On the flexibility of a decision theory-based heuristic for single machine scheduling

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A B S T R A C T

This report extends prior research on the "decision theory" approach for scheduling/sequencing (DTS). Compared to other construction heuristics like priority dispatching (PD) approaches, DTS has the advantage that it is flexible regarding a diverse range of regular and non-regular objectives. Furthermore, multiple decision criteria linearly combined within a single objective function can be addressed.

For sequencing a set of jobs on a single machine, DTS estimates the total effect of selecting the next job in the sequence. To this, the completion times for all jobs resulting from this decision need to be estimated. We provide an estimator for job completion times and prove it to be the expected completion time. We also prove that DTS using this estimator provides optimum solutions for a number of single machine scheduling problems. Finally, we provide an extensive computational study comparing DTS to 38 competing PD approaches for a large variety of objectives (31). The results indicate DTS to be a flexible and viable alternative to PD approaches almost independent of specific objectives and problem instance characteristics.

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Decision theory
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1. Introduction

For more than four decades, the development and use of priority dispatching (PD) rules have played a prominent role in both scheduling theory and the practice of industrial scheduling. The PD approach is easy to understand, simple to apply, and in many cases yields good solutions. However, PD rules suffer from the defect of being aimed at a specific objective function. One rule might perform well when minimizing flow time; another rule might work best when minimizing tardiness is the criterion. As pointed out by Kanet and Zhou (1993), a second drawback of traditional PD is its inherent myopic view when selecting the next job to dispatch. All PD approaches determine a measure of urgency (a priority index) for each job and select the most urgent job (with the highest priority) to be dispatched. The inherent opportunity cost that all other jobs will not be selected is not considered. To overcome this drawback, Chryssolouris et al. (1988, 1991) developed a scheduling approach, which has become known as the "decision theory approach" (DT) for sequencing. The DT approach is based on estimating the (full) consequences (on the objective function value) of selecting a given job to be dispatched next. Thereby, the consequence of selecting one job next includes not only its effect on the objective function value but also the expected effects of all other jobs that are not chosen next. In consequence, the job that provides the most favorable expected "total" consequence is dispatched next (Kanet and Zhou, 1993).

In this paper, we further develop the idea of DT sequencing (DTS). To confirm the worthiness of DTS, we limit the analysis here to a systematic study of single machine sequencing problems in which the objective is the minimization of different functions f on the completion times of a set of jobs ($f(C)$). Beside regular functions (f_{reg}) like total flowtime or total tardiness, we also consider non-regular objective functions f_{nreg} with the restriction that no idle time is allowed. Generally, only non-delay sequences are considered. This restriction to non-delay sequences, even for non-regular objectives, is very common for different single machine sequencing objectives like the minimization of "total weighted earliness and tardiness" (see e.g., Ow and Morton, 1989; Valente and Alves, 2005), "total weighted quadratic earliness and tardiness" (Vila and Pereira, 2013), or "completion time variance" (Srirangacharyulu and Srinivasan, 2010). Focusing on single machine problems is not as restrictive as it may appear because many variants of the problem types $1||f_{reg}(C_j)$ and $1||f_{nreg}(C_j)$, though well studied, are still of theoretical and practical importance. The attention to such single machine problems is well justified as it applies to many industrial settings, e.g., paint shops in a car manufacturing

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facility (Bock and Pinedo, 2010) or any serial production facility or assembly line that is scheduled as a single entity (Pinedo, 2009). Furthermore, with the long-standing industrial focus to move from job-shop process design to work cells, one sees work cells scheduled as a single entity (Wemmerlöv and Hyer, 1989). In addition, for many more scheduling environments, like parallel machines, findings for the $1||f(C_j)$ problem can be used to optimize the sequence on each machine more efficiently (Koulamas, 2010).

Generally, we extend the work of Kanet and Zhou (1993) who demonstrated the viability of the DT approach in a small experimental study. We extend that study in several ways:

- We prove the completion time estimator used here is the expected value.
- We prove for some special cases that DTS yields optimum sequences.
- In our experimental study, we consider exhaustive sets of objective criteria (regular and non-regular) and competitive PD approaches.
- For weighted objective criteria, we carefully compare DTS to PD approaches under different assumptions to the nature of the weights (arbitrary (unrestricted), agreeable, and proportional weights).

The remainder of this paper is organized as follows. In Section 2 we specify how the DT approach can be implemented for single machine sequencing problems along with the aforementioned proofs of the unchosen jobs' completion time estimator and the special cases. Section 3 follows with a description of the experimental study conducted to compare DTS to competing PD approaches. Here, we concentrate our analysis on "conventional" PD approaches, i.e., single-pass construction heuristics. For each considered objective, we searched the literature for the best published approach with which we could compare DTS. Section 4 reports experimental results. Section 5 summarizes and comments on future research directions.

2. Decision theory-based single machine sequencing

Formally, the scope of the problems we address here is as follows. A set N of $n = |N|$ independent jobs (indexed by $j = 1, 2, \ldots, n$) has to be sequenced on a single machine that can process at most one job at a time. Preemption of jobs is not allowed and the machine is continuously available. Each job j has a ready time equal to zero, a processing time p_j, a due date d_j, a weight (tardiness penalty factor) w_j, and an earliness penalty factor h_j,. Based on the completion time C_j of job j, we compare the performance of DTS to PD approaches over a variety of regular objective functions $f_{reg}(C_1, C_2, \ldots, C_n)$ and non-regular objective functions $f_{nreg}(C_1, C_2, \ldots, C_n)$.

Generally, the literature on decision theory based sequencing or scheduling at all is quite rare. As stated above, the basic idea of DTS is attributable to Chryssolouris et al. (1988; 1991) and their MADEMA (MAnufacturing DEcision MAking) framework. Based on MADEMA, Kanet and Zhou (1993) explicitly formulate the DT approach for scheduling problems. Sridharan and Zhou (1996a;b) extend the previous work by considering release dates and the objectives total tardiness minimization and weighted earliness and tardiness minimization, respectively. Mönch et al. (2005) address the problem to schedule jobs on parallel batch machines with incompatible job families and unequal release dates. Within their genetic algorithm, they solve $1|r_j, batch, incompatible|\Sigma w_j T_j$ subproblems via DT. Their results show that the DT approach outperforms all other heuristic methods in terms of solution quality but at costs of higher computation times. DTS might also be seen as a simplification of the "Beam Search" approach of Ow and Morton (1989). Limiting the "beamwidth" parameter to one leads to DTS. For the application of Beam Search see Morton and Pentico (1993) and Sabuncuoglu and Bayiz (1999). Following these authors, the general course of action of DTS is as follows (N_t defines the set of jobs yet to be sequenced at time (decision point) t; $n_t = |N_t|$):

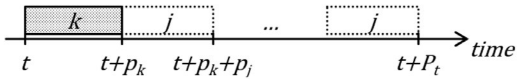

Fig. 1. Illustration of estimated job completion times.

Procedure DTS

Step 0: Set $t = 0$ and $N_t = N$.
Step 1: For each job k from N_t tentatively select k to start at time t and compute $C_k = t + p_k$.
Step 1.1: For each other job $j \in N_t$ $(j \neq k)$, estimate its completion time C_j given that job k starts at time t (via Eq. (1); see below).
Step 1.2: Compute the estimated objective value $Z_k = f(C_1,\ldots, C_n)$.
Step 2: Choose that job k^* to be sequenced next with smallest estimated Z_k.
Step 3: Set $t = t + p_{k^*}$ and remove job k^* from N_t.
Step 4: If N_t is not empty, then go to Step 1.

As can be seen by procedure DTS, the algorithm complexity is $O(n^3)$, which is somewhat higher compared to the complexities of static PD approaches (with $O(n \log n)$) and most dynamic PD approaches (with $O(n^2)$). It should be noted that some more sophisticated dynamic PD approaches also have a complexity of $O(n^3)$. Nevertheless, there are several major advantages of DTS over many PD approaches:

- First, the logic of DTS is easy to follow. It contains no complex formulas as in some PD approaches.
- Second, no a priori parameter estimation is required (e.g., look-ahead parameters or job allowance factors).
- Third, DTS takes a global view in selecting a job to be sequenced next (global in the sense that it considers the opportunity cost of not selecting other jobs), which sets it apart from "myopic" PD approaches that considers only the attributes of each job individually (see e.g., Sridharan and Zhou, 1996a).
- Fourth, and most importantly, it is flexible with regard to the objective criterion. Except for changing the objective function, no other changes regarding the implementation are required.

2.1. Estimating job completion times for DTS

To apply DTS, we need to estimate job completion times as required in Step 1.1 of Procedure DTS shown above. To estimate these completion times given that some job k is scheduled next, we use the following estimator of a job j's completion time $\tilde{E}(C_j)$ as used by Sridharan and Zhou (1996a;b):

$$\tilde{E}(C_j) = t + \frac{p_k + p_j + P_t}{2} \quad \forall j \in N_t, \ \ j \neq k. \tag{1}$$

In expression (1), P_t refers to the total processing time of all jobs $j \in N_t$ to be sequenced at this point in time ($P_t = \sum_{j \in N_t} p_j$). At time t, we tentatively select job k to be sequenced and thus, any other unselected job j will complete processing somewhere in the interval $[t + p_k + p_j, t + P_t]$ (Fig. 1 illustrates).

We prove in the following that this estimator (expression (1)) is the expected value of a job's completion time. Note that when a job k is chosen, then each remaining unchosen job j can, with equal probability, occupy any one of the $u = 1, \ldots, n_t - 1$ (with $n_t = |N_t|$) positions after job k. Given any position u for job j, there are $\binom{n_t - 2}{u - 1}$ ways that $u - 1$ of the remaining $n_t - 2$ jobs can be chosen to reside in the schedule between k and j. Using $\sum_{a=0}^{b}\binom{b}{a} = 2^b$, the total number of outcomes (the event space) for all positions of job j is:

$$\sum_{u=1}^{n_t-1}\binom{n_t-2}{u-1}=2^{n-2}. \tag{2}$$

Consequently, there are 2^{n-2} events defining possible completion times for a job j. In this context, the set $B_j(u)$ contains all jobs assumed to lie between job k and job j when it occupies position u. In any such event, the completion time C_j of job j is

$$C_j(u)=t+p_k+p_j+\sum_{i\in B_j(u)}p_i. \tag{3}$$

As can be seen in (3), p_j and p_k are part of the completion time of j, independent of u, and thus are contained in each of the 2^{n-2} events. Regarding all other jobs ($N_t-\{j\}-\{k\}$), for any position u, each of these jobs could be before j or not. Therefore, they participate in exactly half of the 2^{n-2} events; i.e., they participate in 2^{n-3} events. Thus, the total processing time involved in all of the 2^{n-2} events is:

$$2^{n-2}\left(p_j+p_k\right)+2^{n-3}\sum_{i\in N_t-\{j\}-\{k\}}p_i. \tag{4}$$

In (4), $\sum_{i\in N_t-\{j\}-\{k\}}p_i$ can be replaced by $P_t-p_k-p_j$ (see Fig. 1) and thus, the estimator for $\tilde{E}(C_j)$ as defined by (1) is the expected value $E(C_j)$ for the completion time of job j:

$$\begin{aligned}E\left(C_j\right)&=t+\frac{2^{n-2}\left(p_j+p_k\right)+2^{n-3}\left(P_t-p_k-p_j\right)}{2^{n-2}}\\&=t+p_j+p_k+\frac{1}{2}\left(P_t-p_k-p_j\right)\\&=t+\frac{p_j+p_k+P_t}{2}.\end{aligned} \tag{5}$$

Note that the expected completion time $E(C_j)$ of job j is equal to the mean of its earliest and latest completion time when job k is chosen to be sequenced next.

