The heterogeneous vehicle routing and truck scheduling problem in a multi-door cross-dock system

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ABSTRACT

Cross-docking is a logistics technique applied by many industrial firms to get substantial savings in two warehousing costly functions like storage and order picking. Incoming shipments are unloaded from inbound trucks on a cross-dock terminal with minimal storage space and directly transferred to outbound vehicles that carry them to their destinations. The major decisions at the operational level are the vehicle routing and scheduling, the dock door assignment and the truck scheduling at the cross-dock. Because such decisions are interdependent, all of them are simultaneously considered in the so-called vehicle routing problem with cross-docking (VRPCD). Previous contributions on VRPCD assume that pickup and delivery tasks are accomplished by a homogeneous vehicle fleet, and they mostly ignore the internal transportation of goods through the cross-dock. This work introduces a new rigorous mixed-integer linear programming (MILP) formulation for the VRPCD problem to determine the routing and scheduling of a mixed vehicle fleet, the dock door assignment, the truck docking sequence and the travel time required to move the goods to the assigned stack door all at once. To improve the computational efficiency of the branch-and-cut search, an approximate sweep-based model is developed by also considering a set of constraints mimicking the sweep algorithm for allocating nodes to vehicles. Numerous heterogeneous VRPCD examples involving up to 50 transportation requests and a heterogeneous fleet of 10 vehicles with three different capacities were successfully solved using the proposed approaches in acceptable CPU times.

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1. Introduction

In a competitive global market scenario, process industries must find ways to reduce production and logistics costs and, at the same time, improve responsiveness to better satisfy the needs of their clients. This is why Enterprise-wide Optimization (EWO) has become a research field of great interest in chemical engineering. As stated by Grossmann (2005), EWO is a new emerging area that lies at the interface of process systems engineering and operations research, and focuses on optimizing supply, manufacturing and distribution operations to reduce costs all over the supply chain. Usually, EWO and supply chain management are considered as equivalent terms. However, EWO is more concerned on integrating the information and the decision-making process across various functions of a company (purchasing, manufacturing, distribution, sales), while supply chain management pays more attention on logistics and distribution by assuming that the production plan is already known. Numerous research articles on EWO and supply

chain management have been published in chemical engineering journals in the last ten years. Some of them deal with the optimal operational planning and scheduling of single and multi-echelon distribution networks using fleets of vehicles, and applying warehousing and/or cross-docking strategies (Seferlis and Giannelos, 2004; Cheng and Lee, 2004; Verderame and Floudas, 2009; Dondo et al., 2011; Moghadam et al., 2014; Chu et al., 2015). Other contributions consider the integrated planning of both production sites and distribution networks (Cóccola et al., 2013; Marchetti et al., 2014). In multi-echelon distribution networks, deliveries of products from factories to customers are managed by routing and consolidating incoming shipments in intermediate warehouses carrying long-term inventories. Customer orders are placed at the warehouse and satisfied using the available product inventories, i.e. a warehousing policy. On the other hand, cross-docking is a logistics strategy used by many organizations to gain a competitive advantage over the traditional warehousing by lowering inventories and improving the customer service. Basically, it consists of unloading the incoming freight from suppliers on a distribution docking terminal with a minimal storage space and directly transferred the cargo to outbound vehicles. In this way, two costly functions of warehousing like storage and order picking are eliminated. Storage

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has associated inventory holding costs and order picking is labor intensive. The cross-dock terminal usually presents an I-shape, i.e. a long, narrow rectangle with multiple doors at both sides where trucks are loaded or unloaded. Usually, the goods are staged in front of the outbound doors for a small time period before loading. On a typical cross-docking system, inbound vehicles departing from the cross-dock facility first collect the freight at various pickup locations, return to the cross-dock facility and dock at the assigned strip door where the freight is unloaded and staged in the door area. After sorting the goods by destination or route, they are moved to the assigned stack door and loaded onto outbound trucks. Inside the cross-dock, the freight is moved by workers using forklifts or through a network of conveyor belts. If the goods are to be temporarily stored, they stay on the floor of the cross-dock in front of the designated stack door. No special infrastructure is available at the terminal to stage the freight. After loading the cargo, the outbound trucks carry the goods to multiple locations and return to the cross-dock. When different products are received from multiple suppliers for some common destinations, a consolidating process also takes place at the cross-dock. Therefore, warehousing focuses on holding product stocks while cross-docking is interested on transshipping to achieve smaller inventories and faster deliveries. Goods are most suitable for cross-docking if they have a large and stable demand, high unit value, short life-cycle, and low unit stock-out cost, or there is a large distance between supply points and customer locations. Because not all products present such features, many industrial firms use a combination of cross-docking and warehousing.

Research on cross-docking is rather recently. Most of the papers have been published in the last ten years. A thorough state-of-the-art review on cross-docking can be found in Boysen and Fliedner (2010) and Van Belle et al. (2012). Several cross-docking problems have been defined in the literature. Based on their decision-making level, they can be grouped into three categories: operational, tactical or strategic problems. This paper is focused on short-term operational issues. Different types of decisions should be made at the operational level, including pickup/delivery (P/D) vehicle routing and scheduling, dock door assignment, and truck scheduling (TS) at the cross-dock facility. As these operational decisions are interdependent, better solutions can be identified if they are simultaneously selected. When the number of dock doors is lower than the number of vehicles, several trucks are assigned to the same dock door and stay in the associated line for some time. Then, the usage of each dock door must be carefully scheduled over time. The dock door assignment problem intends to determine the optimal assignment of pickup/delivery vehicles to strip/stack dock doors. A good assignment decision increases the productivity of the cross-dock and reduces the total cross-dock operating time, i.e. it provides faster deliveries. Dock door assignment on a short-term horizon is clearly a part of the truck scheduling problem. In addition, the TS-problem deals with the optimal scheduling of unloading and loading operations at the docks and the exchange of items among inbound and outbound vehicles. In other words, it decides on the sequence of inbound/outbound trucks at the assigned strip/stack door and when they are unloaded or loaded. If several trucks arrive at the same time and are assigned to the same strip door, some bottlenecks at the dock will arise and slow the unloading operations. A good schedule of truck arrivals combined with a suitable dock door assignment allow to unloading vehicles faster with less dock doors. Moreover, an appropriate exchange of loads among pickup and delivery vehicles reduces the average time during which the freight should stay on the cross-dock waiting for loading. Such a temporary storage occurs because the goods do not arrive in the same order that they must be loaded on the assigned truck. In short, the combined dock door assignment and truck scheduling problem not only allocates vehicles to dock doors but also select the freight transported by each outbound truck and establish the time at which it departs from the cross-dock. The TS-problem goal is to complete the cross-dock operations as soon as possible, i.e. at minimum cross-dock operating time.

A significant number of contributions on truck scheduling have been published. The proposed solution approaches are based on mixed-integer (MIP) models and heuristic/meta-heuristic algorithms. Some works consider a cross-dock facility with a single strip and a single stack door. Then, they just schedule the trucks at the dock and find the better exchange of items between inbound and outbound trucks (Yu and Egbelu, 2008; Arabani et al., 2011). Other papers assume a multiple door cross-dock and a number of trucks greater than the number of doors. However, some of them only deal with the assignment and scheduling of inbound trucks (Wang and Regan, 2008; McWilliams et al., 2008; Rosales et al., 2009; Liao et al., 2013). They suppose that the outbound vehicles are already scheduled. Instead, Tsui and Chang (1992), Li et al. (2009), Miao et al. (2012), Kuo (2013) and Van Belle et al. (2013) deal with the scheduling of both inbound and outbound trucks. Li et al. (2009) presented two solution approaches for the planning and scheduling of a single cross-dock with multi-dock doors based on an MILP model and a dependency ranking search heuristic. In turn, Miao et al. (2012) tackled the truck scheduling problem for a multi-crossdock distribution network using two types of meta-heuristics: adaptive tabu search and adaptive genetic algorithms. Kuo (2013) applied a variable neighborhood search to minimize the makespan by optimizing the P/D truck sequencing and the strip/stack door assignment. Van Belle et al. (2013) used a tabu search approach to find the inbound and outbound truck sequencing and the door assignment that minimizes the weighted sum of the total travel time and the total tardiness.

Before the shipments are available at the cross-dock, they have to be collected by inbound trucks at several supply points. After the sorting process, the goods should be delivered by outbound vehicles to multiple customers. The vehicle routing problem seeks to determine the optimal design of pickup and delivery tours and the vehicle stop times at P/D nodes in order to satisfy all the transportation requests within the specified time windows, while minimizing either the total routing cost or the total distribution time. As stated before, the three problems (i.e. dock door assignment, truck scheduling and P/D vehicle routing) are interdependent and should be solved all at once. This is so because the vehicle routing problem provides the set of goods transported by inbound trucks and the vehicle arrival times at the cross-dock, i.e. an important data set for the TS-problem. On the other hand, the departure time from the cross-dock and the freight loaded onto outbound vehicles are defined by the truck scheduling problem. With the knowledge of the cargo to be delivered and the departure times, the vehicle routing problem can determine the customer locations to be visited by each outbound truck and the related stop times, the vehicle return times to the cross-dock and the total operating time. The vehicle routing problem with cross-docking (VRPCD) deals simultaneously with both the vehicle routing and the cross-dock operations. Time windows for starting the service at P/D nodes are usually specified. Few contributions on the VRPCD problem have been published. All of them assume that pickup and delivery tasks are sequentially carried out by a homogeneous fleet of vehicles. Since all the vehicles are identical, the assignment of vehicles to routes is unneeded. Then, it is equivalent to refer to vehicles or tours. Moreover, most papers consider a single cross-dock facility with an unlimited number of dock doors (Lee et al., 2006; Wen et al., 2009; Liao et al., 2010; Santos et al., 2013; Dondo and Cerdá, 2013). As a result, dock door assignment decisions can be omitted and waiting lines in front of the dock doors never arise. Another common assumption is that the
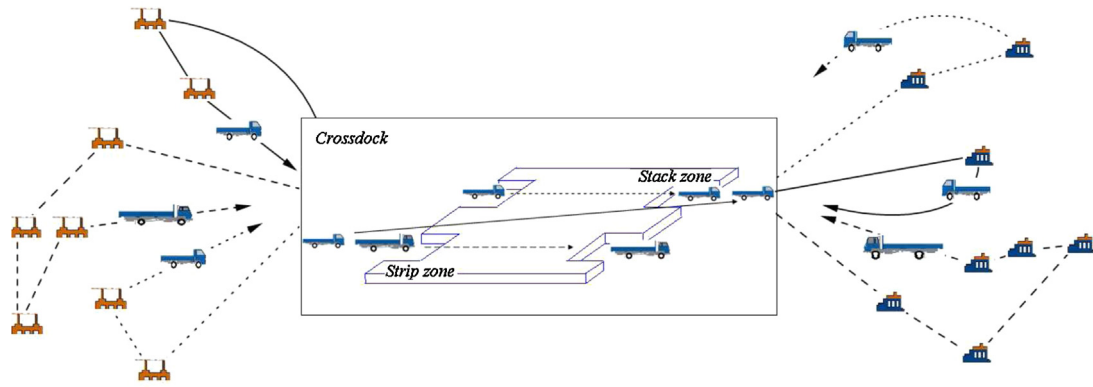
Fig. 1. Illustrating a multi-door cross-docking system.

freight is not interchangeable because each request has a specific destination.

Lee et al. (2006) proposed a mixed-integer linear programming (MILP) formulation that considers both cross-docking operations and the P/D vehicle routing, assuming that all the inbound vehicles arrive at the cross-dock simultaneously. The selected objective function was the sum of transportation and fixed costs of the vehicles. As the problem is NP-hard, a heuristic algorithm based on a tabu search algorithm was applied to solve it. Wen et al. (2009) introduced a mixed integer programming formulation for the VRPCD problem with pickup and delivery tasks to be started within specific time windows in order to minimize the traveled distance. Each transportation request is defined in terms of the pickup node where the freight is loaded and the delivery node to which is destined. To solve large VRPCD problems, an efficient tabu search embedded within an adaptive memory procedure was proposed. Liao et al. (2010) improved the results found by Lee et al. (2006) by applying a new tabu search algorithm to obtain good feasible solutions for the VRPCD problem. Santos et al. (2013) developed an integer programming formulation for the pickup and delivery problem with cross-docking and used a branch-and-price algorithm to solve it. The model allows vehicles collecting and delivering the same set of goods to avoid the stop at the cross-dock in order to reduce the transportation costs. Dondo and Cerdá (2013) developed a monolithic MILP mathematical formulation for the VRPCD that determines the pickup and the delivery routes simultaneously with the truck scheduling at the cross-dock terminal. To improve the computational efficiency of the proposed model, a set of constraints mimicking the widely known sweep heuristic algorithm (Gillett and Miller, 1974) was embedded into the MILP model. By easing the allocation of P/D nodes to routes, the sweep-based approximate formulation finds near-optimal solutions to medium-size problems at very acceptable CPU times. However, dock door assignments and queues of trucks in front of the dock doors were still ignored. Recently, Dondo and Cerdá (2014) developed an MILP approach to tackle VRPCD problems involving a single cross-dock with a limited number of dock doors. The proposed model is able to assign inbound/outbound vehicles to strip/stack dock doors and to schedule the usage of each dock door over time. In other words, it accounts for the temporary stay of vehicles on the waiting lines in front of dock doors. The computational efficiency of the MILP solution algorithm has been greatly improved by incorporating the set of constraints mimicking the VRP-sweep method into the problem formulation.

In their review paper, Van Belle et al. (2012) include an illustrative list of simplifying assumptions of current VRPCD approaches that should be challenged. Among them, they mention: (i) all trucks have the same capacity and cost and they are available at the beginning of the time horizon, (ii) the unloading of vehicles can

start immediately, and (iii) the loading sequence of outbound trucks is neglected. To challenge such assumptions, this paper introduces a new rigorous mathematical model for the VRPCD problem that considers a fleet of vehicles with different capacities and a single cross-dock with a limited number of dock doors. Trucks and routes are no longer equivalent and assignment decisions allocating vehicles to routes should be defined. Unloading of vehicles cannot start immediately because there are more trucks than strip doors. Besides, some vehicles may not be available from the start of the planning horizon, and the loading of an outbound truck cannot start until all the goods it should deliver are available on the cross-dock. In this way, the best loading sequence can always be performed. In addition, the new formulation accounts for the transfer time of the requests from strip to stack dock doors. The problem goal is to minimize either the total routing cost or the total distribution time. As the rigorous model can be applied to heterogeneous VRPCD problem instances with at most 25–30 requests, an efficient approximate approach for discovering good feasible solutions for larger VRPCD problems was also developed. It was obtained by embedding the set of constraints mimicking the sweep algorithm into the problem representation. The resulting sweep-based formulation was validated against the rigorous model by comparing the best solutions obtained by them for different test examples. After validation, a large number of heterogeneous VRPCD problems instances involving up to 50 transportation requests and 10 inbound/outbound vehicles were successfully solved using the sweep-based formulation.

