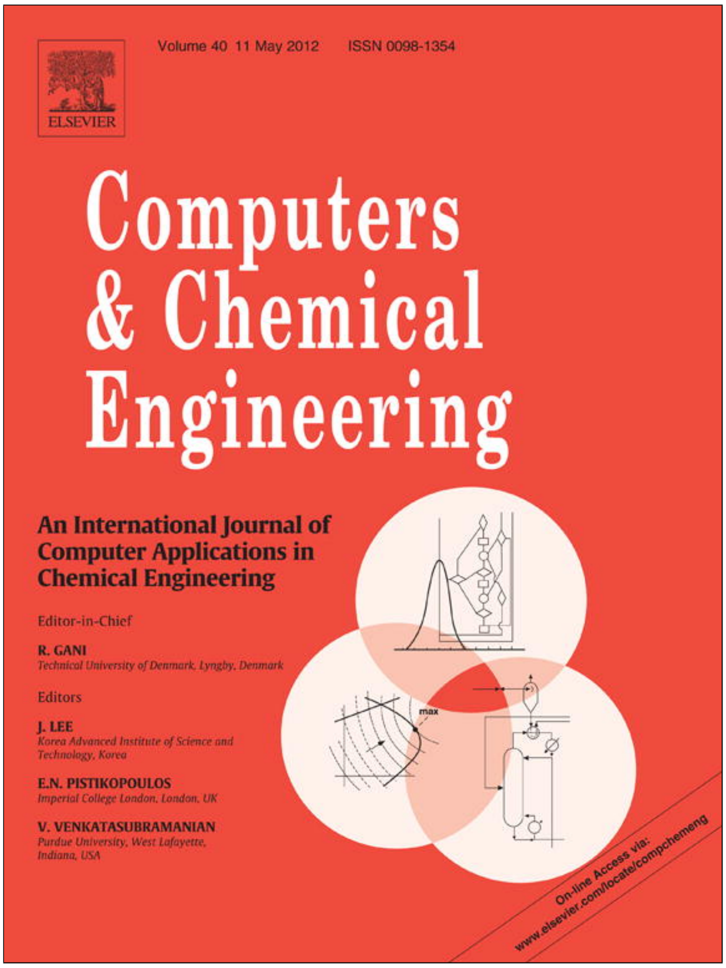
Volume 40 11 May 2012 ISSN 0098-1354
ELSEVIER
Computers & Chemical Engineering
An International Journal of Computer Applications in Chemical Engineering
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Computers and Chemical Engineering 40 (2012) 181–190

Contents lists available at SciVerse ScienceDirect

Computers and Chemical Engineering

journal homepage: www.elsevier.com/locate/compchemeng

Plant Operations, integration, planning / scheduling and supply chain

Multiperiod production planning and design of batch plants under uncertainty

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ARTICLE INFO

Article history:
Received 4 March 2011
Received in revised form 17 January 2012
Accepted 18 January 2012
Available online 28 January 2012

Keywords:
Multiproduct batch plants
Demand uncertainty
Units in series
Design and planning

ABSTRACT

A two-stage stochastic multiperiod LGDP (linear generalized disjunctive programming) model was developed to address the integrated design and production planning of multiproduct batch plants. Both problems are encompassed considering uncertainty in product demands represented by a set of scenarios. The design variables are modeled as here-and-now decisions which are made before the demand realization, while the production planning variables are delayed in a wait-and-see mode to optimize in the face of uncertainty. Specifically, the proposed model determines the structure of the batch plant (duplication of units in series and in parallel) and the unit sizes, together with the production planning decisions in each time period within each scenario. The model also allows the incorporation of new equipment items at different periods. The objective is to maximize the expected net present value of the benefit. To assess the advantages of the proposed formulation, an extraction process that produces oleoresins is solved.

1. Introduction

Batch processes have been widely studied throughout the last years due to their particular suitability for the production of large number of low-volume, high-value products in the same facility (Barbosa-Póvoa, 2007). Usually, at the stage of conceptual design of a batch plant, there are parameters, either external or internal to the process, which are subject to considerable uncertainty. These market and technical parameters include, for instance, product demands, raw materials availability, prices of chemicals, reaction constants, efficiencies, etc.

This work is focused on multiproduct batch plants where several products are produced following the same sequence of processing stages. A special feature of these facilities is their ability to meet production requirements and maximize profits given uncertainties in the market demands for the products.

In dealing with optimization under uncertainty, three research philosophies have been employed over the last years: stochastic programming, fuzzy programming and stochastic dynamic programming (for a short overview see Sahinidis, 2004). Most of the existing approaches that address the effect of uncertainty into batch process optimization have applied stochastic programming (Acevedo & Pistikopoulos, 1998; Aguilar-Lasserre, Bautista Bautista, Ponsich, & González Huerta, 2009; Cao and Yuan, 2002; Cui & Engell, 2010; Ierapetritou and Pistikopoulos, 1996; Liu & Sahinidis, 1996; Maravelias & Grossmann, 2001; Petkov & Maranas, 1997; Subrahmanyam, Pekny, & Reklaitis, 1994; Wu & Ierapetritou, 2007).

Stochastic programming deals with optimization problems whose uncertain parameters are modeled either by continuous probability distributions or by a finite number of scenarios. The approach using scenario analysis has been considerably exploited in the literature and has proven to provide reliable and practical results for optimization under uncertainty (Alonso-Ayuso, Escudero, Garín, Ortuño, & Pérez, 2005; Escudero, Kamesam, King, & Wets, 1993; Gupta & Maranas, 2003; Karuppiah, Martín, & Grossmann, 2010; Liu & Sahinidis, 1996; Shah and Pantelides, 1992; Subrahmanyam et al., 1994). In this paper, the uncertainty in product demands is tackled by the scenario approach.

In general, the two-stage stochastic programming strategy has been considered an effective and widely used method for addressing the optimization problems under uncertainty. In this approach, decision variables are explicitly classified according to whether they are implemented before or after a random event occurs. First-stage (here-and-now) decisions must be made before the uncertain parameters reveal themselves while second-stage (wait-and-see) decisions, also called recourse actions, are made after the outcome of the random events is observed. Thus, through recourse actions, stochastic models consider corrective measures that can be taken after the realization of some uncertain parameters. The two most common objective functions in the literature are the expected cost/profit of the problem.