Summarizing, expression (1) defining a job's expected completion time, can be used to estimate the completion time of all unchosen jobs (see Step 1.1 in Procedure DTS) for any problem of the type $1||f_{reg}(C_j)$ and of the type $1||f_{nreg}(C_j)$ if idle time is not allowed (i.e., only non-delay schedules are considered).

2.2. Problem variants when DTS provides optimum solutions

There are several variants for which we can prove that DTS based on expression (1) leads to optimum solutions:

- Single machine total weighted flowtime with agreeable weights: $1|agreeable\ w_j\ and\ p_j|\Sigma w_jF_j$
- Single machine total flowtime: $1||\Sigma F_j$
- Single machine total weighted flowtime with equal processing times: $1|p_j=p|\Sigma w_jF_j$
- Single machine maximum lateness with equal processing times: $1|p_j=p|max\{L_j\}$
- Single machine maximum tardiness with equal processing times: $1|p_j=p|max\{T_j\}$

For the $1||\Sigma w_jF_j$ problem, it is well known that the shortest weighted processing time (SWPT) dispatching rule generates optimum solutions (see e.g., Baker and Trietsch, 2009). In the following, we prove that DTS is equivalent to the SWPT dispatching approach when job processing times and weights are agreeable (i.e., $p_k<p_j\Rightarrow w_k\geq w_j$) and therefore also generates optimum solutions.

Theorem 1. *For $1|agreeable\ w_j\ and\ p_j|\Sigma w_jF_j$, DTS is optimum.*

Proof:

By assumption we have $p_k<p_j$ and $w_k\geq w_j$. To simplify the notation we omit the flowtime contribution of the current partial schedule that ends at time t; thus P_t simplifies to P.

In step 2 of Procedure DTS (see Section 2), DTS will choose job k to precede job j when

$$\begin{aligned}&w_kp_k+\frac{w_j\left(p_k+p_j+P\right)}{2}+\sum_{i\in N,i\neq j,k}\frac{w_i\left(p_k+p_i+P\right)}{2}\leq w_jp_j\\&+\frac{w_k\left(p_j+p_k+P\right)}{2}+\sum_{i\in N,i\neq j,k}\frac{w_i\left(p_j+p_i+P\right)}{2}.\end{aligned} \tag{6}$$

holds for all j. Since $p_k<p_j$, the third term on the left hand side (lhs) of (6) is smaller than the third term on the right hand side (rhs) of (6) and thus, it is sufficient to test if

$$w_kp_k+\frac{w_j\left(p_k+p_j+P\right)}{2}\leq w_jp_j+\frac{w_k\left(p_j+p_k+P\right)}{2}. \tag{7}$$

This can be simplified to

$$w_kp_k+w_jp_k+w_jP\leq w_jp_j+w_kp_j+w_kP. \tag{8}$$

Because $w_k\geq w_j$, the third term on the lhs of (8) is not greater than the third term on the rhs of (8) and thus, it is sufficient to test if

$$w_kp_k+w_jp_k\leq w_jp_j+w_kp_j. \tag{9}$$

Finally, because $p_k<p_j$ is given, (9) is true and so DTS will choose job k to precede j. This decision is identical to the decision of SWPT: $p_k<p_j$ and $w_k\geq w_j$ lead to $p_k/w_k\leq p_j/w_j$. Since SWPT is optimum, then so is DTS. □

The following corollary follows directly from Theorem 1.

Corollary 1.1. *For $1||\Sigma F_j$, DTS is optimum.*

Proof:

If all weights in (9) are equal to one, then we have $p_k<p_j$ and the resulting sequence is equal to the optimum solution achieved by SPT (as SPT provides optimum solutions for $1||\Sigma F_j$; Baker and Trietsch, 2009). □

Replacing the assumption of agreeable weights by equal processing times in Theorem 1 leads to the following theorems.

Theorem 2. *For $1|p_j=p|\Sigma w_jF_j$, DTS is optimum.*

Proof: Given $p_j=p\ \forall j\in N$, using DTS we have $w_kF_k=w_k(t+p)$ and $w_jF_j=w_j(t+\frac{p+P}{2})$, so that total weighted flowtime is $w_k(t+p)+(t+\frac{p+P}{2})\sum_{j\epsilon N,j\neq k}w_j$. When $w_k\geq w_j\forall j\neq k$, then $\sum_{j\epsilon N,j\neq k}w_j$ is minimum and we have sequencing by SWPT (i.e., smallest p/w_j first). □

Theorem 3. *For $1|p_j=p|max\{L_j\}$, DTS is optimum.*

Proof: Given $p_j=p\ \forall j\in N$, using DTS we have $C_k=t+p$ and $C_j=t+(P+p)/2$. Then $L_k=t+p-d_k$ and $L_j=t+\frac{P+p}{2}-d_j\ \forall j\neq k$. Then $\max\{t+p-d_k,t+\frac{P+p}{2}-d_j\}$ is smallest when $\max\{0,\frac{P}{2}+p+d_k-d_j\}$ is smallest. Clearly $\frac{P}{2}+p+d_k-d_j$ is smallest when $d_k\leq d_j\ \forall j\varepsilon J,j\neq k$. So DTS is equivalent to EDD scheduling which is known to minimize $\max\{L_j\}$ (Baker and Trietsch, 2009). □

Corollary 3.1. *For $1|p_j=p|\max\{T_j\}$, DTS is optimum.*

It is well known that EDD minimizes $\max\{T_j\}$ (Baker and Trietsch, 2009) and thus, by Theorem 3, DTS provides optimum solutions for $1|p_j=p|\max\{T_j\}$. □

Note that even though DTS provides optimal solutions for these problem variants, they are polynomially solvable using variant specific PDs which are more efficient than DTS.

3. Experimental design

To assess the overall performance of DTS applied to problems of the types $1||f_{reg}(C_j)$ and $1||f_{nreg}(C_j)$, we perform an exhaustive computational study comparing DTS to the best performing PD approaches found in literature. Beside the competing PD approaches, the study comprises of several objectives of different complexity and several problem instance sets having different characteristics.

3.1. Considered objective functions

With the goal to assess the performance of DTS particularly with regard to its flexibility concerning decision criteria, a large variety of scheduling objectives are examined. For each objective, we specify its calculation formula, abbreviation, name, and PD approaches providing optimum solutions or PD approaches explicitly designed for this objective function (for details on the PD approaches, see Section 3.2 and the Appendix). For some of the more unusual objectives, we additionally provide literature references (if available).

Regarding the subsequently specified regular objective functions, $F = \Sigma F_j$ refers to total flowtime (with $F_j = C_j - 0$), $T = \Sigma T_j$ refers to total tardiness (with $T_j = \max\{C_j - d_j, 0\}$), and $U_j = 1$ indicates that job j is tardy (else $U_j = 0$).

3.1.1. Regular single-criteria objectives

- ΣF_j, F, total flowtime: SPT (optimum).
- $\Sigma w_j F_j$, WF, total weighted flowtime: SWPT (optimum).
- ΣT_j, T, total tardiness: EDD, MST, MDD, EHD, CR, CoverT, UATC, UMATC, UAR, UBACK.
- $\Sigma w_j T_j$, WT, total weighted tardiness: EHD, WEDD, WMDD, WCR, WCoverT, ATC, MATC, AR, MAR, BACK.
- $\sum T_j^2$, QT, total quadratic tardiness: UQAR, UQB6.
- $\sum w_j T_j^2$, WQT, total weighted quadratic tardiness: QAR, QB6.
- $max\{T_j\}$, maxT, maximum tardiness: EDD (optimum).
- $max\{w_j T_j\}$, maxWT, maximum weighted tardiness: BT31WT (optimum)
- ΣU_j, U, total number of tardy jobs: Moore (optimum)
- $\Sigma w_j U_j$, WU, total weighted number of tardy jobs.

3.1.2. Regular linear composite bi-criteria objectives

To assess the capabilities of DTS solving problems with linear composite bi-criteria objectives forming a regular objective function we use the following objectives (on multi-criteria scheduling in general see e.g., Dileepan and Sen, 1988; Chen and Bulfin, 1993; Nagar et al., 1995; Hoogeveen, 2005):

- $\Sigma F_j + \Sigma T_j$, F + T, total flowtime and total tardiness: Morton and Pentico (1993).
- $\Sigma w_j F_j + \Sigma w_j T_j$, WF + WT, total weighted flowtime and total weighted tardiness: van Wassenhove and Gelders (1978).
- $\sum F_j + \sum T_j^2$, F + QT, total flowtime and quadratic tardiness; here the quadratic tardiness expresses a preference for the tardiness criterion.
- $\sum w_j F_j + \sum w_j T_j^2$, WF + WQT, total weighted flowtime and weighted quadratic tardiness; here the quadratic tardiness expresses a preference for the tardiness criterion.
- $p\Sigma F_j + q\, max\{T_j\}$, F + maxT, weighted sum of total flowtime and maximum tardiness: Sen and Gupta (1983); here, we use equal preference weights ($p = q = 1$).
- $p\Sigma w_j F_j + q\, max\{w_j T_j\}$, WF + maxWT, weighted sum of total weighted flowtime and maximum weighted tardiness: Sen and Gupta (1983); here, we use equal preference weights ($p = q = 1$).