2. Problem definition

The vehicle routing problem with cross-docking is defined as the problem of moving a set of goods from supply points to an intermediate cross-dock terminal w where the loads are consolidated and sorted by route before sending them to their specific destinations (see Fig. 1). A vehicle fleet V composed by trucks $v \in V$ of different capacities (Q_v) is used to transport the goods from the pickup sites to the cross-dock w and immediately after to carry them to the customer locations. Each vehicle first acts as an inbound truck and then performs as an outbound truck. The information about the cargo to be delivered is given by the set of transportation requests R. For each request $r \in R$ is given the shipment size (q_r) and the coordinates $\{(x_r^P, y_r^P); (x_r^D, y_r^D)\}$ of the related pickup and delivery nodes. Based on the coordinates of the P/D nodes, the Euclidean distance between the P/D nodes of every pair of requests $(r, r') \in R$, given by $d_{r,r'}^P / d_{r,r'}^D$ and the Polar coordinates ($r_{w,r}^P / r_{w,r}^D$ and θ_r^P / θ_r^D) of the P/D nodes of each request r can be determined. The Polar coordinates of the P/D nodes are referred to a system with origin at the cross-dock. In addition, it will be assumed that the cross-dock terminal has a limited number of strip doors RD and

stack doors SD. The role of a dock door is fixed, i.e. it does not change with time. It is either a receiving or a loading dock door. Most contributions on the VRPCD problem supposed that there are at least as many dock doors as the number of trucks, i.e. $|RD| \geq |V|$ and $|SD| \geq |V|$. In this way, dock doors are not regarded as scarce resources. Consequently, the assignment of vehicles to dock doors and the scheduling of the dock door usage over time are omitted in the problem formulation. On the contrary, this paper accounts for the possibility of having more trucks than dock doors. Consequently, lines of trucks waiting for service at every dock door are considered. After arriving at the cross-dock, an inbound vehicle usually waits for some time before the assigned strip dock is available for starting the unloading operation. Similarly, an outbound truck that is ready for reloading operations must stay idle for some time until the preceding vehicles on the line have been served. Such waiting times are accounted by the problem formulation. The service time at each pickup/delivery location has two components: a fixed time for shipment-preparation (ft_r^P/ft_r^D) and a variable part that is proportional to the load size q_r. The loading/unloading rate at each pickup/delivery node is given by (lr_r/ur_r). Similar parameters for unloading/loading vehicles at the cross-dock terminal are denoted by (ft_w^P/ft_w^D) and (lr_w/ur_w), respectively. After completing offload operations, the vehicle moves to the shipping door of the terminal, reloads goods and departs to their final destinations. The vehicle transfer-time between an inbound door $d \in RD$ and an outbound door $d' \in SD$ is given by the parameter $ttv_{d,d'}$. In turn, the transfer time of a request between the assigned strip door $d \in RD$ and the assigned stack door $d' \in SD$ per unit size is denoted by the parameter $ttr_{d,d'}$.

The mathematical formulation proposed by Dondo and Cerdá (2014) should be generalized to account for the use of a heterogeneous vehicle fleet carrying the pickup and delivery tasks. As vehicles and routes are no longer equivalent, they are gathered into different groups: the set of vehicles V and the set of pickup/delivery routes S^P/S^D. Moreover, new binary and continuous variables are to be defined. They are: (*i*) the binary variables YP_{rs} and YD_{rs} allocating requests $r \in R$ to pickup/delivery routes $s \in S^P/S^D$; (*ii*) the binary variables JP_{vs} and JD_{vs} assigning vehicles $v \in V$ to pickup/delivery routes $s \in S^P/S^D$; (*iii*) the continuous variables KP_{rv} and KD_{rv} allocating requests $r \in R$ to vehicles $v \in V$. As dock doors are assigned to vehicles, the definition of the variables KP_{rv} and KD_{rv} allows to knowing the strip and stack docks at which the request r is unloaded and reloaded, respectively. Through the variables JP_{vs} and JD_{vs}, it is established the capacity of the vehicle assigned to any route s. Additional constraints defining the values of the new binary/continuous variables are incorporated in the proposed heterogeneous VRPCD model. Other new continuous variables are OC_s^P and OC_s^D standing for the overall routing cost of the pickup/delivery route s. As before, the problem formulation includes the binary variables $DP_{v,d}/DD_{v,d}$ to allocate vehicles to strip/stack dock doors, and the sequencing variables $ZP_{v,v'}/ZD_{v,v'}$ to establish the relative ordering of trucks on the waiting lines of the dock doors. Other important continuous variables already included in the VRPCD formulation of Dondo and Cerdá (2014) are the variables AT_v^P standing for the times at which the pickup vehicles arrive at the cross-dock, the variables RT_v^P denoting the times at which the pickup vehicles are released from their pickup duties after completing the unloading tasks, the variables ST_v^P representing the starting times of the pickup tours by their assigned vehicles and the variables ST_v^D representing the times at which the reloading of outbound vehicles begin.

3. Model assumptions

Contrarily to Dondo and Cerdá (2014), heterogeneous vehicle fleets are now handled and the individual trucks are no longer pre-assigned to pickup/delivery routes. In Dondo and Cerdá (2014), the vehicle v_1 is assigned to tour s_1, vehicle v_2 to tour s_2 and so on. This is not the case in the new problem formulation. Other assumptions are rather similar. Then, the proposed mathematical formulation assumes that:

(i) The cross-docking system comprises a single cross-dock having a known layout with a limited number of strip and stack dock doors.

(ii) The number of strip/stack doors can be lower than the number of vehicles. Then, the dock doors can be regarded as scarce resources that should be scheduled over time.

(iii) Dock doors are exclusively dedicated to either unloading or loading operations, e.g. they are designated as either strip or stack dock doors.

(iv) A heterogeneous vehicle fleet transporting goods from suppliers to destinations through a cross-dock terminal is considered.

(v) The vehicle fleet first accomplishes the required pickup tasks and subsequently performs the delivery tasks.

(vi) Each request has a known size and specific pickup and delivery locations.

(vii) The freight unloaded at the cross-dock is not interchangeable, i.e. each one must be sent to a specific destination.

(viii) Each P/D request must be serviced by a single vehicle, i.e. orders are not splittable.

(ix) Each vehicle can serve more than one pick-up/delivery location.

(x) The pickup and delivery routes must start and finish at the cross-dock.

(xi) The total quantity of goods carried by a vehicle must not exceed its capacity.

(xii) The allocation of vehicles to routes is a model decision.

(xiii) The loading/unloading of a truck at the cross-dock cannot be interrupted, e.g. pre-emption is not allowed.

(xiv) The loading of an outbound truck can start after all the goods to be delivered are available on the cross-dock and the vehicle is docked at the assigned stack door.

(xv) The unloading of an inbound truck can start after the vehicle is docked at the assigned strip door.

(xvi) All activities must be completed within the planning horizon t^{max}.

(xvii) The service time of a truck at either the supply/delivery locations or at the cross-dock is the sum of a fixed stop time and a variable component directly increasing with the size of the cargo to be picked-up/delivered.

(xviii) The goods collected and delivered by the same truck are not unloaded at the cross-dock and remain inside the vehicle.

(xix) The total amount of goods unloaded on the receiving docks and the total freight loaded on outbound trucks at the shipping doors must be equal at the end of the planning horizon. So, there is no final inventory left at the cross-dock.

The proposed formulation can still be applied if the assumptions (v) and (xviii) are relaxed. Therefore, different fleets of inbound and outbound vehicles can also be handled and trucks can be unloaded even if they deliver the same set of goods to their destinations.

4. The problem formulation

4.1. Nomenclature

Sets
R requests
V vehicles
S^P pickup routes

S^D delivery routes
RD receiving (strip) dock doors
SD shipping (stack) dock doors
N unload events

Binary variables
$DP_{v,d}/DD_{v,d}$ denotes that vehicle v has been allocated to the strip/stack dock door d
$WP_{n,v}/WD_{n,v}$ denotes that the unloading/loading activity of vehicle v is associated to the time event n
$XP_{r,r'}/XD_{r,r'}$ establishes the sequencing of nodes (r,r') on pickup/delivery routes
$YP_{r,s}/YD_{r,s}$ denotes that the P/D node r belongs to the route s
$JP_{v,s}/JD_{v,s}$ denotes that the pickup/delivery tour s is traveled by vehicle v
$KP_{r,v}/KD_{r,v}$ denotes that the pickup/delivery node r is visited by vehicle v
$ZP_{v,v'}/ZD_{v,v'}$ sequences vehicles (v,v') waiting for service at the same strip/stack door
U_v^P/U_v^D denotes the usage of the inbound/outbound vehicle v

Nonnegative continuous variables
AT_v^P/AT_v^D arrival time of the inbound/outbound vehicle v at the cross-dock facility
CP_r/CD_r cumulative routing cost from the cross-dock to the P/D node r
$DRS_{v,d,d'}$ denotes that the receiving door $d \in RD$ and the shipping door $d' \in SD$ have been assigned to vehicle v
OC_s^P/OC_s^D overall routing cost for the P/D tour s
RT_v^P time at which the inbound vehicle v is released from its pickup duties
ST_v^P time at which the pickup vehicle v starts the assigned tour
ST_v^D time at which the reloading of the outbound truck v begins
TP_r/TD_r vehicle arrival time at the P/D node of request r
TE_n unload time-event n
$UR_{r,n,v}$ denotes that request r was unloaded on the cross-dock from vehicle v not later than time TE_n
$UT_{r,n}$ denotes that the request r was unloaded on the cross-dock not later than the time event n
$YR_{r,v}$ states that the P/D nodes of request r are both served by vehicle v

Parameters
$d_{r,r'}^P/d_{r,r'}^D$ distance between P/D locations of requests r and r'
$d_{r,w}^P/d_{r,w}^D$ distance between the P/D location of request r and the cross-dock w
ft_r^P/ft_r^P fixed stop time at the P/D site of request r
ft_w^P/ft_w^D fixed stop time for P/D activities at the cross-dock terminal w
lr_r/ur_r loading/unloading rate at P/D sites of request r
lr_w/ur_w loading/unloading rate at the cross-dock terminal w
q_r shipment size for request r
Q_v capacity of vehicle v
sp_v average travel speed of vehicle v
t^{max} length of the planning horizon
$ttv_{d,d'}$ transfer time of vehicle v from the strip door $d \in RD$ to the stack door $d' \in SD$
$ttr_{d,d'}$ transfer time between the strip door $d \in RD$ and the stack door $d' \in SD$ per unit size of the request
uc_v unit distance cost for vehicle v
Δ maximum overlapping width between two adjacent sectors

Non-negative variables for the sweeping-based constraints
φ_s^P lower angular limit of sth-pickup sector

$\Delta\varphi_s^P$ angular width of the sth-pickup sector
ξ_r^P equals to one whenever the pickup/delivery location of request r satisfies the condition $\theta_r^P \in [o, \varphi_r^P]$

4.2. Model constraints

4.2.1. Route building constraints for pickup tours

Allocating requests to pickup routes. The pickup location of each request must be allocated to a single tour. If the assignment variable $YP_{r,s}$ is equal to 1, the pickup node of request r is assigned to the pickup route $s \in S^P$, where the set S^P includes all the pickup tours. $|S^P|$ is usually the number of available pickup vehicles.

$$\sum_{s \in S^P} YP_{r,s} = 1 \quad \forall r \in R \tag{1}$$

Routing cost from the cross-dock to the first visited node on a pickup route. Eq. (2) provides a lower bound on the routing cost CP_r to go from the cross-dock facility to the first visited node r on a pickup tour. The parameter uc represents the routing cost per unit distance and $d_{w,r}^P$ denotes the distance between the cross-dock, identified by the subscript w, and the pickup site of request r. Though written for every request on the tour s, it is only active for the first stop.

$$CP_r \geq uc\, d_{w,r}^P YP_{r,s} \quad \forall r \in R, s \in S^P \tag{2}$$

Cumulative routing cost from the cross-dock to a pickup node not visited on the first place. Sequencing constraints (3a) and (3b) relate the cumulative routing costs from the cross-dock to the pickup sites of a pair of requests $r, r' \in R$ allocated to the same tour s (i.e. $YP_{r,s} = YP_{r',s} = 1$). Such sequencing constraints use a single binary variable $XP_{r,r'}$ (with $r < r'$) to choose the relative order of pickup nodes (r, r') on the same inbound route. If $XP_{r,r'} = 1$ (with $r < r'$), then the request r is served earlier than r'. By Eq. (3a), therefore, $CP_{r'}$ must be larger than CP_r by at least the routing cost over the shortest path connecting the pickup sites of both requests. Otherwise, $XP_{r,r'} = 0$ and node r' is visited before node r. Consequently, $CP_{r'}$ should be lower than CP_r by at least the cost term $(uc\, d_{r,r'}^P)$ as stated by Eq. (3b). One of the Eq. (3) will be active only if both requests (r, r') have been assigned to the same route. The parameter M_C^P is a relatively large number.

$$CP_{r'} \geq CP_r + uc\, d_{r,r'}^P - M_C^P(1 - XP_{r,r'}) - M_C^P(2 - YP_{r,s} - YP_{r,s'}) \tag{3a}$$

$$CP_r \geq CP_{r'} + uc\, d_{r,r'}^P - M_C^P XP_{r,r'} - M_C^P(2 - YP_{r,s} - YP_{r,s'})$$

$$\forall r, r' \in R(r < r'), s \in S^P \tag{3b}$$

As remarked by Van Belle et al. (2012), previous approaches dealing with the VRPCD problem all assume that the loading of a truck along the tour can be done in any order. However, fragile goods should be collected at last to place them on the top. In the proposed model, the truck loading sequence along the route can be controlled through the variables $XP_{r,r'}$ with $r < r'$. If request r' must be picked up after request r, then $XP_{r,r'}$ is set to one.

Overall routing cost for the pickup tour s. Every pickup route should end at the cross-dock facility. As the string of nodes on the pickup route s is unknown before solving the model, Eq. (4) provides a lower bound on the total routing cost of tour s (OC_s^P) by assuming that any node on the route can be the last visited. The largest bound defining the value of OC_s^P is set by the pickup location that is actually last visited on the tour s.

$$OC_s^P \geq CP_r + uc\, d_{r,w}^P - M_C^P(1 - YP_{r,s}) \quad \forall r \in R, s \in S^P \tag{4}$$

Allocating vehicles to pickup tours. Let us define the binary variable $JP_{v,s}$ to denote that vehicle v has been assigned to tour s whenever $JP_{v,s}$ is equal to one. Moreover, it is introduced the continuous variable $KP_{r,v}$ to identify the pickup requests visited by vehicle

v. Eq. (5a) states that a pickup vehicle can at most be assigned to a single tour, and each route must be traveled by only one truck according to Eq. (5b). Besides, Eq. (6a) indicates that the pickup site of request r will be visited by vehicle v only if it belongs to a tour traveled by that vehicle. By Eq. (6b), each pickup request must be served by a single vehicle and, consequently, load splitting is not allowed. Obviously, the value of $KP_{r,v}$ is confined to the interval [0,1].

$$\sum_{s \in S^P} JP_{v,s} \leq 1 \quad v \in V \tag{5a}$$

$$\sum_{v \in V} JP_{v,s} \leq 1 \quad s \in S^P \tag{5b}$$

$$KP_{r,v} \geq YP_{r,s} + JP_{v,s} - 1 \quad \forall r \in R, \ v \in V, \ s \in S^P \tag{6a}$$

$$\sum_{v \in V} KP_{r,v} = 1 \quad r \in R \tag{6b}$$

If the unit routing cost is vehicle-dependent (uc_v), the variable $YP_{r,s}$ should be replaced by $KP_{r,v}$ in Eqs. (2)–(4) and OC_s^P by OC_v^P in Eq. (4).

