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Workforce planning in a lotsizing mail processing problem

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Abstract

The treatment of mail objects in a mail processing centre involves many operations, in particular sorting by destination. Out of the batching problem that we can identify in such a process, there are also staff planning concerns. In this paper, we analyse a treatment area (registered mail) belonging to a mail processing center, where mail objects are treated in a chain production process. The production quantities and the transfer amounts among machines are required to be determined along the daily work period. The objective is to minimize the costs with human resources needed in the process, linked with the lotsizing production plan, by matching staff to work requirements. This leads into a lotsizing and workforce problem, for which we propose an integer programming formulation. A case study of a particular treatment area is also discussed. The formulation is adjusted to the specific constraints of this case study and some computational results are included, considering average, small and high daily amounts of mail arrived to that particular treatment area. © 2004 Elsevier Ltd. All rights reserved.

Keywords: Workforce scheduling; Batching decision process; Mixed integer programming; Postal service

1. Introduction

This work addresses a particular case of a batching decision process in a mail processing center (MPC) in the Portuguese Mail Service “CTT—Correios de Portugal, SA”. One of the crucial concerns of the Staff Management Department is the employees’ assignment plan that is needed to cover all the mail flow in the mail treatment system, where mail-objects are sorted according to their type, final destination and duty evaluation.

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The treatment process is made in MPCs placed in the Portuguese territory. Each MPC is divided into treatment areas (TAs), according to a specific classification of mail-objects: large mail-objects, medium mail-objects, flats, expressed mail and registered mail. In each TA, the employees' assignment plan is established for a week. In this work, we analyse the year 2001 behaviour of a particular TA, in order to evaluate human resources needs from lower to higher daily mail amounts entered in that specific TA. Those observations are important to plan future manpower needs in that TA.

The treatment process made in each TA involves a predefined sequence of treatment units spanning all mail flow. The mail is treated within fixed periods of 15 min, and the sequence of consecutive periods establishes a partition of the whole planning horizon. Among consecutive periods, the mail treated is transferred to a subsequent treatment unit through a well-defined sequence and according to known mail transfer rates. The determination of the amounts treated in each unit and in each period characterizes a batch decision process. All the treatment units require human resources, which must be linked with the feasible working shifts in use in the Company.

Therefore, we may identify in the process a single-item multi-level (machine) lotsizing subproblem, characterizing the batching decision process on an acyclic graph defined by the fixed sequence of treatment units in a single TA, and a shift scheduling subproblem, characterizing the staff planning concerns. It is precisely this last aspect that concentrates the objective of this paper, where workforce overall costs are required to be minimized.

Like in many applications of operations research, this case also combines known and studied central problems. The first of these subproblems is a multi-stage batch decision problem without direct setup, holding or production costs, but using the same flow conservation proprieties appearing in multi-level lotsizing problems. Many researchers have devoted attention to the central version of this optimization problem (with setup, holding and production costs) (see, e.g. [1–6]). Most of these works are motivated by production planning environments, where the multi-level structure is defined by a sequence of machines establishing a gozinto structure. There are different types of gozinto structures (see [6]), most of them describing assembly systems. In our problem we do not assemble items, but we may find in this problem an acyclic gozinto-network of the general type (see [6]), defined by the fixed sequence of treatment units (operations), through which mail-objects (a single item) pass through. A general description of various other lotsizing or batch decision related problems can be found in [7–9]. Formulations for some practical lotsizing based problems are discussed in [10]. Interesting catalogues of selected research are presented in [7,9,11] and various models and optimization methods are discussed in [10,12–15].

The second subproblem is a shift scheduling problem, also known by staff planning or workforce scheduling. It emerges from set covering optimization problems (see, [16]), and comes frequently assigned to other problems in matching staffing to work requirements, characterizing real applications. Various formulations for shift scheduling problems are compared in [17,18]. Optimization methods are discussed in [18,19]. A catalogue of selected research is presented in [20] and a review of applications, methods and models has been recently published in [21]. Applications are commonly described in crew scheduling for bus, rail, air flight and general transportations planning (see, e.g. [22–27]), ground station personnel scheduling at airports or maintenance teams in daily shift schedules (see, e.g. [28–30]), telecommunication services (telephone operator, telemarketing, customer hotline service) (see, e.g. [31,32]), public safety (fire, policy, emergency) (see, e.g. [33]), health care (medical, nursery shift scheduling) (see, e.g. [34,35]), and many other cases where shift scheduling planning has been revealing an important component of ongoing productivity improvement.

Staff scheduling has also been applied to the postal sector in [36–38]. The first work [36] addresses a personnel scheduling problem for postal distribution service in the United States postal distribution stations. The authors propose an integer model for this problem, determining daily assignments of full- and part-time employees to different task categories, considering regular and overtime hours. The paper includes some computational results using real data from a typical week in a large postal distribution station.

The second work [37] addresses a daily personnel scheduling problem to establish weekly tours, involving break time periods and different shift start time periods. The problem considers full- and part-time employees and satisfies some labour requirements. The authors analyse three problems on the system; the first is concerned with the assignment of days off to employees (the days-off scheduling problem); the second determines an assignment of employees to shifts during the day (the shift scheduling problem); and the third involves the construction of weekly tours (the tour scheduling problem). The problem is motivated by general mail facilities (GMFs), which provide for mail treatment in the US Postal Service and analyses a real application in the Providence, Rhode Island GMF.

The third work [38] comes in the trend of the second one. The authors expand on the previous work model and solution techniques and incorporate several new features in their formulation. They consider a linear integer model to solve a weekly tour scheduling problem, determining minimum cost general staffing needs using both full- and part-time employees, respecting labour agreements and governmental regulations. Different scenarios are examined, including different days off, variable start times and the use of part-time flexible workers. This work has also been motivated by the United States Postal Service and applied to the Oklahoma City processing and distribution centre.

Our problem also involves the determination of an assignment of employees to shifts within a daily working time horizon, where different shifts are considered, involving different durations and start time periods. In this case, the model is not solved over known workload requirements but it must cover workloads generated by a batch decision problem running over the same time horizon. Our objective is to determine minimum cost daily workforce needs to treat mail, according to the daily amounts of mail entered in a single TA.

Many papers that consider shift scheduling as a subproblem have work requirements to be accomplished, where adequate staff coverage is required to be planned (see, e.g., the most of the papers on staff scheduling previously mentioned). One of the critics usually pointed to classical lotsizing approaches, is that it ignores, in most of the cases, that the batch decision structure is only a part of a larger global management process, turning harder to infer and analyse alternative strategies (see, e.g. [7]). The present combined approach is sensitive to this aspect, relating the two most relevant process components, namely the treatment lotsizing scheme of TAs and its staff planning concerns.

When looking from the batch decision subproblem perspective, an interesting aspect of the joint system establishes that the decision to treat mail in a certain period on a specific unit does not depend on any direct setup cost, like in lotsizing problems, but in the assigned workforce cost, defining a kind of an indirect setup cost. This means that the employees' constraints impose a workforce upper limit on mail flow in each unit and in each period, characterizing a capacitated version of the multi-level lotsizing subproblem.

In the next section, the company main features and the problem under consideration are described. A mixed integer formulation for a general treatment area on the MPC scheme is proposed in

Section 3. A specific model for a particular treatment area case is described in Section 4, and different scenarios solutions to this case study are discussed in Section 5. Some conclusions are presented in the last section.

2. Company overview and mail processing

2.1. CTT

The company CTT—Correios de Portugal, SA, presents as core business the management and support of postal services over the Portuguese territory, as well as international postal services beginning or ending in Portugal.

CTT is divided into the following main job areas: home and company delivery, logistics and distribution, hybrid mail, financial services and outsourcing services offered at the post-offices all over the country. To accomplish its purposes, the company has approximately 17 600 employees and owns nine mail processing centres, 418 mail distribution centres, 1079 post-offices, 19 000 pillar-boxes and over 4000 vans.

Through the year 2001 over 1370 million postal-objects were processed and nearly 6300 daily postman routes were made. Money transactions exceeded 17 200 million €, which lead the company to have more than 630 million € in revenues with more than 56 million € invested.

As a government-owned company, CTT is committed to accomplish quality standard criteria such as mean time to delivery. The service quality is verified by an independent sector regulator—“ANACOM—Autoridade Nacional de Comunicações”, which pressures CTT to accomplish with the delivery.

To plan the mail distribution process, the company divides the Portuguese territory in approximately 418 areas, each having in average 220 km². Each of these areas has its own mail distribution centre (MDC), which receives all the mail coming from post-offices and pillar-boxes inside that area and sends it to processing centres (MPCs). The MDC also receives mail already treated and having as final destination an address inside that area. Hence, all the mail received in an MDC is sent to be treated in an MPC. MDCs also receive treated mail coming from an MPC, for distribution purposes. So, postal-objects coming from the MDCs of an area are treated (sorted) at the corresponding MPC and are dispatched from there to its destination MDC.

Postal routes, i.e., the daily journey to deliver mail from door to door, begin at the headquarters of the MDC.

The postal process consists of four main steps, namely collection, sorting by destination, transportation and delivery.

