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A conceptual design for a bilateral agent-based land market with heterogeneous economic agents

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ABSTRACT

This paper presents a conceptual design for an agent-based bilateral residential land market. The design includes interactions between multiple buyers and sellers (household agents, developers, and rural land owners) and two local feedbacks to land value—price expectation formation based on local neighborhoods and spatial externalities. To address the methodological challenges inherent in the transition from equilibrium-based analytical models to agent-based simulation, we combine traditional deductive optimization models of behavior at the agent level with inductive models of price expectation formation. Relative to previous models, our proposed model is more closely linked to urban economics; contains a wider range of drivers of land use (LU); and addresses alternative models of division of gains from trade and determination of transaction prices, including models of bid and ask price formation. Our proposed approach is also closely linked to geographic cellular LU models, potentially uniting the strengths of these two disciplinary perspectives.

1. Introduction

Land-use change (LUC) is influenced by interactions between social and biophysical landscapes, with economic development, demographic growth, and shifting social conditions occurring concurrently with land-cover and climate change. Land itself has many roles: property or investment, an administrative unit, soil, a store of mineral resources, or terrain for ecosystems functions (Randall & Castle, 1985). Consequently, different disciplines attempt to explain drivers of LUC from their own perspectives, and within disciplines, LUC is modeled using a variety of approaches: theoretical and empirical, spatial and a-spatial, micro-and-macro-scale. The result is a diversity of explanations of LU development and prescriptions of optimal policies for LU.

The economic perspective investigates how scarce resources such as land can be allocated efficiently between competitive uses, and the land market (LM) is viewed as the main allocation mechanism. Yet, many models of LUC exclude economic drivers and/or LM interactions. This deficit may occur because of the difficulties inherent in integrating static equilibrium-based a-spatial economic land market models (LMM) Drienerlolaan 5, 7522 NB, with the dynamic, heterogeneous spatial environments of LUC models.

This paper presents a conceptual design for an agent-based bilateral residential LM that includes multiple heterogeneous and interacting buyer and seller agents. We outline a proposed set of approaches to address the methodological questions that are raised in the transition from equilibrium-based analytical theoretical models to an agent-based simulation. Relative to previous work in economics and cellular modeling, our proposed model is more closely grounded in urban economics, but moves that perspective further from equilibrium-based modeling. Although we begin from the perspective of economics, our modeling framework emphasizes local spatial interactions and linkages between local processes and heterogeneous patterns of LUC, opening the possibility for coupling the LMM with other spatially explicit, process-based socioeconomic and ecological models. While we focus narrowly on modeling LMs, we hope that the discussion will be of interest to the broader community of LU modelers, whose activities represent and integrate a diversity of disciplinary perspectives and research applications (Benenson & Torrens, 2004b; Brown & Xie, 2006; Crawford, Messina, Manson, & O'Sullivan, 2005; Klosterman & Pettit, 2005; Koomen, Rietveld, & Nijs, 2008; Koomen, Stillwell, Bakema, & Scholten, 2007; Nelson & Geoghegan, 2002; Turner II, Lambin, & Reenberg, 2007; Veldkamp & Verburg, 2004; Walsh & McGinnis, 2008). Our paper lays out a series of open questions and a set of proposed approaches, which we hope will stimulate discussion, debate, and new work by the LU modeling and spatial economics communities.

The paper proceeds as follows. We briefly review related literature, including analytical equilibrium-based and cellular simulation models of urban systems and other agent-based market models. Next, we discuss LMs in the context of agent-based

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modeling (ABM). We then present the conceptual bilateral ABM of residential LMs. First, trading agents and their interactions are defined. Next, approaches to modeling agents' underlying values for buying or selling, the potential deviations between those values and their bid or ask prices, and the determination of a transaction price are discussed. A conceptual model of developers' profit maximization behavior is presented. We conclude with a discussion of a potential path for transferring the model to an empirical context.

2. Previous residential land market models

2.1. Land in economic theory

The concept of *land rent* is central to different economic schools studying land (Randall & Castle, 1985). In theory, in a LM, the transaction price for land reflects the highest value of any agent in the market, the land rent. Under the assumptions of homogeneous land and a representative agent, the amount of land consumed for a particular use and its market price can be modeled as an a-spatial equilibrium of demand and supply (Arnott, Braid, Davidson, & Pines, 1999; Buurman, Rietveld, & Scholten, 2001) (Fig. 1).

Classic economic theory treated land as a factor of production in addition to capital and labor. Later Ricardo (1821/2001) formalized the relationship between the quality (fertility) of land and land rent, with higher rents for higher-quality/productivity land. Following Randall and Castle (1985) the Ricardian rent (Ψ) can be expressed as

$$\Psi_i(F) = p_i \cdot \alpha_i \cdot f(L,F) - \omega \cdot L \tag{1}$$

where F stands for fertility, p_i is the market price for the agricultural good produced using labor input ($f(L,F)$), ω is the wage level per unit of labor, and α_i is a proportionality factor characterizing the particular crop growth.

The model of von Thünen (1826/1966) formalizes the relationship between transportation costs for agricultural goods to the central business district (CBD) and land rents, demonstrating how the location of agricultural activity (in terms of distance (d_i) from the market located at the CBD) depends on the cost of production (c_i), transportation costs (t_i) and market prices (p_i) for an agricultural good (i). The per-acre bid price for land is

$$\Psi_i(d_i) = \frac{(p_i - c_i - t_i \cdot d_i)}{s_i} \tag{2}$$

where s_i is the acre of agricultural land a farmer occupies. The model assumes that land will be allocated to the highest bidder, with the extensive margin at which a bid for one crop exceeds the next-lowest valued crop defining locations of bands of crop types in concentric circles around the CBD.

The Von Thünen model was extended for urban LU by Alonso (1964). According to his bid-rent theory, households choose locations a certain distance from the CBD based on the utility they receive from land and other consumption goods under their budget constraint. The Muth-Mills housing model extends the Alonso model to account for density at each location (by introducing a housing producer who decides the structural density of development) in addition to the rent gradients (see Strazsheim (1987) for review). Other spatial analytical models have been developed to examine the effects of open-space amenities and spatial externalities on land rents (Caruso, Peeters, Cavailhes, & Rounsevell, 2007; Parker, 2007; Wu & Plantinga, 2003).