Vehicle stop times at pickup nodes. Eqs. (7) and (8) define the vehicle stop time at the pickup location of request r (TP_r). The timing constraints (7) and (8) present the same mathematical structures of Eqs. (2) and (3). They are indeed sequencing constraints involving routing time parameters instead of routing cost coefficients. Eq. (7) provides a bound on the first stop time while Eq. (8) provides bounds for later stop times over the route s. The service time at any pickup node is computed as the sum of two terms: a fixed preparation time ft_r^P plus the variable loading time that directly increases with the load size q_r. The proportionality constant lr_r stands for the vehicle loading rate at the pickup node r. Moreover, the routing time along the shortest path connecting the pickup nodes r and r' is given by the ratio between the distance $d_{w,r}^P$ and the vehicle speed sp.

$$TP_r \geq \left(\frac{d_{w,r}^P}{sp}\right) YP_{r,s} \quad \forall r \in R, \ s \in S^P \tag{7}$$

$$TP_{r'} \geq TP_r + ft_r^P + lr_r q_r + \left(\frac{d_{r,r'}^P}{sp}\right) - M_T^P(1 - XP_{r,r'})$$
$$- M_T^P(2 - YP_{r,s} - YP_{r',s}) \tag{8a}$$

$$TP_r \geq TP_{r'} + ft_{r'}^P + lr_{r'} q_{r'} + \left(\frac{d_{r,r'}^P}{sp}\right) - M_T^P XP_{r,r'}$$
$$- M_T^P(2 - YP_{r,s} - YP_{r',s}) \quad \forall r, r' \in R(r < r'), s \in S^P \tag{8b}$$

If some pickup vehicles are not available at the beginning of the planning horizon, Eq. (7) should be substituted by Eq. (7'). The parameter rel_v denotes the release time of vehicle v in Eq. (7').

$$TP_r \geq rel_v + \left(\frac{d_{w,r}^P}{sp}\right) KP_{r,v} \quad \forall r \in R, \ v \in V \tag{7'}$$

Vehicle arrival times at the cross-dock. Eq. (9) defines the vehicle arrival times at the cross-dock facility.

$$AT_v^P \geq TP_r + ft_r^P + lr_r q_r + \left(\frac{d_{w,r}^P}{sp}\right) - M_T^P(1 - KP_{r,v}) \quad \forall r \in R, v \in V \tag{9}$$

If the vehicle speed is truck-dependent (sp_v), the variable $YP_{r,s}$ should be replaced by $KP_{r,v}$ in Eqs. (7) and (8).

4.2.2. Pickup vehicle capacity constraints

Eq. (10) states that the cumulative load collected along the tour $s \in S^P$ cannot exceed the maximum capacity of the vehicle assigned to that tour.

$$\sum_{r \in R} q_r YP_{r,s} \leq \sum_{v \in V} Q_v JP_{v,s} \quad s \in S^P \tag{10}$$

4.2.3. Unloading operations at the receiving dock area

Allocating vehicles to receiving dock doors. By Eq. (11), a vehicle returning to the cross-dock from its pickup trip must perform the unloading operations at just one strip door $d \in RD$. In Eq. (11), the set RD includes all the strip doors available at the cross-dock. Let us define the binary variable $DP_{v,d}$ to denote that the pickup vehicle v has been assigned to the strip door d whenever $DP_{v,d} = 1$.

$$\sum_{d \in RD} DP_{v,d} = 1 \quad \forall v \in V \tag{11}$$

Sequencing pickup vehicles assigned to the same strip door. A pickup truck leaves the strip door after its cargo has been unloaded. Eq. (12) defines a lower bound for the release time RT_v^P at which the pickup vehicle v completes the off-load operations at the cross-dock and is ready to perform delivery tasks. Such a bound is important to set the value of RT_v^P for the vehicle first served at some strip door d. Other vehicles assigned to door d should wait on the line until all the preceding trucks have been served. Constraints (13a) and (13b) relate the times at which the vehicles $(v, v') \in V$ ($v < v'$) assigned to the same receiving door d (i.e. $DP_{v,d} = DP_{v',d} = 1$) finish their unloading tasks. The relative order of a pair of vehicles v and v' on the line of the strip door d is given by a single binary variable $ZP_{v,v'}$ (with $v < v'$). If $ZP_{v,v'} = 1$, then the vehicle v is served before. Otherwise, $ZP_{v,v'} = 0$ and truck v' is unloaded earlier. When the two vehicles are processed at different strip doors, then the constraints (13a) and (13b) both become redundant. The service time of a truck at every dock door is computed as the sum of two components: a fixed preparation time ft_w^P and a variable service time that directly increases with the size of the cargo at a rate ur_w.

$$RT_v^P \geq AT_v^P + ft_w^P + ur_w\left[\sum_{r \in R} q_r (KP_{r,v} - KR_{r,v})\right] \quad v \in V \tag{12}$$

$$RT_{v'}^P \geq RT_v^P + ft_w^P + ur_w\left[\sum_{r \in R} q_r (KP_{r,v'} - KR_{r,v'})\right] - M_T^P(1 - ZP_{v,v'})$$
$$- M_T^P(2 - DP_{v,d} - DP_{v',d}) \tag{13a}$$

$$RT_v^P \geq RT_{v'}^P + ft_w^P + ur_w\left[\sum_{r \in R} q_r (KP_{r,v} - KR_{r,v})\right] - M_T^P ZP_{v,v'}$$
$$- M_T^P(2 - DP_{v,d} - DP_{v',d}) \quad \forall d \in RD, (v, v') \in V(v < v') \tag{13b}$$

$KR_{r,v} = 1$ only if the request r is fully served by vehicle v. If different inbound and outbound vehicle fleets are used, the variables $KR_{r,v}$ are set to zero.

Sequencing unloads events at the cross-dock. An unload event n occurs at the cross-dock whenever a pickup vehicle v just completes the discharge of the cargo to be delivered by other trucks. Therefore, the set N will include as many unloads events as the number of pickup vehicles on duty. Let us define the binary variable $WP_{n,v}$ to allocate pickup vehicles to unload events, and the continuous variable TE_n to represent the time at which the event n occurs. Eqs. (14a) and (14b) state that an inbound vehicle must exactly be assigned to a single event and reciprocally an event should be allocated to only

one vehicle. Events assigned to unused vehicles will never occur, i.e. they are dummy events.

$$\sum_{n\in N} WP_{n,v} \leq 1 \quad \forall v \in V \tag{14a}$$

$$\sum_{v\in V} WP_{n,v} \leq 1 \quad \forall n \in N \tag{14b}$$

By Eq. (15), N is an ordered set with the element n occurring before event n' if $n' > n$. Besides, constraints (16) specify that the event-time TE_n is set by the release time of vehicle v from its pickup assignments (RT_v^P) only if truck v is allocated to event n, i.e. $TE_n = RT_v^P$ when $WP_{n,v} = 1$. By Eq. (16a), the value of RT_v^P is imposed as a lower bound for TE_n whenever vehicle v is assigned to either an earlier event $n' < n$ or to the event n itself. Moreover, a lower bound for the release time of any pickup vehicle v (RT_v^P) is given by the first event time, while the last one provides an upper bound for any RT_v^P (see Eq. (17)).

$$TE_{n'} \geq TE_n \quad \forall n, n' \in N(n < n') \tag{15}$$

$$TE_{n'} \geq RT_v^P + M_T^P(WP_{n,v} - 1) \quad \forall n, n' \in N(n < n'), v \in V \tag{16a}$$

$$RT_v^P \leq TE_{n'} + M_T^P(1 - WP_{n,v}) \quad \forall n, n' \in N(n < n'), v \in V \tag{16b}$$

$$\sum_{n\in N} TE_n = \sum_{v\in V} RT_v^P \tag{16c}$$

$$TE_n \leq RT_v^P \quad n = first(N), \forall v \in V \tag{17a}$$

$$TE_n \geq RT_v^P \quad n = last(N), v \in V \tag{17b}$$

Subset of requests already unloaded at the cross-dock at the event time TE_n. Let $UR_{r,n,v}$ be a continuous variable denoting that request r collected by vehicle v is available on the cross-dock at the event time TE_n only if $WP_{n,v} = KP_{r,v} = 1$. Eqs. (18) and (19) drive $UR_{r,n,v}$ to zero when the request r is not picked up by vehicle v ($KP_{r,v} = 0$) and/or the vehicle v is assigned to an event $n' \neq n$ ($WP_{n,v} = 0$). Otherwise, $UR_{r,n,v} = q_r$ by Eq. (20).

$$\sum_{n\in N} UR_{r,n,v} \leq q_r KP_{r,v} \quad \forall r \in R, v \in V \tag{18}$$

$$UR_{r,n,v} \leq q_r WP_{n,v} \quad \forall n \in N, r \in R, v \in V \tag{19}$$

$$UR_{n,r,v} \geq q_r(WP_{n,v} + KP_{r,v} - 1) \quad \forall n \in N, r \in R, v \in V \tag{20}$$

The requests already unloaded on the cross-dock at time TE_n are provided by the continuous variables $UT_{r,n}$ defined by Eq. (21). Then, $UT_{r,n}$ will be equal to q_r if request r has been discharged from the assigned vehicle not later than time TE_n.

$$UT_{r,n} = \sum_{\substack{n' \in N \\ n' \leq n}} \sum_{v\in V} UR_{r,n',v} \quad \forall n \in N, r \in R \tag{21}$$

Queuing constraints for pickup vehicles allocated to the same receiving door. Let us define the continuous variable $ZP_{v,v'}$ ($v < v'$) with its value confined to the interval [0,1] to denote the relative order of vehicles v and v' on the line of the assigned strip door. If the inbound vehicles v and v' are unloaded at the same strip dock door d (i.e. $DP_{v,d} = DP_{v',d} = 1$) and vehicle v has been assigned to an earlier unload event, then vehicle v must be served before and $ZP_{v,v'} = 1$ by Eq. (22a). In the reverse case, Eq. (22b) indicates that v' is unloaded earlier and $ZP_{v,v'} = 0$. If vehicles v and v' are allocated to different receiving doors, the value of $ZP_{v,v'}$ is meaningless.

$$ZP_{v,v'} \geq WP_{n,v} + \sum_{\substack{n' \in N \\ n' > n}} WP_{n',v'} - 1 \tag{22a}$$

$$ZP_{v,v'} \leq 2 - WP_{n,v} - \sum_{\substack{n' \in N \\ n' < n}} WP_{n',v'} \quad \forall n \in N, v, v' \in V(v < v') \tag{22b}$$

4.2.4. Reloading operations at the shipping dock area

Allocating requests to delivery tours. As stated by Eq. (23), each transportation request must be allocated to a single outbound tour. Let us define the binary variable $YD_{r,s}$ to define the allocation of request r to the outbound trip $s \in S^D$ whenever $YD_{r,s} = 1$. Then,

$$\sum_{s\in S^D} YD_{r,s} = 1 \quad \forall r \in R \tag{23}$$

Allocating vehicles to delivery tours. Similarly to the pickup phase, let us define the binary variable $JD_{v,s}$ to allocate vehicles to delivery tours and the continuous variable $KD_{r,v}$ confined to the interval [0,1] to match up transportation requests and delivery vehicles. If the following condition $YD_{r,s} = JD_{v,s} = 1$ holds, then vehicle v visits the node r and $KD_{r,v} = 1$. In other words, the values of $YD_{r,s}$ and $JD_{v,s}$ fix the value of $KD_{r,v}$. Eq. (24) state that a vehicle can at most be assigned to a single delivery tour and reciprocally a route can be traveled by only one outbound vehicle. In turn, Eq. (25a) allocates the request r to vehicle v only if node r is one of the nodes on the tour traveled by vehicle v. Moreover, Eq. (25b) specifies that each request should be served by a single outbound vehicle. Therefore, load splitting is not allowed.

$$\sum_{s\in S^D} JD_{v,s} \leq 1 \quad v \in V \tag{24a}$$

$$\sum_{v\in V} JD_{v,s} \leq 1 \quad s \in S^D \tag{24b}$$

$$KD_{r,v} \geq YD_{r,s} + JD_{v,s} - 1 \quad \forall r \in R, v \in V, s \in S^D \tag{25a}$$

$$\sum_{v\in V} KD_{r,v} = 1 \quad \forall r \in R \tag{25b}$$

Identifying requests with pickup and delivery locations both visited by the same vehicle. If the pickup and delivery sites of request r are both served by the same vehicle, the related transshipment operations at the cross-dock are not required. In such a case, $KP_{r,v} = KD_{r,v} = 1$ for some vehicle v and the load of request r may not be discharged on the receiving dock, i.e. it remains into the vehicle v. Let $KR_{r,v}$ be a non-negative continuous variable with a domain [0,1] that is defined to identify requests fully served by vehicle v. Eq. (26) drives $KR_{r,v}$ to one whenever $KP_{r,v} = KD_{r,v} = 1$, and drops $KR_{r,v}$ to zero if either of such variables are null. If the vehicles are either inbound or outbound or the collected goods should always be unloaded on the cross-dock, Eq. (26) can be ignored.

$$KD_{r,v} \leq KP_{r,v} \tag{26a}$$

$$KR_{r,v} \leq KD_{r,v} \tag{26b}$$

$$KR_{r,v} \geq KP_{r,v} + KD_{r,v} - 1 \quad \forall r \in R, v \in V \tag{26c}$$

In case the cargos collected and delivered by the same vehicle are unloaded on the cross-dock or different fleets of inbound and outbound vehicles are used, then Eqs. (12) and (13) should be modified as follows,

$$RT_v^P \geq AT_v^P + ft_w^P + ur_w[\sum_{r\in R} q_r KP_{r,v}] \quad v \in V \tag{12'}$$

$$RT_{v'}^P \geq RT_v^P + ft_w^P + ur_w[\sum_{r\in R} q_r KP_{r,v'}] - M_T^P(1 - ZP_{v,v'})$$
$$- M_T^P(2 - DP_{v,d} - DP_{v',d}) \tag{13a'}$$

$$RT_v^P \geq RT_{v'}^P + ft_w^P + ur_w[\sum_{r\in R} q_r KP_{r,v}] - M_T^P ZP_{v,v'}$$
$$- M_T^P(2 - DP_{v,d} - DP_{v',d}) \quad \forall d \in RD, (v, v') \in V(v < v') \tag{13b'}$$

Sometimes, requests with $KR_{r,v} = 1$ must be discharged to allow the unloading of other freights onto the cross-dock. Moreover, the loading of a truck should be done in a certain order to get a better use of its transport capacity or must account for the order in which the

freights are to be delivered (Van Belle et al., 2012). Consequently, every request is usually unloaded onto the cross-dock.