Taking into consideration the expected flow of mail to be treated and the company commitment with the quality criteria, it becomes obvious how important it is for this company to plan carefully the assignment of human resources. Although the MDCs concentrate higher manpower needs, this work focus just in the workforce planning in MPCs, where a correct assignment of employees to workload is also of a great concern, in order to accomplish with the agreed percentage of residual mail at the end of the day, according to the estimated quantity of mail received and to the targets established for quality service.

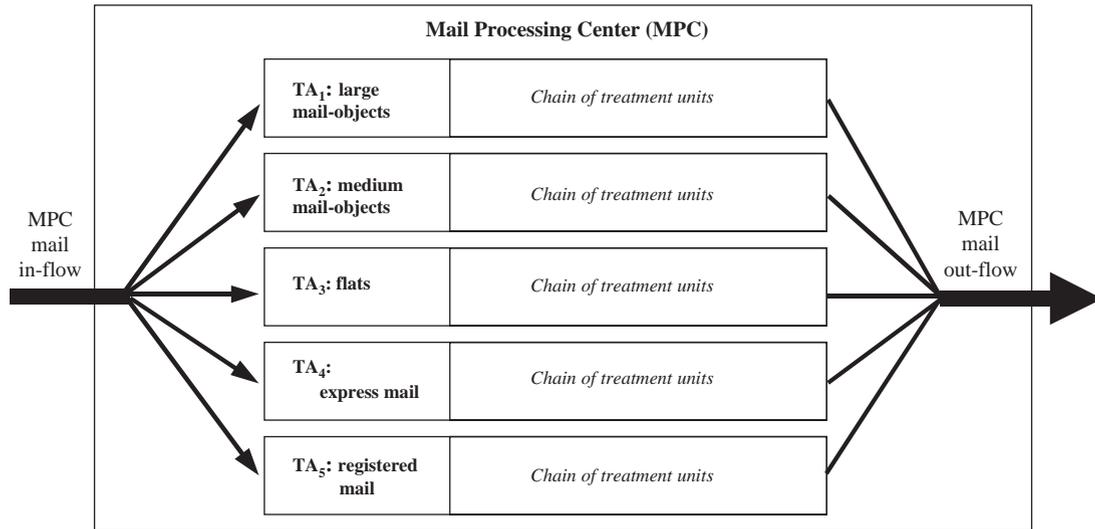


Fig. 1. Scheme of an MPC.

2.2. The MPCs

The main job of an MPC is the mail sorting by type and destination, although it can also work as a point of the course to concentrate and forward mail to other MPCs, where the treatment process takes place.

The mail classification by type is made on a size basis (small, medium and large) or by special service types (express mail and registered mail), being the various types processed in different treatment areas. Each mail type is entirely processed in the same treatment area, even if it includes different delivery speeds.

For some mail types and in some MPCs there are automatic processes to sort by destination. Other mail types (e.g. registered mail) are entirely treated by manual processes.

The mail-objects processed have different characteristics. Each MPC has five TAs (designated by TA_l , $l = 1, \dots, 5$), according to the following features:

- TA_1 : large mail-objects, including packages and parcels,
- TA_2 : medium mail-objects: essentially periodical publications, like magazines and newspapers,
- TA_3 : flats: post-cards and letters up to 20 g (0.71 oz),
- TA_4 : express-mail,
- TA_5 : registered mail.

In each TA, mail is treated (sorted and separated) in a chain of specialized treatment units (TU_i), characterized by manual and automatic operations. This chain defines a productive process, where one or several units feed other units in a well established sequence. A scheme that characterizes an MPC is presented in Fig. 1.

The MPC operates continuously during all the day, although some TAs are closed during a certain period of the day.

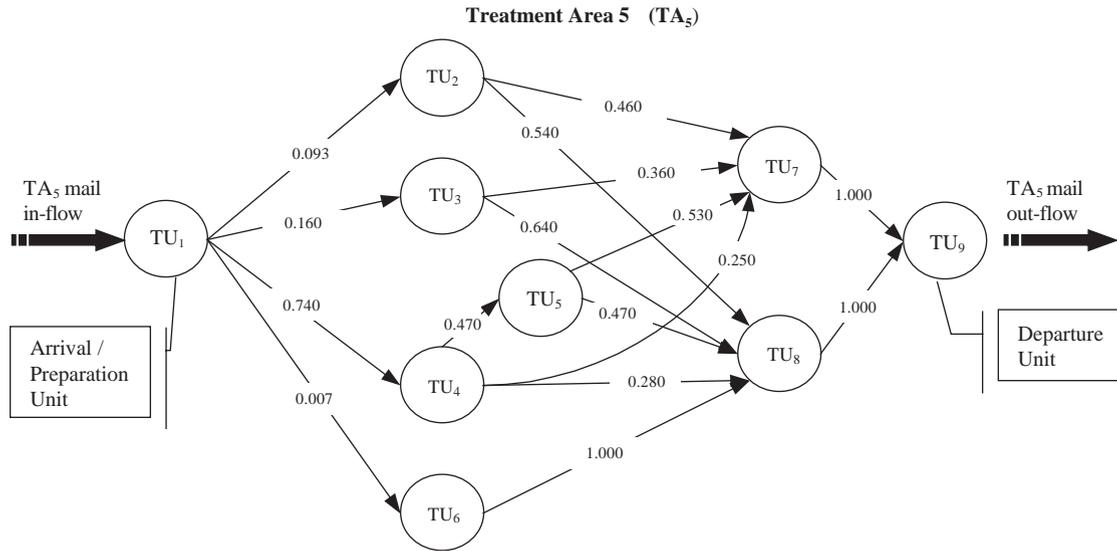


Fig. 2. Scheme of a TA (TA₅).

There is a chain of treatment units (TU_{*i*}) inside each treatment area, which guarantees mail sorting and separation. Mail objects have to pass through a predefined sequence of TUs, where the percentage of mail-objects that is transferred among each pair of TUs is known. Those transfer rates are estimated from samples drawn on a regular basis, revealing very small changes along the time and for this reason being updated in relatively large periods of time. Notice that these transfer rates depend mainly on the specificity of the operations involved in each TU.

We assume no difficulty in the circulation of mail-objects among TUs and all the mail already fully processed goes to a final expedition unit (TU_{*M*}) in each TA (*M* is the total number of TUs).

Mail transfer rates among every pair of TUs is defined in an (*M* × *M*) matrix *P*, where *p_{ri}* is the transfer rate among TU_{*r*} and TU_{*i*}, and *p_{ri}* is equal to 0 when there is no connection between that pair of TUs.

Fig. 2 displays the network describing the sequence of treatment units in TA₅ (registered mail). The label in each line connecting two units is their corresponding transfer rate. If two units are not connected, they do not transfer any mail between them. TU₁ (and other non mentioned TUs) receive all the TA₅ incoming mail and TU₉ sends all the treated mail to outside. For simplicity, the feed up structure is not represented in the scheme.

The treatment process is made during fixed periods of 15 min. The complete sequence of consecutive periods defines the global plan horizon. Each TA has its own plan horizon, corresponding to their daily working time duration, which might be as long as 24 h. We also define a 1 h block as a sequence of four consecutive periods, where each block starts on the hour. Fig. 3 presents a graphical illustration of the various time components used in the treatment process.

Fig. 4 describes the mail flow scheme, through the periods, between two general treatment units TU_{*r*} and TU_{*i*}. In each TU_{*i*} and at each period *j*, there is an incoming mail quantity which is the sum of the mail directly arrived from outside, the mail at the entrance of TU_{*i*} and not treated in

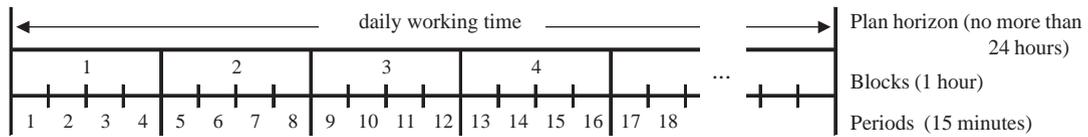


Fig. 3. Graphical illustration of the various time components used in the treatment process.

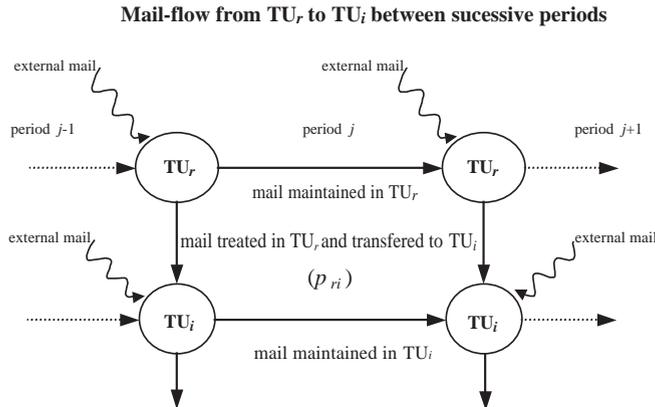


Fig. 4. Mail-flow scheme, through the periods, between two general treatment units TU_r and TU_i . The scheme includes external mail entering in each period in both TUs.

the previous periods, and the mail received from preceding units at the end of period $j - 1$. This global quantity at the entrance of TU_i in the beginning of period j is divided into mail that is treated during period j and mail that does not leave the entrance of TU_i and waits for its treatment in a future period in that TU.