In the area of batch processing, there are significant contributions on design and planning under uncertainty. Approaches tackling the production planning problem include Liu and Sahinidis

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doi:10.1016/j.compchemeng.2012.01.008

Nomenclature

Indices

g	added units in parallel
h	units in series
i	products
k	discrete sizes for the units
m	units in parallel
p	operations
s	scenarios
t, τ	time periods

Deterministic parameters

BM	big-M constants
co_{it}	operating cost coefficient of product i at time period t
cp_{it}	cost coefficient for late delivery of product i in time period t
d^U_{its}	upper bound of demand of product i in time period t for scenario s
d^L_{its}	lower bound of demand of product i in time period t for scenario s
F_{it}	conversion of raw material to produce i at time period t
G_p	set of units in parallel that can be added in operation p
H	time horizon
H_t	net available production time for all products in time period t
H_p	set of possible configurations of units in series in operation p
H^U_p	the maximum number of units in series that can be allocated in operation p
M^U_p	maximum number of units in parallel operating out of phase in operation p
np_{it}	sales price for product i in time period t
NT	maximum number of time periods
SF_{ipt}	size factor of product i in operation p in time period t
SV_p	set of available discrete sizes for the batch units in operation p
p_s	probability of scenario s
pt_{iph}	processing time of product i in operation p with h units in series in period t
r_p	number of discrete sizes available for operation p
wp_{it}	waste disposal cost coefficient per product i in time period t
wr_{it}	waste disposal cost coefficient per raw material i in time period t
α_p	cost coefficient for a batch unit in operation p
β_p	cost exponent for a batch unit in operation p
γ_{pt}	cost coefficient for a batch unit in operation p at time period t
ε_{it}	inventory cost coefficient of raw material i in time period t
κ_{it}	purchasing price for the raw material of product i in time period t
υ_{pk}	standard volume of size k for batch unit in operation p
σ_{it}	inventory cost coefficient of product i in time period t
ζ_i	time periods during which raw materials have to be used
χ_i	time periods during which products have to be used

Boolean variables (first-stage decision variables)

Z_{ph}	true if configuration h is selected in unit operation p
W_{phk}	true if the unit size k is selected in operation p with configuration h
Y_{phmt}	true if there are m units in parallel out-of-phase in operation p with configuration h in time period t
X_{pgt}	true if g units in parallel are added in operation p in time period t

Deterministic variables (first-stage decision variables)

CE_{pt}	investment cost in operation p in time period t
CO_p	batch unit cost in operation p
N_{pt}	number of set of units in parallel in operation p in time period t

Stochastic recourse variables (second-stage decision variables)

C_{its}	amount of raw material for producing i purchased in time period t under scenario s
IM_{its}	inventory of raw material i at the end of a time period t under scenario s
IP_{its}	inventory of final product i at the end of a time period t under scenario s
n_{its}	number of batches of product i in time period t under scenario s
PW_{its}	product i wasted at time period t due to the limited product lifetime under scenario s
q_{its}	amount of product i to be produced in time period t under scenario s
QS_{its}	amount of product i sold at the end of time period t under scenario s
RM_{its}	raw material inventory for product i in time period t under scenario s
RW_{its}	raw material i wasted in time period t due to the limited product lifetime under scenario s
T_{its}	total time for producing product i in time period t under scenario s
ϑ_{its}	late delivery for product i in time period t under scenario s

(1996) who presented a two-stage model for the process planning and process capacity expansion with random variables that assume values from both discrete and continuous probability. Petkov and Maranas (1997) extended the multiperiod planning and scheduling model for multiproduct plants introduced by Birewar and Grossmann (1990) including uncertain product demands. Wu and Ierapetritou (2007) proposed a multi-stage stochastic programming formulation for the simultaneous solution of production planning and scheduling problems using a rolling horizon strategy.

With regard to the batch plant design under uncertainty, Shah and Pantelides (1992) presented a stochastic formulation for the design with uncertain product requirements considering different scenarios. Subrahmanyam et al. (1994) addressed the design and scheduling of batch process through a multiperiod model. The problem is split into two stages: in the first, the design is obtained without considering scheduling constraints, while, in the second stage, a detailed scheduling model is solved. Cao and Yuan (2002) addressed the problem of the optimal design of batch plants with uncertainty in product demands considering different operating modes of parallel units for different products. Alonso-Ayuso et al. (2005) proposed an approach for the product selection and plant sizing problems under uncertainty. Aguilar-Lasserre et al. (2009) developed a multi-objective optimization problem for the design of batch plants with uncertain market demands for products.

Even though many contributions dealing with the design of batch plants under uncertainty have been published, the simultaneous optimization of the design and planning decisions with capacity expansion of the plant has not been sufficiently studied including all the elements considered in this article. Therefore, the goal of this work is to propose a scenario-based approach for the simultaneous design and production planning of multiproduct batch plants under uncertain demands over a multiperiod context. From the design perspective, both kinds of unit duplications, in series and in parallel are considered. A two-stage stochastic model is proposed, where capacity expansion is admitted. New in parallel units working out-of-phase can be added in different time periods. The selection of the number of units in series can be only made in the first time period. First-stage decisions consist of design variables (mainly Boolean variables) that allow determining the batch plant structure. Second-stage decisions consist of planning variables (continuous variables) to determine the production, purchases, and inventories of raw materials and products for each period throughout the time horizon under each scenario, given the plant structure decided at the first-stage.

The design alternative of duplicating units in series in a unit operation has been recently introduced in general models for multiproduct batch plants by Moreno and Montagna (2007). As they remarked, this kind of duplication is only used in specific unit operations and the trade-offs introduced in the process depend on the operation.

One important characteristic of this work is that generalized disjunctive programming (GDP) has been employed in order to formulate the multiperiod stochastic linear model. GDP has been introduced as an alternative model to the mixed-integer programming (MIP), where discrete decisions and constraints are represented through disjunctions and logical propositions (Lee & Grossmann, 2000; Vecchietti, Lee, & Grossmann, 2003).

The remaining parts of this article are organized as follows. First, the problem of design and planning with capacity expansion under uncertainty is described in Section 2. The two-stage stochastic linear generalized disjunctive programming (LGDP) formulation is developed in Section 3. Section 4 is devoted to the LGDP model reformulation into a mixed-integer linear programming MILP model. Considering a batch plant producing oleoresins as a motivating example, numerical results are presented using the proposed model in Section 5. Finally, some concluding remarks are summarized in Section 6.

2. Problem description

The problem addressed in this work can be stated as the simultaneous optimization of design and production planning decisions under uncertainty of multiproduct batch plants over a multiperiod context. Capacity expansions are allowed at different time periods. In this study, uncertainty is considered in the market demands.

Consider a multiproduct batch plant with a set P of unit operations that processes a set I of products over a time horizon H. Since this is a multiperiod problem, the time horizon is divided into $t = 1, 2, \ldots, NT$ specified time periods H_t, not necessarily of the same length.

Each operation p can be performed by different configurations of units in series. Let H_p denote the set of possible configurations of units in series h for each operation p. The number of units in series is selected in the first time period and do not change thereafter. The selected configuration of units in series can be also duplicated in parallel operating out-of-phase. The duplication of units in parallel can be different in each time period t allowing the capacity expansion of the plant. Let M_p be the set $\{1, 2, \ldots, M_p^U\}$ of possible number of equal units that can be allocated in parallel in each operation p. Parameter M_p^U denotes the maximum number of elements of this set. Thus, in each time period t, m identical sets of units in parallel operate out-of-phase. Each set corresponds to the units arranged in series in that operation.

The variable N_{pt} represents the number of sets of units in parallel in operation p at each time period t. This value is modified taking into account that $g \in G_p = \{0, 1, \ldots, M_p^U\}$ set of units in parallel can be added in every time period t.