2.2. Cellular spatial simulation models

The limitations of analytical models for representing neighborhood effects and two-dimensional patterns have led to the development of cellular spatial simulations. These models (including cellular automata, spatial econometrics models and ABMs) represent economic and market influences to varying degrees.

Cellular automaton models (Benenson & Torrens, 2004a) represent transportation and neighborhood influences through calibrated parameters, which reflect socioeconomic influences only implicitly (Batty, Xie, & Sun, 1999; Jantz, Goetz, & Shelley, 2003; White & Engelen, 1993). Econometric models calibrate transition coefficients based on relationships between socioeconomic drivers and land prices, and may use these calibrated models for simulation modeling to produce spatially explicit outcome maps (Irwin & Bockstael, 2002). The estimated coefficients of such models reflect but do not directly represent interactions between supply and demand measured at some point in time.

Several cellular models include hypothetical LMs, but with primary emphasis on the demand side. The SOME and SLUCE models allow agents to choose the property that maximizes their utility without competition from other sellers and assume that the locating agent will outbid the current use (Brown et al., 2008). Caruso et al. (2007) develop a sophisticated model of residential demand, allowing relocation by renters and a competitive rental market. However, the supply price of rural parcels is taken as fixed, and renters are assumed to capture all gains from trade. Parker and Meretsky (2004) represent demand a-spatially through a fixed demand curve, and model the land conversion decisions of a hypothetical parcel manager. Benenson (1998) uses a simple adaptation mechanism to establish the price of houses, in which the price of an occupied house adapts to reflect the wealth of the occupant and the average value of neighboring houses. These factors, along with the cultural identity of neighborhoods, affect the dissonance of residents, which in turn may motivate them to move. Diappi and Bolchi (2008) model supply-side redevelopment decisions of landlords and developers, using an exogenous potential land rent function, but endogenous capitalized land rents based on the state of upkeep of the property. Miller, Hunt, Abraham, and Salvini (2004) propose two approaches to modeling commercial and residential LMs. In each, supply and demand offers are made by heterogeneous buyers and sellers. The first adjusts zonally based prices when markets do not clear. The second determines prices through bilateral transactions. Price expectations in the next

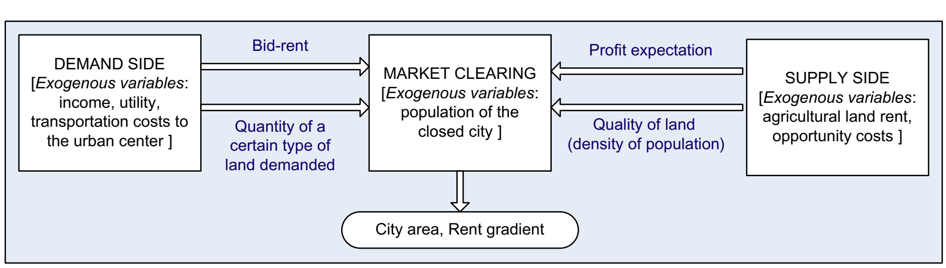

Fig. 1. Schematic structure of the land market model in economics.

round are influenced through an estimated hedonic price function. McNamara and Werner (2008) model the hotel property market using an adaptive model of profit expectation formation by developer agents who supply and sell hotels. Hotel owners bid to acquire properties based on expected profits, with the highest bidder winning the property. Torrens (2007) models LM interactions with dynamic price updating on both the demand and supply side, but the offer prices of residences are not formed based on market conditions or agent preferences.

Several models of agricultural LMs model both demand and supply decisions (Berger, 2001; Happe, Kellermann, & Balmann, 2006; Polhill, Parker, & Gotts, 2008). While these models are becoming increasingly more sophisticated, they do not model differences between the buyer's willingness to pay (underlying utility or payoffs for the land) and her bid or offer price for the land; nor do they model differences between the seller's willingness to accept (opportunity cost of the sale) and his ask price.

3. Agent-based models, markets, and land-use change

3.1. Why model markets with ABMs?

Driven by the desire to better represent and explore complex economic systems, applications of ABM to economics and market interactions are increasing (Arthur, Durlauf, & Lane, 1997; Epstein & Axtell, 1996; Kirman & Vriend, 2001; LeBaron, 2006; Lux, 1998; Tesfatsion, 2006). In spite of rapid growth, the field of ABM market modeling is still relatively new. Current application areas include financial markets, markets for pollution emissions, auctions for the electro-magnetic spectrum, electricity markets, and on-line e-markets (Marks, 2006). These ABMs relax traditional restrictive assumptions of economic models:

- The concept of equilibrium is central to most economic models. However, economic markets are dynamic adaptive systems (Tesfatsion, 2006) and may be out of equilibrium (Arthur, 2006). The dynamic path to equilibrium can be modeled in greater detail and out-of-equilibrium properties more fully explored using agent-based market modeling.
- Many economic models take a representative agent approach, in which the demand curve of one agent is extrapolated to represent the demand for the particular good in the whole economy. The limitations of this approach, discussed by Kirman (1992), can be overcome through ABMs' ability to represent diverse agent types.
- In standard economic models, agents are assumed to be rational and have perfect information about environment. In reality agents have bounded computational ability, memory, and perception (Marks, 2006).
- Standard economic models exclude most agent–agent and agent-environment interactions (Epstein & Axtell, 1996; Tesfatsion & Judd, 2006). Market interactions in ABMs occur during price formation and price negotiation. Non-market interactions include externalities, information transfer, and social networks.

3.2. Agent-based market models in practice

Market design for ABM is discussed at length by Marks (2006). The logic of the ABM market mechanism is described by Mackie-Mason and Wellman (2006) in three steps (Fig. 2).

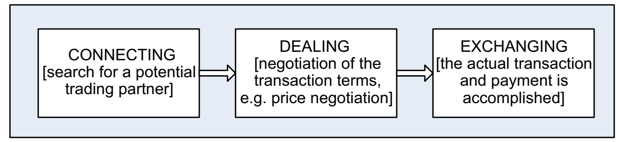

Fig. 2. Main steps of a market transaction, the core of market mechanism (Summarized from Mackie-Mason and Wellman (2006)).