Each TA receives their corresponding mail during the daily working time. Mail entered in period $j - 1$ is ready to start the treatment process in period j . The mail arriving instants depend on the closing time of the MDCs and from their distance to the MPC, implying, in general, very small variations in those arriving instants. Hence, the amounts of mail arrived during each individual period is strongly dependent on the global daily mail flow generated at MDCs and not from the particular behaviour of those MDCs. Those daily amounts change along the year, with peak weeks in some periods of the year. Variations on these amounts will be discussed for the particular case study proposed in this paper.

In order to give the chance to any mail-object to go through on the treatment process, each TU_i is not allowed to receive mail α_i periods before the end of the plan horizon, where α_i defines a certain number of periods and is established by the maximum number of TUs that a mail-object entered from outside to TU_i may need to cover until reaching the last TU. Mail arrived to an already closed TU will be treated on the next day.

An upper limit on the quantity of mail that can remain in each TU_i at the end of the daily working time is assumed. We also impose a standard quality criteria of 99% on the total volume of mail that must be treated at the end of that same period, i.e. ready to leave the corresponding TA. The remaining 1% includes mail that is to be treated in the next day.

The Staff Management Department intends to plan shift scheduling schemes, to face average and extreme scenarios of daily income mail amounts to each individual TA. The working shifts in use in the company range between 3 and 8 h shifts, corresponding to 3–8 consecutive 1 h blocks. All these working shifts must be included in the overall TA daily working time. Each shift has a fixed cost per employee, according to its duration. The purpose of this problem is to minimize the overall workforce salaries, according to the global assignment of employees to each working shift determined.

In each TA, the employees' assignment plan is established for each week. So, the problem is solved prior to each week, planning the workforce needs, according to the amounts of mail expected to be received in that week. This expectation is based on empirical evaluation, based on historical data and on typical seasonal behaviour. In fact, we can observe peak weeks, corresponding to lower and higher income flow amounts. This behaviour varies according to the type of mail considered in each particular TA.

In general, workforce availability inside the MPC can be shared between TAs and used for individual TAs weekly workforce planning. On the other hand, during each week plan established, the employees cannot move from one TA to another. We also assume that any employee can work in any TU inside each TA, at different periods of time. This change between TUs can only occur on the hour, so that in each 1 h block we have a permanent number of employees in each TU. Out of TU_M , where we assume no need for manpower assignment, we consider a limit on the number of employees in each TU_i and in each block.

3. A mixed integer linear formulation

In this section, we propose a mixed integer linear formulation for the problem introduced in the previous section. This formulation combines two sets of constraints, namely lotsizing constraints to characterize the batching structure of the problem, and shift scheduling constraints, defining its staff planning structure. As stated before, the objective of this study is concentrated in this last aspect. We start by presenting all the variables, parameters and constraints for the model. A few explanations of the main aspects of the formulation are given next.

Parameters:

M	total number of TUs, (TU_M is the last unit in the treatment area)
N	total number of periods in the time operating interval of the TA
H	number of 1 h blocks
K	total number of shifts
c_k	shift k employees individual salary, $k = 1, \dots, K$
q_{ij}	quantity of mail that enters the TA and directly arrives at TU_i during period $j - 1$, $i = 1, \dots, M - 1$, $j = 1, \dots, N$
p_{ri}	transfer rate of mail-objects from TU_r to TU_i , $r, i = 1, \dots, M$ and $r \neq i$
b_i	quantity of mail-objects that a employee can handle in each period in TU_i , $i = 1, \dots, M - 1$
l_i	upper limit on the number of employees at TU_i in each 1 h block, $i = 1, \dots, M - 1$
d_k	upper limit on the number of employees in shift k , $k = 1, \dots, K$
e_i	upper limit on mail quantity allowed to stay in TU_i at the end of the process, $i = 1, \dots, M - 1$

a_{kh} 1 if shift k covers the h th 1 h block, 0 otherwise, $k = 1, \dots, K$, $h = 1, \dots, H$
 α_i number of periods that each TU_i is closed for mail entrance, before the end of the daily working time, $i = 1, \dots, M - 1$

Variables:

x_{ij} nonnegative variable, representing the quantity of mail-objects treated in TU_i during period j , $i = 1, \dots, M$, $j = 1, \dots, N$
 s_{ij} nonnegative variable, representing the number of mail-objects kept at the end of period j at TU_i , $i = 1, \dots, M$, $j = 1, \dots, N$
 w_{ih} integer variable, defining the number of employees in TU_i during block h , $i = 1, \dots, M - 1$, $h = 1, \dots, H$
 v_k integer variable, defining the number of employees assigned to shift k , $k = 1, \dots, K$

In this formulation we consider $x_{i0} = 0$ and $s_{i0} = 0$ ($i = 1, \dots, M$), $x_{Mj} = 0$ ($j = 1, \dots, N$) and $w_{Mh} = 0$ ($h = 1, \dots, H$); so we can ignore those variables from the model. In order to guarantee the treatment of the majority of mail-objects that enters the TA, we close each TU_i entrance of mail at the $(N - \alpha_i)$ th period, considering α_i as the maximum number of TUs that a mail-object entered from outside to TU_i may need to cover until reaching TU_M . This means that all q_{ij} , for $i = 1, \dots, M$ and $j = N - \alpha_i + 1, \dots, N$, should be ignored from the model. The same happens to q_{Mj} , because no mail arrives directly to TU_M .

Formulation 1:

$$\text{Minimize } \sum_{k=1}^K c_k v_k \tag{1}$$

$$\text{Subject to } q_{ij} + s_{i,j-1} + \sum_{\substack{r=1 \\ r \neq i}}^{M-1} (p_{ri} x_{r,j-1}) = x_{ij} + s_{ij}, \quad i = 1, \dots, M, \quad j = 1, \dots, N, \tag{2}$$

$$s_{iN} + \sum_{\substack{r=1 \\ r \neq i}}^{M-1} p_{ri} x_{rN} \leq e_i, \quad i = 1, \dots, M - 1, \tag{3}$$

$$s_{MN} + \sum_{r=1}^{M-1} p_{rM} x_{rN} \geq 0.99 \sum_{i=1}^{M-1} \sum_{j=1}^{N-\alpha_i} q_{ij} \tag{4}$$

$$x_{ij} \leq b_i w_{ih}, \quad i = 1, \dots, M - 1, \quad j = 1, \dots, N, \quad h = \lceil j/4 \rceil, \tag{5}$$

$$w_{ih} \leq l_i, \quad i = 1, \dots, M - 1, \quad h = 1, \dots, H, \tag{6}$$

$$\sum_{i=1}^{M-1} w_{ih} = \sum_{k=1}^K a_{kh} v_k, \quad h = 1, \dots, H, \tag{7}$$

$$v_k \leq d_k, \quad k = 1, \dots, K, \tag{8}$$

$$x_{ij} \geq 0, \quad i = 1, \dots, M, \quad j = 1, \dots, N, \quad (9a)$$

$$s_{ij} \geq 0, \quad i = 1, \dots, M, \quad j = 1, \dots, N, \quad (9b)$$

$$w_{ih} \in N_0, \quad i = 1, \dots, M - 1, \quad h = 1, \dots, H, \quad (10a)$$

$$v_k \in N_0, \quad k = 1, \dots, K. \quad (10b)$$

Constraints (2) are the flow conservation constraints in each TU_i and in each period j , and they establish the equilibrium between the incoming and the outgoing mail quantity in the mentioned TU in period j . The incoming mail is defined as the sum of the mail arrived during the $(j - 1)$ th period (q_{ij}), the mail at the entrance of TU_i and not treated in the previous periods ($s_{i,j-1}$), and the mail received from preceding units at the end of period $j - 1$ ($\sum_{r=1, r \neq i}^M p_{ri} x_{r,j-1}$). This quantity at the entrance of TU_i in the beginning of period j must be equal to the mail treated in period j (x_{ij}) plus the mail that does not leave the entrance of TU_i (s_{ij}) and waits to be treated in a future period in TU_i . Constraints (3) impose an upper limit on the number of mail-objects allowed to stay in each TU at the end of the process. The $(p_{ri} x_{rN})$ terms represent mail treated in preceding TUs, which are transferred to TU_i at the end of the N th period. Inequality (4) guarantees that 99% of the overall mail that enters in TA is assembled in the final TU_i at the end of the process. Notice that $x_{Mj} = 0$ ($j = 1, \dots, N$), so the final arrival TU simply accumulates (almost) all the mail flow and it does not require any employees. These three sets of constraints characterize the multi-level lotsizing structure of the problem. On the other hand, inequalities (6)–(8) describe the shift scheduling subproblem. Considering this separation, inequalities (5) can be seen as coupling constraints, relating the treatment variables (x_{ij}) with the staff slot variables (w_{ih}). They guarantee a sufficient number of employees to cover the mail treatment in each period in TU_i . Constraints (6) and (8) impose upper limits to the number of employees in each TU_i and in each shift k , respectively. We relate these two sets of variables (w_{ih} and v_k) in constraints (7), guaranteeing the employees assignment among shifts and their corresponding 1 h blocks, according to the various shift intervals allowed. The objective is to minimize the overall workforce cost in this TA, which is stated in the objective function (1).