Furthermore, the design problem involves the selection of equipment sizes for batch units in each operation p, among a set $SV_p = \{v_{p1}, v_{p2}, \ldots, v_{p,r_p}\}$ of available discrete sizes. Here, v_{pk} represents a discrete size k for the batch unit in operation p and r_p is the given number of available discrete sizes from the commercial point of view for that operation. The basic data for representing the operations are the size factors SF_{ipt} and processing time pt_{ipt} required for each product i in each operation p at every time period t.

The multiproduct batch plant operates with single-product campaigns (SPC) and zero wait (ZW) transfer policy. In SPC, each campaign is devoted to produce only one product until fulfill its demand. In ZW policy, a batch, after being processed in a unit, must be transferred without delay to the next one in the process.

The product demands along the planning horizon are not known to the decision maker with certainty, but it is assumed that the uncertainty can be represented by a set of scenarios S. Moreover, each scenario $s \in S$ has a known probability p_s that reflects the likelihood of each scenario to take place with $\sum_{s \in S} p_s = 1$. Besides, the different possible scenarios are described through lower and upper bounds on product demand levels in each time period d_{its}^L and d_{its}^U.

It is important to note that only some unit operations admit the structural option of duplicating units in series. In addition, this type of duplication not only affects the operation itself but also the rest of the operations of the process. In order to maintain the posynomial structure of the model (Grossmann & Sargent, 1979), the yield for all the configurations in series in a given operation is assumed to be constant, through appropriate size factors values. Thus, the formulation with fixed size and time factors, widely used in the literature, is preserved. In consequence, the size factor for product i in operation p remains equal regardless of the selected configuration of units in series. However, each configuration of units in series h has a different operation time pt_{ipht}. A detailed description of this assumption can be found in Moreno and Montagna (2007).

In every scenario s, production planning decisions allow to determine at each period t and for each product i, the amount to be produced q_{its}, the number of batches n_{its}, and the total time T_{its} to produce product i. Furthermore, at the end of every period t, the levels of both final product IP_{its} and raw material inventories IM_{its} are obtained. Moreover, the total sales QS_{its}, the amount of raw material purchased C_{its}, and the raw material to be used for the production RM_{its} of product i in each time period t are determined with this formulation. In this model, it is assumed that each product requires a unique raw material that it is not shared by other products. This assumption is valid for the oleoresins plant example solved below. However, more sophisticated transformation processes can be easily incorporated. If time periods are equal, wastes due to the expired product shelf life PW_{its} and due to the limited raw material lifetime RW_{its} are also added in the formulation. Also, late deliveries ϑ_{its} that take place in each period are determined. The goal of the problem is to determine the optimal design over all possible scenarios and the optimal production planning and capacity expansion in each scenario, in the face of uncertainties in market demand for products.

2.1. Scenario generation

According to Zimmerman (2000), the choice of the appropriate method for modeling uncertainty depends on the context. Also, this author argued that no single method is capable of modeling all kinds of uncertainty. At present, there exist a considerable number of theories or methods to model uncertainty, i.e., probabilistic theories, scenario-based methods, fuzzy set theories, etc. Each of these approaches makes assumptions about available information. In this paper, the scenario planning approach is adopted for dealing with uncertainty in product demands.

The scenario planning approach is a technique for analyzing alternative futures and developing business strategies. Scenario planning postulates a set of plausible futures, or scenarios originated from the present state (Mobasheri, Orren, & Sioshansi, 1989). Uncertainty is represented by a moderate number of discrete realizations of stochastic quantities constituting distinct scenarios (Mulvey, Rosenbaum, & Shetty, 1997). Thus, a scenario is a complete, consistent, and plausible future state of the world that could take place if one or more major events were to occur.

As proposed by Vanston, Frisbie, Lopreato, and Poston (1977), the scenarios can be generated in a number of ways with the method chosen depending on the intended use of the scenario, the nature of the organization, and the personal preference of the planning group involved. In this first work, the purpose is to generate a representative set of scenarios that are both optimistic and pessimistic within a risk analysis framework (Mulvey, 1996).

Previous works considering uncertainty assume that the demand of each product is an independent random parameter. Nonetheless, demands for similar products, like the ones manufactured in multiproduct batch plants, tend to be correlated and controlled by a small number of factors (economic growth, competitor actions, etc.). Thus, it is assumed that a moderate number of scenarios S is generated in order to appropriately describe the trends of estimates of product demands along the whole horizon time.

3. Linear disjunctive programming model

Based on the above definitions, the two-stage stochastic optimization LGDP model to determine the optimal production planning policy and the plant structure in a multiproduct batch plant is as follows.

3.1. Objective function

The objective function (1) maximizes the expected net present value (*ENPV*) over a set of scenarios S along the entire time horizon.

$$\max ENPV = \sum_{s\in S}\sum_{t\in T}\sum_{i\in I} p_s np_{it} QS_{its} - \sum_{s\in S}\sum_{t\in T}\sum_{i\in I} p_s \times \left\{ \kappa_{it} C_{its} + \left[\varepsilon_{it}\left(\frac{IM_{i,t-1,s}+IM_{its}}{2}\right) H_t + \sigma_{it}\left(\frac{IP_{i,t-1,s}+IP_{its}}{2}\right) H_t \right] + (wp_{it} PW_{its} + wr_{it} RW_{its}) + (co_{it} q_{its} + cp_{it}\vartheta_{its}) \right\} - \sum_{t\in T}\sum_{p\in P} CE_{pt} \quad (1)$$

The economic criterion in Eq. (1) is calculated by the probabilistic average of the difference between the revenue due to product sales and the overall costs in each scenario, minus the last term corresponding to the capital investment cost for batch units in the plant. To determine the revenues, the product price, np_{it}, is multiplied by the amount sold in each time period. The overall costs in each scenario include costs of raw materials, inventory costs and waste disposal costs for both raw materials and final products, operating costs, and penalty costs for late delivery. Parameter κ_{it} denotes the price of raw material used to manufacture product i in time period t, while ε_{it} and σ_{it} are the inventory costs per unit of final product and raw material, respectively. Furthermore, wp_{it} and wr_{it} are the unit costs due to expired product and raw material, respectively. Parameter co_{it} denotes the operating cost coefficient and cp_{it} represent the late delivery cost coefficient. Note that all the above cost coefficients take into account the time value of money, in other words, they are discounted prices for each time period with a specified interest rate.