3.1.3. Non-regular single-criteria objectives

The following non-regular objective functions are studied (with $L_j = C_j - d_j$ referring to the lateness of job j, $E_j = \max\{d_j - C_j, 0\}$ referring to earliness of job j and *Var* referring to variance):

- $\frac{\sum T_j}{\sum U_j}$, CMT, conditional mean tardiness: Kanet and Hayya (1982) and Baker (1984).
- $\sqrt{\frac{1}{n}\sum T_j^2}$, RMST, root-mean-squared tardiness: Mebarki and Shahzad (2013).
- $\sum L_j^2$, QL, total quadratic lateness: Gupta and Sen (1983)and Schaller (2002).
- $\sum w_j L_j^2$, WQL, total weighted quadratic lateness: Sen et al. (1995).
- $\frac{1}{n}\sum (C_j - \bar{C})^2$ with $\bar{C} = \frac{1}{n}\sum C_j$, CTV, completion time variance: SMV; Merten and Muller (1972).
- $\frac{1}{n}\sum w_j (C_j - \bar{C}^w)^2$ with $\bar{C}^w = \frac{1}{\sum w_j}\sum w_j C_j$, WCTV, weighted completion time variance: WSMV; Cai (1995) and Nessah and Chu (2010).
- $\frac{1}{n}\sum (T_j - \bar{T})^2$ with $\bar{T} = \frac{1}{n}\sum T_j$, TV, tardiness variance: Haupt (1989).
- $\frac{1}{n}\sum w_j (T_j - \bar{T}^w)^2$ with $\bar{T}^w = \frac{1}{\sum w_j}\sum w_j T_j$, WTV, weighted tardiness variance.
- $\frac{1}{n}\sum (L_j - \bar{L})^2$ with $\bar{L} = \frac{1}{n}\sum L_j$, LV, lateness variance: Haupt (1989).
- $\frac{1}{n}\sum w_j (L_j - \bar{L}^w)^2$ with $\bar{L}^w = \frac{1}{\sum w_j}\sum w_j L_j$, WLV, weighted lateness variance.

3.1.4. Non-regular linear composite bi-criteria objectives

The following linear composite bi-criteria objectives forming non-regular objective functions are studied to assess the flexibility of DTS:

- $\Sigma(h_j E_j + w_j T_j)$, WE + WT, total weighted earliness and weighted tardiness: LIN-ET, EXP-ET, WPT-MS.
- $\sum(E_j + T_j^2)$, E + QT, total earliness and quadratic tardiness: EQTP_EXP.
- $\sum(h_j E_j^2 + w_j T_j^2)$, WQE + WQT, total weighted quadratic earliness and weighted quadratic tardiness: ETP_v2, ETP_LIN_vk.
- $\sum(F_j + L_j^2)$, F + QL, total flowtime and quadratic lateness; here the quadratic lateness expresses a preference for the lateness criterion.
- $\sum(w_j F_j + w_j L_j^2)$, WF + WQL, total weighted flowtime and weighted quadratic lateness; here the quadratic lateness expresses a preference for the lateness criterion.

3.2. Competing PD approaches

For each of the objective functions, we searched the literature to find (most) competitive PD rules to compare with DTS. The following Table 1 provides a complete list of the PD approaches we consider (a more detailed description of each PD approach is given in the Appendix).

Beside the PD approaches listed above, we use two other PD-like solution methods for three specific objectives because no suitable PD approaches exist.

First, for the objective total number of tardy jobs (U), we use Moore's Algorithm ("Moore") also known as Hodgson's Algorithm for comparison (Moore, 1968). This algorithm is quite similar to a PD approach and as Moore's Algorithm calculates optimum solution, no PD approaches have been developed for this objective.

Second, for the objectives completion time variance (CTV) and weighted completion time variance (WCTV), we use the SMV procedure as described in Kanet (1981) because this approach is the most competitive PD-like approach (cf. Manna and Prasad, 1999;

Table 1
Considered PD approaches.

Abb.	Short description	Reference
SPT	Shortest processing time	Baker and Trietsch (2009)
SWPT	Shortest weighted processing time	Baker and Trietsch (2009)
EDD	Earliest due date	Baker and Trietsch (2009)
WEDD	Weighted earliest due date	Kanet and Li (2004)
EHD	–	Arkin and Roundy (1991)
MST	Minimum slack time	Baker and Trietsch (2009)
MDD	Modified due date	Baker and Bertrand (1982)
WMDD	Weighted modified due date	Kanet and Li (2004)
CR	Critical ratio	Kanet and Li (2004)
WCR	Weighted critical ratio	Kanet and Li (2004)
CoverT	Cost over time	Carroll (1965); adapted by Morton and Pentico (1993) and Kanet and Li (2004)
WCoverT	Weighted cost over time	Morton and Pentico (1993)
ATC (UATC)	Apparent tardiness costs (unweighted)	Rachamadugu and Morton (1983), Vepsalainen and Morton (1987)
MATC (UMATC)	Modified apparent tardiness costs (unweighted)	Alidaee and Ramakrishnan (1996)
AR (UAR)	(unweighted)	Alidaee and Ramakrishnan (1996), Valente and Schaller (2012)
MAR (UMAR)	Modified AR (unweighted)	Valente and Schaller (2012)
QAR (UQAR)	Quadratic AR (unweighted)	Valente and Schaller (2012)
BT31T	Backward dispatching to minimize maximum cost (e.g., tardiness)	Lawler (1973), Baker and Trietsch (2009)
BT31WT	Backward dispatching to minimize maximum cost (e.g., weighted tardiness)	Lawler (1973), Baker and Trietsch (2009)
BACK (UBACK)	Backward dispatching (unweighted)	Valente and Schaller (2012)
QB6 (UQB6)	Quadratic backward version 6 (unweighted)	Valente and Schaller (2012)
LIN-ET	Linear extended local optimum earliness/tardiness	Ow and Morton (1989)
EXP-ET	Exponential extended local optimum earliness/tardiness	Ow and Morton (1989), Valente and Alves (2005)
EXP-ET-VA	EXP-ET version with adaptive κ	Valente and Alves (2005)
WPT-MS	Weighted processing time and minimum slack	Valente and Alves (2005)
EQTP-EXP	Earliness and quadratic tardiness penalty with an exponential function"	Valente (2007)
ETP-v2	Earliness/tardiness priority dispatching	Valente and Alves (2008)
ETP_LIN_vk	Earliness/tardiness priority dispatching with dynamic look ahead	Valente and Alves (2008)

Srirangacharyulu and Srinivasan, 2010). For the WCTV objective, we slightly adapted the basic SMV procedure by using the WCTV (abbreviated WSMV) criteria to decide if we insert the largest unscheduled job on the immediate left or on the immediate right of the smallest (scheduled) job (cf. Kanet, 1981; p. 1456).

3.3. Problem instances

After analyzing the considered objective functions (see Section 3.1), it is obvious that problem instances must be differentiated according to processing times, weights, due dates, and earliness penalties. To provide suitable instances, we generally follow the approach used to generate problem instances for the single machine total tardiness problem as provided by the OR-Library (http://people.brunel.ac.uk/~mastjjb/jeb/orlib/wtinfo.html) but introduce further generation parameters. Like Vila and Pereira (2013) and others (e.g., Valente, 2010), we distinguish between processing times with a low (PTVL) and a high variability (PTVH). To that end, processing times are drawn from two discrete uniform distribution restricted by [45, 55] and [1, 100], respectively. With regard to weights (and tardiness penalty factor), we also distinguish between a low (WVL) and a high variability (WVH) and additionally distinguish between unrestricted (arbitrary) weights (UW), proportional weights (PW; with $w_j = p_j$), and agreeable weights (AW; with $p_k \leq p_j$ then $w_k \geq w_j$). For the weights, we use the discrete uniform distributions restricted by [45, 55] and [1, 100] for WVL and WVH, respectively. For earliness penalties h_j, we differentiate between identical earliness penalties (IEP; with $h_j = w_j$), half tardiness penalties (HEP; with $h_j = {}^1\!/_2 w_j$), and unrestricted (arbitrary) earliness penalties with a high variability (UEPVH) and a low variability (UEPVL; drawn from the same uniform distribution as the weights). To determine due dates, we follow the approach widely used in literature and draw integer due dates from the discrete uniform distribution restricted by $[P\ (1 - tf - \frac{rdd}{2}),\ P(1 - tf + \frac{rdd}{2})]$ with $P = \sum_{j \in N} p_j$, mean tardiness (tightness) factor *tf*, and relative due date range factor *rdd* (first authors using such kind of due date generation parameters are Srinivasan, 1971; Wilkerson and Irwin, 1971). Both, tardiness factor and due date range factor are set to 0.2, 0.4, 0.6, 0.8, and 1.0. Concerning the number of jobs per instance, we set n to 25, 50, 100, 200, 400, and 800. Table 2 lists the five generated instance sets S1 to S5 according to their basic characteristics. For each combination of characteristics, we randomly generate 10 instances and the resulting instance subsets are named e.g., S1−1 to S1−12.

Altogether, 93,840 instances within the five instance sets and 9384 subsets are generated.

3.4. Experimental design summary

To keep the experimental study manageable, we limit the comparison of DTS to only those PD approaches that we judged to be relevant for a specific objective. In doing so, we aggregated the experimental study into six clusters C1 to C6 whereby each cluster is defined by a subset of objectives, a subset of solution methods, and a corresponding instance set. Table 3 summarizes the experimental design.

It is important to note that we included also those objectives in our analysis where an optimum polynomial algorithm is known (i.e., F, WF, maxT, maxWT, and U). Of course, DTS is certain to be outperformed by these algorithms but as we intend to assess the flexibility of DTS regarding all "standard" objectives, these are still considered within the experimental study.

Table 2
Generated instance sets, parameters, and total number of (TNO) instances.

Name	Num. of jobs	Proc. time variability	Weight variability	Weight type	Due date tardiness factor (tf)	Due date range factor (rdd)	Earliness penalty type	TNO subsets/instances
S1	25, 50, 100, 200, 400, 800	PTVL, PTVH						12/120
S2	25, 50, 100, 200, 400, 800	PTVL, PTVH	WVL, WVH	UW, PW, AW				72/720
S3	25, 50, 100, 200, 400, 800	PTVL, PTVH			0.2, 0.4, 0.6, 0.8, 1.0	0.2, 0.4, 0.6, 0.8, 1.0		300/3000
S4	25, 50, 100, 200, 400, 800	PTVL, PTVH	WVL, WVH	UW, PW, AW	0.2, 0.4, 0.6, 0.8, 1.0	0.2, 0.4, 0.6, 0.8, 1.0		1800/18,000
S5	25, 50, 100, 200, 400, 800	PTVL, PTVH	WVL, WVH	UW, PW, AW	0.2, 0.4, 0.6, 0.8, 1.0	0.2, 0.4, 0.6, 0.8, 1.0	IEP, HEP, UEPVL, UEPVH	7200/ 72,000

Table 3
Experiment cluster, characteristics, and total number of (TNO) experiments.