Allocating delivery vehicles to shipping dock doors. Let $DD_{v,d}$ denote a binary variable allocating outbound vehicles to shipping doors. If $DD_{v,d} = 1$, then the loading operations for vehicle v will occur at the shipping door $d \in SD$. By Eq. (27), an outbound vehicle on duty must be loaded at just one stack dock door. The SD comprises all the shipping doors available at the cross-dock.

$$\sum_{d \in SD} DD_{v,d} = 1 \quad \forall v \in V \qquad (27)$$

Identifying the strip and stack dock doors assigned to each vehicle. The continuous variable $DRS_{v,d,d'}$ with domain [0,1] is introduced to indicate that vehicle v should move from the strip door $d \in RD$ to the stack door $d' \in SD$ before starting the reloading operations. Eq. (28) drive the variable $DRS_{v,d,d'}$ to one whenever $DP_{v,d} = DD_{v,d'} = 1$, and drops $DRS_{v,d,d'}$ to zero if either of such variables is null. Moreover, a vehicle v should at most be assigned to a single pair of strip/stack dock door.

$$DRS_{v,d,d'} \leq DP_{v,d} \qquad (28a)$$

$$DRS_{v,d,d'} \leq DD_{v,d'} \qquad (28b)$$

$$DRS_{v,d,d'} \geq DP_{v,d} + DD_{v,d'} - 1 \quad \forall v \in V, \, d \in RD, \, d' \in SD \qquad (28c)$$

$$\sum_{d \in RD} \sum_{d' \in SD} DRS_{v,d,d'} = 1 \quad \forall v \in V \qquad (28d)$$

If inbound and outbound vehicle fleets are different, then Eq. (28) should be omitted.

Sequencing outbound vehicles assigned to the same shipping door. Let the continuous variable ST_v^D be the earliest time at which the outbound vehicle v can start reloading the cargo at the cross-dock. In addition, it is defined the continuous variable $ZD_{v,v'}$ to control the relative order of vehicles v and v' on the line of the common stack door. Assuming that the same fleet of vehicles is used to perform pickup and delivery tasks, constraints (29) and (30) should be considered to determine the value of ST_v^D. On one hand, the loading of a delivery vehicle v cannot start before truck v completes its pickup assignments and moves to the assigned stack door, i.e. not earlier than RT_v^P. This constraint (29) is important for the vehicles that are first served at the stack doors. On the other hand, the reloading of vehicle v cannot begin until all the preceding trucks on the line of the assigned stack door $d \in SD$ have been served. To this end, Eqs. (30a) and (30b) relate the times ST_v^D and ST'^D_v at which the pair of vehicles $(v,v') \in V$ (with $v < v'$) sharing the same shipping door d (i.e. $DD_{v,d} = DD_{v',d} = 1$) can start the reloading operations at the cross-dock. If vehicle v precedes v' on the line of the stack door d, then the sequencing variable $ZD_{v,v'}$ must be equal to one and Eq. (30a) applies. Otherwise, $ZD_{v,v'} = 0$ and Eq. (30b) becomes the relevant constraint. When the two vehicles are allocated to different stack doors, constraints (30) are both redundant and the value of $ZD_{v,v'}$ is meaningless. In Eq. (29), the parameter $ttv_{d,d'}$ denotes the time spent by a vehicle to move from the receiving door $d \in RD$ to the shipping door $d' \in SD$. When the fleets of inbound and outbound vehicles are different, constraint (29) should be omitted. Then, the model can still be applied even if the vehicles are either inbound or outbound trucks.

$$ST_v^D \geq RT_v^P + \sum_{d \in RD} \sum_{d' \in SD} ttv_{d,d'} DRS_{v,d,d'} \quad \forall v \in V \qquad (29)$$

$$ST_{v'}^D \geq ST_v^D + ft_w^D + ur_w \left[\sum_{r' \in R} q_{r'}(KD_{r',v} - KR_{r',v}) \right] - M_T^D (1 - ZP_{v,v'})$$
$$- M_T^D (2 - DP_{v,d} - DD_{v',d}) \qquad (30a)$$

$$ST_v^D \geq ST_{v'}^D + ft_w^D + ur_w \left[\sum_{r' \in R} q_{r'}(KD_{r',v'} - KR_{r',v'}) \right] - M_T^D ZD_{v,v'}$$
$$- M_T^D (2 - DP_{v,d} - DD_{v',d}) \quad \forall d \in RD, \, d' \in SD, \, v, \, v' \in V(v < v') \qquad (30b)$$

In general, the travel time between the docks is small compared with the time during which the freights will remain temporarily on the cross-dock waiting for the arrival of other loads delivered by the same outbound truck.

Another constraint on the value of ST_v^D. An outbound vehicle does not start loading operations until every request r that should deliver is available at the assigned shipping door. In Eq. (31), the parameter $ttr_{d,d'}$ denotes the transfer time between the strip door d and the stack door d' per unit size of the request. If q_r is the size of request r, then its transfer time between docks d and d' is equal to $q_r ttr_{d,d'}$. Eq. (31) is enforced only if request r is collected by vehicle v' and delivered by vehicle v and, in addition, the truck v' is unloaded at the strip door d' and the truck v is loaded at the stack door d.

$$ST_v^D \geq RT_{v'}^P + q_r ttr_{d,d'} - M_T^D (2 - KP_{r,v'} - KD_{r,v})$$
$$- M_T^D (2 - DP_{v,d} - DD_{v',d'}) \quad \forall r \in R, \, v, \, v' \in V(v \neq v'),$$
$$d' \in RD, \, d \in SD \qquad (31)$$

When every request transported by an inbound vehicle should be unloaded even if some freight is delivered by the same truck, the condition $v \neq v'$ in Eq. (31) must be omitted. If the transfer time of a request r across the cross-dock is the same whatever is the selected pair of assigned strip and stack doors, Eq. (31) reduces to Eq. (32).

$$ST_v^D \geq RT_{v'}^P + q_r ttr - M_T^D (2 - KP_{r,v'} - KD_{r,v}) \quad \forall r \in R, \, v, \, v' \in V(v \neq v') \qquad (32)$$

Allocating delivery vehicles to unloads events. Let us define the binary variable $WD_{n,v}$ to denote that the outbound vehicle v has been assigned to the unload event $n \in N$ only if $WD_{n,v} = 1$. Eq. (33) asserts that each outbound vehicle must be assigned to a single event $n \in N$. However, several vehicles can be allocated to the same unload event. The allocation of the outbound vehicle v to event n means that the requests assigned to that truck have already been unloaded at the cross-dock at time TE_n. Therefore, such requests all feature $UT_{r,n} = 1$. If request r has been allocated to vehicle v (i.e. $KD_{r,v} = 1$), then Eq. (34) assigns truck v to event n only if $UT_{r,n} = 1$. Otherwise, $WD_{n,v}$ is equal to zero. In addition, Eq. (35) drives the variable $WD_{n,v}$ to zero if the unloading of vehicle v occurs at some later event $n' > n$, i.e. $WP_{n',v} = 1$. Eq. (35) should be omitted if every truck is either inbound or outbound.

$$\sum_{n \in N} WD_{n,v} = 1 \quad \forall v \in V \qquad (33)$$

$$UT_{r,n} \geq WD_{n,v} + KD_{r,v} - 1 \quad \forall n \in N, r \in R, v \in V \qquad (34)$$

$$WD_{n,v} \leq \sum_{\substack{n' \in N \\ n' \geq n}} WP_{n',v} \quad \forall n \in N, v \in V \qquad (35)$$

Queuing constraints for outbound vehicles assigned to the same shipping door. If the outbound vehicles v and v' are loaded at the same stack door and vehicle v has been allocated to an earlier event, then the vehicle v should be served before and the sequencing variable $ZD_{v,v'}$ is equal to 1 by Eq. (36a). In the reverse case, Eq. (36b) states that vehicle v' is reloaded earlier and $ZD_{v,v'} = 0$. If vehicles v

and v' are assigned to different stack doors, the value of $ZD_{v,v'}$ is meaningless.

$$ZD_{v,v'} \leq 2 - WD_{n,v} \sum_{\substack{n' \in N \\ n' < n}} WD_{n',v'} \tag{36a}$$

$$ZD_{v,v'} \geq WD_{n,v} + \sum_{\substack{n' \in N \\ n' \geq n}} WD_{n',v'} - 1 \quad \forall n \in N, v, v' \in V(v < v') \tag{36b}$$

4.2.5. Route building constraints for the delivery phase

Route building constraints with mathematical structures similar to those proposed for the pickup phase can be written for the delivery routes. Their formulations can be derived from Eqs. (2)–(8) by simply replacing the assignment variable $YP_{r,s}$ by $YD_{r,s}$, the routing cost CP_r by CD_r, the visiting time TP_r by TD_r, the sequencing variable $XP_{r,r'}$ by $XD_{r,r'}$ (with $r < r'$), and the superscript P by D. Outbound routing cost constraints are given by Eqs. (37)–(39), while Eqs. (40)–(43) state for the vehicle stop time constraints.

Routing cost from the cross-dock to the first visited node on a delivery route.

$$CD_r \geq uc \; d_{w,r}^D \; YD_{r,s} \quad \forall r \in R, \; s \in S^D \tag{37}$$

Cumulative routing cost from the cross-dock to a delivery node not visited on the first place

$$CD_{r'} \geq CD_r + uc \; d_{r,r'}^D - M_C^D(1 - XD_{r,r'}) - M_C^D(2 - YD_{r,s} - YD_{r,s'}) \tag{38a}$$

$$CD_r \geq CD_{r'} + uc \; d_{r,r'}^D - M_C^D XD_{r,r'} - M_C^D(2 - YD_{r,s} - YD_{r,s'})$$
$$\forall r, r' \in R(r < r'), s \in S^D \tag{38b}$$

Overall routing cost for delivery tours

$$OC_s^D \geq CD_r + uc \; d_{w,r}^D - M_C^D(1 - YD_{r,s}) \quad \forall r \in R, s \in S^D \tag{39}$$

Vehicle stop time at the delivery node first visited

$$ST_v^D \geq TE_n - M_T^D(1 - WD_{n,v}) \quad \forall n \in N, v \in V \tag{40}$$

$$TD_r \geq ST_v^D + ft_w^D + ur_w[\sum_{r' \in R} q_r, KD_{r',v}] + \left(\frac{d_{w,r}^D}{sp}\right) - M_T^D(1 - KD_{r,v})$$
$$\forall r \in R, v \in V \tag{41}$$

Vehicle stop times at delivery nodes that are not first visited

$$TD_{r'} \geq TD_r + ft_r^D + ur_r q_r + \left(\frac{d_{r,r'}^D}{sp}\right) - M_T^D(1 - XD_{r,r'})$$
$$- M_T^D(2 - YD_{r,s} - YD_{r',s}) \tag{42a}$$

$$TD_r \geq TD_{r'} + ft_{r'}^D + ur_{r'} q_{r'} + \left(\frac{d_{r,r'}^D}{sp}\right)$$
$$- M_T^D XD_{r,r'} - M_T^D(2 - YD_{r,s} - YD_{r',s}) \quad \forall r, r' \in R(r < r'), s \in S^D \tag{42b}$$

Arrival times of outbound vehicles at the cross-dock facility

$$AT_v^D \geq TD_r + ft_r^D + ur_r q_r + \left(\frac{d_{w,r}^D}{sp}\right) - M_T^P(1 - KD_{r,v}) \quad \forall r \in R, v \in V \tag{43}$$

4.2.6. Outbound vehicle capacity constraints

Eq. (44) states that the cumulative load collected along the tour $s \in S^D$ cannot exceed the maximum capacity of the vehicle assigned to that tour.

$$\sum_{r \in R} q_r YD_{r,s} \leq \sum_{v \in V} Q_v \; JD_{v,s} \quad s \in S^D \tag{44}$$

4.2.7. Time window constraints

When time windows within which the service of P/D nodes should start are specified, then the constraints (45a) and (45b) should be complied.

$$TP_r^{\min} \leq TP_r \leq TP_r^{\max} \tag{45a}$$

$$TD_r^{\min} \leq TD_r \leq TD_r^{\max} \quad r \in R \tag{45b}$$

4.2.8. Valid inequalities to accelerate the convergence to the optimal solution

Symmetric breaking constraints for the assignment of inbound vehicles to tours. To avoid symmetrical solutions for pickup tours, the constraint (46) is incorporated in the model. If vehicles $(v,v') \in V^Q$ have the same capacity Q and $v < v'$, then vehicle v should be assigned before v' to a route requiring a capacity equal to or lower than Q. Let us consider a pair of tours s and s' (with $s < s'$) that can be traveled by vehicles $(v,v') \in V^Q$ $(v < v')$ with the same capacity Q. Then, the assignment of vehicles (v,v') to tours (s,s') implies that $YP_{v,s} = YP_{v',s'} = 1$. In other words, Eq. (46) excludes the solution with $YP_{v,s'} = YP_{v',s} = 1$. In contrast, the allocation of vehicles of a similar type to outbound routes is decided by the model.

$$JP_{v',s} \leq \sum_{\substack{s' \in S \\ s' < s}} JP_{v,s'} \quad \forall v, v' \in V^Q(v < v'), s \in S^P \tag{46}$$

By letting the model choose the best assignment of non-similar vehicles to pickup/delivery tours, a good synchronization between inbound vehicle arrivals and outbound vehicle departures and consequently a lower total distribution time can both be achieved.

Valid inequalities restricting the allocation of vehicles to strip dock doors. To partially eliminate symmetric solutions, the constraint (47) is added to the mathematical model just to solve large problems. If the set RD comprises three elements $\{d_1, d_2, d_3\}$, then the constraint (47) allocates the dock door d_1 to the vehicle that first unloads the cargo on the cross-dock terminal and assigned to the first event ($n = 1$). Similarly, the dock door d_2 is allocated to the vehicle completing the unloading operation in the second place and assigned to the second event ($n = 2$); dock d_3 to the truck finishing the pickup duties on third place and so on. The constraint (47) does not exclude the optimal solution from the feasible region but just avoid symmetrical assignments.

$$\sum_{\substack{d \in RD \\ d \leq n}} DP_{v,d} \geq \sum_{\substack{n' \in N \\ n' \leq n}} WP_{n',v} \quad \forall n \in N(n \leq |RD|), v \in V \tag{47}$$

4.3. The objective function

Depending on the relative magnitude of the major costs involved in the problem, alternative objective functions (48) can be used. All of them can be handled by the proposed formulation.

(a) Minimizing the total vehicle routing cost

$$Min \, z = \sum_{s \in S} OC_s^P + \sum_{s \in S^D} OC_s^D \tag{48a}$$

(b) Minimizing the total distribution time

$$Min \, z = \sum_{v \in V} AT_v^D \tag{48b}$$

(c) Minimizing the total makespan

$$Min \quad z = MK \quad subject \ to \quad MK \quad \geq AT_v^D, v \in V \quad (48c)$$

(d) Minimizing a weighted combination of objectives (a) and (b)

$$Min \ z = \tau \sum\nolimits_{v \in V} AT_v^D + \sum\nolimits_{s \in S^P} OC_s^P + \sum\nolimits_{s \in S^D} OC_s^D \quad (48d)$$

The coefficient τ in Eq. (48d) represents the cost per unit time spent in wages and other time-dependent expenses. In this way, it has been defined a rigorous mathematical formulation of the heterogeneous VRPCD that includes the constraints {(1)–(31), (33)–(44), (46)–(47)} and one of the alternative objective functions (48).

4.4. Allocating nodes to vehicles using the sweep heuristics algorithm

Dondo and Cerdá (2013) developed a set of equations shown in Appendix A to speed-up the allocation of pickup/delivery nodes to vehicles by mimicking the VRP sweep algorithm introduced by Gillett and Miller (1974). By applying the sweep-based constraints, pickup/delivery nodes are grouped into a number of angular sectors each one assigned to a different vehicle. The cross-dock facility is at the origin of a polar coordinate system used to describe the location of a P/D node in terms of the radial ($d_{w,r}$) and the angular (θ_r) coordinates. The customer nodes are arranged by increasing θ_r and the nodes are assigned to the selected vehicle as the angular coordinate continually rises while the truck is not overloaded. Otherwise, a new vehicle is chosen and the procedure is continued until every site has been assigned to exactly one vehicle. The width of the angular areas is adjusted in order to minimize the value of the objective function while accounting for the capacity of the assigned vehicle. Moreover, the best polar angle for starting the procedure is also optimized (see Appendix B). By including the constraints (B1)–(B10) mimicking the sweep algorithm into the problem formulation, the portion of the solution space just containing petal-shape routes is explored.