3.2. Modeling structural decisions

The discrete structural decisions, corresponding to the first-stage decisions, are modeled through Boolean variables in disjunctions (2)–(4) and logical propositions (5).

$$\bigvee_{h\in H_p}\left[\begin{array}{c} Z_{ph} \\ \bigvee_{k\in SV_p}\left[\begin{array}{c} W_{phk} \\ n_{its} \geq \left(\dfrac{SF_{ipt}}{\upsilon_{pk}}\right) q_{its} \quad \forall i\in I,\ t\in T,\ s\in S \\ CO_p = h(\alpha_p \upsilon_{pk}^{\beta_p}) \end{array}\right] \end{array}\right] \quad \forall p\in P \quad (2)$$

$$\bigvee_{m\in M_p}\left[\begin{array}{c} Y_{phmt} \\ N_{pt} = m \\ T_{its} \geq \dfrac{pt_{ipht}}{m} n_{its} \quad \forall i\in I, \forall s\in S \end{array}\right] \quad \forall t\in T,\quad \forall p\in P \quad (3)$$

$$\bigvee_{g\in G_p}\left[\begin{array}{c} X_{pgt} \\ N_{pt} = N_{p,t-1} + g \\ CE_{pt} = gCO_p\gamma_{pt} \end{array}\right] \quad \forall t\in T,\quad \forall p\in P \quad (4)$$

$$Z_{ph} \Leftrightarrow \left(\bigvee_{m\in M_p} Y_{phmt}\right) \quad \forall h\in H_p,\quad p\in P,\quad t\in T \quad (5)$$

Two decision levels are posed in nested disjunctions in Eq. (2) for each unit operation p. In the first level, Boolean variable Z_{ph} is true when configuration of units in series h is selected in unit operation p and is false in the opposite case. In the next level, variable W_{phk} is true when discrete unit size k is selected to carry out operation p with configuration h. The first constraint into this disjunction corresponds to the sizing equation for batch units in the plant (Moreno & Montagna, 2007). This expression relates the number of batches, n_{its}, and the amount of product i elaborated, q_{its}, with the unit size, υ_{pk}, through the size factor, SF_{ipt}, which specifies the volume (or mass) of material which must be processed in operation p to produce a unit volume (or mass) of final product i. The second constraint calculates the equipment cost, CO_p, as a power function of the selected discrete size, υ_{pk}, multiplied by the number h of units in series for each term.

As mentioned earlier, the units in every operation can be duplicated in parallel working out-of-phase. In order to include this structural option in the formulation, a set of disjunctions in Eq. (3) is added. Here, Boolean variable Y_{phmt} is true when there are m identical units in parallel in operation p with configuration h at time period t. Each term of these disjunctions includes constraints that determine the number of set of units in parallel N_{pt} at each period t and the total time to produce each product T_{its}.

The following disjunctions, Eq. (4), are associated with the discrete choice of the units to be added at each time period in every operation, i.e., the capacity expansion of the plant. This is accomplished with the Boolean variable X_{pgt} which is true if g units in parallel are added at time period t in operation p with configuration h. The first constraint in these disjunctions determines the number of units in each period t considering the number in the previous one plus the units included in the corresponding time period. The last constraint determines the expansion cost in each time period, CE_{pt}, by multiplying the cost in operation p, CO_p, by the number g of units added at period t and the parameter γ_{pt}, which is a cost coefficient for operation p taking into account the time period t involved.

Finally, logical constraint (5) establishes that m units in parallel operating out-of-phase will be selected in operation p with h units in series at time period t, if and only if, the configuration in series h is selected to carry out operation p (i.e., Z_{ph} is true).

3.3. Production planning constraints

The following planning constraints involve the second-stage variables explicitly associated with each demand scenario. In these constraints, it is assumed that each product is manufactured with a unique raw material that is not shared with other products. This assumption is valid for the oleoresins plant example solved below. Nevertheless, the generalization for producing product i from several raw materials can be easily made (see, Moreno, Montagna, & Iribarren, 2007).

$$IP_{its} = IP_{i,t-1,s} + q_{its} - QS_{its} - PW_{its} \quad \forall i \in I, \quad t \in T, \quad s \in S \tag{6}$$

$$IM_{its} = IM_{i,t-1,s} + C_{its} - RM_{its} - RW_{its} \quad \forall i \in I, \quad t \in T, \quad s \in S \tag{7}$$

$$IP_{its} \le \sum_{\tau=t+1}^{t+\chi_i} QS_{i\tau s} \quad \forall i \in I, \quad t \in T, \quad s \in S \tag{8}$$

$$IM_{its} \le \sum_{\tau=t+1}^{t+\zeta_i} RM_{i\tau s} \quad \forall i \in I, \quad t \in T, \quad s \in S \tag{9}$$

$$IP_{its} \le IP_{it}^{U} \quad \forall i \in I, \quad t \in T, \quad s \in S \tag{10}$$

$$IM_{its} \le IM_{it}^{U} \quad \forall i \in I, \quad t \in T, \quad s \in S \tag{11}$$

$$\vartheta_{its} \ge \vartheta_{i,t-1,s} + d_{its}^{L} - QS_{its} \quad \forall i \in I, \quad t \in T, \quad s \in S \tag{12}$$

$$RM_{its} = F_{it} q_{its} \quad \forall i \in I, \quad t \in T, \quad s \in S \tag{13}$$

Eq. (6) states that the inventory of final product i at the end of time period t under scenario s, IP_{its}, is equal to the inventory at the end of the preceding time period $t-1$, $IP_{i,t-1,s}$ plus the quantity produced, q_{its}, minus the amount sold, QS_{its}, and the amount of product wasted, PW_{its} in the current time period. In the same way, Eq. (7) calculates the inventory of raw material for manufacturing product i at the end of time period t, IM_{its}, as the inventory in the previous time period, $IM_{i,t-1,s}$, plus the amount purchased, C_{its}, minus the quantity utilized in the production, RM_{its}, and the amount of raw material wasted, RW_{its}, in each time period. The initial inventories for raw material, IM_{i0s}, and product final, IP_{i0s}, are assumed to be given. The last term in Eqs. (6) and (7) are added only when the time periods in the problem have the same length.

In Eqs. (8) and (9), the amount of final product and raw material in stock, are constrained by limited lifetimes. Let ζ_i and χ_i be the time periods during which final products and raw materials have to be used. Therefore, any product or raw material stored during time period t cannot be sold or processed after the next ζ_i or χ_i time periods, respectively. Also, constraints (10) and (11) force the total quantities of both raw material and product in stock to be lower than the available storage capacities in every time period t.

By using appropriate penalty constraints (12), failures to fulfill commitments can be quantified. If a given batch of product i in scenario s meets a minimum product demand d_{its}^{L} with delay, then a late delivery, ϑ_{its}, takes place in that time period.

The mass balance in Eq. (13) determines the amount of raw material necessary for the production of product i in each time period t, RM_{its}, where the parameter F_{it} is the process conversion of product i in time period t assuming that only one main raw material is used for producing product i.

3.4. Time constraint

Considering the SPC-ZW policy, the time constraint (14) imposes that the total time required for producing all products cannot exceed the available time horizon.

$$\sum_{i} T_{its} \le H_t \quad \forall t \in T, \quad s \in S \tag{14}$$

In short, the general stochastic LGDP model optimizes the capacity expansion, production planning and investment decisions for batch plants over NT time periods under a set S of scenarios. The proposed LGDP model will be reformulated as a mixed integer linear program (MILP) through the big-M reformulation (see Lee & Grossmann, 2000).

4. Reformulation of LGDP model

As Grossmann (2002) remarked, any problem posed as LGDP can always be reformulated as a mixed-integer linear programming (MILP) problem. For modeling purposes, it is advantageous to start with LGDP models since it allows for an easy and compact representation and visualization of the discrete choices posed in the problem (Vecchietti & Grossmann, 2000; Vecchietti et al., 2003).

In order to solve the above LGDP model, a reformulation has to be made to obtain a format compatible with the optimization program solvers. The most straightforward way of transforming LGDP into MILP is to replace Boolean variables by binary variables, and the disjunctions by "big-M" constraints. The logical constraints are converted into linear inequalities. This transformation is termed big-M (BM) reformulation (Sawaya and Grossmann, 2005).