LeBaron (2006) outlines fundamental questions that need to be answered when designing markets, including what kind of good will be traded, how the preferences of individuals will be formalized, what kind of mechanism will be used to determine prices, whether agents can learn, whether information is private or public, how information is presented and processed, and finally what benchmark/criteria will be used to track the operation of a market. Reviewing previous work in financial ABM markets, LeBaron identifies several approaches to modeling determination of the market-clearing price, including price adaptation based on the difference between supply and demand, numerical clearing, auction mechanisms, and random connection of trading partners, with trades occurring when gains from trade are positive. In many ABM market models, reinforcement learning algorithms at the individual agent level are used to establish price expectations and bid/ask prices for individual agents (Arthur, 2006; LeBaron, 2006; Tesfatsion, 2006).

3.3. Why model land markets using ABM?

The advantages of applying ABM in ecological-economic systems are widely discussed (Bousquet & Le Page, 2004; Grimm & Railsback, 2006). The rationale for modeling LUC using ABM laid out by Parker, Manson, Janssen, Hoffmann, and Deadman (2003) can be carried forward to argue that ABMs are appropriate for modeling LMs. Because land differs from other market goods, ABM market models developed for other applications must be further adapted to model LMs. Drawing on previous research on LMs and our own analysis, we summarize the unique features of LMs that motivate development of a new variety of agent-based market model.

A heterogeneous commodity traded by heterogeneous agents: Every property (land parcel/house) is *immobile* and has unique attributes (soil, slope, neighborhood characteristics, and accessibility) (Buurman et al., 2001). There are *several types of buyers and sellers* participating in the LM. For example, potential sellers include farmers selling agricultural land, developers supplying new residences, and relocating households.[1] These types of sellers may have different motivations, opportunity costs, types of behavior, and pricing strategies.[2] Within the same type, buyers and sellers differ in their location preferences, motivations, resources, and information.

Spatial and agent–agent interactions: The use of a property affects the use and value of the surrounding properties through spatial externalities and local price expectation feedbacks. Agents operating in an LM are involved in both market and non-market interactions (Grevers, 2007).

Importance of non-equilibrium dynamics: LMs are cyclic and are rarely in equilibrium. Housing market growth, decline, and bubbles are everyday news. These out-of-equilibrium dynamics can be effectively explored with an ABM LM. However, LMs have *slower dynamics* than other markets. The *supply of land is fixed* or

[1] In practice government often plays an important influential role in the LM. Spatial planning policy and zoning regulations directly affect the elasticity of land and housing supply. Taxes and subsidies applied to the area under local government jurisdiction exert influence upon buyers and sellers choices.

[2] There is a distinction between behavior of buyers and sellers in land ownership market and tenants and landlords in rental markets. These two types of LMs are interconnected because the market price of land and houses influence the rental price of those. However, different models to explain location choice and market prices are used for rental and ownership markets. In our paper we focus on ownership LMs rather than on rental markets. We also do not distinguish between land and property markets; implicitly, LMs refer to the market for individual residences.

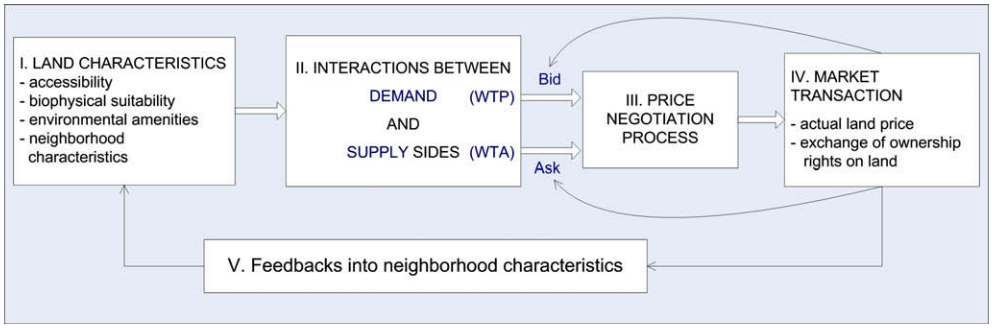

Fig. 3. Conceptual scheme of the agent-based land market.

restricted by regulation in the short run (Smith, Rosen, & Fallis, 1988), limiting adjustment of the LM. Moreover, the purchase of land is an infrequent, high transaction cost, *long-term investment*,[3] often requiring an extensive search for buyers. As a result, relatively few market transactions occur as compared, for example to financial markets. Thus, price expectation formation and learning about the behavior of other traders are slower, and traders usually form strategic responses based on dynamic market conditions. Sellers and buyers leave the market after a successful trade, and rarely interact again with the same trading partner. This complicates the implementation of price learning algorithms used in commodity ABM markets.

An ABM LM that incorporates the heterogeneity, interactions, and non-equilibrium dynamics of real-world LMs can be useful in many ways. ABM LMs can be used to explore the effects of heterogeneous agent-level drivers of LUC, such as incomes, interest rates, social preferences, and credit availability. As discussed by Polhill et al. (2008), by providing information about heterogeneous land rents (private shadow values), ABM LMs may reveal areas where growth pressure is high and deviations between private and public shadow values of land.[4] Finally, ABM LMs can be designed to reflect buyer and sellers' boundedly rational price expectations and explore the effects of adaptive price expectation updating mechanisms.

4. Designed land markets

4.1. Conceptual scheme: tradable good and traders in the land market

Our conceptual model of an artificial LM combines rules adapted from standard urban economics with a cellular spatial simulation model. We move beyond previous work by implementing *agents' heterogeneity*, a *spatially explicit* setup, and *direct modeling of price formation and market transactions*. Both demand and supply sides are represented in detail, facilitating model experiments focused on the drivers of each. Our approach leads to the emergence of heterogeneous land rent patterns without restrictive assumptions to identify prices in equilibrium. Results from the first implementation are reported in Filatova, Parker, and van der Veen (2007), and further model analysis is ongoing.

Fig. 3 represents the logic of our model. Essentially, the agent level interactions illustrated in Boxes III and IV replace the top-down market clearing conditions that define equilibrium in traditional models (Fig. 1). Based on land characteristics (Box I) and individual preferences, buyers and sellers form bid and ask prices for properties, which are functions of their willingness to pay (WTP) and willingness to accept (WTA) (Box II), and negotiate with potential traders over a transaction price (Box III). If negotiation is successful, then the market transaction takes place (Box IV). Current transaction prices influence bid and ask prices in the next time period. Moreover, as a property is converted, the altered LU feeds back into the spatial neighborhood (Box V), through, for example, changes in density, availability of open space, or the social characteristics of the neighborhood.