We consider variables x_{ij} and s_{ij} as nonnegative, although representing nondivisible units (mail-objects). This option simplifies the problem and represents a negligible lost to the correct description of the problem, due to the discrepancy of a unit in the total amount of mail treated. The other variables represent employees and have to be defined as integer variables.

An interesting aspect of this model is characterized by the coupling constraints (7). Contrary to a staff decision problem, we do not simply impose a cover from a selection of shifts over the total workforce in each one hour block, defined by the inequalities $\sum_{i=1}^{M-1} w_{ih} \leq \sum_{k=1}^K a_{kh} v_k$ to each $h \in \{1, \dots, H\}$. Instead, we further demand the two quantities in each term to be equal, which implies Eqs. (7). The solutions to be dropped (those verifying $\sum_{i=1}^{M-1} w_{ih} < \sum_{k=1}^K a_{kh} v_k$), are concerned with cases where we have a smaller number of employees assigned to TUs than those assigned to the shifts covering a certain block h . We can show that any of those solutions can be substituted by another feasible solution, obtained from the previous one by just increasing the values of the variables w_{ih} until we reach the equality defined by constraint (7). Both solutions have the same objective function value. By using the equality constraints a significant number of dominated solutions are dropped from

necessary to separate both types of mail and to sort them by destination. There are two demands in the treatment of the mail:

- (i) TU₇ simply treats inter-regional mail, which has to be ready for expedition before 23:00. This means that inter-regional mail (which comes from TU₇) should be ready in TU₉ before the end of the first time interval. The regional mail processed during this time interval is kept at TU₈. Although there are two different items (regional and inter-regional mail) in this problem, we consider it as a single-item case.
- (ii) Regional mail must be treated before 04:00 and should be in TU₉ before that time.

Time interval 2—from 23:00 to 04:00: In this period only regional mail (addressed to the same MPC area) arrives at TA₅. This mail does not require any kind of preparation, hence node 1 is eliminated from the network.

Therefore, the original matrix $P = [p_{ri}]_{r,i=1,\dots,M}$ is splitted into two matrices $P1 = [p1_{ri}]_{r,i=1,\dots,M}$ and $P2 = [p2_{ri}]_{r,i=1,\dots,M}$, where M is the number of TUs and $P1$ and $P2$ are concerned with the time intervals 1 and 2, respectively.

TU_{M-2} and TU_{M-1} can share their human resources availability, so we consider their manpower assignment in conjunction. We simply demand mail to be kept separated (regional and inter-regional), while treated by the same team.

Considering these further characteristics, the following mixed integer programming model can be associated to the treatment area TA₅, where $N1$ represents the number of periods until 23:00.

Formulation 2:

$$\text{Minimize } \sum_{k=1}^K c_k v_k \tag{1}$$

$$\text{Subject to } q_{ij} + s_{ij-1} + \sum_{\substack{r=1 \\ r \neq i}}^{M-1} (p1_{ri} x_{rj-1}) = x_{ij} + s_{ij}, \quad i = 1, \dots, M, \quad j = 1, \dots, N1, \tag{11a}$$

$$q_{ij} + s_{ij-1} + \sum_{\substack{r=1 \\ r \neq i}}^{M-1} (p2_{ri} x_{rj-1}) = x_{ij} + s_{ij}, \quad i = 1, \dots, M, \quad j = N1+1, \dots, N, \tag{11b}$$

$$s_{iN1} + \sum_{\substack{r=1 \\ r \neq i}}^{M-1} p1_{ri} x_{rN1} \leq 0.01 \left(\sum_{j=1}^{N1-\alpha_i} q_{ij} + \sum_{j=1}^{N1} \sum_{\substack{r=1 \\ r \neq i}}^{M-1} p1_{ri} x_{rj} \right),$$

$$i = 1, \dots, M - 2, \tag{12a}$$

$$s_{iN} + \sum_{\substack{r=1 \\ r \neq i}}^{M-1} p2_{ri} x_{rN} \leq 0.01 \left(\sum_{j=1}^{N-\alpha_i} q_{ij} + \sum_{j=1}^{N1} \sum_{\substack{r=1 \\ r \neq i}}^{M-1} p1_{ri} x_{rj} + \sum_{j=N1+1}^N \sum_{\substack{r=1 \\ r \neq i}}^{M-1} p2_{ri} x_{rj} \right),$$

$$i = 1, \dots, M - 1, \tag{12b}$$

$$\sum_{i=M-1}^M \left(s_{iN1} + \sum_{\substack{r=1 \\ r \neq i}}^{M-1} p1_{ri} x_{rN1} \right) \geq 0.99 \sum_{i=1}^{M-1} \sum_{j=1}^{N1-\alpha_i} q_{ij}, \tag{13a}$$

$$s_{MN} \sum_{\substack{r=1 \\ r \neq i}}^{M-1} P2_{rM} x_{rN} \geq 0.99 \sum_{i=1}^{M-1} \sum_{j=1}^{N-\alpha_i} q_{ij}, \tag{13b}$$

$$x_{ij} \leq b_i w_{ih}, \quad i = 1, \dots, M-3, \quad j = 1, \dots, N, \quad h = \lceil j/4 \rceil, \tag{14a}$$

$$x_{M-2j} + x_{M-1j} \leq b_{M-2} w_{M-2h}, \quad j = 1, \dots, N, \quad h = \lceil j/4 \rceil, \tag{14b}$$

$$w_{ih} \leq l_i, \quad i = 1, \dots, M-2, \quad h = 1, \dots, H, \tag{6}$$

$$\sum_{i=1}^{M-2} w_{ih} = \sum_{k=1}^K a_{kh} v_k, \quad h = 1, \dots, H, \tag{7}$$

$$x_{ij} \geq 0, \quad i = 1, \dots, M, \quad j = 1, \dots, N, \tag{9a}$$

$$s_{ij} \geq 0, \quad i = 1, \dots, M, \quad j = 1, \dots, N, \tag{9b}$$

$$w_{ih} \in N_0, \quad i = 1, \dots, M-2, \quad h = 1, \dots, H, \tag{10a}$$

$$v_k \in N_0, \quad k = 1, \dots, K. \tag{10b}$$

As before, variables x_{i0} , s_{i0} ($i = 1, \dots, M$) and $x_{M,j}$ ($j = 1, \dots, N$) and parameters q_{ij} ($i = 1, \dots, M$; $j = N - \alpha_i + 1, \dots, N$) can be ignored from the model. Furthermore, it follows from the network presented in Fig. 2 that there are $M = 9$ TU units and $\alpha_1 = 4$, $\alpha_4 = 3$, $\alpha_2 = \alpha_3 = \alpha_5 = \alpha_6 = 2$, $\alpha_7 = \alpha_8 = 1$. Remember that TU₉ does not receive mail from outside.

This formulation differs from the previous one in some constraints, that are discussed next. Those differences are directly related with the mentioned features of the treatment area TA₅. In the set of flow conservation constraints (11a) and (11b), matrices $P1$ and $P2$ define the new transfer rate network structure in both time intervals. Constraints (12a) and (12b) substitute inequalities (3), which define upper limits to the end process residual number of mail-objects that can be left in each TU and are directly dependent on the global volume of mail arrived to each of those TUs (no more than 1% of this quantity), in both end time periods, $N1$ and N , respectively. The new constraints establish a non constant upper limit on the amount of mail allowed to remain in each TU, differing from inequalities (3) proposed for the general case. Out of the 99% of mail to be treated at the end of the process and defined by the constraint (13b), we must guarantee that all the mail entered until the $(N1 - \alpha_i)$ th period should be at TU _{$M-1$} (regional mail) and at TU _{M} (inter-regional mail) until the end of period $N1$. This is defined in the constraint (13a), allowing a 1% tolerance of unsent inter-regional mail plus non accumulated regional mail at TU _{$M-1$} , at the end of the $N1$ th period. Notice that this 1% tolerance of undelivered mail is established by the standard quality criteria, while the previous 1% TUs residual mail tolerance is an internal organizational criteria, so both concepts cannot be self-substituted. The lower limit on the number of employees (w_{ih}) imposed by the amount of mail treated (x_{ij}) is the same as before. We consider apart the TU _{$M-2$} and TU _{$M-1$} cases, where

both units amount of mail treated is covered by the number of employees assigned to TU_{M-2} . This leads into constraints (14b). Notice that there is no need of human resources assigned to TU_M . As mentioned at the end of Section 2.2, workforce availability inside an MPC can be shared among TAs and used for individual TAs weekly workforce planning. TA_5 is not very demanding in human resources needs, when compared with other TAs. For this reason, no upper limits to the number of employees in each shift are assumed, hence inequalities (8) have been deleted from the present model.

5. Computational experiments to the case study

Consider the case study presented in the previous section and based in Fig. 2 treatment scheme. Furthermore, let the daily working time of TA_5 be the one settled before, from 17:00 to 4:00. Hence, the following parameters are set in Formulation 2:

- (i) $M = 9$ —number of TUs,
 $N = 44$ —number of periods ($N1 = 24$),
 $H = 11$ —there are 11 blocks in the 17:00 to 04:00 time interval,
 $K = 39$ —number of working shifts of 3–8 h, as defined in Table 1.
- (ii) Costs for each working shift of 3, 4, 5, 6, 7 and 8 h: ($c_k, k = 1, \dots, K$)

Shifts duration (h)	3	4	5	6	7	8
Cost (€)	18.66	24.88	31.10	37.32	43.54	49.76

All the costs are proportional to 6.22 €, defining the hourly cost of work. Nonproportional costs, according to shifts duration, do not impose any further difficulties to the problem.