Name	Objectives	TNO objectives	Solution methods	TNO solution methods	Inst. set	TNO instances	TNO experiments
C1	F, CTV	2	DTS, SPT, SMV	3	S1	120	720
C2	WF, WCTV	2	DTS, SWPT, WSMV	3	S2	720	4320
C3	T, QT, maxT, U, F + T, F + QT, F + maxT, CMT, RMST, QL, TV, LV, F + QL	13	DTS, SPT, EDD, EHD, MST, MDD, CR, CoverT, UATC, UMATC, UAR, UMAR, BT31T, UQAR, UBACK, UQB6, Moore	17	S3	3000	663,000
C4	WT, WQT, maxWT, WU, WF + WT, WF + WQT, WF + maxWT, WQL, WTV, WLV	10	DTS, SWPT, WEDD, EHD, WMDD, WCR, WCoverT, ATC, MATC, AR, MAR, BT31WT, QAR, BACK, QB6	15	S4	18,000	2,700,000
C5	E + QT	1	DTS, SPT, EDD, EHD, MST, MDD, CR, CoverT, UATC, UMATC, UAR, UMAR, BT31T, UQAR, UBACK, UQB6, LIN-ET, EXP-ET, EXP-ET-VA, WPT-MS, EQTP_EXP, ETP_v2, ETP_LIN_vk, SMV	24	S3	3000	72,000
C6	WE + WT, WQE + WQT, WF + WQL	3	DTS, SWPT, WEDD, EHD, WMDD, WCR, WCoverT, ATC, MATC, AR, MAR, BT31WT, QAR, BACK, QB6, LIN-ET, EXP-ET, EXP-ET-VA, WPT-MS, EQTP_EXP, ETP_v2, ETP_LIN_vk, WSMV	23	S5	72,000	4,968,000

Altogether, the experiment clusters define 8,408,040 experiments. Of course, for all solution methods other than DTS, the sequencing for a problem instance is only done once and the objectives given by a cluster are evaluated by this sequence. This reduces computational efforts remarkably. Instance generation, DTS and PD approaches, and experiment configuration and management are implemented within a comprehensive Java framework. Experiments are executed on four workstations with an Intel Core^{TM} i7-2600 CPU @ 3.40 GHz and 16 GB RAM.

4. Experimental results

The main goal of the following analysis is to assess the solution quality achieved with DTS and particularly, to assess the flexibility of DTS to tackle regular and non-regular objectives with different (single and linear composite bi-criteria) criteria. After the definition of the central key performance indicator, we present general insights and aggregated results before comparing DTS to the best performing PD approaches (with regard to a specific objective).

Table 4
Influence of the weight variability on the solution quality (with regard to the weight type).

	WF	WT	WQT	maxWT	WU	WF + WT	WF + WQT	WF + maxWT	WCTV	WQL	WTV	WLV	WE + WT	WQE + WQT	WF + WQL
UW	+	+	+	+	+	++	+	++	+	+	0	0	+	+	+
AW	+	+	+	++	+	++	+	++	+	+	0	0	+	+	+

4.1. Key performance indicator

To assess the solution quality of DTS and the PD approaches, we use the key performance indicator “mean relative improvement versus the worst objective function value” (MRIW; Valente and Schaller, 2012). MRIW is used since objective function values could be equal to zero for some instances and thus, relative improvements versus the best objective function value are troublesome (since division by zero is undefined). The MRIW is specified as follows: For a given problem instance and objective function, the relative improvement versus the worst result for a solution method M_s (with $s = 1, \ldots, z$) is indicated by RIW_s. Let v_{best} and v_{worst} indicate the best (smallest) and worst (largest) objective function value of all z solution methods (with regard to a single problem instance and a single objective function), respectively. If $v_{best} = v_{worst}$, then RIW_s is set to zero. Otherwise, $RIW_s = |(v_{worst} - v_s)/v_{worst} \bullet 100|$, where v_s is the objective function value of solution method M_s. Based on this definition of RIW_s, mean values (MRIW) for each solution method per objective function (see Section 3.1) and instance set or subset (see Section 3.3) can be calculated.

4.2. General insights

To keep the analysis manageable, we omit a detailed description of the results concerning individual objectives, solution methods, and instance characteristics (e.g., number of jobs, processing time variability …) but give some general insights before presenting aggregated results.

- The number of jobs per instance has only a small impact on the solution quality of all solution methods, independent of the objective.
- Concerning the processing time variability (PTVL vs PTVH), their influence on the solution quality is
- Noteworthy for the objectives F, CTV, WF, WCTV, F + T, F + maxT, F + QL, WT, WQT, maxWT, WF + WT, WF + maxWT, and
- Minor for the objectives T, QT, maxT, U, F + QT, CMT, RMST, QL, TV, LV, WU, WF + WQT, WQL, WTV, WLV, E + QT, WE + WT, WQE + WQT, WF + WQL.
- The influence of the weight variability (WVL vs WVH) on the solution quality is analyzed with regard to the weight types UW and AW; weight type PW is not considered here because $w_j = p_j$ holds for all jobs and influences are minimal in general). In Table 4, “++” means a high influence, “+” means a small influence, and “0′’ depicts a minimal influence.
- Regarding the influence of the earliness penalty characteristics (IEP, HEP, UEPVH, UEPVL) on the solution quality, we can state that the influence is generally small for the objectives WE + WT, WQE + WQT, and WF + WQL. The influence slightly increases for WE + WT and WQE + WQT if the processing time variability is high and/or earliness penalties are unrestricted and have a high variability (PTVH and/or UEPVH).

The detailed results forming the data basis of these insights can be found in the supplementary material (file “Detailed-Analysis.xlsx”).

Computation times for different (groups of) solution methods (cf. Section 3.2 and the Appendix) are listed in Table 5. Each cell entry reports mean computation times for all objectives a solution method is applied to (cf. Table 3) depending on the number of jobs n. The mean (with regard to n and all corresponding computation times) relative standard deviation (coefficient of variation) of 0.77% shows that the computation time measurements are almost undistorted.

As the computation times in Table 5 show, DTS is outperformed by all solution methods (as to be expected), even by the PD approaches having a complexity of $O(n^3)$. This is due to the fact that the objective value calculation has larger computational efforts than just determining a parameter of a solution method (e.g., the parameter p_j^{mod} for BACK with $O(n^3)$). Observe that computation times of DTS increase faster with increasing problem instance size than the other solution methods.

4.3. Aggregated results

To assess the overall performance of DTS and particularly its flexibility, we present the MRIWs of DTS and all competing PD approaches (with regard to the considered objective functions) in the following tables (Tables 6–9). Each cell entry (except in the “MEAN” columns) represents the mean values for all instances of the corresponding instance set (cf. Tables 2 and 3). Bold cell entries mark the best (largest) MRIW per objective and italic and underlined cell entries mark the second best MRIW when DTS is best.

The MRWIs in Table 6 shows that DTS is generally suitable and competitive for regular, unweighted objectives except for the objective maxT. For linear composite bi-criteria objectives, DTS even achieves best results for two objectives. Moreover, the flexibility of DTS is substantiated by the largest mean MRWIs (see columns 7 and 11).

Analyzing the MRWIs for regular, weighted objectives (cf. Table 7), it can also be stated that DTS is suitable and competitive (with some limitations regarding the objective maxWT). In turn, DTS achieves best results for two regular, weighted linear composite bi-criteria objectives. Again, the flexibility of DTS is substantiated by the largest mean MRWIs (see columns 6 and 10).

Also for non-regular, unweighted objectives, DTS is suitable and competitive (as can be seen by the MRWIs in Table 8) but regarding the CTV objective, it must be stated that DTS is clearly outperformed by SMV. Concerning the flexibility, DTS is minimal outperformed by CR (see columns 7 and 10).

Analyzing the MRWIs in Table 9, DTS outperforms all other solution methods regarding the non-regular, weighted objectives WQL, WTV, and WLV. Regarding the other non-regular, weighted objectives, DTS is outperformed but competitive (even for the objective WCTV). Regarding the flexibility, DTS is most flexible for non-regular, weighted, single-criteria objectives and second best for linear composite bi-criteria objectives (see columns 5 and 9).

Summarizing the analysis of MRWIs in Tables 6–9 (and particularly the MEAN columns) with regard to the flexibility of DTS to tackle different objectives, we can report that DTS is suitable to solve almost any objective competitively and that no other PD approach shows similar flexibility when considering all types of objectives.

4.4. Comparing DTS to best performing PD approaches

In this section, we present a more detailed analysis by comparing the solution quality of DTS to a reference solution method

Table 5
Mean computation times (in milliseconds).

n	DTS	Static PD approaches with $O(n \cdot \log n)$	Dynamic PD approaches with $O(n^2)$	Dynamic PD approaches with $O(n^3)$	Explicit earliness-tardiness PD approaches	(W)SMV	Moore
25	0.23	0.01	0.03	0.13	0.02	0.04	0.01
50	1.42	0.02	0.05	0.47	0.05	0.04	0.06
100	12.58	0.02	0.13	2.85	0.13	0.13	0.09
200	98.19	0.04	0.47	21.70	0.47	0.33	0.68
400	878.74	0.08	1.96	199.56	1.84	1.49	2.53
800	9682.21	0.18	8.29	2173.49	7.99	8.09	13.39

Table 6
MRIWs by solution method and regular, unweighted objectives.