Extensive previous theoretical and empirical research has been conducted to identify the drivers of land value as outlined in Boxes I and V (Anas, Arnott, & Small, 1998; Irwin & Bockstael, 2002; Lambin & Geist, 2006). However, many *open questions* remain regarding translation of the assumptions of analytical theoretical models into spatially explicit dynamics as represented in Boxes II–IV. We therefore focus on these dynamics and refer to previous literature for the other aspects of the conceptual model.

The first step in analysis of a market is to define the participants. Several agent types participate on both the supply and demand side (Box II, Fig. 3), including households, developers and rural land owners as seen in Fig. 4. The market behavior of each actor in the LM is discussed below.

4.2. Reservation prices, bid and ask prices, and gains from trade

WTP and WTA for land are reservation prices for land—the maximum price a buyer is willing to pay for a good, and the minimum price at which a seller is willing to sell (Fig. 5). Economic theory suggests that reservation prices depend on preferences for characteristics of the spatial good (accessibility, availability of environmental amenities, neighborhood characteristics, etc.) and agents' financial resources.

The difference between the WTP and WTA defines the gains from trade (GFT)—the economic surplus that can be captured from the market transaction—and the realized transaction price defines the division of the GFT between the buyer and seller. The realized transaction price depends on bid and ask strategies and perceived market conditions. Current theory simply bounds, but does not directly identify, the transaction price. In representative agent models of a homogeneous good, the equilibrium market-clearing price is assumed to be the price for all realized transactions. However, residential land is generally sold through bilateral bidding and negotiation. In this case, a clear distinction should be made between WTP and bid price, and WTA and ask price. Since economic agents try to maximize their GFT, a buyer tends to set a bid price lower than her WTP (by ε_b), and a seller sets his ask price higher than his WTA (by ε_s) (Formula (3) and Fig. 5).

[3] The fact that housing is a long-term investment implies that agents' discount rates and access to capital affect land purchase decisions. While we do not include discounting explicitly in the framework presented here, the equations could easily be modified to include intertemporal considerations.

[4] The shadow value of a resource reflects the increase in payoffs at the margin that would be provided by an additional unit of the resource. The private shadow value reflects the increase in individual utility or profits; while the public shadow value reflects the value to society as a whole.

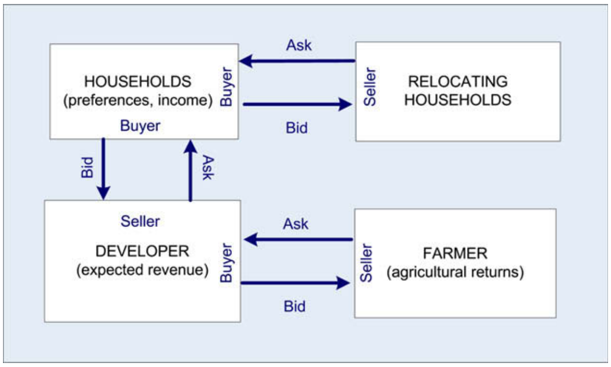

Fig. 4. Interaction between demand and supply side, Box II of Fig. 3.

$$P_{\mathrm{bid}} = \mathrm{WTP} - \varepsilon_{\mathrm{b}}; \quad P_{\mathrm{ask}} = \mathrm{WTA} + \varepsilon_{\mathrm{s}} \tag{3}$$

Neither ε_b nor ε_s can exceed the total GFT $\bar{\varepsilon}_b$, if a transaction is to be feasible. For example, in Fig. 5, if the buyer sets her bid price lower than the seller's WTA, the transaction will not occur. The buyer's strategic incentive is to set her bid price as close to the seller's WTA as possible, but still above. The seller has complementary incentives; he wants to set the ask price as close as possible to the buyer's WTP, but still below.

In the LM, agents are heterogeneous according to their behavior (e.g. the goal is to maximize or satisfy utility vs. profit), their resources (income-constrained households vs. financial capital constrained developers), and in the type of land they seek to buy or sell (existing dwellings for individual households and rural residential parcels for developers and rural land owners). Thus, agents' WTP and WTA formation varies by sector (household/developer/rural land owner). In the following sections, we review theoretical research regarding reservation and bid/ask price formation for different types of agents and propose strategies for calculation of reservation prices.

4.2.1. Buyer households (residential use)

As a starting point to model a household's reservation price, we return to the classical theoretical models of residential location based on the framework proposed by Alonso (Alonso, 1964; Strazsheim, 1987). The conventional economic approach to find a willingness to pay for a housing unit is to solve the budget-constrained utility maximization problem: max $U(z,s,d)$, s.t. $z + s \cdot R(d) = Y - T(d)$, where, z is a composite good, s is the quantity of land/housing purchased, d is the distance from the city centre, $R(d)$ is the distance-dependent land/housing price, Y is the household's budget, and $T(d)$ is commuting cost at distance d. The demands for the composite good and for housing as well as land rent ($R^*(d,u)$, Formula (4)) are derived simultaneously by applying market-clearing conditions (assuming that demand is equal to supply at equilibrium) and assuming that utility is equal for all agents in the city.

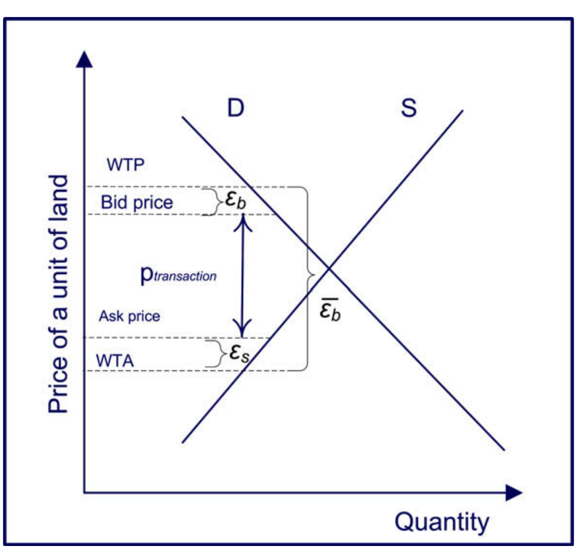

Fig. 5. Price negotiation and division of gains from trade.

$$\begin{aligned} R^*(d,u) &\equiv R^*[Y - T(d), u] \\ &= \max_{z,s} \left\{ \frac{(Y - T(d) - z)}{s} \quad \text{s.t. } U(z,s) = u \right\} \end{aligned} \tag{4}$$

This traditional method to identify the households' willingness to pay at each point in space, or bid-rent function, relies on assumptions of *representative agents* and *market equilibrium*.