4.5. Valid cuts for large heterogeneous VRPCD problems

If Eq. (46) is applied to all inbound vehicles by replacing V^Q (set of vehicles having the same capacity Q) by the entire set of pickup vehicles V^P, then the model will still have the chance to select the most convenient allocation of vehicles to delivery tours that better suited to the pickup vehicle arrival times. As shown in Section 5, good solutions are still found by considering Eq. (49) but at lower computational cost.

$$JP_{v',s} \leq \sum\nolimits_{\substack{s' \in S \\ s' < s}} JP_{v,s'} \quad \forall v, v' \in V(v < v'), \quad s \in S^P \quad (49)$$

For larger heterogeneous VRPCD problems, competitive feasible solutions can be discovered at reasonable CPU times by considering constraints similar to Eq. (49) to also pre-assign outbound vehicles to delivery tours.

$$JD_{v',s} \leq \sum\nolimits_{\substack{s' \in S \\ s' < s}} JD_{v,s'} \quad \forall v, v' \in V(v < v'), s \in S^D \quad (50)$$

5. Results and discussion

The new rigorous formulation for the heterogeneous vehicle routing problem with cross-docking not only deals with fleets of vehicles of different capacities but also no longer assumes a pre-defined assignment of inbound/outbound trucks to pickup/delivery routes. In contrast to the approach of Dondo and Cerdá (2014) that

pre-allocates vehicle v_1 to tour s_1, truck v_2 to tour s_2 and so on, the assignment of vehicles to tours is now decided by the proposed model. Therefore, improved results may be obtained even for the homogeneous fleet case.

To illustrate the advantages of the new approach in providing high-quality solutions to the heterogeneous VRP problems with cross-docking within bounded CPU times, a series of 48 medium-size heterogeneous VRPCD examples have been solved. Each one is characterized by the number of customer requests $|R|$ to be satisfied, the cross-dock layout, the number of available vehicles $|V|$ and their corresponding capacities (i.e. the vehicle fleet composition). In all of them, it is assumed that the same vehicle fleet sequentially accomplishes both pickup and delivery tasks. An example consisting of 20 requests, 4 vehicles, and a cross-dock facility with 2 receiving doors and 2 shipping doors will be labeled 20R-4V-2RD-2SD. Besides, it is necessary to define the fleet composition accomplishing the pickup and delivery tasks. A fleet composed by one truck with a capacity of 90 volume units, 2 vehicles with 75 units of capacity and one truck with 60 units is characterized by {90^1-75^2-60^1}, i.e. {$cap^{|V_{cap}|}$}. The examples solved in this paper involve up to 50 transportation requests, a vehicle fleet with at most 10 trucks featuring three different capacities and cross-dock layouts with up to 10 dock doors. The data for the 50 customer requests including the shipment sizes, the Cartesian coordinates of the related P/D nodes and the node time windows are all reported in Table A1 of the Appendix A. When time windows are considered, the service of a P/D node by the assigned truck must begin within the time interval delimited by the specified earliest and latest service times given in Table A1. Moreover, the time at which the service begins it is a decision variable.

The 48 examples were generated by considering the first R requests of Table A1 with $|R|$ varying from 10 to 50, and a rising number of vehicles and dock doors as the value of $|R|$ grows. For each example, three types of vehicles fleets are considered: a homogeneous vehicle fleet and a pair of heterogeneous fleets with trucks of three different capacities. Despite the fleet composition is varied, the total fleet transport capacity for each example is always the same. It is higher than the overall load to be transported by less than 10%. Taking into account that the loading of an outbound truck cannot usually be done in any order (Van Belle et al., 2012), it will be assumed that the cargo transported by any pickup vehicle is fully unloaded at the cross-dock, i.e. $KR_{r,v}$ is set to zero for any r and v. Moreover, all the vehicles are available at the start of the planning horizon ($rel_v = 0$ for any $v \in V$) and the customer orders must be fulfilled within a planning time-horizon going from $t = 0$ to $t^{max} = 400$ time-units. Two types of problem targets were considered: the least total routing cost given by Eq. (48a) and the minimum total distribution time computed by Eq. (48b). When using the first target, the best solution can be found in a much shorter CPU time. If it is adopted the minimum distribution time target, the larger computational cost somewhat decreases by minimizing $\sum_{v \in V}(AT_v^P + AT_v^D)$ instead of $\sum_{v \in V}(AT_v^D)$. The parameter Δ denoting the maximum angular overlapping between pickup or delivery tours has been fixed to zero, unless no feasible solution is found or time window constraints are considered. In such cases, the value of Δ is increased to 0.3–1.0.

Vehicle transfer times between inbound and outbound dock doors ($ttv_{d,d'}$) are reported in Table A2 of the Appendix A. The selected values for the remaining model parameters are $lr_r = ur_r = 0.2$; $ft_r^P = ft_r^D = 0.5$; $lr_w = ur_w = 0.5$; $ft_w^P = ft_w^D = 0.5$, $sp = 1$, $uc = 1$ and $ttr_{d,d'} = 0$. For some problem instances, a positive value for $ttr_{d,d'}$ was chosen. All the examples were solved using GAMS 23.7.3 in a 2.66 MHz two-processor PC with 24 MB RAM and 8 cores-per-processor. The relative gap tolerance has been fixed at 10^{-2} and a maximum CPU time of 3600 s was allowed for problems with less than 40 requests. When $|R| \geq 40$ or time

Table 1
Comparing the best routing cost solutions found for heterogeneous VRPCD examples with the rigorous formulation and the sweep-based model.

$\|R\|$	Fleet composition (cap$^{\|V\|}$)	$\|RD\|$	$\|SD\|$	Rigorous formulation			Sweeping-based formulation		
				Best cost solution[a]	CPU time to find it (s)	Optimality gap (%)	Best cost solution[a]	CPU time to find it	Optimality gap (%)
10	90^1-60^1	1	1	398.6	50	–	398.6	16	–
12	90^1-75^1-60^1	1	1	443.9	205	9.9	443.9	141	–
15	90^1-75^1-60^1	2	2	559.0	3441	19.9	562.5	307	–
18	90^1-75^2-60^1	2	2	668.3	1041	36.8	631.0	625	2.5
20	90^1-75^2-60^1	2	2	742.3	2629	56.0	676.7	123	2.9
22	90^2-75^1-60^2	2	2	[b]	–	–	728.6	1931	9.2
24	90^2-75^1-60^2	2	2	[b]	–	–	826.2	982	5.4

[a] After the CPU time limit of 3600 s.
[b] No feasible solution found.

windows are considered, the CPU time limit was increased to 5000–7200 s.

5.1. Validating the sweep-based model for heterogeneous VRPCD problems

The rigorous formulation and the sweep-based model without constraints (49) and (50) both seek the best allocation of inbound/outbound vehicles to pickup/delivery tours in order to minimize the selected objective function. To compare the quality of the solutions found and the CPU time required to discover them and prove their optimality, both approaches were applied to a series of examples involving 10–24 transportation requests and a heterogeneous fleet comprising 2–5 vehicles with three different capacities {60, 75, and 90 volume units}. Besides, the cross-dock presents 1–2 strip and stack dock doors. Here, the word "optimality" denotes that the best alternative within the solution space being explored has been encountered. It was adopted the total routing cost as the objective function and a CPU time limit of 3600 s. Computational results are shown in Table 1.

From the results shown in Table 1, it can be concluded that both approaches almost provide the same best solution for problems with 15 requests or less. They consist of P/D petal-shaped routes. However, the sweep-based formulation found them at lower computational cost. This is so because the approximate formulation explores a more compact feasible region that still contains the best pickup/delivery tours. For larger examples with 18–20 requests, the best solutions found by the rigorous approach after 3600 s of CPU time are worse than the ones discovered by the sweep-based model, and obviously present larger optimality gaps that sharply increase with the problem size. No feasible solution is found by the rigorous model after 3600 s for examples with more than 20 requests. Contrarily, the sweep-based model discovers good solutions for such examples in acceptable CPU times.

Therefore, it can be concluded that the approximate model based on the sweep heuristics algorithm appears as a powerful, accurate tool to use instead of the rigorous formulation for solving medium-size heterogeneous VRPCD examples.

5.2. Improving the computational efficiency of the sweep-based approach

For a heterogeneous fleet, the design of pickup/delivery routes certainly depends on the order the trucks are loaded by the sweep-based model. Longer routes are surely designed for trucks with a larger capacity and vice versa. If the sweep model starts assigning nodes to a large truck, the first generated route will include a higher number of visited nodes. An opposite result would be obtained if the selected vehicle has the lowest capacity. Moreover, the set of nodes visited by each vehicle will also depend on the order that the trucks are loaded by the sweep constraints. The best truck loading

sequence is chosen by the sweep-based model to generate the pickup and delivery routes for a heterogeneous VRPCD problem. On the other hand, every outbound truck must return to the base within the cross-dock time window $[0, t^{max}]$. Then, a good exchange of loads between inbound and outbound vehicles will be important to meet such a time constraint. To this purpose, the model accounts for the times at which the loads collected by the pickup vehicles are available on the cross-dock. Though the truck loading sequence (TLS) is a key decision for the design of pickup and delivery tours, the cross-dock time window constraint makes the TLS for delivery tours more crucial. For a pre-defined TLS for inbound tours like the one prescribed by Eq. (49), the sweep-based formulation will still have the chance to choose the most convenient TLS for delivery tours that better synchronize with the resulting pickup tours. Usually, the incorporation of Eq. (49) in the sweep-based formulation produces a substantial improvement on its computational efficiency at the expense of some minor deterioration on the solution quality. The feasible region of the augmented sweep model is a subspace of the solution space explored by the rigorous formulation that only contains P/D petal-shaped tours.

To analyze the impact of Eq. (49) on both the solution quality and the computational cost, a series of examples consisting of 10–35 requests, a heterogeneous fleet with 2 to 7 vehicles of three different capacities {60, 75, and 90 volume units} and a cross-dock with 1 to 3 strip and stack dock doors has been tackled using the sweep-based model with and without Eq. (49). Computational results are shown in Table 2. It includes the best solutions found by both formulations, the CPU times required to find them and the related optimality gaps. The optimality gap is measured with regards to the lower bound on the objective value of each formulation after 3600 s of CPU time. It is observed that the sweep-based formulation without constraint (49) provides better solutions while the number of requests does not exceed 30. However, the inclusion of Eq. (49) in the sweep model reduces the CPU time by an average factor of 3.3 at the expense of an average deterioration of the objective value less than 2%. For larger examples, the "pure" sweep-based model finds either worse solutions or no feasible solution at all after 3600 s of CPU time. In contrast, the augmented sweep formulation still finds good solutions in reasonable CPU times. Therefore, the augmented sweep model with Eq. (49) looks like an efficient, accurate tool to deal with larger heterogeneous VRPCD problems.

After validating the augmented sweep-based approach, a series of 36 VRPCD-examples involving 15 to 40 transportation requests, a vehicle fleet composed by 3 to 8 trucks with three different compositions {60, 75, and 90 volume units} and a cross-dock with 2 to 4 receiving/shipping dock doors have been solved. The total P/D routing cost was selected as the objective function to be minimized. Once the best cost solution has been found, the P/D routes are fixed and the total distribution time is minimized. Then, the total distribution time will play the role of a secondary target. In other words, a two-level approach is performed with the total routing cost being

Table 2
The effect of including the constraint (49) on the best routing cost solution and the computational time when using the sweep-based formulation.

$\|R\|$	Fleet composition (cap$^{\|V\|}$)	$\|RD\|$	$\|SD\|$	Sweep-based formulation			Augmented sweep-based formulation with constraint (49)		
				Best cost solution[a]	CPU time to find it (s)	Optimality gap (%)	Best cost solution[a]	CPU time to find it (s)	Optimality gap (%)
10	90^1-60^1	1	1	398.6	16	–	402.9	20	–
12	90^1-75^1-60^1	1	1	443.9	141	–	461.1	129	–
15	90^1-75^1-60^1	2	2	562.5	307	–	577.7	116	–
18	90^1-75^2-60^1	2	2	631.0	625	2.5	648.6	123	–
20	90^1-75^2-60^1	2	2	676.7	123	2.9	681.9	37	–
22	90^2-75^1-60^2	2	2	728.6	1931	9.2	732.5	455	–
24	90^2-75^1-60^2	2	2	826.2	982	5.4	842.9	132	–
27	90^2-75^2-60^2	3	3	867.2	1083	6.5	886.4	1204	3.3
30	90^2-75^2-60^2	3	3	906.7	3139	22.7	923.7	290	1.6
33	90^2-75^3-60^2	3	3	1008.4	3355	57.1	1000.1	666	3.2
35	90^2-75^3-60^2	3	3	[b]	–	–	1045.6	1207	2.0

[a] After the CPU time limit of 3600 s.
[b] No feasible solution found.

minimized at the upper level and the least total distribution time is sought at the lower level. Table 3 reports the best routing cost solutions, the best lower-level distribution times, the CPU times required to find them and the related optimality gaps in case the gap tolerance of 1% is not reached in 3600 s of CPU time.

For problem instances with 35 requests or less, the augmented sweep-based approach is able to explore the model feasible space and discovers the best possible solution in acceptable CPU times. In many cases, the computer run is promptly stopped because

the optimality gap drops below 1% in a CPU time much lower than 3600 s. The additional CPU time required to minimize the total cross-dock operating time at the lower level is usually very short. Fig. 2 shows the sketches of the best petal-shaped P/D tours found by the augmented sweep-based model for examples 20R-4V-2RD-2SD, 30R-6V-3RD-3SD and 40R-8V-4RD-4SD all involving a heterogeneous fleet of vehicles with three different capacities (90, 75 and 60 volume units). In turn, Tables 4–6 present detailed descriptions of such solutions including: (a) the sequence

Table 3
Best routing cost solutions for examples with 15 to 40 transport requests using the augmented sweep-based model.