	Single criterion						Bi-criteria			
	F	T	QT	maxT	U	MEAN*	F+T	F+QT	F+maxT	MEAN**
DTS	**18.4**	49.1	63.4	35.8	54.5	**50.7**	**21.0**	62.8	**20.5**	**34.8**
SPT	**18.4**	9.9	11.2	5.7	25.4	13.1	18.0	11.1	*20.3*	16.5
EDD		39.2	57.1	**58.9**	28.2	45.9	7.4	56.5	5.6	23.2
EHD		36.0	54.3	**58.9**	27.8	44.2	4.8	53.7	3.3	20.6
MST		35.1	53.3	58.6	27.3	43.6	4.3	52.7	2.9	20.0
MDD		**51.6**	55.1	36.5	36.1	44.8	14.9	54.5	11.1	26.8
CR		46.4	63.6	55.1	23.5	47.2	10.0	62.9	7.1	26.7
CoverT		44.0	48.8	29.4	16.4	34.7	19.0	48.3	16.7	28.0
UATC		51.5	54.9	36.2	40.4	45.7	15.6	54.2	11.9	27.2
UMATC		51.5	54.7	36.1	40.9	45.8	15.7	54.1	12.0	27.3
UAR		51.2	54.2	35.5	40.3	45.3	19.8	53.7	16.5	30.0
UMAR		51.1	54.0	35.2	41.0	45.3	19.9	53.4	16.6	30.0
BT31T		36.8	55.1	**58.9**	28.2	44.8	2.2	54.4	0.5	19.0
UQAR		47.4	63.8	55.2	30.8	*49.3*	17.4	63.3	14.9	*31.9*
UBACK		45.0	39.2	20.9	52.3	39.4	*20.0*	38.9	17.3	25.4
UQB6		47.4	**64.0**	55.7	28.9	49.0	16.8	**63.4**	14.0	31.4
Moore		38.5	32.0	30.6	**64.2**	41.3	12.0	31.5	10.5	18.0
SMV	0.0									
DTS - best PD approach	0.0	−2.5	−0.6	−23.1	−9.6	1.4	1.0	−0.6	0.2	2.9
Instance set	S1	S3	S3	S3	S3		S3	S3	S3	
Cluster	C1	C3	C3	C3	C3		C3	C3	C3	

* Mean values with regard to the objectives T, QT, maxT, and U.
** Mean values with regard to the objectives F+T, F+QT, and F+maxT.

(RSM). This RSM (best performing PD approach) is determined by the largest MRWI with regard to a specific objective (as highlighted in Tables 6–9).

Generally, we compute all following key figures (Tables 10–13) for each instance subset individually and then aggregate them by their mean values with regard to all instance subsets of an instance set (cf. Table 2). The detailed data and statistical analysis can be found in the supplementary material (file "DTSvsRSM - Statistical-Analysis.xlsx").

For comparing the solution quality of the two solution methods RSM and DTS, we report the mean relative difference of MRWIs (MRdiff = (MRIW(DTS)-MRIW(RSM))/max{MRIW(DTS), MRIW(RSM)}) and the mean coefficient of variation of the relative difference of these MRWIs (CVdiff). Additionally, we performed two-sided, pairwise t-tests on each instance subset to test whether the difference of the MRIWs (MRIW(DTS)-MRIW(RSM)) is statistically significant (<0.05; with degrees-of-freedom df = 9) or not. Because of the different number of instance subset in each instance set, we report the mean relative number of statistically significant different MRIWs ("Sig. different MRWIs") in addition to the mean p-value and the mean t-value (these mean values are with regard to all instance subsets of the corresponding instance set).

Since the key figures presented in Tables 10–13 speak for themselves, we omit their discussion in detail but report further insights regarding the solution quality of DTS compared to RSM whenever noteworthy.

As stated above, DTS is not competitive for maxT. Concerning this objective, we can additionally state that the solution quality of DTS deteriorates with increasing *rdd* and *tf*. A similar observation can be reported for objective U: if *rdd* increases, the performance of DTS worsens. Regarding objective F + T, the good performance of DTS improves with increasing *rdd* if the processing time variability is high (PTVH) and deteriorates with increasing *rdd* if the processing time variability is low (PTVL). For the objective F + maxT, DTS outperforms SPT for instance subsets of any type.

For WF, SWPT outperforms DTS negligible for instance subsets with unrestricted weights except subsets having a high processing time variability. Regarding the objective WT, we can report that ATC has largest benefits for instances with $tf = 0.6$ and that a high due date range factors slightly deteriorates the performance of DTS. In contrast, for WQT, high due date range factors improves the solution quality of DTS. Concerning the objective maxWT, we can state that the performance of DTS deteriorates with increasing *rdd* and *tf* and that for WU, the solution quality of DTS remarkably improves with increasing *rdd* and *tf*. For the objective WF + WQT DTS is only outperformed by QB6 for instance subsets with a high processing time variability, proportional weights, and high due date tightness factor ($tf \geq 0.6$). An interesting observation can be reported for WF + maxWT, SWPT achieves for instance subsets with unrestricted or agreeable weights almost identical good results as DTS but provides the worst solution quality for most instance subsets having proportional weights.

Table 7
MRIWs by solution method and regular, weighted objectives.

	Single criterion						Bi-criteria			
	WF	WT	WQT	maxWT	WU	MEAN*	WF + WT	WF + WQT	WF + maxWT	MEAN**
DTS	20.5	53.8	66.5	51.1	**56.8**	**57.1**	**25.7**	65.8	**23.8**	**38.5**
SWPT	**20.8**	16.1	14.7	17.0	28.9	19.2	22.7	14.7	23.7	20.4
WEDD		39.4	52.0	59.1	30.3	45.2	17.0	51.6	14.5	27.7
EHD		34.7	49.7	50.0	27.9	40.6	5.5	48.8	3.3	19.2
WMDD		41.3	38.6	45.5	45.4	42.7	21.3	38.3	18.7	26.1
WCR		44.4	59.9	52.5	23.9	45.2	10.4	58.8	7.0	25.4
WCoverT		43.7	46.2	31.8	16.1	34.5	19.4	45.6	16.2	27.1
ATC		**56.2**	57.9	47.9	42.2	51.1	19.2	57.0	14.6	30.3
MATC		56.1	57.8	47.8	42.6	51.1	19.4	56.9	14.7	30.3
AR		56.1	57.0	47.0	43.4	50.9	24.4	56.4	20.1	33.6
MAR		55.9	56.6	46.5	43.8	50.7	24.5	56.0	20.2	33.6
BT31WT		45.3	59.1	**71.5**	31.6	50.0	8.6	57.9	5.6	24.0
QAR		53.0	67.4	65.4	32.2	54.5	22.2	66.7	18.6	35.9
BACK		52.3	47.2	48.3	54.8	50.6	23.6	46.6	20.1	30.1
QB6		53.2	**67.7**	65.1	31.4	54.3	20.5	**66.9**	16.7	34.7
WSMV	0.0									
DTS - best PD approach	−0.2	−2.4	−1.2	−20.3	2.0	2.5	1.3	−1.1	0.2	2.6
Instance set	S2	S4	S4	S4	S4		S4	S4	S4	
Cluster	C2	C4	C4	C4	C4		C4	C4	C4	

* Mean values with regard to the objectives WT, WQT, maxWT, and WU.
** Mean values with regard to the objectives WF + WT, WF + WQT, and WF + maxWT.

Table 8
MRIWs by solution method and non-regular, unweighted objectives.

	Single criterion							Bi-criteria		
	CTV	CMT	RMST	QL	TV	LV	MEAN*	F + QL	E + QT	MEAN**
DTS	3.6	67.2	49.3	57.3	71.4	70.9	63.2	57.3	65.6	61.4
SPT	0.0	30.7	6.1	7.7	12.0	5.0	12.3	7.7	21.5	14.6
EDD		61.7	45.3	52.3	71.5	71.2	60.4	52.2	60.0	56.1
EHD		59.6	43.7	50.1	74.8	74.6	60.6	50.0	57.3	53.7
MST		59.2	43.0	49.7	74.3	**74.6**	60.2	49.7	56.5	53.1
MDD		66.0	43.2	50.3	51.0	55.9	53.3	50.3	58.0	54.1
CR		**67.3**	49.4	**59.3**	70.9	70.3	**63.4**	**59.2**	66.1	**62.7**
CoverT		65.7	36.1	23.1	45.3	22.4	38.5	23.1	52.7	37.9
UATC		63.5	42.9	49.2	50.7	54.6	52.2	49.2	57.8	53.5
UMATC		63.0	42.8	49.0	50.5	54.4	52.0	49.0	57.7	53.3
UAR		63.2	42.3	39.2	49.9	44.4	47.8	39.2	57.2	48.2
UMAR		62.5	42.1	38.9	49.6	44.0	47.4	38.9	57.0	47.9
BT31T		59.9	44.2	46.8	**75.2**	64.2	58.1	46.8	58.1	52.5
UQAR		65.4	49.6	46.8	70.6	59.6	58.4	46.8	66.1	56.4
UBACK		43.8	31.3	23.2	32.6	23.9	31.0	23.1	44.1	33.6
UQB6		66.6	**49.8**	42.2	71.1	52.0	56.3	42.1	66.2	54.2
Moore		16.1	25.5	36.1	28.6	40.7	29.4	36.1		
LIN-ET									49.8	
EXP-ET									49.9	
EXP-ET-VA									48.7	
WPT-MS									51.1	
EQTP_EXP									66.0	
ETP_v2									50.3	
ETP_LIN_vk									**66.2**	
SMV	**25.1**								4.8	
DTS - best PD approach	−21.5	−0.2	−0.5	−1.9	−3.8	−3.7	−0.2	−1.9	−0.6	−1.2
Instance set	S1	S3	S3	S3	S3	S3		S3	S3	
Cluster	C1	C3	C3	C3	C3	C3		C3	C5	

* Mean values with regard to the objectives CMT, RMST, QL, TV, and LV.
** Mean values with regard to the objectives F + QL and E + QT.

Regarding the objective CMT, the PD approach CR is most competitive for instance subsets with a high processing time variability and a low due date range factor ($rdd = 0.2$). For the objectives RMST, QL, TV, and LV, DTS's solution quality deteriorates if the processing time variability is high and the due date range factor is low ($rdd \leq 0.4$). Concerning objective F + QL, the performance of DTS is most inferior if the processing time variability is high, $rdd \leq 0.6$, and $tf = 0.6$; in almost all other cases achieves DTS competitive results.

Relating DTS's poor performance for WCTV, we would like to state that DTS particularly fails for all instance subsets with proportional weights but that DTS clearly outperforms WSMV for unrestricted weights and a high weight variability (independent of the processing time variability). Regarding the objectives WQL and WTV, the superiority of DTS improves with increasing due date tightness factor. Concerning WLV, EHD is only competitive for instance subsets with $rdd = 0.2$ or a due date range factor $tf = 1.0$.

Summarizing the comparison of DTS to RSMs, we can report that DTS is outperformed (with a mean MRdiff of −6.45%) by the reference method for many of the considered objectives (21 out of 31), however, it outperforms the reference method for 10 objectives.

Table 9
MRIWs by solution method and non-regular, weighted objectives.