Economic agent-based LU models in agricultural contexts often represent the choice problem of agents as a resource-constrained maximization problem, with agents substituting boundedly rational inductively estimated or dynamically evolving prices for expected prices formed through some rational expectations mechanism (Berger, 2001; Happe et al., 2006). We pursue a similar approach, staying conceptually close to the analytical budget-constrained utility maximization Alonso modifications that include open-space amenities, but noting that, in the absence of restrictive equilibrium conditions that identify land rent, individual households cannot solve for their WTP. We therefore arrive at:

Open question 1: How can the standard equilibrium-based Alonso model be translated for application to heterogeneous agents in a dynamic spatial environment?

We propose four potential approaches to the problem, moving from the least to most complex.

Approach 1: Following Gode and Sunder's (1993) ABM markets for homogeneous goods, assume a WTP function for each agent type, and draw randomly from that function to define a population of trading agents. Their "Zero-Intelligence traders" form bids by subtracting some random value from their exogenously assigned WTP. The disadvantage of this approach for a good with heterogeneous characteristics, such a spatial good, is that WTP is the same for goods with different quality. Thus, the bid price does not reflect location-specific amenities. This approach may suffice for highly abstract computational experiments, but the potential for translation into a real-world context is limited.

Approach 2: As in approach 1, heterogeneous WTP functions, which depend on agent's preferences and income, are assigned to agents. Approach 2 essentially assumes a set of exogenous individual-level demand curves for housing, allowing property-specific values to reflect agent-level income and preferences.

Approach 3: This approach assumes an explicit utility function for housing that reflects preferences for proximity to the CBD (P) and green amenities (A) and a fixed, exogenous optimal expenditure share for the property $Y = Y_{\text{housing}} - T \cdot d$ (the share of an exogenous total housing and transport cost budget spent on the property). Preferences can be formalized using a Cobb-Douglas utility function (Wu & Plantinga, 2003). We then define the following WTP function:

$$\mathrm{WTP} = \frac{Y \cdot U^n}{b^n + U^n} \tag{5}$$

which bounds WTP by the budget constraint, and exhibits the qualitative properties of a traditional demand function. The parameter b is a proxy for the prices of other goods. A buyer decides which house to bid on by randomly selecting a subset of houses that are affordable under her budget constraint. She then chooses to bid on the house that gives her maximum utility, with her WTP given by Eq. (5). In applications of the first model implementation, we implement agent heterogeneity in this context by varying the utility weights of each good or the budget constraint for housing, and we implement spatial heterogeneity by varying amenity levels across space (Filatova et al., 2007). This approach may be appropriate in highly regulated housing markets, where loans are made based on monthly payments that represent a fixed proportion of income, and where this constraint is binding for most agents.

Approach 4: We propose to implement an Alonso-style approach by replacing the land price defined through equilibrium assumptions with a parametric adaptively estimated land rent $\hat{r}$, whose estimation may reflect the boundedly rational price expectations of heterogeneous buyer agents. Following traditional models, each agent solves a budget-constrained utility maximization problem (Eq. (6)), in which the value of a residence s increases with amenity level A and decreases with distance from the CBD d.

$$\max U(z, s(A,d)) \quad \text{s.t. } Y = p_z \cdot z + \hat{r} \cdot s(A,d) + T \cdot d \tag{6}$$

The solution to this maximization problem provides optimal demands for the composite good and attributes of the spatial good (amenity levels and accessibility). These optimal solutions can be used to derive an analytical demand curve (WTP) for housing as a function of estimated land rent and, from that, an optimal housing budget.

The challenge in this approach comes from the need to initialize $\hat{r}$. We propose two solutions. First, the model could be initialized using the bid-rent values from analytical solutions to a simplified version of the model.[5] Second, the landscape could be initialized with plausible selling prices, given agent utility functions. Agents would form an expected rent by sampling properties selling for between 25% and 40% of their income (a standard budget metric for lenders) and estimating prices through a hedonic regression model (similarly to the model proposed by Miller et al. (2004)). Either method would be likely to induce some path dependence in model outcomes, which should be formally explored.

Expected rent must be dynamically updated. A new class of "Real Estate Agents," who might possess differential levels of knowledge and estimation strategies, could estimate $\hat{r}$ through learning algorithms (Arthur, 2006; Tesfatsion, 2006). As the model runs and price levels change, expected land rent could be updated either according to a fixed (each time the agent is active) or event driven time schedule. (For example, a buyer may update price expectations following a given number of unsuccessful bids.) Each time price expectations are updated, she will re-solve her utility maximization problem, alter her optimal budget share for housing and shift her demand curve. This approach promises to endogenously model expectation-driven price dynamics that are a prominent feature of real-world housing markets, in which budget shares on housing increase even as incomes and preferences remain fixed. This is accomplished while still maintaining a theoretically grounded WTP and budget share for housing.

4.2.2. Modeling a buyer's bid price

A buyer's strategy for setting her bid price will likely depend on her WTP and expectations for future prices. To our knowledge, few theoretical models of these prices expectations exist. The relationship between bid and ask prices are often dependent on the state of the housing market. In perceived "sellers'" markets (when demand exceeds supply), bid prices are often higher than ask prices. Buyers compete against each other, raising their bids in the hope capturing a desirable property. In "buyers'" markets, bids are often below ask prices, and ask prices are often lowered over time in the hopes of attracting a buyer. These dynamic relationships between bid and ask prices likely drive cyclical housing market dynamics, but also ideally should be endogenous to our model.

These dynamics lead us to the first of several questions:

Open question 2: Given a theoretical WTP, how are bid prices set?