- (iii) Matrices $P1$ and $P2$ are defined in Appendix A, where $p1_{ri}$ is the mail-flow transfer rate from TU_r to TU_i ($r, i = 1, \dots, M$) in the 17:00–23:00 time interval, and $p2_{ri}$ is the mail-flow transfer rate from TU_r to TU_i ($r, i = 1, \dots, M$) in the 23:00–04:00 time interval.
- (iv) The parameters q_{ij} ($i = 1, \dots, M - 1, j = 1, \dots, N$) are defined in Appendix A, representing the number of mail-objects arrived directly to each TU per period.
- (v) We also consider processing speed rate values, defined by b_i ($i = 1, \dots, M$) parameters. These are presented in the following table, by mail-objects/man per period:

TU	1	2	3	4	5	6	7 and 8
Mail-objects/man	1500	250	280	313	313	25	350

We assume that no more than 15 employees can simultaneously be in the same TU, so $l_i = 15$ for $i = 1, \dots, M$. As stated before, we have ignored the upper limit on the number of employees on the same working shift, which means that constraints (8) have been dropped from the model.

We have used the ILOG/CPLEX 7.0 package LP and mixed integer (MIP) solver to process the mixed integer linear programming Formulation 2 with all the mentioned parameters. The branch-and-cut version of the MIP solver has been chosen, which allows the generation of

general feasible cuts, provided by the software. The optimization model has been run in a Pentium III 800 MHz processor machine.

Next some scenarios are analysed, showing the problem sensitivity to some of its input parameters. These changes focus on the daily amount of mail arrived to TA₅ and on the quality criteria level. As mentioned in Section 2.2, the mail arriving instants to this area, and also to the other ones, depend on the closing time of the MDCs and on their distance to the MPC, implying, in general, very small variations in those arriving instants. Hence, the amounts of mail arrived during each individual period are strongly dependent on the global daily mail flow generated at MDCs and not to the particular behaviour of those MDCs. Those daily amounts change along the year, with peak weeks in some periods of the year. In this paper, year 2001 daily income mail-flow is analysed for TA₅, in order to evaluate human resources needs from lower to higher amounts of mail entered in that specific TA. Those observations are important to plan future manpower needs at TA₅.

Consider the total amount of mail arrived to TA₅ in a specific day, defined by $Q = \sum_{i=1}^{M-1} \sum_{j=1}^{N-\alpha_i} q_{ij}$. Then, for the year 2001, the 251 daily observed Q values conduct to the following statistical parameters: $\bar{x} = 46\,925$, $s = 13\,715.923$, skewness = -0.29 (with std. error of skewness = 0.148) and kurtosis = 0.458 (with std. error of kurtosis = 0.148). Although both quotients (skewness and kurtosis divided by their respective std. errors) fall between -2 and 2 , the Kolmogorov–Smirnov (with Lilliefors correction) goodness-of-fit test gives no evidence that the sample comes from the Normal distribution with the estimated sample parameters, presenting a p -value equal to 0.002 . We used the SPSS statistical software, version 11.0 (see [40]).

Based on these observations, the following four points are to be addressed next:

- (i) the average case ($Q = 46\,925$), including some aspects related with Formulation 2 resolution and some changes on the model parameters;
- (ii) a comparison between small, average and high mail amount cases, considering $Q = 36\,709$ (first quartile), $Q = 46\,925$ and $Q = 55\,006$ (third quartile), respectively;
- (iii) the analysis of the weeks with small and high average amounts of mail arrived, considering the median observation in those weeks, with $Q = 25\,947$ and $Q = 69\,039$, respectively;
- (iv) the analysis of the weeks with highest and shortest differences between the maximum and the median observations.

Due to some privacy options of the Company, the comparison with the practical solutions in use are simply provided for the aggregated average case.

The q_{ij} parameters table, presented in Appendix A, defines the average case income mail quantities arrived per period, so for $Q = 46\,925$. For different Q values, the q_{ij} quantities are proportional to those presented in the mentioned table.

(i) *Average case*: The linear relaxation optimal solution value of Formulation 2 defines a lower bound equal to 610.36 €, and was obtained after 0.28 s of CPU time.

The MIP solver of CPLEX 7.0 was unable to reach optimality after 12 h of running time. Nevertheless, some interesting feasible solutions have been obtained during the branch-and-bound execution. The best lower and upper bounds found at the end of this experience are 638.69 € and 646.88 €, respectively. Hence there is a duality gap of 1.27% . In the first node of the branch-and-bound tree, the MIP solver strength the model with 84 Gomory fractional cuts and four MIR cuts, leading the lower bound to 615.05 €. After some experiences on node and variable selection strategies of the

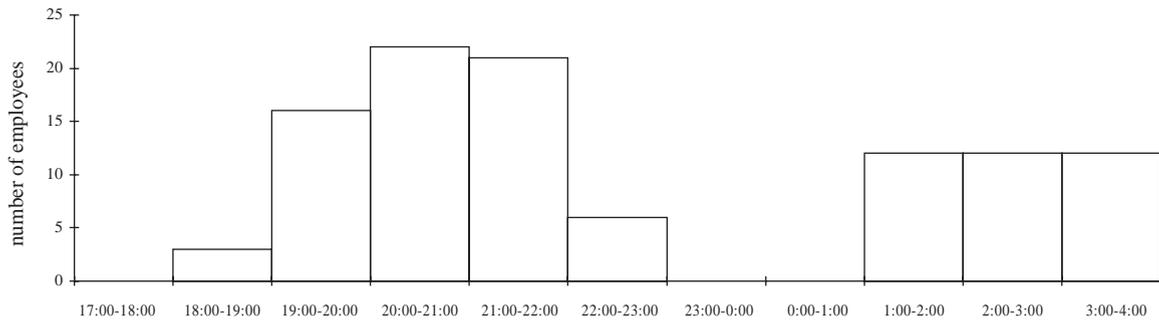


Fig. 5. Number of employees determined per block.

MIP solver, we have chosen to use the best-bound search strategy for node selection and the variable selection based on pseudo-costs (see [39]).

In this case, the normal daily treatment process of this TA is guaranteed by a 104 h workforce solution, corresponding to a 646.88 € daily cost. The 1.27% duality gap simply corresponds to 1 h workforce, that is, either the best feasible solution is optimal or there is an optimal solution requiring one less hour.

We have also tried to solve the problem starting the branch-and-bound method from Formulation 2 with constraints (7) defined as less or equal inequalities, characterizing the shifts covering over the hourly assignment of employees to TUs. In this case, we reached the same 104 h feasible solution, although the lower bound obtained after the same 12 h of running time was equal to 636.41 €, defining a 1.62% duality gap, that is superior to the one obtained in the previous case.

The best feasible solution found is presented in Table 10 in Appendix B, which presents the number of employees determined per block and assigned to each TU: variables w_{ih} values.

Fig. 5 shows the total number of employees per block, according to the 104 h solution found.

Fig. 6 presents the amount of mail-objects in the system and not yet treated (line) per period j ($j = 1, \dots, 44$), defined by the difference between the total amount entered before period j and the quantity already arrived at TU₉. The figure also shows the total amounts entered per period j (stars) (arrived during the $(j - 1)$ th period).

We cannot establish a fair comparison between both charts, because the speed rate (in mail-objects per period) among TUs is different. In any case, if we try to lean Fig. 6 against Fig. 5, it gives an idea of how workforce adapts to the mail amount being treated, per block. That comparison shows, for instance, that the 12 employees assigned to the 3 h shift (1:00–4:00), which allows for a significant decrease of mail-objects in the (1:00–2:00) block, may reflect some spare capacity in manpower usage in the time interval from 2:00 to 4:00. This spare capacity might be appropriately used if an augmented amount of mail (namely regional mail) arrives to this TA. Notice that our minimum duration shifts are three hours time intervals.

Table 2 presents the number of employees determined per shift type, and described by the solution values of the variables v_k . No employees have been assigned to the remaining shifts that are not included in this table.

We may think in many alternative scenarios, still centred in the average arrived amount case ($Q = 46925$). Some of these alternative cases are discussed in the next paragraphs, obtained through appropriate changes in some parameters.

