solutions for examples 1 and 2, the usage of pipeline l_2 is rather low (60%) compared with line l_1. In example 2, only product *P2* is injected at the input station *N2* during the ten-day planning horizon because the initial volume of *P4* in pipeline l_2 plus the starting stock of *P4* in tanks of depot *N4* are large enough to meet the expected demands at the receiving terminal *N6*. Under these circumstances, the pipeline scheduler analyzes the possibility of activating a delivering line (l_6) connecting depot *N4* with a new client (*N8*) demanding product *P4* (see Fig. 12). Supplies of product *P4* to *N8* could be much less expensive in terms of operating costs by using pipeline transportation instead of trucks.

Pipeline l_6 contains 12,000 m^3 (12 in. diameter, 164 km long) and can operate at a flow rate varying between 200 and 400 m^3/h. The pumping cost is $cp_{N5,l5,P4}$ = \$0.61 for moving a unit volume of *P4* from *N4* to *N8*, and product *P4* can be dispatched from *N8* to customer locations at a maximum rate of 700 m^3/h. Moreover, minimum/maximum and initial inventory levels as well as the demand of product *P4* at node *N8* are all given in Table 6. The pipeline scheduler wonders if the transport capacity of lines l_2 and l_6 would be large enough for fulfilling the new client's demands, and how this issue would affect the operation of the whole pipeline network.

By adopting $|I^{new}|$ = 5, the optimal sequence of pumping operations involves the execution of five composite runs and 28 batch injections, i.e. 6 more injections compared with example 2. Moreover, the six pipelines remain active throughout the whole time horizon. Three of the new batch injections are pumped at the origin of the new line l_6 and destined for node *N8*. The best solution found in 174.5 CPU s is shown in Fig. 13. As reported in Table 4, even though a new destination is considered, all product demands were fulfilled in the same overall time found for example 2, i.e. a makespan of 150 h. In contrast to example 2, however, line l_2 operates during the five composite runs to pump new batches of products $P2_{40,000}$, $P4_{16,000}$, and $P2_{50,000}$ (the subscripts indicate the batch sizes in m^3). As expected, the usage of pipeline l_2 rises from 60% to 70.7%, and the available stocks of products *P2* and *P4* at node *N2* reach the minimum allowed levels at the end of the planning horizon.

The new pipeline l_6 pumps product $P4_{8000}$ at a low rate during the first composite run and then it is turned off. At time t = 99.2 h, after injecting a new batch of $P4_{20,000}$ into line l_4, the inventory level of *P4* at *N4* reaches the minimum admissible value (see Fig. 14). However, a new batch of *P4* coming from line l_2 is discharged into tanks of *N4* during the next runs, thus permitting line l_6 to operate at

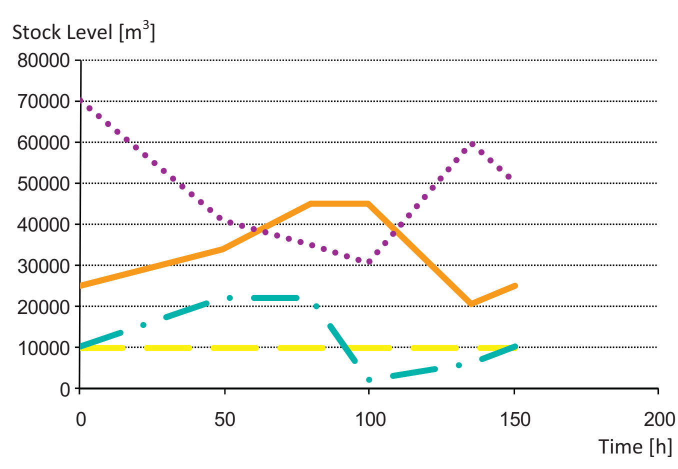

Fig. 14. Inventory levels at node N4 (example 3).

the maximum pumping rate during the last 15 h. Inventory profiles for the other products ($P1$, $P2$, and $P3$) look quite similar to the ones found for example 2.

Finally, no change in the optimal solution is observed if $|I^{new}|$ is increased by one. Therefore, Fig. 13 depicts the best pumping and delivery schedule for example 3.

6. Conclusions

A novel monolithic approach for the scheduling of mesh-structure pipeline networks has been developed. The proposed MILP continuous-time model can be applied to complex transport systems composed by an arrangement of interconnected pipelines with several entry and exit points. Each pipeline has a single source at its origin and one or several terminals over the line.

The MILP formulation is the first monolithic problem representation whose solution simultaneously provides the pumping run times at every input station, the product sequence and batch sizes at every pipeline, and the volumes diverted from in-transit batches to terminals all at once. Besides, the formulation can rigorously track the variation of product inventories with time at every depot, and trace batch movements through every pipeline. Because alternative paths between some nodes can exist, a critical model task is the choice of the best route for every shipment. The pipeline network operational plan consists of a sequence of composite pumping runs of variable length. Over a composite run, a single batch injection can at most be executed at the origin of every individual pipeline. Moreover, every batch injection should take place within the time slot assigned to the related composite run. To guarantee the discovery of the best pipeline network schedule, a sufficient number of runs are to be defined. The problem goal is to timely meet all product demands at distribution terminals within the planning horizon at minimum total cost including pumping, interface, pipeline utilization, and inventory carrying contributions.

The approach has been illustrated by successfully solving three case studies of growing complexity. The difficulty level has been raised by including more pipelines and/or logistic nodes in the transportation network, thus generating alternative routes between input and output terminals. In this way, some congested routes can be debottlenecked. Moreover, the model could also be used to evaluate the convenience of adding new pipelines to the network by simply comparing the operational costs with and without the existence of additional lines. Accounting for the expected future demands and the estimated cost-savings obtained through the network design reformulation, the pipeline planner can readily estimate the payback period of the investment in new pipelines.

The largest example tackled in this work deals with a mesh pipeline network transporting four products through seven pipelines from a pair of refineries to nine terminals (two of them are dual-purpose terminals). By adopting a rather low number of composite runs, the optimal solution has been found in all cases at quite acceptable CPU times. Moreover, the increase of the computational time as the network complexity grows remains quite reasonable. For the largest example, the optimal pipeline plan specifies the execution of a total of 28 batch injections at the input stations over a ten-day time horizon, i.e. a real-world case study. An interesting model feature is the clever handling of ingoing and outgoing flows at transfer terminals to always keep product inventory levels within the permissible ranges. In a next work, the approach will be generalized to also consider reversible pipelines where product batches can move in both directions.

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