In this way, disjunctions in Eqs. (2)–(4) must be transformed to the following constraints with positive big-M constants, which are used to represent sufficient large bounds, in order to obtain a MILP model.

$$\sum_{h \in H_p} z_{ph} = 1 \quad \forall p \in P \tag{15}$$

$$\sum_{s \in S} w_{phk} = z_{ph} \quad \forall p \in P, \quad h \in H_p \tag{16}$$

$$n_{its} \ge \left(\frac{SF_{ipt}}{v_{pk}}\right) q_{its} - BM1_{it}(1 - w_{phk}) \quad \forall i \in I, \quad p \in P, \quad h \in H_p, \quad k \in SV_p, \quad t \in T, \quad s \in S \tag{17}$$

$$CO_p \ge h(\alpha_p v_{pk}^{\beta_p}) - BM2_p(1 - w_{phk}) \quad \forall p \in P, \quad h \in H_p, \quad k \in SV_p \tag{18}$$

$$CO_p \le h(\alpha_p v_{pk}^{\beta_p}) + BM2_p(1 - w_{phk}) \quad \forall p \in P, \quad h \in H_p, \quad k \in SV_p \tag{19}$$

$$N_{pt} \ge m - BM3_{pt}(1 - y_{phmt}) \quad \forall p \in P, \quad h \in H_p, \quad m \in M_p, \quad t \in T \tag{20}$$

$$N_{pt} \le m + BM3_{pt}(1 - y_{phmt}) \quad \forall p \in P, \quad h \in H_p, \quad m \in M_p, \quad t \in T \tag{21}$$

$$T_{its} \ge \frac{pt_{ipht}}{m} n_{its} - BM4_{it}(1 - y_{phmt}) \quad \forall i \in I, \quad p \in P, \quad h \in H_p, \quad m \in M_p, \quad t \in T, \quad s \in S \tag{22}$$

$$\sum_{g \in G_p} x_{pgt} = 1 \quad \forall p \in P, \quad g \in G_p, \quad t \in T \tag{23}$$

$$N_{pt} \ge N_{p,t-1} + g - BM5_{pt}(1 - x_{pgt}) \quad \forall p \in P, \quad g \in G_p, \quad t \in T \tag{24}$$

$$N_{pt} \leq N_{p,t-1} + g + BM5_{pt}(1 - x_{pgt}) \quad \forall p \in P, \quad g \in G_p, \quad t \in T \tag{25}$$

$$CE_{pt} \geq gCO_p\gamma_{pt} - BM6_p(1 - x_{pgt}) \quad \forall p \in P, \quad g \in G_p, \quad t \in T \tag{26}$$

$$CE_{pt} \leq gCO_p\gamma_{pt} + BM6_p(1 - x_{pgt}) \quad \forall p \in P, \quad g \in G_p, \quad t \in T \tag{27}$$

Finally, the logical propositions are converted into linear mixed-integer constraints by applying a systematic procedure. Firstly, the logical propositions are converted into the conjunctive normal form (CNF) or clausal form by removing implications in each of the clauses and applying De Morgan's theorem (Grossmann & Biegler, 2004). Thus, a CNF is a conjunction of logical clauses, i.e., each clause is connected with the AND operator. Also, each clause consists of only OR operators, i.e., disjunctions of Boolean variables or their negations. This representation allows transforming the logical expressions into an equivalent mathematical representation (Cavalier, Pardalos, & Soyster, 1990; Cavalier & Soyster, 1987). Thus, the double implication in the logical propositions in Eq. (5) can be represented by the following form:

$$\sum_{m \in M_p} y_{phmt} = z_{ph} \quad \forall p \in P, \quad h \in H_p, \quad t \in T \tag{28}$$

In summary, the big-M reformulation to the original problem consists of objective function (1) subject to constraints (15)–(28), plus planning constraints (6)–(13), and constraint (14) about the time horizon. Bounds on the involved variables must be also added.

The tightest values for big-M constants ($BM1$–$BM6$) for the above equations are calculated by means of the following expressions:

$$BM1_{it} = n_{it}^U \quad \forall i \in I, \quad t \in T \tag{29}$$

$$BM2_p = H_p^U \alpha_p (\max v_{pk}^{\beta_p}) \quad \forall p \in P \tag{30}$$

$$BM3_p = M_p^U \quad \forall p \in P \tag{31}$$

$$BM4_{it} = T_{it}^U \quad \forall i \in I, \quad t \in T \tag{32}$$

$$BM5_p = M_p^U \quad \forall p \in P \tag{33}$$

$$BM6_{pt} = M_p^U H_p^U \gamma_{pt} (\max v_{pk}^{\beta_p}) \quad \forall p \in P, \quad t \in T \tag{34}$$

where the parameter H_p^U is the maximum number of units in series that can be allocated in operation p.

5. Motivating example

In this section a motivating example based on a multiproduct batch plant that produces oleoresins is considered. Furthermore, in order to verify whether the posed formulation captures the interactions between different design options taking into account uncertain demands, five cases of the original problem has been analyzed. This example and the derived cases were implemented in GAMS using the GUROBI/MILP solver with a 0% margin of optimality and were all performed on a Intel(R) Core(TM)2 Duo CPU, 2.40 GHz with a 1.98GB of RAM.

In order to illustrate the proposed stochastic LGDP model approach, a multiproduct batch plant that produces four oleoresins is considered. The products are sweet bay (A), pepper (B), thyme (C) and rosemary (D) oleoresins. The batch plant consists of four unit operations: (1) extraction, (2) expression, (3) evaporation, and (4) blending. A global planning horizon of 2 years has been considered, i.e., H = 12,000 h. A discretization interval of 3 months is used for the multiperiod model, resulting in 8 time periods, i.e., H_t = 1500 h. The process data for this example are shown in Table 1, where the processing time of each operation corresponds to the option with only one unit.

It is assumed that a maximum number of five units in series with a countercurrent arrangement may be assigned in the extraction operation, i.e., $H_1^U = 5$. In other words, operation 1 can be performed by either one (h_1), two (h_2), three (h_3), four (h_4) or five (h_5) units in series. Processing times for each product i at the extraction operation take smaller values as the number of units in series grows. In Table 2 extraction processing times for each configuration in series h, t_{ipht}, are summarized.

All the unit operations in this process can be duplicated in parallel up to 3 sets of units operating out-of-phase ($M_p^U = 3$). A set of 5 discrete sizes is provided to select process units. Table 3 shows the available discrete sizes for each operation and cost coefficients associated. Coefficients γ_{pt} are obtained by using the values of α_p (cost coefficient of operation p at the beginning of the project) and taking into account the time period involved.

Prices of raw materials and final products are given in Table 4. As can be seen, it is assumed that both prices increase after the first year due to the annual inflation rate of 5%.

Demand for the products is uncertain and 3 scenarios with probabilities $p_1 = 0.5$, $p_2 = 0.3$, and $p_3 = 0.2$ are considered. In the first time period all the scenarios show the following upper demands, i.e., 6000, 5000, 7000 and 8000 kg, for product A, B, C, and D, respectively. It is assumed that product demand for each time period will increase in comparison with the given conditions. Thus, the expected growth rates per time period in each scenario are 20%, 10%, and 5%, respectively. Minimum product demands in every time period for all scenarios are assumed as 50% of maximum product demands.