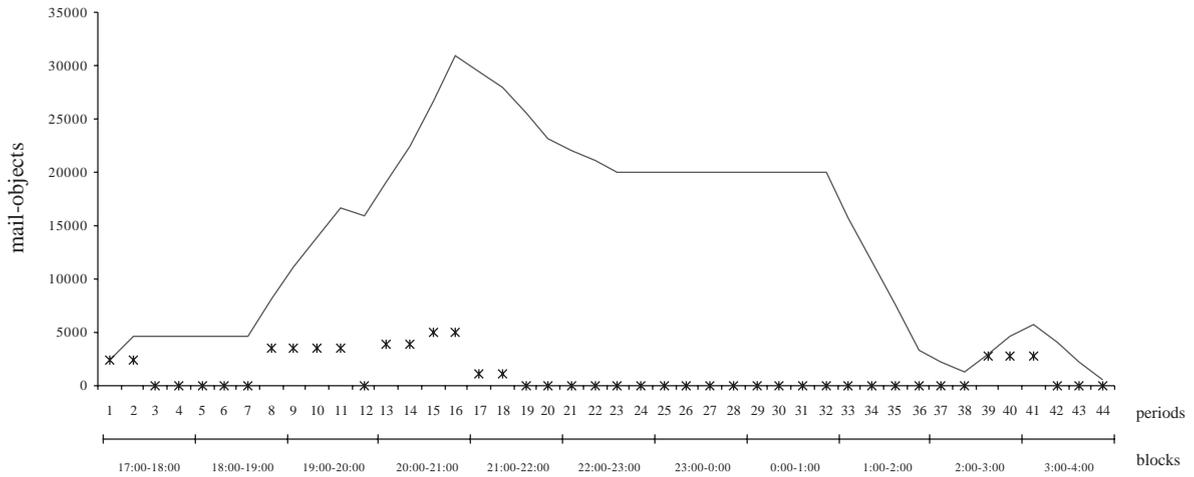


Fig. 6. Amount of mail-objects in the system and not yet treated (line) per period j , and the total amounts entered per period j (stars) (arrived during the $(j - 1)$ th period), for $j = 1, \dots, 44$.

Table 2
Number of employees determined per shift type: variables v_k solution values

Shifts	Number of employees
18:00–21:00	1
18:00–22:00	2
19:00–22:00	13
20:00–23:00	6
01:00–04:00	12

It can be showed that for the same amount of mail-objects arrived ($Q = 46\,925$), the 104 h workforce can treat up to 46 742 mail-objects, corresponding to 99.61% of the total amount entered. This means that the proposed workforce can go beyond the 99% standard for the quality criteria. Notice that the previous reported best feasible solution, described in Table 10 in Appendix B, only treat 46 456 mail-objects, corresponding to 99% of the total amount. The 99.61% solution has been obtained from a changed version of Formulation 2. The new model involves the maximization of the nonnegative variable ϕ , representing the standard quality criteria, also appearing in constraints (13a) and (13b). The model includes the following additional constraint:

$$\sum_{k=1}^K \delta_k v_k = 104 \tag{15}$$

to force the use of only 104 h of workforce (δ_k is the time duration, in hours, of shift k).

In a different perspective, we also looked for the maximum increase on the total amount of mail that TA₅ can receive, keeping the same 99% quality criteria and forcing again for a 104 h of manpower usage. This case involves a different changed version of Formulation 2, which includes a

Table 3

MIP solver information related with the three quality criteria scenarios: 97%, 95% and 90%

Quality criteria (%)	Lower bound (€)	Upper bound	Duality gap (%)
97	617.25	628.25 € = 101 h	1.751
95	609.55	622.04 € = 100 h	2.008
90	591.85	603.38 € = 97 h	1.911

new nonnegative variable β , defining the proportional increase in the previous amount of 46 925 mail-objects. So the new formulation involves the maximization of β , subject to Formulation 2 constraints, added by equality (15) and substituting all the parameters q_{ij} by $q_{ij}\beta$, in constraints (11a)–(13b). After 12 h of the branch-and-bound execution, the best feasible solution found was $\beta = 1.0044$, with an upper bound defined by $\beta = 1.0258$. This means that we can treat at least 47 131 mail-objects with the same workforce resources with no damage to the service quality standards, corresponding to more 206 mail-objects when compared with the initial case (for proportional increases). This increase can not exceed 1210 mail-objects, considering the upper bound value obtained. Remember that it is assumed that an increase on the daily mail amount entered is established by proportional increases in each period mail amounts arrived, according to the q_{ij} values presented in Appendix A.

Another interesting aspect is related with the 99% standard quality criteria on the total amount of mail treated, and the 1% of mail that is allowed to stay in each TU and treated in the next day. As mentioned in Section 2.1, an independent sector regulator follows the time series movement of delayed mail. Those limits are agreed with the Company and are frequently a matter of discussion. At this point, we may discuss two scenarios: the 100% quality criteria; and relaxed standard quality criteria's. We start by the first scenario.

From an optimizer perspective, these tolerances seem to be an imperfection of the system, which should not be assumed by the model. From the manager perspective, these tolerance limits have long been studied, and reflect a good compromise between an admissible negligible failure and the additional workforce cost needed to accomplish this failure. If we try to answer the optimizer expectations by solving Formulation 2 with 100% quality criteria and 0% mail left on TUs (providing the appropriate changes in constraints (12a), (12b), (13a) and (13b)), we obtain (again after 12 h of the branch-and-bound execution) a lower bound equal to 676.43 € and an upper bound equal to 690.46 €. This means that the optimal solution to that specific problem involves between 109 and 111 workforce hours. So to close this residual volume of nontreated mail, we must spend at least more 5 h, representing an increase on the overall manpower effort. This solution is interesting to compare with the previous reported case, which showed that we are able to treat 99.61% of the global amount entered using only the 104 h workforce. So the mentioned five additional hours required to meet the 100% case reflect only 0.39% (183 mail-objects), when keeping the same global amount entered (46 925 mail-objects).

For the relaxed scenarios, we have considered the following standard quality criteria parameters: 97%, 95% and 90%, keeping the 1% residual mail tolerance at TUs and all the other initial parameters. In this case, the changes are only present in constraints (13a) and (13b). After 12 h of running time, the MIP solver gave the solutions presented in Table 3.

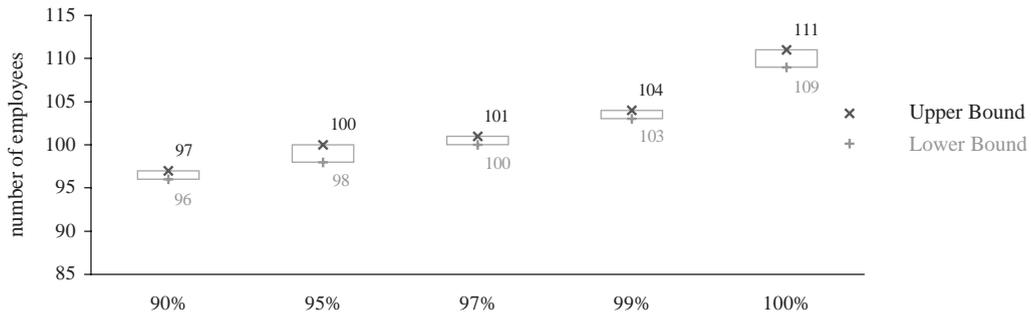


Fig. 7. Lower and upper bound number of employees determined to the five quality criteria scenarios: 90%, 95%, 97%, 99% and 100%.

These three scenarios and the previous 99% and 100% cases are compared in Fig. 7. All the cases are defined in workforce hours, including the lower bound transformed values, which are obtained from the expression $\lceil (\text{lower bound})/6.22 \rceil$ (remember that the hourly cost of work is equal to 6.22 €), defining a lower limit on manpower usage.

Fig. 7 shows that, when placed at the 99% case, the additional workforce effort needed to increase the quality criteria is much superior than the benefit obtained from relaxing that same quality criteria. In fact, comparing only the upper bound values, the 1% quality criteria increase over the 99% case requires seven additional workforce hours, which coincides with the manpower spared from relaxing the quality criteria to 90%. This subject is a frequent theme of discussion in the Company, to which the previous analysis indicate that a decrease in the delivery quality standards may not correspond to a significant decrease in manpower usage.

In order to compare with the practice in this TA₅, we applied to the MPC manager for a practical workforce solution to the 46 925 mail-objects case, keeping all the parameters initially introduced. This MPC has recently been installed and the current manager has been adapting its workforce availability to the various TAs, making successive experiences on employees' local changes. For the average case considered to TA₅, his proposal corresponds to a 112 h workforce, representing 696.64 €, using the same kind of 3–8 h shifts and verifying the same quality criteria described and used to build Formulation 2. This represents a gap of 7.14%, when compared with the best solution proposed in this paper.

This comparison shows not only the great interest of the optimization formulation presented in this paper but also the confirmation that the solution proposed by the manager to this TA₅ was not far from the best he could afford to, although the 104 h solution shows that he could have gone further.

In a different perspective, it is also interesting to analyse the case where only 8 h working shifts are in operation. By adapting Formulation 2 to that particular situation, we obtain an optimal solution of 945.44 € on daily salaries, using 152 h of total workforce. This means that by using the 3–8 h shifts solutions, we can reduce significantly the human resources needs in this particular TA.