	Single criterion					Bi-criteria			
	WCTV	WQL	WTV	WLV	MEAN*	WE + WT	WQE + WQT	WF + WQL	MEAN**
DTS	13.7	**60.7**	**73.9**	**74.0**	**69.6**	45.0	63.9	63.3	57.4
SWPT	1.4	8.5	11.1	6.6	8.7	12.7	17.4	16.2	15.5
WEDD		44.6	60.0	62.3	55.6	31.5	49.6	48.1	43.1
EHD		48.4	70.6	72.4	63.8	31.8	52.1	52.4	45.5
WMDD		37.1	30.6	43.3	37.0	32.6	42.8	41.4	39.0
WCR		57.0	67.6	68.7	64.4	41.8	60.1	60.0	54.0
WCoverT		24.1	40.0	21.1	28.4	30.8	29.1	29.2	29.7
ATC		51.4	50.1	55.3	52.3	43.5	55.1	54.9	51.2
MATC		51.2	50.0	55.1	52.1	43.3	54.9	54.7	51.0
AR		41.1	49.0	46.8	45.6	37.4	46.3	45.3	43.0
MAR		40.9	48.5	46.6	45.3	37.2	46.2	45.1	42.8
BT31WT		50.4	70.7	61.0	60.7	44.9	56.4	56.2	52.5
QAR		49.1	72.2	62.4	61.3	36.1	53.1	52.0	47.1
BACK		28.5	37.4	28.4	31.4	34.1	33.7	33.1	33.6
QB6		45.6	72.9	54.0	57.5	37.4	48.6	48.7	44.9
LIN-ET						48.4	51.3	48.7	49.5
EXP-ET						48.1	51.3	48.7	49.4
EXP-ET-VA						47.7	50.7	48.0	48.8
WPT-MS						48.8	51.9	49.3	50.0
EQTP_EXP						41.0	57.7	57.7	52.1
ETP_v2						**49.4**	53.6	51.6	51.5
ETP_LIN_vk						48.1	**65.7**	**63.6**	**59.1**
WSMV	**18.2**					11.2	16.8	16.7	14.9
DTS - best PD approach	−4.6	3.7	1.1	1.7	5.1	−4.4	−1.7	−0.2	−1.7
Instance set	S2	S4	S4	S4		S5	S5	S5	
Cluster	C2	C4	C4	C4		C6	C6	C6	

* Mean values with regard to the objectives WQL, WTV, and WLV.
** Mean values with regard to the objectives WE + WT, WQE + WQT, and WF + WQL.

Table 10
Comparison regarding regular, unweighted objectives.

	Single criterion					Bi-criteria		
Objective	F	T	QT	maxT	U	F + T	F + QT	F + maxT
RSM	SPT	MDD	UQB6	EDD	Moore	UBACK	UQB6	SPT
MRdiff [%]	0.00	−5.80	−1.26	−43.24	−14.61	5.20	−1.27	3.36
CVdiff	–	−1.40	−1.64	−0.71	−0.56	3.08	−1.61	2.30
Mean p-value	–	0.06	0.05	0.02	0.02	0.07	0.03	0.04
Mean t-value	–	12.47	10.57	16.41	16.08	10.65	12.45	4.58
Sig. different MRWIs [%]	0.00	68.00	69.67	89.00	91.33	81.67	92.00	72.67
Instance set	S1	S3	S3	S3	S3	S3	S3	S3
TNO subsets	12	300	300	300	300	300	300	300

Table 11
Comparison regarding regular, weighted objectives.

	Single criterion					Bi-criteria		
Objective	WF	WT	WQT	maxWT	WU	WF + WT	WF + WQT	WF + maxWT
RSM	SWPT	ATC	QB6	BT31WT	BACK	MAR	QB6	SWPT
MRdiff [%]	−0.57	−4.48	−2.35	−32.67	15.06	2.99	−2.29	30.00
CVdiff	−2.98	−1.79	−1.98	−0.95	2.39	3.82	−1.98	1.51
Mean p-value	0.00	0.07	0.04	0.03	0.05	0.05	0.03	0.08
Mean t-value	18.93	10.70	10.96	11.79	12.39	12.58	11.73	8.60
Sig. different MRWIs [%]	33.33	62.17	76.33	82.28	80.72	72.39	87.61	73.22
Instance set	S2	S4	S4	S4	S4	S4	S4	S4
TNO subsets	72	1800	1800	1800	1800	1800	1800	1800

Table 12.
Comparison regarding non-regular, unweighted objectives.

	Single criterion						Bi-criteria	
Objective	CTV	CMT	RMST	QL	TV	LV	F + QL	E + QT
RSM	SMV	CR	UQB6	CR	BT31T	MST	CR	ETP_ LIN_vk
MRdiff [%]	−86.26	0.54	−1.60	−3.50	−5.72	−6.74	−3.50	−1.04
CVdiff	−0.15	14.20	−1.51	−2.49	−3.17	−1.99	−2.49	−1.58
Mean p-value	0.00	0.07	0.05	0.02	0.10	0.16	0.02	0.03
Mean t-value	58.06	15.09	11.18	14.87	12.10	10.84	14.88	11.74
Sig. different MRWIs [%]	100.00	75.33	72.33	95.00	61.00	61.33	95.00	89.67
Instance set	S1	S3	S3	S3	S3	S3	S3	S3
TNO subsets	12	300	300	300	300	300	300	300

Table 13
Comparison regarding non-regular, weighted objectives.

Objective	Single criterion				Bi-criteria		
Objective	WCTV	WQL	WTV	WLV	WE + WT	WQE + WQT	WF + WQL
RSM	WSMV	WCR	QB6	EHD	ETP_v2	ETP_LIN_vk	ETP_LIN_vk
MRdiff [%]	−39.97	7.92	2.10	1.96	−5.42	−2.90	−0.53
CVdiff	−1.52	2.43	7.18	9.26	−2.40	−2.45	−20.21
Mean p-value	0.03	0.03	0.06	0.04	0.05	0.01	0.02
Mean t-value	37.61	15.77	10.75	13.15	14.09	16.01	15.89
Sig. different MRWIs [%]	91.67	91.61	68.17	89.00	75.28	96.69	95.31
Instance set	S2	S4	S4	S4	S5	S5	S5
TNO subsets	72	1800	1800	1800	7200	7200	7200

5. Summary

In this paper, we extended prior research on the "decision theory" approach to scheduling. To this end, the completion time estimator used here was proven to be the expected value and for several simple cases, DTS was proven to yield optimum solutions.

To assess the flexibility of DTS, we conducted an exhaustive experimental study comparing the solution quality of altogether 39 solution methods (DTS, 35 PD approaches and three PD like approaches) with regard to specific objective functions (altogether 31 different objective functions were considered). These objectives comprise eight groups: single-criterion or linear composite bi-criteria, regular or non-regular, weighted or unweighted. In three of these objective groups (cf. Tables 8 and 9), the mean performance of DTS was slightly worse (never more than −1.7%) compared to the best mean of all PD approaches but in five of these objective groups (cf. Tables 6, 7, and 9), DTS mean performance was better than that of all competing PD approaches (at least by 1.4%). The results show the great flexibility of DTS almost independent of the objective function and specific problem instance characteristics.

Aside from its flexibility, DTS has several other advantages. The logic of DTS is easy to follow, it contains no complex formulas and no parameters are required. The basic search structure remains constant regardless of the objective and we do not need a different search procedure for different problems. Furthermore, because of its demonstrated flexibility particularly for linear composite bi-criteria objectives, we can infer that DTS would perform well for more complex, yet to be determined objectives (like linear combinations of earliness, tardiness, and/or flowtime).

Further research topics could be the consideration of ready times, sequence dependent setup times and inserted idle-time (delayed/active schedules). Beside the investigation of further problem variants, a performance comparison of DTS to more complex solution methods like meta-heuristics (e.g., Tabu Search or Genetic Algorithms) could be of interest.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.cor.2018.09.005.

Appendix

In the following, we briefly describe all PD approaches used for comparison with DTS. For each approach we list its abbreviation, name, originally considered objective(s), formula and parameters to calculate the priority index π_j of job j, the "selection (sorting) strategy" indicating the job to be allocated next (min or max), reference literature, and remarks.

For PD approaches having one or more parameters, we use the best parameters proposed in literature. In the case of identical priority indices, we break ties first by non-decreasing p_j, then by non-decreasing d_j, and finally by job number as given by the instance data.

Static PD approaches with O(n • log n)

- SPT, "shortest processing time", F, $\pi_j = p_j$, min, Baker and Trietsch (2009).
- SWPT, "shortest weighted processing time", WF, $\pi_j = p_j/w_j$, min, Baker and Trietsch (2009); this rule is also abbreviated WSPT and defined by $\pi_j = w_j/p_j$ and max — with identical results.
- EDD, "earliest due date", T, $\pi_j = d_j$, min, Baker and Trietsch (2009).
- WEDD, "weighted earliest due date", WT, $\pi_j = {}^{d_j}/_{w_j}$, min, Kanet and Li (2004).
- EHD, -, $\pi_j = d_j - \frac{1}{2}p_j$, WT (T), min, Arkin and Roundy (1991).
- MST, "minimum slack time", T, $\pi_j(t) = sl_j = d_j - (t + p_j)$, min, Baker and Trietsch (2009); updating the slack after each sequencing decision does not influence the final sequence.

Dynamic PD approaches with $\mathbf{O(n^2)}$

- MDD, "modified due date", T, $\pi_j(t) = \max\{d_j, t + p_j\}$, min, Baker and Bertrand (1982).
- WMDD, "weighted modified due date", WT, $\pi_j(t) = {}^{1}/_{w_j}(\max\{p_j,\ d_t - t\})$, min, Kanet and Li (2004).
- CR, "critical ratio", T, $\pi_j(t) = {}^{(d_j - t)}/_{p_j}$, min, Kanet and Li (2004).
- WCR, "weighted critical ratio", WT, $\pi_j(t) = {}^{(d_j - t)}/_{w_j p_j}$, min, Kanet and Li (2004).
- CoverT, "cost over time", T,

$$\pi_j(t,\ \alpha) = {}^{c_j}/_{p_j} \text{ with } c_j = \begin{cases} 1 & \text{if } t \geq u_j \\ {}^{(t - u_j)}/_{(u_j - n_j)} & \text{if } n_j \leq t < u_j \\ 0 & \text{if } t\ > n_j \end{cases}$$

with $n_j = d_j - \alpha\, p_j$ (with $\alpha = \sum_{j\in N} d_j / \sum_{j\in N} p_j$) and $u_j = d_j - p_j$, max, Carroll (1965); adapted by Morton and Pentico (1993) and Kanet and Li (2004) for single machine sequencing.

- WCoverT, "weighted cost over time", WT,

$$\pi_j(t) = {}^{w_j c_j}/_{p_j}$$

with c_j as defined for COVERT, max, Morton and Pentico (1993).