Products and raw materials lifetimes in time periods are 3 and 2, respectively. The inventory coefficient costs per ton of both final products and raw materials for all the products are 1.5$/(ton h) and 0.2$/(ton h), respectively. These values are assumed to be the same for all time periods. Cost coefficients for late delivery it is assumed as 50% of product prices. An annual discount rate of 10% is employed here.

Table 1
Process data for the motivating example.

i	Size factors, S_{ipt} (L/kg)				Processing time, t_{ipt} (h)				Conversion factor
	1	2	3	4	1 (h_1)	2	3	4	F_{it}
A	20	15	12	1.5	25.95	1.0	2.5	0.5	11.11
B	23	15	12	1.5	39.46	2.0	1.5	2.0	11.11
C	30	20	17	1.5	27.93	1.0	2.0	1.0	15.87
D	40	25	24	1.5	34.09	1.0	3.0	1.0	22.22

Table 2
Extraction times t_{ipht} (h) for different configurations in series.

i	Number of unit in series				
	1	2	3	4	5
A	25.95	9.28	5.35	3.47	2.37
B	39.46	9.76	5.55	3.59	2.44
C	27.93	9.41	5.41	3.51	2.39
D	34.09	9.63	5.50	3.56	2.42

Table 3
Available standard sizes.

Option	Discrete volumes, v_{pk} (L) operation			
	1	2	3	4
1	500	500	250	50
2	1000	700	500	100
3	1500	1000	750	150
4	2500	1500	1000	200
5	3000	2000	1500	250
Cost coef. α_p	1350	1750	1200	975
Cost exp. β_p	0.6	0.6	0.6	0.6

Table 4
Economic data for the motivating example.

i	Costs of raw materials, κ_{it} ($/kg)								Prices of products, np_{it} ($/kg)							
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
A	1.50	1.50	2.20	2.20	1.58	1.58	2.31	2.31	36.00	36.00	38.00	38.00	37.80	37.80	39.90	39.90
B	2.50	2.50	2.50	2.50	2.63	2.63	2.63	2.63	40.00	40.00	40.00	40.00	42.00	42.00	42.00	42.00
C	1.00	0.80	1.00	0.80	1.05	0.84	1.05	0.84	37.00	37.00	37.00	35.00	38.85	36.75	38.85	36.75
D	1.80	1.80	0.60	0.60	1.89	1.89	0.63	0.63	40.00	40.00	35.00	35.00	42.00	42.00	36.75	36.75

The resulting mathematical model, which comprises 7661 equations, 368 binary variables, and 1125 continuous variables, was solved in a CPU time of 85.45 s. The expected net present value (*ENPV*) in the optimal solution has a value of 4.41 M$.

Fig. 1 shows the optimal structure of the batch plant obtained in every time period. In this figure, units in dotted line are included in different time periods. As can be seen, in time period 1, there is only one unit in all operations except in operation 1 which has five units in series. Later, in time period 4, a set of 5 units in series is incorporated in the operation of extraction. These sets of units are working in parallel out-of-phase. The operation 3 has one unit during the three first time periods. Then, in the fourth time period, a new unit is added in parallel out-of-phase, what allows reducing idle times. Finally, in the sixth time period another unit is allocated in this operation, where three units work in parallel out-of-phase. The other unit operations have a unique equipment item in all time periods. The unit sizes selected for each operation are 2500 L, 2000 L, 1500 L, and 200 L, respectively.

Production planning decisions for each product are different for each scenario in the optimal solution. Tables 5–7 summarize the results of planning decisions for scenarios 1–3.

The following conclusions can be drawn from Tables 4–7. In all scenarios, the purchases of material to manufacture product A are made only in time periods 1, 2, 5 and 6 because the costs have the lowest values. The excess of raw material bought in time intervals 2 and 6 are held as raw material inventory allowing the production in subsequent time periods. Moreover, note that production of product A in time periods 2, 3, 6 and 7 are higher than the amount sold in the same interval. For that reason, the extra amount is carried out forward as inventory for satisfying demands in the following time intervals. On the other hand, for scenarios 2 and 3, the amounts produced of product B, in each time period, are sold in the same time interval. Therefore, the corresponding inventories are zero. In scenario 1, the excess made in time periods 4 and 5 for this product is held as a product inventory to satisfy the demands in intervals 6 and 7 where production is reduced.

For product C, the purchases reach maximum values in time periods 2, 4, 6 and 8 because of the lower prices of raw material. When prices rise the purchases are stopped as well as the production of this product. Likewise, in all scenarios, purchases for product D are stopped in time intervals 5 and 6, and its production take places in almost all of the time periods.

5.1. Study of different cases

In order to comprehend the implications of the inclusion of uncertainty in this formulation, in this subsection several cases has been solved. First, a deterministic problem with no product demand increase during the global horizon is presented. Additionally, in the following three cases below, each scenario presented in the motivating example has been considered in isolation by formulating and solving the corresponding deterministic problem. Finally, to assess the interactions and benefits of including several design options together with the possibility of capacity expansion, another case is solved without the inclusion of new units in different time periods.

5.1.1. Case (a)

First, in order to analyze the capacity of the proposed approach, the previous problem is solved considering a hypothetical case where no increase in product demands occurs. This deterministic multiperiod problem has the same upper demand for the four products (i.e., 6000, 5000, 7000, and 8000 kg) in each time period. The optimal solution corresponds to a net present value (*NPV*) of 2.48 M$. The optimal configuration of the batch plant is considerably changed compared to the original problem. Even more, the size requirements for each operation are smaller than in the previous problem. As a result, in operation 1 there are five equal units in series and only one unit in the other operations. The unit sizes selected for operations 1, 2, 3 and 4 are 1500 L, 1000 L, 1000 L, and 100 L, respectively. As expected, the plant maintains this structure along the planning horizon because there is no change in product demands.

5.1.2. Case (b)

In this case, the motivating example is solved considering that product demands in scenario 1 are known with certainty. As was mentioned above, in scenario 1 a 20% increase in product demands in every time period over their values in the first time interval is considered.

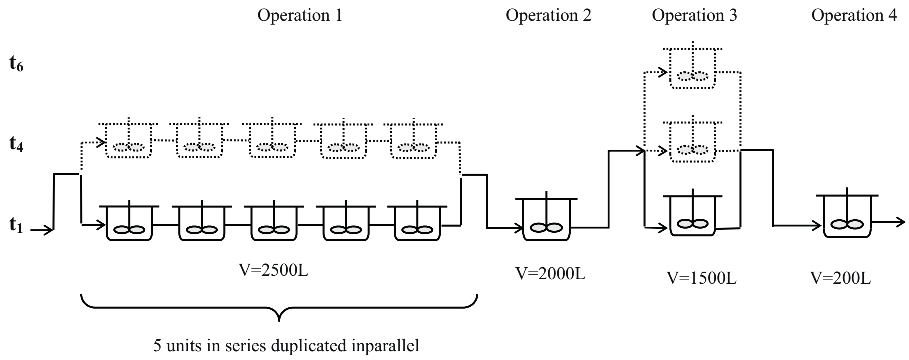

Fig. 1. Optimal structure of the batch plant for the motivating example.