(ii) *Comparison between small, average and high amount cases:* In this point, we analyse the average case: $Q = 46\,925$, and also small and high daily amounts of mail-objects entered at TA₅. The small and high amount cases are the first and the third quartiles, namely $Q = 36\,709$ and $55\,006$,

Table 4

Workforce solutions per block determined to the Q_1 , \bar{Q} and Q_3 mail arrival scenarios

Blocks	17:00– 18:00	18:00– 19:00	19:00– 20:00	20:00– 21:00	21:00– 22:00	22:00– 23:00	23:00– 00:00	0:00– 1:00	1:00– 2:00	2:00– 3:00	3:00– 4:00	Total
Q_1	0	2	11	17	16	7	0	0	10	10	10	83
\bar{Q}	0	3	16	22	21	6	0	0	12	12	12	104
Q_3	1	1	15	25	24	11	0	0	15	15	15	122

Table 5

MIP solver information related with the three mail arrival scenarios: Q_1 , \bar{Q} and Q_3

Amounts entered	Lower bound (€)	Upper bound	Duality gap (%)
Q_1	501.80	516.30 € = 83 h	2.89
\bar{Q}	638.69	646.88 € = 104 h	1.27
Q_3	739.52	758.88 € = 122 h	2.55

respectively. We designate the three mentioned cases by $\bar{Q} = 46\,925$, $Q_1 = 36\,709$ and $Q_3 = 55\,006$, respectively.

Table 4 presents the number of employees per block in the best feasible solution obtained by the MIP solver, after 12 h of running time. The standard quality criterion has been kept at a 99% level. Like the \bar{Q} case, the other two quantities analysed (Q_1 and Q_3) require only 3 and 4 h shifts.

In Table 5, the lower and upper bound limits obtained after 12 h of the branch-and-bound execution, and the associated duality gaps, are reported.

These results show that a workforce ranging between 83 and 122 h can answer 50% of the 251 observations of daily mail income amounts. This means that the 122 h workforce solution is sufficient to treat 75% of the mentioned 251 observations, when the quality criteria is established at a 99% standard. This result may introduce a new discussion, that is, how far could we go, answering to the 251 daily observations, with the same 122 h of workforce, if the quality criteria standard is decreased? To answer this question, we have analysed unitary decreases on the quality criteria value, from 99% to 88%, where the objective is to determine the maximum amount of mail-objects allowed to enter to TA₅ and treated, at the quality standard established, by a 122 h workforce. This analysis is similar to a previous one, proposed in point (i). It also involves a changed version of Formulation 2, which includes the same non-negative variable β , defining the proportional increase in the 55 006 amount of mail-objects. The modified formulation involves the maximization of β , subject to Formulation 2 constraints, together with equation $\sum_{k=1}^K \delta_k v_k = 122$ (δ_k is the time duration, in hours, of shift k) and substituting all the parameters q_{ij} by $q_{ij}\beta$, in constraints (11a)–(13b). Table 6 presents the best feasible solutions found to each of the approaches, from 99% to 88% cases, after 12 h of the branch-and-bound execution. The accomplishment columns represent the percentage of accomplishment achieved over the 251 daily observations. To each daily amount observation Q , the mentioned accomplishment is considered at the quality criteria level (ξ) established, so considering that Q^ξ mail-objects are treated.

Table 6

MIP solver information related with the twelve quality criteria standards: 99–88%, imposing the use of 122 h of workforce. Mail-objects (m-o) are rounded to the closest integer value

Workforce=122 h, quality criteria (ξ) (%)	Lower bound	Accomplishment (%)	Upper bound	Accomplishment (%)	Duality gap (%)
99	$\beta \geq 1.008 = 55466$ m-o	76	$\beta \leq 1.018 = 55996$ m-o	77	0.98
98	$\beta \geq 1.014 = 55776$ m-o	77	$\beta \leq 1.059 = 58251$ m-o	83	4.25
97	$\beta \geq 1.041 = 57249$ m-o	79	$\beta \leq 1.083 = 59595$ m-o	86	3.88
96	$\beta \geq 1.061 = 58361$ m-o	83	$\beta \leq 1.106 = 60837$ m-o	89	4.07
95	$\beta \geq 1.092 = 60064$ m-o	86	$\beta \leq 1.126 = 61941$ m-o	90	3.02
94	$\beta \geq 1.101 = 60562$ m-o	88	$\beta \leq 1.142 = 62817$ m-o	91	3.59
93	$\beta \geq 1.118 = 61497$ m-o	90	$\beta \leq 1.158 = 63697$ m-o	92	3.45
92	$\beta \geq 1.153 = 63447$ m-o	91	$\beta \leq 1.180 = 64907$ m-o	93	2.29
91	$\beta \geq 1.176 = 64687$ m-o	93	$\beta \leq 1.194 = 65677$ m-o	94	1.51
90	$\beta \geq 1.194 = 65695$ m-o	94	$\beta \leq 1.210 = 66557$ m-o	94	1.32
89	$\beta \geq 1.212 = 66667$ m-o	94	$\beta \leq 1.224 = 67327$ m-o	95	0.98
88	$\beta \geq 1.224 = 67327$ m-o	95	$\beta \leq 1.242 = 68317$ m-o	95	1.45

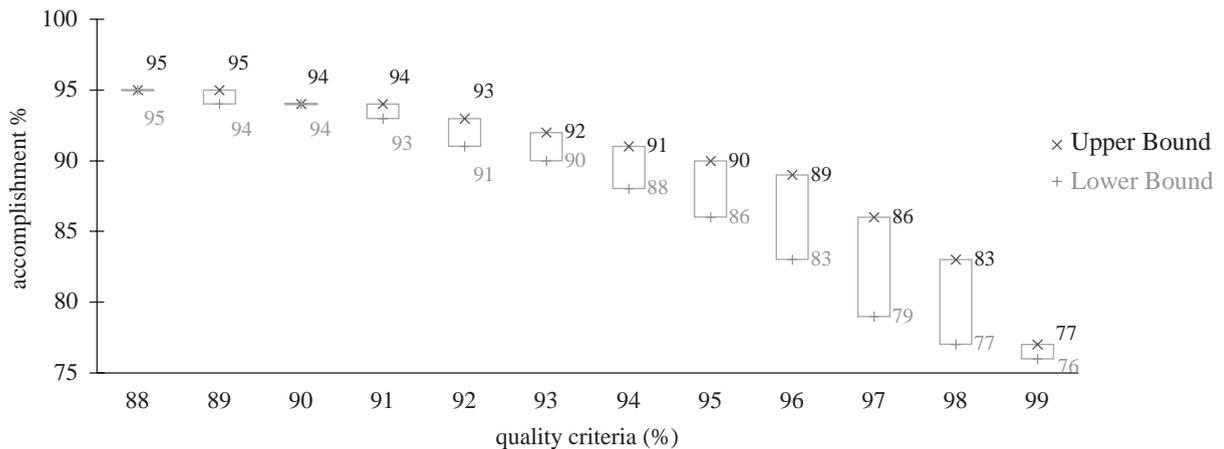


Fig. 8. Lower and upper bound values of the accomplishment to each quality criteria level, ranging from 99% to 88%, with a fixed workforce of 122 h.

Fig. 8 presents the lower and upper bound values of the accomplishment achieved according to unitary variations in the quality criteria level, considering the results in Table 6.

As expected, when the quality criteria level is allowed to be decreased, the 122 h of workforce are sufficient to answer to a higher number of days, in the 251 universe. Namely, for a quality criteria level of 99%, the 122 workforce hours are sufficient to answer to at least 76% of the days, while if that quality standard is allowed to be relaxed to 93%, then the same workforce can answer up to 90% of the days. If only 5% of the days are allowed to fail with the quality criteria established, than this standard must be relaxed to 88%, still using the same workforce.

Table 7

MIP solver information related with the two scenarios: Q^- and Q^+

Daily mail amounts	Lower bound	Upper bound	Duality gap (%)
$Q^- = 25\,947$	376.50 € → 61 h	379.42 € = 61 h	0.77
$Q^+ = 69\,039$	914.82 € → 147 h	933.00 € = 150 h	1.95

We recall that the 122 workforce hours solution has been suggested to answer the Q_3 amount of mail case, being sufficient to guarantee a 99% quality criteria on 75% of the days. The previous result shows that we can go further, establishing that the same manpower level can answer at least 76% of the days, still keeping the same quality criteria standard.

(iii) *Weeks with small and high average amounts of mail arrived:* The previous analyses are related with daily income mail amounts, based on the 251 daily observations of the year 2001. As mentioned in Section 2.2, in practice, TA₅ employees' assignment plan is established for each week, according to the amounts of mail expected to be received during that week. In this point, and also in the next one, year 2001 data observations are going to be analysed again, but this time looking for its weekly behaviour.

In those data values, the median observation in the weeks with smaller and higher average mail income amounts are $Q^- = 25\,947$ and $Q^+ = 69\,039$ mail-objects, respectively. These two peak weeks may be seen as low and hard workforce weeks in human resources needs. The low week case can be observed in August during the typical vacations period, when most Public Institutions are closed. The high week peak occurs in December and may be related with Christmas time and with the proximity of the end of the year. Remember that TA₅ only treats registered mail.

In Table 7, the lower and upper bound limits obtained after 12 h of the branch-and-bound execution and the associated duality gaps, are reported. A lower limit on manpower usage has been calculated through the expression $\lceil (\text{lower bound})/6.22 \rceil$ (using the hourly cost of work, equal to 6.22 €).

The results define extreme values in human resources needs, according to small and high weekly income mail amounts. Those limits indicate that only 61 workforce hours are needed in low peak periods (e.g., in August), while in high peak periods the system may require 150 h of workforce. These solutions refer to the 99% quality criteria standard. Resorting to an analysis described in point (ii), the $Q^+ = 69\,039$ daily mail arrived case belongs to the 5% worse observations which, for a workforce of 122 h, does not even allow to reach an 88% level on the quality criteria.