- ATC, "apparent tardiness costs" (also called "apparent urgency – AU"), WT (T),

$$\pi_j(t,\ \kappa) = {}^{w_j}/_{p_j} \exp\left(-\frac{\max\{0,\ sl_j\}}{\kappa\bar{p}}\right)$$

with the static mean processing time $\bar{p} = {}^{P}\!/_{n}$ (with $P = \sum_{j \in N} p_j$), look ahead parameter $\kappa = 2.0$, and in the unweighted case $w_j = 1$ (UATC), max, Rachamadugu and Morton (1983) and Vepsalainen and Morton (1987).

- MATC, "modified apparent tardiness costs", WT (T),

$$\pi_j(t,\ \kappa) = {}^{w_j}\!/_{p_j} \exp\left(-\frac{\max\{0,\ sl_j\}}{\kappa\ \bar{p}_t} \right)$$

with the dynamic mean processing time $\bar{p}_t = {}^{P_t}\!/_{n_t}$, look ahead parameter $\kappa = 2.0$, and in the unweighted case $w_j = 1$ (UMATC), max, Alidaee and Ramakrishnan (1996).

- AR, "-", WT (T)

$$\pi_j(t,\ \kappa) = \begin{cases} {}^{w_j}\!/_{p_j} & \text{if } sl_j \le 0 \\ {}^{w_j}\!/_{p_j} \left(\frac{\kappa\ \bar{p}}{\kappa\ \bar{p} + sl_j} \right) & \text{otherwise} \end{cases}$$

with look ahead parameter $\kappa = 2.0$ and in the unweighted case (UAR) $w_j = 1$, max, originally described in Alidaee and Ramakrishnan (1996) – the version depicted here is from Valente and Schaller (2012).

- MAR, "modified AR", WT (T)

$$\pi_j(t,\ \kappa) = \begin{cases} {}^{w_j}\!/_{p_j} & \text{if } sl_j \le 0 \\ {}^{w_j}\!/_{p_j} \left(\frac{\kappa\ \bar{p}_t}{\kappa\ \bar{p}_t + sl_j} \right) & \text{otherwise} \end{cases}$$

with look ahead parameter $\kappa = 2.0$ and in the unweighted case (UMAR) $w_j = 1$, max, Valente and Schaller (2012).

- QAR, "quadratic AR", WQT (QT)

$$\pi_j(t, \sigma) = \begin{cases} {}^{w_j}\!/_{p_j} \left(\bar{p}_t + 2 \max\left\{ t + p_j - d_j,\ 0 \right\} \right) & \text{if } sl_j \le 0 \\ {}^{w_j}\!/_{p_j} \bar{p}_t \left(\frac{\kappa\ \bar{p}_t}{\kappa\ \bar{p}_t + sl_j} \right) & \text{otherwise} \end{cases}$$

with a dynamic parameter κ depending on the number of critical jobs:$\kappa = \max\{0.5,\ |C|\}$ (with set $C = \{j \in N_t | 0 < sl_j \le sl_{crit}\}$, whereby $sl_{crit} = \sigma\ P_t$ and $\sigma = 0.1$) and in the unweighted case (UQAR) $w_j = 1$, max, Valente and Schaller (2012).

- BT31T, "backward dispatching to minimize maximum cost (e.g., tardiness) – Theorem 3.1″, maxT, $\pi_j(t^B) = T_j = \max\{t^B - d_j, 0\}$ with t^B denoting the decision point for backward dispatching (i.e., the completion time of the job to be scheduled next), min, Lawler (1973) or Baker and Trietsch (2009).
- BT31WT, "backward dispatching to minimize maximum cost (e.g., weighted tardiness) – Theorem 3.1″, maxWT, $\pi_j(t^B) = w_j T_j = w_j \max\{t^B - d_j, 0\}$, min, Lawler (1973) or Baker and Trietsch (2009).

Dynamic PD approaches with $\mathbf{O(n^3)}$

- BACK, "backward dispatching", WT (T)

$$\pi_j\left(t^B\right) = \begin{cases} p_j & \text{if } sl_j^B \le 0 \\ -\left(w_j / p_j^{mod}\right) & \text{otherwise} \end{cases}$$

with $sl_j^B = t^B - d_j$, and $p_j^{mod} = \min\{p_j, \min\{T_i | i \in N_{t^B} \wedge i \ne j \wedge T_i \rangle 0\}\}$ (version three for p_j^{mod}) with N_{t^B} denoting the set of unscheduled jobs at decision point t^B and in the unweighted case (UBACK) $w_j = 1$, max, Valente and Schaller (2012).

- QB6, "quadratic backward version 6", WQT (QT)

$$\pi_j\left(t^B,\ \omega\right) = \begin{cases} p_j & \text{if } sl_j^B \le 0 \\ -\left(w_j / p_j^{mod}\right)\left[\left(sl_j^B\right)^2 -\ \nu\left(\max\left\{t^B - p_t^{max} - d_j,\ 0\right\}^2\right)\right] & \text{otherwise} \end{cases}$$

with $p_t^{max} = \max\{p_j | j \in N_{t^B}),\ p_j^{mod} = \min\{p_j, \min\{T_i | i \in N_{t^B} \wedge T_i \rangle 0\}\}$ (version two for p_j^{mod}), and

$$\nu = \begin{cases} 0 & \text{if } \bar{p}_t \ge \overline{sl}^B \\ 1 & \text{if } \bar{p}_t \left\langle \overline{sl}^B \wedge \frac{\overline{sl}^B}{t^B} \right\rangle \omega \\ \frac{\overline{sl}^B - \bar{p}_t}{\overline{sl}^B} & \bar{p}_t < \overline{sl}^B \wedge \frac{\overline{sl}^B}{t^B} \le \omega \end{cases}$$

(with $\overline{sl}^B = \frac{1}{|N_{t^B}|} \sum sl_j^B$ referring to the mean slack of all jobs in N_{t^B} and parameter $\omega = 0.5$) and in the unweighted case (UQB6) $w_j = 1$, max, Valente and Schaller (2012).

Explicit earliness-tardiness PD approaches

- LIN-ET, "linear extended local optimum earliness/tardiness", WE + WT,

$$\pi_j(t,\ \kappa) = \begin{cases} W_j & \text{if } sl_j \le 0 \\ W_j - \frac{sl_j (W_j + H_j)}{\kappa\ \bar{p}_t} & \text{if } 0 \le sl_j \le\ \kappa \bar{p}_t \\ -H_j & \text{otherwise} \end{cases}$$

with $W_j = \frac{w_j}{p_j}$, and $H_j = \frac{h_j}{p_j}$ (h_j representing the earliness penalty factor), and $\kappa = 3$, max, Ow and Morton (1989).

- EXP-ET, "exponential extended local optimum earliness/tardiness", WE + WT,

$$\pi_j(t, \kappa) = \begin{cases} W_j & \text{if } sl_j \le 0, \\ W_j\ \exp\left(-\frac{H_j + W_j}{H_j} \left(\frac{sl_j}{\kappa\ \bar{p}_t} \right) \right) & \text{if } 0 \le sl_j \le \left(\frac{W_j}{H_j + W_j} \right) \kappa\ \bar{p}_t \\ H_j^{-2} \left(W_j - \frac{(H_j + W_j) sl_j}{\kappa\ \bar{p}_t} \right)^3 & \text{if } \left(\frac{W_j}{H_j + W_j} \right) \kappa\ \bar{p}_t < sl_j \le \kappa\ \bar{p}_t \\ -H_j & otherwise \end{cases}$$

with parameter as defined for LIN-ET, max, Ow and Morton (1989); the version depicted here is from Valente and Alves (2005).

- EXP-ET-VA, "EXP-ET version with adaptive κ as defined by Valente and Alves (2005)", WE + WT,

$$\pi_j(t, \kappa) = \begin{cases} W_j & \text{if } sl_j \le 0, \\ W_j\ \exp\left(-\frac{H_j + W_j}{H_j} \left(\frac{sl_j}{\kappa\ \bar{p}_t} \right) \right) & \text{if } 0 \le sl_j \le \left(\frac{W_j}{H_j + W_j} \right) \kappa\ \bar{p}_t \\ H_j^{-2} \left(W_j - \frac{(H_j + W_j) sl_j}{\kappa\ \bar{p}_t} \right)^3 & \text{if } \left(\frac{W_j}{H_j + W_j} \right) \kappa\ \bar{p}_t < sl_j \le \kappa\ \bar{p}_t \\ -H_j & otherwise \end{cases}$$

with instance dependent (adaptive) parameter κ, max, Valente and Alves (2005).

- WPT-MS, "weighted processing time and minimum slack", WE + WT

$$\pi_j(t, \kappa) = \begin{cases} W_j & \text{if } sl_j \le 1 \\ \frac{W_j}{sl_j} & \text{if } 1 < sl_j \le \left(\frac{W_j}{H_j + W_j} \right) \kappa\ \bar{p}_t \\ -H_j \left(\frac{1 - (\kappa\ \bar{p}_t -\ sl_j)}{\kappa\ \bar{p}_t - \frac{W_j}{H_j + W_j} \kappa\ \bar{p}_t} \right)^2 & \text{if } \left(\frac{W_j}{H_j + W_j} \right) \kappa\ \bar{p}_t < sl_j \le \kappa\ \bar{p}_t \\ -H_j & \text{otherwise} \end{cases}$$

with instance dependent (adaptive) parameter κ, max, Valente and Alves (2005).

- EQTP-EXP, "earliness and quadratic tardiness penalty with an exponential function", E + QT,

$$\pi_j(t, \sigma) = \begin{cases} \frac{1}{p_j} \left(\bar{p}_t + 2\left(t + p_j - d_j \right) \right) & \text{if } sl_j \le 0 \\ \frac{\bar{p}_t}{p_j} exp\left(-(\bar{p}_t + 1) \frac{sl_j}{\kappa\ \bar{p}_t} \right) & \text{if } 0 < sl_j < \left(\frac{\bar{p}_t}{\bar{p}_t + 1} \right) \kappa\ \bar{p}_t \\ \left(\frac{1}{p_j} \right)^{-2} \left(\frac{\bar{p}_t}{p_j} - \frac{1}{p_j} (\bar{p}_t + 1) \frac{sl_j}{\kappa\ \bar{p}_t} \right)^3 & \text{if } \left(\frac{\bar{p}_t}{\bar{p}_t + 1} \right) \kappa\ \bar{p}_t \le sl_j \le \kappa\ \bar{p}_t \\ -\frac{1}{p_j} & \text{otherwise} \end{cases}$$

with a dynamic parameter κ depending on the number of critical jobs:$\kappa = |C|$ (with set $C = \{j \in N_t | 0 < sl_j \leq sl_{crit}\}$, whereby $sl_{crit} = \sigma\ n_t\ \bar{p}_t$ and $\sigma = 0.6$, max, Valente (2007).