Table 5
Optimal production planning variables for scenario 1.

t	A ($\times10^4$ kg)					B ($\times10^4$ kg)					C ($\times10^4$ kg)					D ($\times10^4$ kg)				
	q_{it}	QS_{it}	IP_{it}	C_{it}	IM_{it}	q_{it}	QS_{it}	IP_{it}	C_{it}	IM_{it}	q_{it}	QS_{it}	IP_{it}	C_{it}	IM_{it}	q_{it}	QS_{it}	IP_{it}	C_{it}	IM_{it}
1	0.60	0.60	0.00	6.67	0.00	0.50	0.50	0.00	5.56	0.00	0.70	0.70	0.00	11.1	0.00	0.40	0.40	0.00	8.89	0.00
2	1.58	0.72	0.86	29.1	11.5	0.60	0.60	0.00	6.67	0.00	1.85	0.84	1.01	29.3	0.00	0.48	0.48	0.00	10.7	0.00
3	0.72	0.86	0.72	0.00	3.58	0.72	0.72	0.00	8.00	0.00	0.00	1.01	0.00	0.00	0.00	2.53	1.15	1.38	56.3	0.00
4	0.32	1.04	0.00	0.00	0.00	1.54	0.89	0.65	17.1	0.00	2.66	1.21	1.45	42.2	0.00	3.04	1.38	3.04	176	108
5	1.24	1.24	0.00	13.8	0.00	2.28	1.04	1.89	25.3	0.00	0.00	1.45	0.00	0.00	0.00	2.27	1.66	3.65	0.00	57.8
6	3.28	1.49	1.79	60.4	23.9	0.85	1.24	1.49	9.44	0.00	3.83	1.74	2.09	60.8	0.00	2.60	1.99	4.26	0.00	0.00
7	0.00	1.79	0.00	0.00	23.9	0.00	1.49	0.00	0.00	0.00	0.00	2.09	0.00	0.00	0.00	7.74	12.0	0.00	172	0.00
8	2.15	2.15	0.00	0.00	0.00	1.79	1.79	0.00	19.9	0.00	2.51	2.51	0.00	39.85	0.00	2.87	2.87	0.00	63.7	0.00

Table 6
Optimal production planning variables for scenario 2.

t	A ($\times10^4$ kg)					B ($\times10^4$ kg)					C ($\times10^4$ kg)					D ($\times10^4$ kg)				
	q_{it}	QS_{it}	IP_{it}	C_{it}	IM_{it}	q_{it}	QS_{it}	IP_{it}	C_{it}	IM_{it}	q_{it}	QS_{it}	IP_{it}	C_{it}	IM_{it}	q_{it}	QS_{it}	IP_{it}	C_{it}	IM_{it}
1	0.60	0.60	0.00	6.67	0.00	0.50	0.50	0.00	5.56	0.00	0.70	0.70	0.00	11.1	0.00	0.40	0.40	0.00	8.89	0.00
2	1.39	0.66	0.73	24.3	8.87	0.55	0.55	0.00	6.11	0.00	1.62	0.77	0.85	25.7	0.00	0.44	0.44	0.00	9.78	0.00
3	0.80	0.73	0.80	0.00	0.00	0.61	0.61	0.00	6.72	0.00	0.00	0.85	0.00	0.00	0.00	2.03	0.97	1.06	45.2	0.00
4	0.00	0.80	0.00	0.00	0.00	0.67	0.67	0.00	7.39	0.00	1.96	0.93	1.03	31.1	0.00	2.24	1.06	2.24	54.7	4.97
5	0.88	0.88	0.00	9.76	0.00	0.73	0.73	0.00	8.13	0.00	0.00	1.03	0.00	0.00	0.00	0.22	1.17	1.29	0.00	0.00
6	2.03	0.97	1.06	35.5	13.0	0.81	0.81	0.00	8.95	0.00	2.37	1.13	1.24	37.6	0.00	0.00	1.29	0.00	0.00	0.00
7	1.17	1.06	1.17	0.00	0.00	0.89	0.89	0.00	9.84	0.00	0.00	1.24	0.00	0.00	0.00	1.42	1.42	0.00	31.5	0.00
8	0.00	1.17	0.00	0.00	0.00	0.97	0.97	0.00	10.8	0.00	1.36	1.36	0.00	21.7	0.00	1.56	1.56	0.00	34.6	0.00

Table 7
Optimal production planning variables for scenario 3.

t	A ($\times10^4$ kg)					B ($\times10^4$ kg)					C ($\times10^4$ kg)					D ($\times10^4$ kg)				
	q_{it}	QS_{it}	IP_{it}	C_{it}	IM_{it}	q_{it}	QS_{it}	IP_{it}	C_{it}	IM_{it}	q_{it}	QS_{it}	IP_{it}	C_{it}	IM_{it}	q_{it}	QS_{it}	IP_{it}	C_{it}	IM_{it}
1	0.60	0.60	0.00	6.67	0.00	0.50	0.50	0.00	5.56	0.00	0.70	0.70	0.00	11.1	0.00	0.40	0.40	0.00	8.89	0.00
2	1.29	0.63	0.66	22.1	7.72	0.52	0.52	0.00	5.83	0.00	1.51	0.74	0.77	23.9	0.00	0.42	0.42	0.00	9.33	0.00
3	0.70	0.66	0.70	0.00	0.00	0.55	0.55	0.00	6.13	0.00	0.00	0.77	0.00	0.00	0.00	1.81	0.88	0.93	40.2	0.00
4	0.00	0.70	0.00	0.00	0.00	0.58	0.58	0.00	6.43	0.00	1.66	0.81	0.85	26.4	0.00	1.90	0.93	1.90	44.3	2.11
5	0.73	0.73	0.00	8.10	0.00	0.61	0.61	0.00	6.75	0.00	0.00	0.85	0.00	0.00	0.00	0.09	0.97	1.02	0.00	0.00
6	1.57	0.77	0.80	26.8	9.38	0.64	0.64	0.00	7.09	0.00	1.83	0.89	0.94	29.1	0.00	0.00	1.02	0.00	0.00	0.00
7	0.84	0.80	0.84	0.00	0.00	0.67	0.67	0.00	7.44	0.00	0.00	0.94	0.00	0.00	0.00	1.07	1.07	0.00	23.8	0.00
8	0.00	0.84	0.00	0.00	0.00	0.70	0.70	0.00	7.82	0.00	0.98	0.98	0.00	15.6	0.00	1.13	1.13	0.00	25.0	0.00

For this case, an optimal *NPV* of 6.23 M$ was obtained. The optimal solution selected five units in series in operation 1 which are duplicated in parallel in the fourth time period. Operation 3 has one unit until the third time interval where a new unit is added. Finally, in time period 6 another unit is allocated in this operation. Unit operations 2 and 4 have only one equipment item during the planning horizon. After the sixth time period, the optimal plant structure for this case is the same as that shown in Fig. 1, obtained for the original two-stage stochastic problem.