Ex.[a]	$\|R\|$	$\|RD\|$	$\|SD\|$	Fleet composition (cap$^{\|V\|}$)	Best routing cost	CPU time (s)		Optimality gap (%)	Total operating time[b] (h)	CPU time to find it (s)
						To find it	To prove optimality			
1	15	2	2	75^3	583.0	14	42	–	983.2	0.3
2				80^1-75^1-70^1	569.9	12	28	–	952.1	0.2
3				90^1-75^1-60^1	577.7	10	116	–	1023.5	0.2
4	18	2	2	75^4	655.1	27	78	–	1138.4	0.7
5				80^1-75^2-70^1	645.5	130	2707	–	1149.0	0.5
6				90^1-75^2-60^1	648.6	123	278	–	1091.1	0.7
7	20	2	2	75^4	774.4	15	41	–	1304.5	0.6
8				80^1-75^2-70^1	695.9	25	2100	–	1246.1	0.6
9				90^1-75^2-60^1	681.9	37	309	–	1198.7	0.6
10	21	2	2	75^4	805.6	48	149	–	1349.2	1
11				80^1-75^2-70^1	762.9	69	632	–	1244.7	0.4
12				90^1-75^2-60^1	729.5	262	2078	–	1314.6	0.4
13	24	2	2	75^5	909.8	21	45	–	1610.9	1
14				80^2-75^1-70^2	910.9	43	133	–	1581.7	1
15				90^2-75^1-60^2	842.9	132	631	–	1584.9	1
16	27	3	3	75^6	954.8	57	93	–	1764.5	1
17				80^2-75^2-70^2	898.3	132	1013	–	1676.1	1
18				90^2-75^2-60^2	886.4	1204	3600[a]	3.3	1600.7	6
19	28	3	3	75^6	963.9	439	740	–	1746.8	1
20				80^2-75^2-70^2	927.1	2768	3600[a]	2.0	1747.1	2
21				90^2-75^2-60^2	900.7	2142	3600[a]	3.1	1698.5	5
22	30	3	3	75^6	1003.3	196	271	–	1833.9	35
23				80^2-75^2-70^2	1007.5	80	620	–	1817.4	5
24				90^2-75^2-60^2	923.7	290	3600[a]	1.6	1775.1	8
25	33	3	3	75^7	1037.3	789	1080	–	1981.1	212
26				80^2-75^3-70^2	1017.3	2464	3600[a]	2.6	1998.7	14
27				90^2-75^3-60^2	1000.1	666	3600[a]	3.2	1909.3	172
28	35	3	3	75^7	1080.3	250	815	–	2093.8	389
29				80^2-75^3-70^2	1050.5	2947	3600[a]	2.5	2088.0	89
30				90^2-75^3-60^2	1045.6	1207	3600[a]	2.0	2146.3	568
31	38	4	4	75^8	1264.1	3123	3600[a]	3.1	2262.6	14
32				80^3-75^2-70^3	1220.3	2676	3600[a]	15.1	2440.4	43
33				90^3-75^2-60^3	1216.6	1369	3600[a]	7.8	2370.2	5
34	40	4	4	75^8	1329.1	239	1580	1.1	2456.0	1948
35				80^3-75^2-70^3	1335.4	2110	3600[a]	4.3	2455.4	3164
36				90^3-75^2-60^3	1303.7	6256	7200[a]	30.2	2502.6	98

[a] CPU time limit.
[b] Using the total routing time as secondary target.

Table 4
Detailed description of the best solution found for example 20R-4V-2RD-2SD using the heterogeneous fleet $\{90^1\text{-}75^2\text{-}60^1\}$.

Vehicle	Tour	Load collected	Vehicle returning time	Tour cost
Pick-up routes				
V1^{90}	r12-r3-r9-r20-r1	67	100.0	84.1
V2^{75}	r10-r19-r7-r8-r11-r18	71	108.9	91.7
V3^{75}	r6-r5-r17-r16-r14	70	102.1	85.6
V4^{60}	r13-r2-r15-r4	57	101.2	87.8

Receiving dock door	Vehicle	Service start time	Drop-off requests	Vehicle leaving time
Unloading operations				
RD1	V4	101.2	r13-r2-r15-r4	130.2
	V3	130.2	r6-r5-r17-r16-r14	165.7
RD2	V1	100.0	r12-r3-r9-r20-r1	134.0
	V2	134.0	r10-r19-r7-r8-r11-r18	170.0

Shipping dock door	Vehicle	Vehicle arrival time	Ship-on requests	Service start-time	Service completion time
Shipping operations					
SD1	V4	132.2	r1-r9-r3	134.0	154.0
	V3	167.7	r4-r2-r16-r12-r17	167.7	201.2
SD2	V2	174.0	r7-r11-r10-r19-r18-r8	201.2	237.2
	V1	136.0	r13-r20-r6-r14-r5	165.7	211.2

Vehicle	Load to deliver	Vehicle departure time	Tour	Vehicle returning time	Tour cost
Delivery routes					
V1^{90}	89	211.2	r13-r20-r6-r14-r5	328.1	96.7
V2^{75}	71	237.2	r7-r11-r10-r19-r18-r8	354.7	100.3
V3^{75}	66	201.2	r4-r2-r16-r12-r17	281.9	65.0
V4^{60}	39	154.0	r1-r9-r3	234.0	70.7
Total P/D vehicle routing cost					681.9
Total P/D vehicle usage time				1198.7	

Table 5
Detailed description of the best solution found for Example 30R-6V-3RD-3SD using the heterogeneous fleet $\{90^2\text{-}75^2\text{-}60^2\}$.

Vehicle	Tour	Load collected	Vehicle returning time	Tour cost
Pick-up routes				
V1^{90}	r30-r9-r12-r3-r20-r1-r26	90	106.9	85.4
V2^{90}	r10-r19-r7-r8-r11-r18	71	108.9	91.7
V3^{75}	r28-r29-r16-r17-r5-r6	68	88.7	72.1
V4^{75}	r13-r25-r14-r15-r2	66	95.5	79.8
V5^{60}	r22-r23-r21	58	87.3	74.2
V6^{60}	r27-r4-r24	53	64.4	52.3

Receiving dock door	Vehicle	Service start time	Drop-off requests	Vehicle leaving time
Unloading operations				
RD1	V6	64.4	r27-r4-r24	91.4
	V3	91.4	r28-r29-r16-r17-r5-r6	125.9
	V2	125.9	r10-r19-r7-r8-r11-r18	161.9
RD2	V5	87.3	r22-r23-r21	116.8
	V4	116.8	r13-r25-r14-r15-r2	150.3
RD3	V1	106.9	r30-r9-r12-r3-r20-r1-r26	152.4

Shipping dock door	Vehicle	Vehicle arrival time	Ship-on requests	Service start-time	Service completion time
Shipping operations					
SD1	V5	120.8	r12-r23-r29	152.4	182.4
	V4	154.3	r7-r30-r11-r10-r19-r28-r8	182.4	219.9
SD2	V6	95.4	r26-r25	152.4	164.4
	V3	129.9	r27-r1-r9-r3-r18	164.4	201.4
SD3	V1	154.4	r17-r24-r16-r22-r21-r2-r4	154.4	198.9
	V2	169.9	r5-r14-r6-r20-r13-r15	198.9	243.9

| Vehicle	Load to deliver	Vehicle departure time	Tour	Vehicle returning time	Tour cost
Delivery routes					
V1^{90}	88	198.9	r17-r24-r16-r22-r21-r2-r4	309.6	89.6
V2^{90}	89	243.9	r5-r14-r6-r20-r13-r15	361.4	96.7
V3^{75}	73	201.4	r27-r1-r9-r3-r18	314.7	96.2
V4^{75}	74	219.9	r7-r30-r11-r10-r19-r28-r8	327.0	88.8
V5^{60}	59	182.4	r12-r23-r29	265.1	69.5
V6^{60}	23	164.4	r26-r25	197.3	27.4
Total P/D vehicle routing cost					923.7
Total P/D vehicle usage time				1775.1	

Pickup routes

Delivery routes
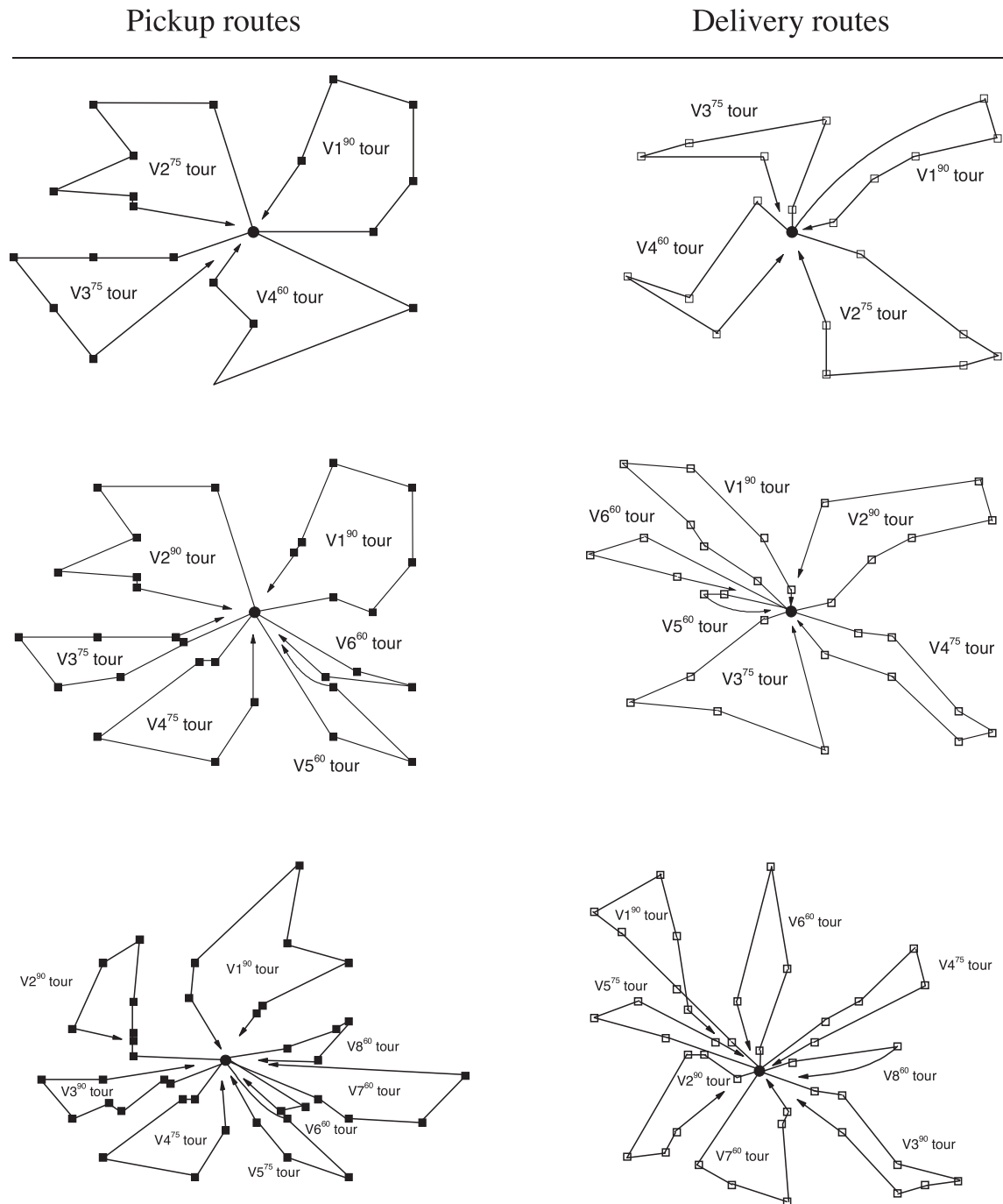
Fig. 2. The best routing cost solutions for examples 20R-3V-2RD-2SD, 30R-6V-3RD-3SD and 40R-8V-4RD-4SD involving heterogeneous fleets.

of nodes in every P/D tour; (b) the arrival times of inbound vehicles at the cross-dock; (c) the total cargo collected by each inbound vehicle; (d) the lines of trucks at receiving and shipping dock doors; (e) the times at which inbound and outbound trucks start and finish their unloading and loading operations, respectively; (f) the total load to be delivered by outbound trucks and the times at which they depart from the base, and (g) the return times of outbound trucks to the base. Analyzing the results shown in Table 4 for Example 20R-4V-2RD-2SD, several conclusions are drawn:

(1) Some loads are immediately transferred from inbound to outbound trucks without any temporary stay on the

cross-dock. This is the case for requests r_1–r_3 that are immediately transferred to truck V_4 after they are unloaded on the cross-dock from vehicle V_1.

(2) In contrast, most of the cargo remains on the cross-dock for some time period because the outbound vehicle that delivers them must wait for its turn on the queue of the assigned stack dock door. For instance, the request r_{12} must stay on the cross-dock from $t = 134.0$ to $t = 167.7$ because the assigned delivery truck V_3 is not available at the assigned stack door SD_1. In other words, the designated outbound vehicle is not ready to start loading operations until time $t = 167.7$.

(3) Sometimes, however, the truck is waiting at the stack door for the arrival of some assigned cargo not still available on the

Table 6
Detailed description of the best solution found for Example 40R-8V-4RD-4SD using the heterogeneous fleet $\{90^3\text{-}75^2\text{-}60^3\}$.

Vehicle	Tour	Load collected	Vehicle returning time	Tour cost
Pick-up routes				
$V1^{90}$	r33-r10-r38-r20-r9-r1-r26	89	142.0	120.7
$V2^{90}$	r34-r18-r11-r7-r39-r19-r8	88	118.6	97.5
$V3^{90}$	r28-r6-r29-r37-r16-r17-r5	85	95.2	74.7
$V4^{75}$	r13-r25-r14-r15-r2	66	93.5	79.8
$V5^{75}$	r21-r23-r22-r32	67	89.7	74.3
$V6^{60}$	r27-r24	34	45.4	37.6
$V7^{60}$	r35-r4-r36-r40	57	103.6	90.2
$V8^{60}$	r30-r31-r3-r12	54	61.7	48.9

Receiving dock door	Vehicle	Service start time	Drop-off requests	Vehicle leaving time
Unloading operations				
RD1	V6	45.4	r27-r24	62.9
	V3	95.2	r28-r6-r29-r37-r16-r17-r5	138.2
RD2	V8	61.7	r30-r31-r3-r12	89.2
	V7	103.6	r35-r4-r36-r40	132.6
RD3	V5	89.7	r21-r23-r22-r32	123.7
	V2	123.7	r34-r18-r11-r7-r39-r19-r8	168.2
RD4	V4	93.5	r13-r25-r14-r15-r2	127.0
	V1	142.0	r33-r10-r38-r20-r9-r1-r26	187.0

Shipping dock door	Vehicle	Vehicle arrival time	Ship-on requests	Service start-time	Service completion time
Shipping operations					
SD1	V5	130.7	r29-r23-r12-r32	138.2	172.7
	V4	133.0	r14-r6-r13-r20-r35	187.0	222.0
	V8	93.2	r5-r33	222.0	244.0
SD2	V3	142.2	r7-r30-r11-r37-r10-r19-r28	187.0	228.5
SD3	V2	170.2	r27-r25-r26-r9-r31-r1	187.0	230.0
SD4	V6	65.9	r4-r15-r36-r2	132.6	155.1
	V7	138.6	r3-r40-r18-r34-r8	168.2	198.7
	V1	189.0	r17-r16-r22-r39-r38-r21-r24	198.7	243.7

Vehicle	Load to deliver	Vehicle departure time	Tour	Vehicle returning time	Tour cost
Delivery routes					
$V1^{90}$	89	243.7	r17-r16-r22-r39-r38-r21-r24	382.5	117.5
$V2^{90}$	85	230.0	r27-r25-r26-r9-r31-r1	326.9	76.9
$V3^{90}$	82	228.5	r7-r30-r11-r37-r10-r19-r28	346.8	98.4
$V4^{75}$	69	222.0	r14-r6-r13-r20-r35	325.3	87.0
$V5^{75}$	68	172.7	r29-r23-r12-r32	257.8	69.5
$V6^{60}$	44	155.1	r4-r15-r36-r2	267.7	101.8
$V7^{60}$	60	198.7	r3-r40-r18-r34-r8	290.6	77.4
$V8^{60}$	43	244.0	r5-r33	305.0	51.5
Total P/D vehicle routing cost					1303.7
Total P/D vehicle usage time				2502.6	

cross-dock. This is the case of vehicle V_1 that stays idle at the stack door SD_2 from $t = 136.0$ to $t = 165.7$ waiting for the unloading of request r_{14} picked up by truck V_3.