Drawing in part on ABMs of financial markets, we propose two approaches for setting bid prices, which could be combined. However, they should be tested independently as well as in combination, since they may have comparable effects on market dynamics. If effects are comparable, the simplest approach is justified.

Approach 1: In the current model implementation, the WTP of a buyer is adjusted depending on whether it is a buyers' or a sellers' market. We introduce a variable ε, which serves as a proxy for the state of the market (Eq. (7)).

$$P_{\text{bid}} = P_{\text{WTP}} \cdot (1 + \varepsilon), \tag{7}$$

where $\varepsilon = (NB-NS)/(NB+NS)$; NB = number of buyers and NS = number of sellers. If the number of buyers and sellers are equal, $\varepsilon = 0$ and buyers bid their willingness to pay. If there are more buyers than sellers, buyers are in a less favorable situation, and bids will increase. Conversely, bids will decrease when sellers exceed buyers.

This approach is easy to implement in a programming context and potentially leads to endogenous cyclical price variations. Based on information available through the web, Realtors and housing market analyses, real-world agents are likely to perceive the state of the market and the direction of market trends. A disadvantage of this approach is the assumption that agents' bids may exceed their theoretical WTP. This could be remedied by normalizing the bid to fall between WTP and an estimated WTA (consistent with Fig. 5).

Approach 2: Modification to WTP can be based on information about recent comparable sales, average days on the market, and the rate of change of prices. These three pieces of information are readily available to buyers. Information on recent sales could be used to estimate an expected sales price for the property, which could be used to modify the agent's bid relative to the ask price. The bid could also be modified up or down depending on whether prices were rising or falling. Days on the market and the rate of change of prices would be proxies for the buyer's perception of competition among buyers. Essentially, Approach 2 can be viewed as a strategic approach to pricing by the buyer. She forms her best estimate of the highest other bid that the seller is likely to receive, and bids no higher than this amount.

4.2.3. Modeling the seller's decision to relocate

In the case of a single residential seller, his willingness to accept will be determined by his opportunity cost of keeping the residence—the utility that he might gain by selling and moving. Several studies investigate household mobility (Clark, Huang, & Withers, 2003; Kan, 2002; van der Vlist, Gorter, Nijkamp, & Rietveld, 2002). In a comprehensive overview of the theory of household relocation behavior, Clark and Van Lierop (1987) distinguish between inter-urban and intra-urban migration. They argue that main reasons for intra-urban migration are transitions to a new life cycle stage, development of more attractive housing options elsewhere, changes in the neighborhood, and a desire for greater accessibility to central locations. Inter-urban migration is mainly motivated by employment changes. Clark and van Lierop describe relocation behavior as a two-stage process. A household

[5] In the case of homogeneous agents and a homogeneous landscape, this initialization should imply that the model would quickly converge to that analytical solution. This could be an important robustness test for the model. However, we anticipate that this model will be used primarily to analyze heterogeneous agents and landscapes.

first decides to move (for one of the reasons mentioned above), and then it searches for the location for which the expected utility net of moving costs exceeds the expected utility of staying. Other spatially explicit urban simulation models focus in more detail on households' mobility due to ethnic sorting (segregation with respect to the nationality or ethnic group) (Schelling, 1978) and wealth preferences (clustering with respect to income) (Torrens, 2007). This background leads us to:

Open question 3: How should the decision to relocate be modeled?

We propose three approaches (consistent with urban economic theory) to modeling relocation behavior, each of which is likely to apply to different agent types.

Approach 1: An agent becomes dissatisfied with his neighborhood (due to a change in racial/ethnic balance, income disparities, a decline in green amenities or public services (for example school quality), or an increase in congestion or commute times, with two possible action thresholds for relocation:

1. An agent's current utility level (U_{it}) has fallen below the level that he had when he first purchased the house (U_i^*), (probably by a certain threshold) $U_{it} < U_i^*$.
2. The utility of some other location ($\tilde{U}_i$) that household i could purchase given his housing budget is higher than the utility of the current location (U_{it}) net of the utility of moving (U_m), $\tilde{U}_i > U_{it} - U_m$.

Approach 2: Household life-cycle: At thresholds defined by household age and size, agents may form their own households, seek an independent residence, then seek a larger residence (more bedrooms) or higher levels of open space, neighborhood safety, or school quality. Agents at later life stages may seek smaller residences, easier access to local amenities, high-amenity locations, or proximity to extended family.

Approach 3: Job-following migration: If employment locations are included in the model, then household workers may follow shifts in employment. (Note that if employment locations shift within the city for a given agent, the commuting times faced by that agent may change, leading to dissatisfaction with the neighborhood as discussed in point one.)

4.2.4. Modeling seller's WTA and ask price

Open question 4: How should WTA be determined for selling households?

Compensation for the costs paid for the current residence is likely to determine the minimum reservation price for most sellers, barring bankruptcy. Households relocating in the same area, however, wish to increase their utility. Thus, their WTA will be given by the sales price that allows them to purchase the house that achieves the minimum utility increase required to relocate; their WTA, thus, is derived from their WTP for another house. A seller agent has an incentive to set an ask price as high above his WTA as the market will bear, leading to:

Open question 5: How do seller households set their ask prices?

Approach 1: Sellers may also respond to the perceived market power of buyers by adjusting their ask price upwards in the case of a sellers' market, and downward in a buyers' market. (Eq. (8) with ε from Eq. (7))

$$P_{\text{ask}} = \text{WTA} \cdot (1 + \varepsilon) \tag{8}$$

Approach 2: The WTA is adjusted upward or downward based on the difference between the WTA and prices in the seller's neighborhood, dependent on a coefficient of sensitivity $\chi \in [0;1]$). (Eq. (9))

$$P_{\text{ask}} = \text{WTA} + \chi \cdot \Delta, \quad \text{where } \Delta = P_{\text{averageNeighborhood}} - \text{WTA} \tag{9}$$

This approach, implemented in Filatova et al. (2007) using Moore neighborhoods, incorporates local spatial price feedbacks, and will reinforce the positive price effects of location-specific amenities. As in Approach 1 in Section 4.2.2, a disadvantage is that sellers may price their houses below their WTA, which contradicts economic theory. A modified approach maintains links to economic theory, while incorporating local price feedbacks. (Eq. (10))

$$P_{\text{ask}} = \text{Max}(\text{WTA}, P_{\text{averageNeighborhood}}) \tag{10}$$

Approach 3: Rather than basing their decision on current sales prices, sellers (or their real estate agents) may attempt to forecast a probable sales price, as would buyers, using information about recent comparable sales, average days on the market, and the rate of change of prices. A variety of reinforcement learning or inductive statistical models could be used to represent this process. Obviously, if the same methods were used by both buyers and sellers, bid and ask prices would be identical, and the dynamics of bidding up and falling prices would not occur unless driven by pure differences in reservation prices (which are likely to occur given heterogeneous agents). This leads to:

Open question 6: How can differences in bid and ask prices be modeled?