Considering the 61 and 150 workforce hours proposed to both cases Q^- and Q^+ , respectively, it is interesting to analyse the way the system reacts when facing the highest amount observations occurring inside each one of those weeks. We recall that the values initially considered correspond to the median among each week observations. In the first case (in Q^- week) the highest daily income amount is equal to 28 366 mail-objects, while in the second case (in Q^+ week) that value is equal to 77 950 mail-objects. Considering appropriate changes in Formulation 2 and solving the adapted model, we can state that the 61 workforce hours can treat up to 27 581 mail-objects, corresponding to a 98% quality criteria; while in the second case, the 150 workforce hours allow to treat 74 888 mail-objects, corresponding to a 96% quality criteria.

Table 8

MIP solver information related with the weeks with shortest and highest differences between the maximum and the median observations, respectively 1003 and 18363 mail-objects. (In both cases the model has been solved for the median observation)

	Mail-objects			Considering the median observation for mail income amount	
	Median	Maximum	Difference	Lower bound	Upper bound
Shortest difference	58 818	59 821	1003	782.05 € → 126 h	802.38 € = 129 h
Highest difference	46 421	64 784	18 363	632.57 € → 102 h	646.88 € = 104 h

(iv) *Weeks with high and short error plans, in workforce assignment:* Still centred in the weekly behaviour of year 2001 mail income observations, we analyse the errors that may be committed when trying to plan human resources needs for a week period. With this purpose and using the mentioned data observations, we analyse the weeks with highest and shortest differences between the maximum and the median observations. Four outliers are excluded from the discussion.

This analysis starts by determining the workforce required to answer the median amount case, to both weeks under consideration. Table 8, presents the lower and upper bound limits obtained after 12 h of the branch-and-bound execution. Again, the lower limit on manpower usage has been calculated through the expression $\lceil (\text{lower bound})/6.22 \rceil$ (considering the hourly cost of work, equal to 6.22 €).

These results show that in both weeks, if the human resources intended to assigned to this TA are based on the median mail amount observation, then there should be proposed 129 workforce hours for the first case (shortest difference) and 104 h of workforce to the second one (highest difference). Both workforce resources should be kept during the whole week. The problem comes when we have to face the maximum amounts in that week with the proposed human requirements, and a question comes immediately: what is the quality criteria achieved if we use the proposed workforce (129 h to the first case and 104 h to the second one) to treat the maximum amount expected in the correspondent week? To answer this question, we considered a modified version of Formulation 2, similar to another one used before, involving the maximization of the nonnegative variable ϕ , representing the standard quality criteria, also appearing in constraints (13a) and (13b). The two cases under consideration (shortest and highest difference) require different formulations. For the first case (shortest difference), the model includes the constraint $\sum_{k=1}^K \delta_k v_k = 129$, while for the second one (highest difference) the model considers $\sum_{k=1}^K \delta_k v_k = 104$, where δ_k is the time duration, in hours, of shift k . Both formulations use different daily mail income amounts, namely $Q = 59\,821$ and $Q = 64\,784$, respectively, for the shortest and the highest difference cases.

Table 9 presents results related with both modified formulations, after 12 h of the branch-and-bound execution.

The results show that in the shortest difference case, the 99% quality criteria is still guaranteed, while in the highest difference case we may only achieve a quality criteria level of 81%. This means that the Company may face a shortage in the delivery quality standard if high differences in daily mail amounts received are neglected, when the plan is established for an expected median scenario in manpower resources proposed to each week.

Table 9

MIP solver information related with the two modified formulations, proposed to maximize the quality criteria achieved when facing maximum mail amount quantities with a workforce established according to the median observation mail amount, to both shortest and highest difference scenarios

	Maximum	Workforce (h)	Lower bound	Upper bound
Shortest difference	59 821 m-o	129	$\phi \geq 0.99 = 59\,256$ m-o	$\phi \leq 0.995 = 59\,500$ m-o
Highest difference	64 784 m-o	104	$\phi \geq 0.81 = 52\,442$ m-o	$\phi \leq 0.815 = 52\,773$ m-o

It can be showed that the 64 784 mail-objects can be treated by a workforce solution of 141 h (lower limit of 139 h), with a guaranteed quality criteria of 99%.

6. Conclusions

This paper describes an application to a batching decision process, in which the daily workforce cost of a mail processing center TA is minimized. A formulation to this problem has been proposed. A branch-and-bound algorithm has processed the resulting optimization problem and has been able to produce solutions, answering to many proposed scenarios. Those solutions have been considered important for decision making support to the Company Staff Department. Based in the year 2001 behaviour, workforce requirements were discussed for different daily mail amounts entered in TA₅, and considering different levels for the quality criteria on daily total mail treated.

Other changes on the parameters could have been considered, namely variations on the type and duration of working shifts, periods of failure in some TUs and other scenarios that have not been discussed in this work.

The next step of this collaboration with CTT should involve the study of the other TAs, by adapting Formulation 1 to those specific treatment areas. The last step should incorporate in a single model all the TAs staff planning structure, characterizing a global environment and considering the employees interchange among TAs. This model is more challenging, especially from the optimization point of view, eventually requiring the use of decomposition techniques to obtain good solutions in a reasonable amount of time.

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Appendix A

P1 matrix, where $p_{1_{ri}}$ is the mail-flow transfer rate from TU_r to TU_i ($r, i = 1, \dots, M$) in the 17:00–23:00 period

$$P1 = \begin{bmatrix} 0 & 0.09 & 0.16 & 0.74 & 0 & 0.007 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.46 & 0.54 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.36 & 0.64 & 0 \\ 0 & 0 & 0 & 0 & 0.47 & 0 & 0.25 & 0.28 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.53 & 0.47 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} .$$

P2 matrix, where $P2_{ri}$ is the mail-flow transfer rate from TU_r to TU_i ($r, i = 1, \dots, M$) in the 23:00–04:00 period

$$P2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0.47 & 0 & 0 & 0.53 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} .$$

Parameters q_{ij} ($i = 1, \dots, M - 1, j = 1, \dots, N$) representing the number of mail-objects arrived directly to each TU per period are given below:

Period	TU1	TU2	TU3	TU4	TU5	TU6	TU7	TU8
1 → 17:00–17:15	2345	0	0	0	0	0	0	0
2 → 17:15–17:30	2345	0	0	0	0	0	0	0
3 → 17:30–17:45	0	0	0	0	0	0	0	0
4 → 17:45–18:00	0	0	0	0	0	0	0	0
5 → 18:00–18:15	0	0	0	0	0	0	0	0
6 → 18:15–18:30	0	0	0	0	0	0	0	0
7 → 18:30–18:45	0	0	0	0	0	0	0	0
8 → 18:45–19:00	3518	0	0	0	0	0	0	0
9 → 19:00–19:15	3518	0	0	0	0	0	0	0

Period	TU1	TU2	TU3	TU4	TU5	TU6	TU7	TU8
10 → 19:15–19:30	3518	0	0	0	0	0	0	0
11 → 19:30–19:45	3518	0	0	0	0	0	0	0
12 → 19:45–20:00	0	0	0	0	0	0	0	0
13 → 20:00–20:15	3881	0	0	0	0	0	0	0
14 → 20:15–20:30	3881	0	0	0	0	0	0	0
15 → 20:30–20:45	4951	0	0	0	0	0	0	0
16 → 20:45–21:00	4952	0	0	0	0	0	0	0
17 → 21:00–21:15	1055	0	0	0	0	0	0	0
18 → 21:15–21:30	1055	0	0	0	0	0	0	0
19 → 21:30–21:45	0	0	0	0	0	0	0	0
...
37 → 02:00–02:15	0	0	0	0	0	0	0	0
38 → 02:15–02:30	0	0	0	0	0	0	0	0
39 → 02:30–02:45	0	0	450	2078	0	8	260	0
40 → 02:45–03:00	0	0	450	2078	0	8	260	0
41 → 03:00–03:15	0	0	450	2078	0	8	260	0
42 → 03:15–03:30	0	0	0	0	0	0	0	0
43 → 03:30–03:45	0	0	0	0	0	0	0	0
44 → 03:45–04:00	0	0	0	0	0	0	0	0

Appendix B

Table 10 presents the number of employees determined per block and assigned to each TU: variables w_{ih} values.

Table 10

Number of employees determined per block and assigned to each TU: variables w_{ih} values

One hour blocks	Employees in each TU (w_{ih})							
	TU1	TU2	TU3	TU4	TU5	TU6	TU7, 8	Total
17:00–18:00	0	0	0	0	0	0	0	0
18:00–19:00	1	0	1	1	0	0	0	3
19:00–20:00	2	1	1	7	2	1	2	16
20:00–21:00	3	2	0	9	5	1	2	22
21:00–22:00	1	1	3	6	3	0	7	21
22:00–23:00	0	0	1	0	1	1	3	6
23:00–00:00	0	0	0	0	0	0	0	0
00:00–01:00	0	0	0	0	0	0	0	0
01:00–02:00	0	0	0	0	0	0	12	12
02:00–03:00	0	0	1	6	2	0	3	12
03:00–04:00	0	0	1	3	2	1	5	12

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