(4) Another reason for the temporary stay of some cargo on the cross-dock is that some requests to be delivered by an outbound truck arrive later. For instance, the request r_{13} is unloaded at time $t = 130.2$ and stays on the cross-dock until $t = 165.7$. The delay is due to the late arrival of request r_{14} to be delivered by the same assigned truck V_1.

(5) The use of dedicated inbound and outbound fleets instead of a single fleet carrying both tasks can only produce a minor reduction in the total distribution time. This conclusion can be inferred by comparing the times at which the vehicles arrive at the shipping dock doors and the beginning of the loading operations. Vehicle arrival times are almost always lower than their service starting times.

(6) When the problem size grows and the number of transportation requests to be serviced by a heterogeneous fleet rises to 40, the computational cost shows a substantial increase. This is the case for the example 40R-8V-4RD-4SD involving a heterogeneous fleet composed by 8 vehicles with three different capacities. It is an indication that a more efficient approach should be used for larger problems.

5.3. A compact augmented sweep-based model

The truck loading sequence for delivery tours can also be predefined by including Eq. (50) in the augmented sweep-based model. By so doing, outbound vehicle v_1 is first loaded by the sweep-based approach to generate the first delivery route s_1, vehicle v_2 is next loaded giving rise to route s_2 and so on. By considering Eq. (50), therefore, a more compact solution space enclosed within the one of the rigorous approach is generated. It still contains only P/D petal-shaped tours. Changing the order of the vehicles in the set V, another solution space may be generated. When the compact augmented sweep-based model with constraints (49) and (50) is applied to the example 40R-8V-4RD-4SD and the P/D tasks are accomplished by the heterogeneous vehicle fleet $V = \{V1^{90}, V2^{90}, V3^{90}, V4^{75}, V5^{75}, V6^{60}, V7^{60}, V8^{60}\}$, the best solution found after 3600 s presents a total routing cost equal to 1341.0 discovered in 1036 s, an optimality gap of 2.33% and a lower-level distribution time of 2506.1 time units. With the previous augmented sweep-based model without Eq. (50), the best total routing cost amounts to 1303.7 (see Table 3). Then, the inclusion of Eq. (50) produces a deviation of 2.86% but a substantial reduction on the CPU time to find the best solution. Moreover, the lower-level total distribution time slightly rises from 2502.6 to 2506.1. Let us assume that the

Table 7
Computational results for examples involving 42-to-50 requests using the compact augmented sweep-based model.

Ex. [#]	$|R|$	$|RD|$	$|SD|$	Fleet composition $(cap^{|V|})$	Best routing cost	CPU time (s)		Gap (%)	Total routing time[**] (h)	CPU time to find it (s)
						To find it	To prove optimality			
37	42	4	4	75^8	1409.0	3290	3600[#]	8.8	2679.9	627
38				$80^3\text{-}75^2\text{-}70^3$	1413.9	1612	2765	1.6	2657.8	38
39				$90^3\text{-}75^2\text{-}60^3$	1384.0	2237	3600[#]	3.4	2582.5	289
40	45	4	4	75^9	1490.7	902	3600[#]	1.2	2775.4	94
41				$80^3\text{-}75^3\text{-}70^3$	1494.3	2687	3600[#]	1.8	2745.8	395
42				$90^3\text{-}75^3\text{-}60^3$	1499.5	3315	3600[#]	4.0	2834.8	582
43	48	5	5	75^9	1617.5	4831	5000[#]	7.2	2988.6	381
44				$80^3\text{-}75^3\text{-}70^3$	1618.8	3898	5000[#]	4.6	3016.9	1293
45				$90^3\text{-}75^3\text{-}60^3$	1634.6	4925	5000[#]	7.9	3159.1	868
46	50	5	5	75^{10}	1707.0	2167	5000[#]	6.4	3157.8	122
47				$80^3\text{-}75^4\text{-}70^3$	1707.2	4094	5000[#]	2.7	3203.6	3076
48				$90^3\text{-}75^4\text{-}60^3$	1716.8	3554	5000[#]	8.3	3212.1	1329

[#] CPU time-limit.
[**] As secondary target.

order of vehicles in the set V has been changed and V is now given by: $\{V1^{60}, V2^{75}, V3^{90}, V4^{60}, V5^{75}, V6^{90}, V7^{60}, V8^{90}\}$. For the new set V, the best solution is discovered by the compact augmented sweep model with Eqs. (49) and (50) in 1037 s and it features a total routing cost equal to 1331.4 and an optimality gap of 2.63%. Besides, the lower-level total distribution time drops from 2502.6 to 2389.2.

Therefore, the inclusion of Eq. (50) in the augmented sweep model allows to discovering very attractive P/D route designs in acceptable CPU times at the expense of a small deterioration of the solution quality whatever the order of the elements in V is selected.

The compact augmented sweep model was applied to a series of examples involving 42–50 transportation requests, a homogeneous/heterogeneous fleet with 8–10 trucks and 4–5 receiving and shipping dock doors. The best solutions found after the CPU time limit of 3600 s for $|R| < 45$ and 5000 s for $|R| \geq 45$ are informed in Table 7. A sketch of the P/D tour designs for the example 50R-10V-5RD-5SD using a vehicle fleet given by: $V = \{V1^{90}, V2^{90}, V3^{90}, V4^{75}, V5^{75}, V6^{60}, V7^{60}, V8^{60}\}$ is shown in Fig. 3. In all the examples, the branch-and-cut algorithm produces a series of feasible solutions with a decreasing routing cost but the CPU time reported in Table 7 is the one needed to discover the best one.

5.4. The total distribution time as the primary objective function

When the total distribution time (TDT) given by Eq. (48b) is adopted as the problem objective function instead of the total routing cost, the computational efficiency of the branch and cut solution algorithm is worsened. A larger CPU time is needed and the optimality gaps after 3600 s are much greater than those reported for the total routing cost. Nonetheless, the CPU time needed to discover the best set of P/D tour designs is usually much smaller than the maximum allowed CPU time. The best TDT solutions found after 3600 s for examples involving 15 to 35 requests and 3 to 7 vehicles with three different compositions are shown in Table 8. This table also presents a comparison with the results obtained when the TDT plays the role of a secondary target. For heterogeneous fleets, the lower-level TDT solutions look very promising for most of the examples. Sketches of the best solution for the example 30R-6V-3RD-3SD using a homogeneous and a heterogeneous fleet are shown in Fig. 4.

5.5. Handling P/D node time windows

Time windows within which the service of P/D nodes should be started are also given in Table A1 of the Appendix A. Usually, the time window constraints prevent from reaching feasible solutions featuring non-overlapping P/D tours. Then, the augmented sweep-based model will probably be unable to find a feasible solution unless it allows some overlapping between pickup/delivery tours. In other words, a larger feasible space including solutions with overlapping petal-shaped P/D tours is to be explored. To this end, the parameter Δ in the sweep constraints is fixed equal to a positive value ranging from 0.3 to 1.0.

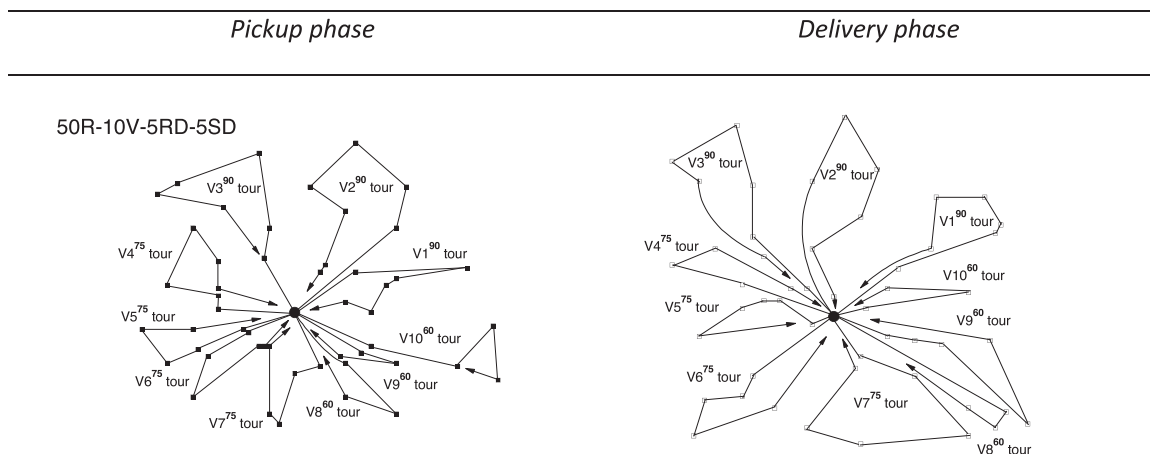| *Pickup phase*	*Delivery phase*

Fig. 3. The best P/D vehicle routes found for the example 50R-10V-5RD-5SD using a heterogeneous fleet.

Table 8
The best total distribution time (TDT) solutions found for examples with 15-to-35 requests using the TDT as the primary and the secondary target.

$|R|$	Fleet composition ($cap^{|V|}$)	$|RD|$	$|SD|$	Best total distribution time (TDT) after 3600 s		
				Using the TDT as a primary target		Best TDT as secondary target
				Best travel time	CPU time to find it (s)	
15	75^3	2	2	937.5	727	983.2
	90^1-75^1-60^1	2	2	901.4	307	1023.5
18	75^4	2	2	1067.8	1258	1138.4
	90^1-75^2-60^1	2	2	1091.1	1297	1091.1
20	75^4	2	2	1287.9	605	1304.5
	90^1-75^2-60^1	2	2	1198.7	2555	1198.7
21	75^4	2	2	1343.8	833	1349.2
	90^1-75^2-60^1	2	2	1217.5	1433	1314.6
24	75^5	2	2	1571.9	1603	1610.9
	90^2-75^1-60^2	2	2	1575.0	733	1584.9
27	75^6	3	3	1653.7	3227	1764.5
	90^2-75^2-60^2	3	3	1523.7	790	1600.7
30	75^6	3	3	1829.2	2759	1833.9
	90^2-75^2-60^2	3	3	1763.8	2274	1775.1
35	75^7	3	3	2080.6	2737	2093.8
	90^3-75^2-60^3	3	3	2111.5	2318	2146.3

Table 9
Computational results for examples with time window constraints involving 20–40 requests using the augmented sweep-based model.

$|R|$	$|V|$	Fleet composition ($cap^{|V|}$)	$|RD|$	$|SD|$	Best routing cost[b]	CPU time to find it (s)	Gap (%)	Total distribution time[a]
20	4	90^1-75^2-60^1	2	2	681.9	918	7.3	1347.4
24	5	90^2-75^1-60^2	2	2	845.0	2142	12.1	1658.2
30	6	90^2-75^2-60^2	3	3	923.6	2623	29.3	2028.1
35	7	90^2-75^3-60^2	3	3	1077.2	1893	2.0	2253.3
40	8	90^3-75^2-60^3	4	4	1386.8	1944	38.5	2705.9

[a] As secondary target.
[b] After the CPU time limit of 3600 s.
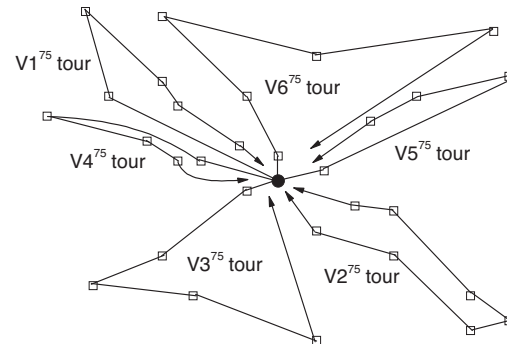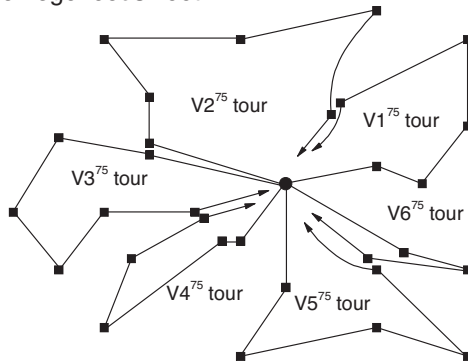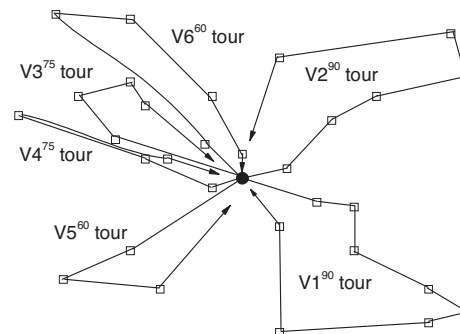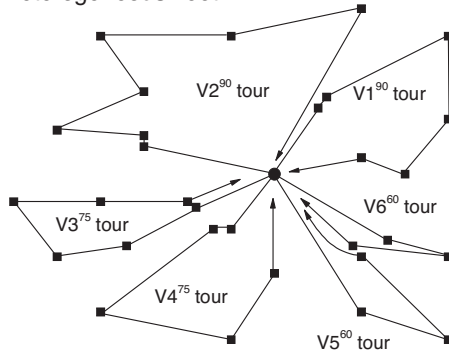
Fig. 4. The best distribution time solutions for example 30R-6V-3RD-3SD using a homogeneous and a heterogeneous vehicle fleet.

Table 10
Computational results for examples with finite transfer times of requests between strip and stack dock doors using the augmented sweep-based model.

$|R|$	$|V|$	Fleet composition (cap$^{|V|}$)	$|RD|$	$|SD|$	Best routing cost[b]	CPU time to find it (s)	Gap[b] (%)	Total distribution time[a]
20	4	90^1-75^2-60^1	2	2	681.9	68	–	1212.3
24	5	90^2-75^1-60^2	2	2	842.9	1624	–	1635.5
30	6	90^2-75^2-60^2	3	3	925.3	757	4.0	1794.6
40	8	90^3-75^2-60^3	4	4	1326.4	1794	1.4	2473.3

[a] As secondary target.
[b] After 3600 s of CPU time.

Δ denotes the maximum allowed tour overlapping. In addition to overlapping, the fulfillment of the time-window constraints may also produce some crossing of route legs in the best solutions.

The augmented sweep-based formulation was applied to a series of heterogeneous VRPCD examples consisting of 20 to 40 requests and 4 to 8 vehicles of different capacities. Computational results and the best P/D tour designs for the examples

20R-4V-2RD-2SD-TW, 30R-6V-3RD-3SD-TW and 40R-8V-4RD-4SD-TW are shown in Table 9 and Fig. 5.