Ask price formation might rely most heavily on recent activity in the physical neighborhood of the residence, whereas bid prices formation might be more dependent on opportunity costs of bidding on residences in *different* neighborhoods. Sellers might have private information that they try to conceal through a too-high ask price. Sellers and buyers may have heterogeneous degrees of urgency for achieving a transaction. Finally, differences in bid and ask prices may arise from the differential experience of boundedly rational real estate agents who advise buyers and sellers. If however, theoretical and empirical models indicate that bid and ask prices can be assumed to be the same, then the process of negotiation need not be modeled explicitly; rather, in trades where GFT are positive, the estimated bid/ask price can be used as the transaction price, thereby defining the division of GFT.

4.3. New home production – developer agents

4.3.1. Developers

Developers (housing producers) serve as an intermediary between farmers willing to sell agricultural land and households willing to buy a house. They *buy* undeveloped land, convert it to residential land, and *sell* housing (see Fig. 4), meaning that they form an interdependent WTP for the agricultural land and WTA for new residential units. In the Muth-Mills modification of the Alonso model (Brueckner, 1987; Strazsheim, 1987) developers act as housing producers, maximizing profit by combining land $L(d)$ and capital $K(d)$ to supply housing $H(d)$ at location d (Eq. (11)).

$$H(d) = H[L(d), K(d)] \tag{11}$$

The land price $R(d)$ is determined endogenously as a solution of maximization problem of a developer (Eq. (12)) and defines the developer's WTP for agricultural land in the analytical problem. The developer's profit maximization problem is

$$\max \pi_{dx} = p(d) \cdot H(d) - R_{\text{ag}}(d) \cdot L(d) - i \cdot K(d) \tag{12}$$

where π_{dx} is the profit of developer x at location d, $p(d)$ is the willingness to pay of households for a unit of housing at location d, $R_{\text{ag}}(d)$ is a price for agricultural land, and i is the interest rate. Developers are assumed to be price takers with respect to the price for housing. Thus, their price expectations are based on derived demand from the households, which itself comes from the theoretical bid-rent functions derived from an Alonso-type model (Eq. (6)).

In theory, the increased WTP of households nearer to CBD resulting from lower commutes leads to a higher optimal density of residences (size and proximity of residences and height of buildings) for developers closer to the CBD (Brueckner, 1987; Kraus, 2006). Assuming constant returns to scale, the housing producer's theoretically optimal structural density (SD = K/L, capital-land ratio) and the developer's WTP for agricultural land (R_{ag}) are derived by maximizing profit (Eq. (13)).

$$\max \pi_{dx}^{\text{per unit of land}} = p(d) \cdot H(\text{SD};1) - R - i \cdot \text{SD} \qquad (13)$$

Since the price for housing ($p(d)$) is a function of a households' preferences, income, distance from CBD and travel costs, the structural density also depends on these drivers (Brueckner, 1987).

Empirical research suggests that developers are motivated by market demand for housing (preferences of new-home buyers) and are constrained by policy regulations (Levine, 2006; Levine & Inam, 2004). Evidence also exists that developers specialize in particular kinds of development, each of which provide different levels of private and public open-space amenities (Vigmostad, 2003).

Regardless of the complexity with which developers' profit maximization decisions are modeled, again a familiar challenge presents itself—that of modeling price expectation formation for households' WTP when developers face heterogeneous buyers, leading to a pair of questions:

Open question 7: How should the profit-maximizing choice of development type be modeled?

Approach 1: For abstract theoretical models, the translation of the Muth-Mills model with optimal density could be used to differentiate housing types by density, with a pool of developer agents created who specialize in particular development densities. This approach does not, a-priori, account for agent heterogeneity.

Approach 2: Models of market segmentation under monopolistic competition (in which sellers offer slightly specialized versions of a good that is homogeneous in some basic characteristics) could be adapted to reflect heterogeneous preferences of agents for open-space amenities, commute times, and property characteristics (Dixit & Stiglitz, 1977; Singh & Xavier, 1984).

Either of these approaches would require developers to estimate consumers' heterogeneous WTP functions for different development types, leading to:

Open question 8: How should the willingness to pay of particular groups of individual buyers that appears in developers' profit function be estimated?

Approach 1: The urban economics literature provides some examples of equilibrium-based models of a developer who is able to differentiate among different groups of housing consumers (Henderson & Thisse, 1999). The underlying assumption is that potential customers have different incomes and WTP for housing, which the developer may be able to exploit by providing both private and public goods.[6]

Approach 2: Similarly to Approach 3 for seller ask price formation, boundedly rational developers estimate inductive hedonic demand curves based on information about agent characteristics and recent sales. Developers likely have access to a wide range of resources and information to estimate demand—in fact, they may have a staff of economists dedicated to that task. While the WTP of individual buyers is private information, accepted bid prices are public information. Thus, a hedonic demand curve for each homogeneous housing product could be estimated through recent sales data. Given that developers would know these estimates reflect interactions between supply and demand, they may set ask prices higher than estimated WTP, then reduce housing prices if they remain unsold.

[6] We do not discuss migration motivated by local public goods (Tiebout, 1956) here. A discussion of pricing decision of developer in the framework of Tiebout model can be found elsewhere (Pines, 1991).

4.3.2. Developers' WTP for agricultural land

Open question 9: How should developers' WTP (expected price) for agricultural land be determined?

Approach 1: An exogenously set intertemporal opportunity cost for agricultural production can be used as the developer's expected price for rural land, assuming that the developer will be able to capture all GFT from the transaction, perhaps accurate only when the supply of conversion land is abundant. This approach is simple, but also grounded in economic theory.