5.1.3. Case (c)

In the next case, the deterministic problem is solved with the demands for the products corresponding to scenario 2, i.e., considering a 10% increase in product demands in each time interval. Here, the value of the objective function (*NPV*) for this problem is 3.71 M$. Fig. 2 illustrates the optimal configuration obtained for the batch plant. It can be seen that the optimal configuration is drastically changed compared to the original problem. Here, four units in series are selected in the operation of extraction and there is one unit in the remaining operations of the plant. It is important to highlight that all units are added in the first time period for this case.

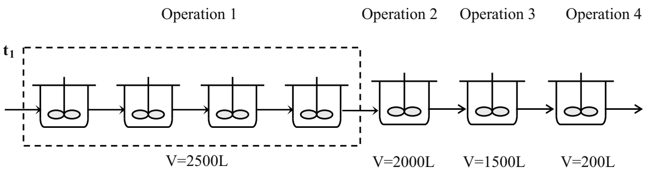

Fig. 2. Optimal structure of the batch plant for case (c).

5.1.4. Case (d)

In this last deterministic case, the problem was solved considering that product demands for scenario 3 are known with certainty. The resulting total benefit for this case is 3.06 M$. In the first time period, the optimal structure consists in one unit in all operations except for operation 1 where five units in series are selected. Later, in the time period 3, a second equipment item is added in operation 3 that works out-of-phase with the first one to reduce the limiting cycle time. Here, the unit sizes selected for each operation

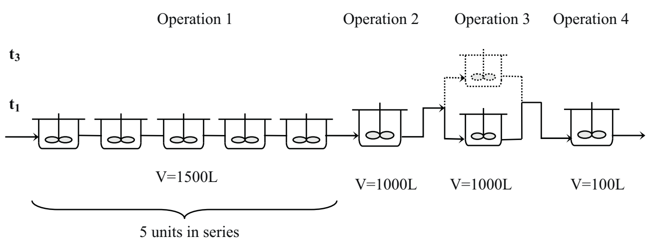

Fig. 3. Optimal structure of the batch plant for case (d).

are smaller than those obtained in previous cases as can be seen from Fig. 3.

5.1.5. Case (e)

Now, the motivating example was solved without considering the structural option of adding units in parallel at different time periods. In other words, the structure and the number of units in the batch plant are selected at the beginning and remain the same along the time horizon. Here, the optimal solution selected two sets of five units in series that are working in parallel in the operation of extraction; three units in parallel were allocated in the third operation, and only one unit for the remaining operations. Optimal unit sizes are the same as those shown in Fig. 1. The optimal solution in this case was 4.25 M$, which is lower than the best value of 4.41 M$ attained in the original problem. Comparing these results, it can be concluded that the approach proposed considering the capacity expansion of the plant during the time horizon yields better solutions.

Interesting observations can be concluded from the results of every case solved. Since product demands are time invariant for case (a) the plant structure is selected at the beginning of the time horizon and no new unit is installed in subsequent periods. The resulting plant is considerably different from the solution of the stochastic formulation for the motivating example. As a result, this case does not take advantage of the proposed multiperiod formulation which accounts for the variability of cost and demands due seasonal or market changes.

In cases (b)–(d) each scenario of the motivating example is considered separately. They are based on the on the assumption of perfect knowledge of product demands in every time period. Note that the plant structure obtained solving these cases, with exception of case (b), are quite different from the optimal configuration found for the original example. Since they are deterministic problems, the net present value *NPV* was calculated and their values range from 6.23 for case (b) to 3.06 M$ for case (d). In order to compare the numerical results of each case solved with the optimal solution of the initial stochastic example, the *ENPV* has been obtained by fixing the structure of the plant to those obtained in each case and optimizing, in the original stochastic formulation, the production planning variables only. The resulting *ENPVs* are 4.40, 2.45, and 1.79 M$ for cases (b)–(d), respectively. It should be noted that the slight difference of case (b) from the solution of the original problem is due to the plant structures are highly similar allowing to meet the production targets of the scenarios. For the other two cases the solution has an associated significant penalty cost because the minimum product demands in all scenarios cannot be fulfilled with those plant configurations.

This analysis shows that the decomposition of the problem in two levels, design and planning, obtains suboptimal solutions. Note that cases (b)–(d) do not consider the simultaneous impact of design and planning for several scenarios. It is not possible to design the plant considering only one scenario and assume that the other ones will achieve an appropriate performance in that facility. Only the proposed stochastic formulation allows obtaining a result that balances the impact of various scenarios simultaneously as well as design and planning decisions.

Finally, the last case posed includes the uncertainty in product demands through different scenarios but no unit addition is allowed at different time periods. Here, the plant configuration is selected in the first time period. Such configuration is the same as the final structure of the original example after the sixth period, where the last unit is added in operation 3, but with a higher *ENPV*. This difference is due to the fact that equipment installed in the first periods has a larger impact in the objective function than those included later.

6. Conclusions

In this work, a two-stage stochastic LGDP model has been formulated to address the design and production planning of multiproduct batch plants in presence of demand uncertainty. Several contributions can be emphasized in this article.

The proposed model considered a dynamic context where variations in prices, product demands, costs, and raw materials availability due to seasonal or market fluctuations are taken into account. Therefore, the optimal structure initially adopted cannot be maintained during the plant lifetime. So, in this work, the plant configuration can be modified and new units can be added in order to fulfill new product requirements considering all the fluctuations.

Design and production planning decisions are simultaneously considered. Many previous approaches prioritize decomposition of the problem in two levels: while in the first one the design is attained, in the second one the production is planned using the plant configuration previously obtained. Taking into account the proposed approach, critical trade-offs between design and production decisions are appropriately assessed as was shown in the example.

The proposed approach using a LGDP optimization model is capable of handling different levels of decisions. Structural decisions (the duplication of units in parallel working out-of-phase), design decisions (unit sizes) and planning decisions (production, inventory, purchases, etc.) are appropriately represented using linear disjunctions. The disjunctive formulation of this problem allows for an easy and compact representation and visualization of the discrete choices posed. In order to obtain a linear model the size units are considered available in discrete sizes which correspond to the real procurement of equipment. In order to solve the LGDP model, the “big-M” reformulation was adopted to transform the LGDP model into a MILP one which can be solved to global optimality.

Finally, the proposed model considers uncertainty in product demands represented by a set of scenarios. The formulation of the problem through scenarios allows for the simultaneous treatment of several variable elements: demands not only change along the time horizon but also they fluctuate taking into account the uncertain context.

The design variables, such as the selection of equipment of standard size and the addition of new units in parallel in each time period, are independent of the scenarios, i.e., they are first-stage variables. On the other hand, production planning variables, which include working levels of the plants for each time period, are scenario-dependent variables, i.e., they are second-stage variables.

The performance of the proposed formulation has been assessed through a motivating example dealing with a batch plant that produces vegetable extracts, particularly oleoresins. Several cases have been solved in reasonable computation times, showing the advantages of the presented model.

Acknowledgments

The authors acknowledge the financial support from Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET) and from Agencia Nacional de Promoción Científica y Tecnológica (ANPCyT) in Argentina.

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