5.6. Finite transfer time of requests across the cross-dock facility

In order to analyze the impact of the internal transportation of requests from strip to stack dock doors on the best solution found, finite values have been assigned to the model parameters $ttr_{d,d'}$.
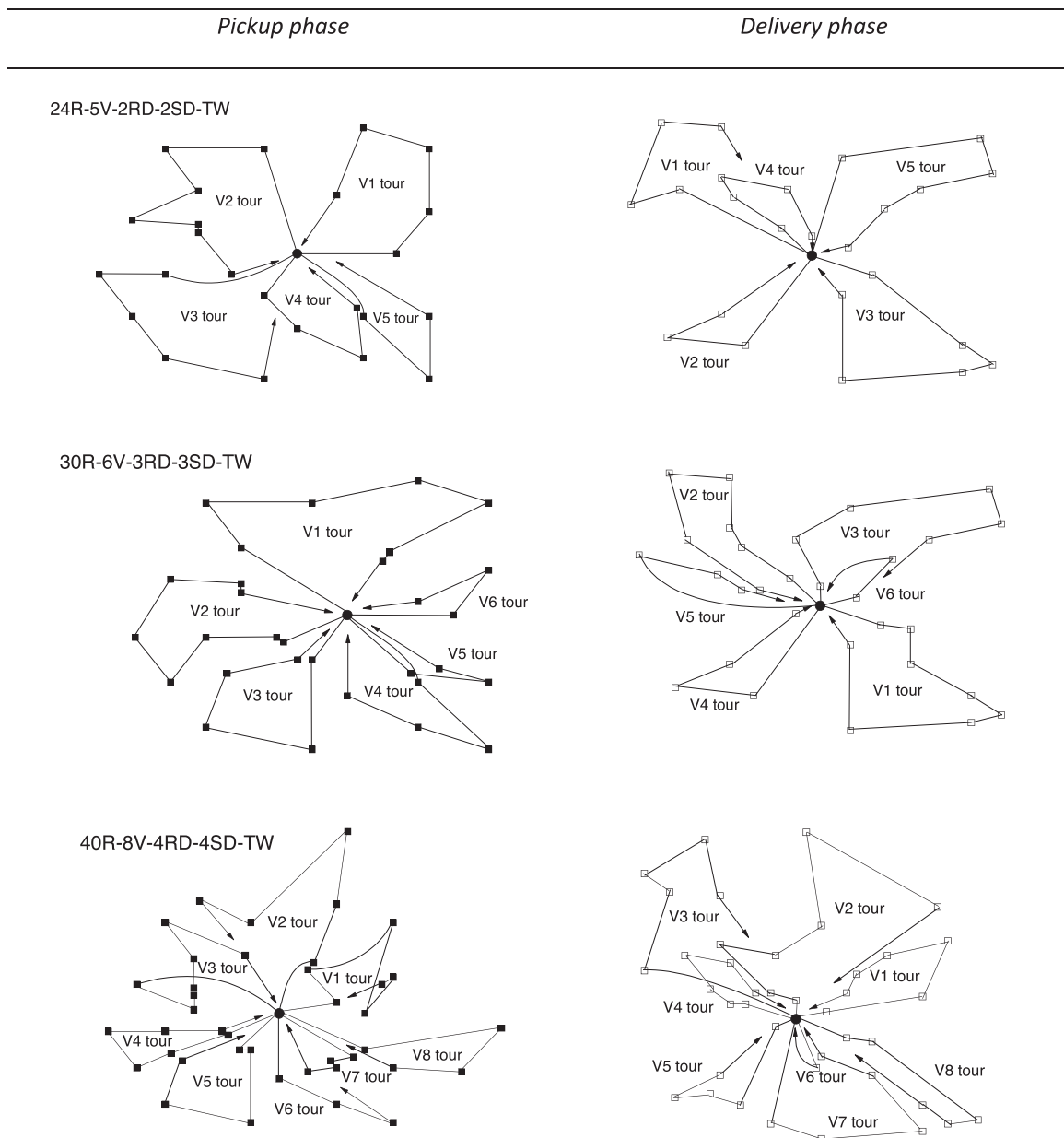

Fig. 5. The best routing cost solutions for the examples 24R-5V-2RD-2SD-TW, 30R-6V-3RD-3SD-TW and 40R-8V-4RD-4SD-TW with time windows.

They are given in Table A3 of the Appendix. With the new values of $ttr_{d,d'}$, the examples 20R-4V-2RD-2SD, 24R-5V-2RD-2SD, 30R-6V-3RD-3SD and 40R-8V-4RD-4SD with a heterogeneous vehicle fleet were solved again. The total routing cost was selected as the objective function to be minimized, while the least distribution time plays the role of secondary target. Computational results are given in Table 10. It is observed a slight increase of the lower-level total distribution time with regards to that obtained with $ttr_{d,d'} = 0$. In contrast, the P/D route designs and the best routing cost both remain practically the same.

6. Conclusions

New solution approaches for the heterogeneous vehicle routing problem with cross-docking have been presented. They all assume a single cross-dock with a limited number of dock doors and a heterogeneous vehicle fleet carrying out the required pickup and delivery tasks in a sequential manner. Because there are more trucks than dock doors, queues of vehicles waiting for loading or unloading goods on the cross-dock are accounted by the proposed formulation. Moreover, the internal transportation of requests through the cross-dock from strip to stack dock doors is also considered. One of the approaches is based on a rigorous MILP model while the others are obtained by incorporating a set of constraints mimicking the widely known VRP sweep algorithm into the exact formulation. In this way, the resulting sweep-based model explores a more compact solution space containing P/D petal-shape routes with/without overlapping. Three variants of the approximate sweep-based methodology have been developed to deal with VRPCD case studies of increasing size. They differ on how the loading order of inbound/outbound trucks by the sweep constraints is defined. The larger solution space is obtained when the vehicle loading sequence is a model decision. More compact feasible regions are generated if the loading sequence just for pickup trucks or for both pickup and delivery trucks are pre-assigned and given by the ordering of them in the vehicle set *V*. To fix the loading sequence, new constraints must be included in the MILP to generate the so-called augmented sweep-based formulation. The rigorous approach can solve rather small examples with at most 20–25 transportation requests. However, their results serve as the reference to evaluate the performance of sweep-based formulations and validate their use for solving larger problem instances. Two alternative objective functions were adopted: the total routing cost and the total distribution time with the first one producing results at much lower computational cost. A two-level optimization scheme with the least routing cost as the primary goal and the minimum distribution time as the lower-level target allows to efficiently discovering minimum routing cost solutions with near-optimal distribution times.

A substantial number of examples comprising 15 to 50 transportation requests, 3 to 10 vehicles and up to 10 dock doors were successfully solved. The rigorous and the sweep-based formulations were applied to examples with up to 25 requests and their best routing cost solutions were compared to validate the approximate approach. Very good solutions to larger examples were found in very acceptable CPU times by using the augmented sweep-based model. VRPCD problems with time windows within which the service of P/D nodes must start were also tackled using the sweep-based approach. To obtain good feasible solutions, the overlapping of pickup and/or delivery routes was allowed by choosing a finite value for the parameter Δ of the sweep constraints representing the maximum angular overlapping. Obviously, the TW-constraints produce some deterioration in both the value of the objective functions and a sharp increase of the computational cost. The influence of the internal transportation of requests through the cross-dock was also studied. From the results, it was concluded that the impact on the

P/D tour designs is almost negligible while the total distribution time presents a minor increase.

Appendix A.

Table A1
Data for the 50 customer transportation requests.

Request	Load	X coord	Y coord	Time windows *a*	Time windows *b*	X coord	Y coord	Time windows *a*	Time windows *b*
				Pick-up stage				Delivery stage	
r1	10	41	49	10	70	20	20	240	240
r2	7	35	17	10	70	31	52	280	240
r3	13	55	45	10	70	24	12	210	320
r4	19	55	20	20	80	35	40	230	260
r5	26	15	30	40	100	41	37	230	260
r6	3	25	30	20	80	53	52	260	260
r7	5	20	50	40	100	45	30	210	340
r8	9	10	43	20	80	40	25	320	280
r9	16	55	60	30	90	11	14	250	320
r10	16	30	60	30	90	65	7	280	340
r11	12	20	42	20	80	60	12	270	340
r12	19	50	35	30	90	13	52	220	300
r13	23	30	25	20	80	63	65	300	260
r14	20	15	10	40	100	47	47	260	230
r15	8	30	5	0	60	40	60	280	290
r16	19	10	20	20	80	20	55	240	260
r17	2	5	30	20	80	30	42	220	320
r18	12	20	40	30	90	40	3	290	340
r19	17	15	60	40	100	60	5	290	260
r20	9	45	65	30	90	65	56	260	260
r21	11	45	20	0	60	20	68	270	240
r22	18	45	10	40	100	10	69	260	280
r23	29	55	5	40	100	5	48	240	260
r24	12	44	22	10	70	22	50	220	240
r25	8	28	25	0	60	25	39	200	300
r26	15	40	47	20	80	22	39	170	240
r27	22	48	23	0	60	31	33	220	260
r28	7	26	29	10	70	50	20	300	300
r29	11	18	22	20	80	18	43	240	240
r30	8	45	38	10	70	50	29	210	340
r31	14	53	43	30	90	28	15	260	320
r32	9	40	19	10	70	27	42	220	280
r33	17	29	51	40	100	60	41	240	300
r34	12	20	36	40	100	39	22	280	340
r35	14	50	25	10	70	45	42	280	340
r36	10	67	19	20	80	37	85	270	330
r37	17	16	24	20	80	71	8	280	340
r38	6	47	85	10	70	17	83	280	340
r39	21	21	66	60	120	5	74	280	340
r40	14	74	31	0	60	30	7	220	280
r41	19	8	70	–	–	66	58	–	–
r42	11	47	47	–	–	18	37	–	–
r43	20	29	25	–	–	9	5	–	–
r44	13	75	15	–	–	55	28	–	–
r45	9	12	73	–	–	31	69	–	–
r46	26	32	2	–	–	67	11	–	–
r47	8	28	82	–	–	10	30	–	–
r48	12	38	72	–	–	64	29	–	–
r49	15	57	72	–	–	43	72	–	–
r50	10	69	48	–	–	54	65	–	–

Cross-dock Cartesian coordinates: $X_w = 35$, $Y_w = 35$.

Table A2
Vehicle transfer times ($ttv_{d,d'}$) between strip and stack dock doors.

Strip/stack door	SD_1	SD_2	SD_3	SD_4	SD_5
RD_1	2	4	8	3	1
RD_2	4	2	5	6	6
RD_3	7	6	2	4	3
RD_4	6	4	3	2	4
RD_5	6	9	5	4	2

Table A3
Request transfer times ($ttr_{d,d'}$) between strip and stack dock doors per unit size.

Strip/Stack door	SD_1	SD_2	SD_3	SD_4
RD_1	0.10	0.30	0.40	0.60
RD_2	1.40	0.15	0.25	0.55
RD_3	0.50	0.30	0.10	0.20
RD_4	0.80	0.40	1.00	0.20

Appendix B. The set of constraints mimicking the sweeping algorithm

In order to mimic the sweep algorithm of Gillett and Miller (1974), the following set of constraints has been added to the problem formulation.

Angular limits and width of the sth-circular sector. As stated by Eq. (B1), the upper angular limit of sector s is the lower limit of sector $(s+1)$. Moreover, the set of zones defined by the model should cover the whole region to be served. By Eq. (B2), the sum of their angular widths must be equal to 2π.

$$\varphi_{s+1}^P = \varphi_s^P + \Delta\varphi_s^P \quad \forall s \in S(s < |S|) \tag{B1}$$

$$\sum_{s \in S} \Delta\varphi_s^P = 2\pi \tag{B2}$$

Unused sectors arising first in the set S. A number of angular zones equal to the number of available vehicles should be predefined but some zones could be fictitious because not all the vehicles might be used. The binary variable U_s^P has a zero value for a fictitious zone. The constraint (B3) drives the angular width of any fictitious sector to zero. On the other hand, Eq. (B4) ensures that fictitious sectors, if any, will arise first.

$$\Delta\varphi_s^P \leq 2\pi U_s^P \quad \forall s \in S \tag{B3}$$

$$U_{s+1}^P \geq U_s^P \quad \forall s \in S(s < |S|) \tag{B4}$$

Allocating nodes to vehicles. Through Eq. (1) each pickup location must be assigned to exactly one tour. If tour s is not used ($U_s^P = 0$), then Eq. (B5) does not allow to assign customer locations to that tour.

$$YP_{r,s} \leq U_s^P \quad \forall r \in R, s \in S \tag{B5}$$

Feasible allocation of nodes to the sector s. For every zone before the last one, all pickup nodes featuring an angular coordinate θ_r^P within the sector s, i.e. $\theta_r^P \in [\varphi_s^P, \varphi_{s+1}^P]$, must be allocated to sector s. This condition is enforced by Eqs. (B6) and (B7). The tour assignment for nodes located just on the boundary between sectors s and $s+1$ is left to the model.

$$\varphi_s^P \leq \theta_r^P + 2\pi(1 - YP_{r,s}) \tag{B6}$$

$$\varphi_{s+1}^P + \Delta \geq \theta_s^P YP_{r,s} \quad \forall r \in T, s \in S(s < |S|) \tag{B7}$$

The tuning parameter Δ allows an overlap of magnitude Δ between two adjacent sectors and it is used in time-windows constrained problems. In that case, nodes located within the Δ-sized overlapped area can be allocated to the sector s or $s+1$.

Allowing the first used angular sector to start at the best angular position. The last zone requires a special constraint because the rotating ray may start its movement from an initial polar angle φ_1^P

larger than ($\min \theta_r^P$). If so, the pickup locations with an angular coordinate $\theta_r^P \in [0, \varphi_1^P]$ must be allocated to the last sector $s = |S|$. The binary variable ε_r^P is defined to optimize the initial polar angle at which the rotating ray must start its movement. The variable ε_r^P takes the value 1 whenever the rotating ray starts its movement from an initial polar angle greater than ($\min \theta_r^P$). If $\varepsilon_r^P = 1$ and the pickup location of request r satisfies the condition: $\theta_r^P \in [0, \varphi_1^P]$, then Eqs. (B8) and (B9) assign request r to the last sector. If ε_r^P is set to zero, the sweeping procedure starts from an angle equal to ($\min \theta_r^P$) By optimizing the value of ε_r^P, better solutions can be found at the expense of a larger computational cost.

$$YP_{r,s} \geq \varepsilon_r^P \quad \forall r \in R, s = |S| \tag{B8}$$

$$\theta_r^P(\varepsilon_r^P + U_s^P - 1) \leq \varphi_s^P \quad \forall r \in R, s \in S \tag{B9}$$

Eq. (B9) reduces to: $\theta_r^P \varepsilon_r^P \leq \varphi_s^P$ for every existent sector s. If $\varepsilon_r^P = 1$, then Eq. (B8) becomes: $\theta_r^P \leq \varphi_s^P$. For fictitious sectors, the constraint (B9) becomes redundant. Eq. (B10) is incorporated into the problem formulation to speed-up the convergence rate.

$$\theta_r^P \geq \varphi_s^P = 2\pi\varepsilon_r^P \quad \forall r \in R, s = first(S) \tag{B10}$$

Besides, Eq. (B11) plays the role of Eq. (B6) for the last sector. This constraint forces a request assigned to the last sector $s = |S|$ to have an angular coordinate $\theta_r^P \geq \varphi_{|S|}^P$, but no longer applies if ε_r^P is equal to one. When $\varepsilon_r^P = 0$, Eq. (B11) looks similar to Eq. (B6). The proposed set of constraints is just written for pickup routes but an identical set can be proposed for the delivery tours.

$$\varphi_s^P \leq \theta_r^P + 2\pi(1 + \varepsilon_r^P - YP_{rs}) \quad \forall r \in R, s = |S| \tag{B11}$$

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