Approach 2: In equilibrium-based models, the price for undeveloped land is assumed to be derived from developers' profit function (Eq. (12)).[7] Again, the equilibrium problem could be translated into an ABM context by implementing an inductively estimated expected price for agricultural land, based on recent sales.

4.4. Price negotiation and the land transaction price

The process of determining sales prices and executing trades (Boxes III and IV in Fig. 3) raises several questions. First:

Open question 10: How should sellers decide which bid to accept?

Sellers status and ask prices are public information, available easily to all buyers. Buyer's bids may be above or below the ask price, as discussed earlier. Two approaches are possible:

Approach 1: The seller can accept the first bid that is at or above his ask price.

Approach 2: The seller can gather bids over a certain time frame, then accept the highest bid that is above his WTA. That time frame may also be endogenous, depending on average local time on the market and rates of change of prices. To avoid the complication of buyers withdrawing during this interval, the seller agent could collect bid prices simultaneously at a "Sunday open house," then decide which, if any, bid to accept at the end of the round. A buyer then may issue a revised bid based updated estimation of WTP and/or a bid price in the next round, if her first bid were rejected. In either of these approaches, the accepted price defines the transaction price. However, when both buyers and sellers offer their true reservations prices (WTP/WTA), another question must be answered:

Open question 11: How should the gains from trade from the transaction be divided?

Researchers have taken several approaches to this problem, most involving some algorithmic division of GFT. Happe et al. (2006) divide the GFTs using the geometric mean of WTP and WTA, and Polhill et al. (2008) impose a Vickery auction so that the auction winner pays the bid of the second-highest bidder. (An overview of types of auctions and their applicability to ABM is provided in Wooldridge (2002).) Arsenault, Nolan, and Schoney (2007) compare the results of several alternative auction mechanisms in their rural LM model, and find that the model results are not sensitive to the auction mechanism, suggesting that in simple circumstances, models may not be sensitive to assumptions regarding division of GFT.

A decision about whether/how to model bid and ask prices and division of gains from trade may depend on whether cyclical housing dynamics are an important part of the research question. In the case of irreversible conversion of open space driven by spikes in housing demand, they may be essential to explaining observed dynamics. However, if the purpose of the model is comparison to

[7] The theoretical literature presents more complex strategic models of price negotiation with landowners from whom housing producers buy undeveloped land. A game-theoretic approach (Asami & Teraki, 1991) analyzes the outcome of sequential pairwise negotiations between a single developer and several landowners over the price to be paid for land.

other abstract, theoretical models of effects of open space on property values, a simpler approach may suffice.

5. Confronting the conceptual model with the real world: next steps

5.1. Benchmarks for land markets

Replication of benchmark theoretical models through simplified versions of an ABM is an important strategy for structural model validation (LeBaron, 2006), one that we pursue in related work (Filatova et al., 2007; Parker & Meretsky, 2004). We used a set of economic and spatial metrics to compare conventional model and ABM LM. Strategies for model calibration, verification, and validation appropriate for our model are discussed elsewhere (Grimm et al., 2005; Lambin & Geist, 2006; Parker et al., 2003).

5.2. Empirical modeling

In principal empirical ABM offer two advantages over traditional reduced-form statistical empirical models of land conversion. The first is that demand and supply can be modeled separately, based on structural representations of utility and profit functions, to which multiple statistical models may contribute. The second is that the dynamics of price formation are explicitly represented, allowing for endogenous evolution of land rents in response to shifts in factors that influence supply and demand, relocation decisions, and in-migration by new agents.

The major challenge to translating this framework into an empirical context, however, is to empirically parameterize those structural utility and profit functions, so that WTP and WTA functions can be derived and dynamically updated. The problem is compounded by the fact that utility functions are not observable or directly measurable. Further, while data are available on bid and ask prices, we expect that those prices will be lower (higher) than actual WTP and WTA. Finally, while data on real estate transactions are available, these data represent the result of interactions between demand and supply[8] and are rarely easily matched to the demographic characteristics of buyers or sellers.

Several approaches are possible, however, to construct empirical analogs of theoretical willingness to pay functions for buyers and willingness to accept functions for sellers.

Approach 1: *Experimental/conjoint analysis*: Although there are fundamental problems related to parameter and functional form identification, experimental approaches have been used to identify potential parameter weights for utility functions. In an experimental setting, agents could be endowed with budgets based on fixed prices and allowed to trade housing "goods" with particular characteristics. Alternatively, in a survey setting agents could be allowed to choose among housing options based on their actual budgets.

Approach 2: *Revealed preference approaches through statistical models that combine household and spatial survey data*: In theory, demand and supply curves can be estimated if survey data are available that link the characteristics and preferences of buyers/sellers to actual sales transactions. Data on the characteristics of the residence are often linked to transaction/tax assessment records, and data on the spatial characteristics of the neighborhood of the house could be derived through GIS. Resident surveys can capture information about buyers who currently reside in the house. These data provide sufficient information to estimate a demand curve. Obtaining similar data on sellers, and linking that data to homes purchased as well as homes sold, would require a seller survey and would pose greater challenges for gathering GIS data on sellers' new residences.

[8] Thanks to Nancy Bockstael for making this point.

6. Conclusions

In this paper, we have outlined a detailed conceptual model of a LM with interactions between heterogeneous agents—buyer households, relocating seller households, and developers. Our proposed model moves beyond existing work by modeling interactions between multiple agent types, modeling the process of bid and ask price formation, and proposing agent decision models that combine deductive optimization with inductive models of price expectation formation. Our discussion is well grounded in economic theory, but also is closely linked to previous cellular models of LU originating in geography. Thus, we hope that our presentation will be of interest to both urban/environmental economists and cellular spatial modelers and will serve to bring these two groups closer together in knowledge and perspective.

Because restrictive assumptions and equilibrium solutions need not be imposed on ABMs, many—perhaps too many!—choices are available to modelers. We have outlined a series of open questions that are inherent in making the transition for theoretical equilibrium-based urban economic models to agent-based residential LMM, and we have proposed solutions to each of them. We plan next to compare the effects of the alternative proposed solutions within our simulation model. We also welcome feedback on these proposed solutions, comparative modeling to explore their implications, and suggestions for additional alternatives.

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