- ETP-v2, "earliness/tardiness priority dispatching", WQE + WQT,

$$\pi_j(t) = \begin{cases} \frac{w_j}{p_j}\left(\bar{p}_t + 2\max\left\{t + p_j + d_j,\ 0\right\}\right) & \text{if } sl_j \leq 0 \\ \min\left\{\ \frac{w_j}{p_j}\left(\bar{p}_t + 2\max\left\{t + p_j + d_j,\ 0\right\}\right),\right. & \\ \left.\ \frac{h_j}{p_j}\left(\bar{p}_t - 2\max\left\{d_j - t - p_j,\ 0\right\}\right)\ \right\} & otherwise \end{cases}$$

, max, Valente and Alves (2008).

- ETP_LIN_vk, "earliness/tardiness priority dispatching with dynamic look ahead", WQE + WQT,

$$\pi_j(t,\ \kappa_H,\ \kappa_L,\ \omega) = \begin{cases} \frac{w_j}{p_j}\left(\bar{p}_t + 2\left(t + p_j - d_j\right)\right) & \text{if } sl_j \leq 0 \\ T_j^0 - \frac{sl_j\left(T_j^0 - E_j^{\kappa\bar{p}_t}\right)}{\kappa\bar{p}_t} & \text{if } 0 < sl_j < \kappa\bar{p}_t \\ \frac{h_j}{p_j}\left(\bar{p}_t - 2\left(d_j - t - p_j\right)\right) & \text{if } sl_j \geq \kappa\bar{p}_t \end{cases}$$

with $T_j^0 = \bar{p}_t(\frac{w_j}{p_j})$, $E_j^{\kappa\bar{p}_t} = (\bar{p}_t - 2\kappa\bar{p}_t)(\frac{h_j}{p_j})$, and $\kappa = p^{crit}\kappa_H + (1 - p^{crit})\kappa_L$ with $p^{crit} = \frac{|C|}{n_t}$ (with set $C = \{j \in N_t | 0 \leq sl_j \leq sl_{max}\}$ and $sl_{max} = \omega\ n_t\ \bar{p}_t$) and the parameter $\kappa_H = 8.5$, $\kappa_L = 0.5$, and $\omega = 0.25$, max, Valente and Alves (2008).

References

Alidaee, B., Ramakrishnan, K.R., 1996. A computational experiment of COVERT-AU class of rules for single machine tardiness scheduling problem. Comput. Indus. Eng. 30 (2), 201–209. doi:10.1016/0360-8352(95)00166-2.

Arkin, E.M., Roundy, R.O., 1991. Weighted-tardiness scheduling on parallel machines with proportional weights. Oper. Res. 39 (1), 64–81. doi:10.1287/opre.39.1.64.

Baker, K.R., 1984. Sequencing rules and due-date assignments in a job shop. Manag. Sci. 30 (9), 1093–1104. doi:10.1287/mnsc.30.9.1093.

Baker, K.R., Bertrand, J.W.M., 1982. A dynamic priority rule for scheduling against due-dates. J. Oper. Manage. 3 (1), 37–42. doi:10.1016/0272-6963(82)90020-1.

Baker, K.R., Trietsch, D., 2009. Principles of Sequencing and Scheduling. John Wiley, Hoboken, N.J..

Bock, S., Pinedo, M.L., 2010. A decomposition scheme for single stage scheduling problems. J. Sched. 13 (3), 203–212. doi:10.1007/s10951-010-0165-1.

Cai, X., 1995. Minimization of agreeably weighted variance in single machine systems. Eur. J. Oper. Res. 85 (3), 576–592. doi:10.1016/0377-2217(93)E0367-7.

Carroll, D.C., 1965. Heuristic Sequencing of Single and Multiple Component Jobs. MIT, Cambridge, MA PhD Dissertation.

Chen, C.-L., Bulfin, R.L., 1993. Complexity of single machine, multi-criteria scheduling problems. Eur. J. Oper. Res. 70 (1), 115–125. doi:10.1016/0377-2217(93)90236-G.

Chryssolouris, G., Pierce, J., Dicke, K., 1991. An approach for allocating manufacturing resources to production tasks. J. Manuf. Syst. 10 (5), 368–382. doi:10.1016/0278-6125(91)90055-7.

Chryssolouris, G., Wright, K., Pierce, J., Cobb, W., 1988. Manufacturing systems operation: dispatch rules versus intelligent control. Rob. Comput. Integr. Manuf. 4 (No. 3-4), 531–544.

Dileepan, P., Sen, T., 1988. Bicriterion static scheduling research for a single machine. Omega 16 (1), 53–59. doi:10.1016/0305-0483(88)90008-4.

Gupta, S.K., Sen, T., 1983. Minimizing a quadratic function of job lateness on a single machine. Eng. Costs Prod. Econ. 7 (3), 187–194. doi:10.1016/0167-188X(83)90012-5.

Haupt, R., 1989. A survey of priority rule-based scheduling. OR Spectrum 11 (1), 3–16. doi:10.1007/BF01721162.

Hoogeveen, H., 2005. Multicriteria scheduling. Eur. J. Oper. Res. 167 (3), 592–623. doi:10.1016/j.ejor.2004.07.011.

Kanet, J.J., 1981. Minimizing variation of flow time in single machine systems. Manag. Sci. 27 (12), 1453–1459. doi:10.1287/mnsc.27.12.1453.

Kanet, J.J., Hayya, J.C., 1982. Priority dispatching with operation due dates in a job shop. J. Oper. Manage. 2 (3), 167–175. doi:10.1016/0272-6963(82)90004-3.

Kanet, J.J., Li, X., 2004. A weighted modified due date rule for sequencing to minimize weighted tardiness. J. Sched. 7 (4), 261–276. doi:10.1023/B:JOSH.0000031421.64487.95.

Kanet, J.J., Zhou, Z., 1993. A decision theory approach to priority dispatching for job shop scheduling. Prod. Oper. Manag. 2 (1), 2–14. doi:10.1111/j.1937-5956.1993.tb00035.x.

Koulamas, C.P., 2010. The single-machine total tardiness scheduling problem: review and extensions. Eur. J. Oper. Res. 202 (1), 1–7. doi:10.1016/j.ejor.2009.04.007.

Lawler, E.L., 1973. Optimal sequencing of a single machine subject to precedence constraints. Manag. Sci. 19 (5), 544–546. doi:10.1287/mnsc.19.5.544.

Manna, D.K., Prasad, V.R., 1999. Bounds for the position of the smallest job in completion time variance minimization. Eur. J. Oper. Res. 114 (2), 411–419. doi:10.1016/S0377-2217(98)00002-2.

Mebarki, N., Shahzad, A., 2013. Correlation among tardiness-based measures for scheduling using priority dispatching rules. Int. J. Prod. Res. 51 (12), 3688–3697. doi:10.1080/00207543.2012.762131.

Merten, A.G., Muller, M.E., 1972. Variance minimization in single machine sequencing problems. Manag. Sci. 18 (9), 518–528. doi:10.1287/mnsc.18.9.518.

Mönch, L., Balasubramanian, H., Fowler, J.W., Pfund, M.E., 2005. Heuristic scheduling of jobs on parallel batch machines with incompatible job families and unequal ready times. Comput. Oper. Res. 32 (11), 2731–2750. doi:10.1016/j.cor.2004.04.001.

Moore, J.M., 1968. An *n* job, one machine sequencing algorithm for minimizing the number of late jobs. Manag. Sci. 15 (1), 102–109. doi:10.1287/mnsc.15.1.102.

Morton, T.E., Pentico, D.W., 1993. Heuristic Scheduling Systems: With Applications to Production Systems and Project Management, second ed. Wiley, New York, NY.

Nagar, A., Haddock, J., Heragu, S., 1995. Multiple and bicriteria scheduling: a literature survey. Eur. J. Oper. Res. 81 (1), 88–104. doi:10.1016/0377-2217(93)E0140-S.

Nessah, R., Chu, C., 2010. A lower bound for weighted completion time variance. Eur. J. Oper. Res. 207 (3), 1221–1226. doi:10.1016/j.ejor.2010.05.050.

Ow, P.S., Morton, T.E., 1989. The single machine early/tardy problem. Manag. Sci. 35 (2), 177–191.

Pinedo, M.L., 2009. Planning and Scheduling in Manufacturing and Services, second ed. Springer, Dordrecht.

Rachamadugu, R.M.V., Morton, T.E., 1983. Myopic Heuristics for the Weighted Tardiness Problem on Identical Parallel Machines Report CMU-R I-TR-83-17.

Sabuncuoglu, I., Bayiz, M., 1999. Job shop scheduling with beam search. Eur. J. Oper. Res. 118 (2), 390–412. doi:10.1016/S0377-2217(98)00319-1.

Schaller, J.E., 2002. Minimizing the sum of squares lateness on a single machine. Eur. J. Oper. Res. 143 (1), 64–79. doi:10.1016/S0377-2217(01)00322-8.

Sen, T., Dileepan, P., Lind, M.R., 1995. Minimizing a weighted quadratic function of job lateness in the single machine system. Int. J. Prod. Econ. 42 (3), 237–243. doi:10.1016/0925-5273(95)00201-4.

Sen, T., Gupta, S.K., 1983. A branch-and-bound procedure to solve a bicriterion scheduling problem. IIE Trans. 15 (1), 84–88. doi:10.1080/05695558308974617.

Sridharan, S.V., Zhou, Z., 1996a. A decision theory based scheduling procedure for single-machine weighted earliness and tardiness problems. Eur. J. Oper. Res. 94 (2), 292–301. doi:10.1016/0377-2217(96)00133-6.

Sridharan, S.V., Zhou, Z., 1996b. Dynamic non-preemptive single machine scheduling. Comput. Oper. Res. 23 (12), 1183–1190. doi:10.1016/S0305-0548(96)00025-1.

Srinivasan, V., 1971. A hybrid algorithm for the one machine sequencing problem to minimize total tardiness. Nav. Res. Logist. 18 (3), 317–327. doi:10.1002/nav.3800180304.

Srirangacharyulu, B., Srinivasan, G., 2010. Completion time variance minimization in single machine and multi-machine systems. Comput. Oper. Res. 37 (1), 62–71. doi:10.1016/J.COR.2009.03.007.

Valente, J.M.S., 2007. Heuristics for the single machine scheduling problem with early and quadratic tardy penalties. Eur. J. Ind. Eng. 1 (4), 431. doi:10.1504/EJIE.2007.015391.

Valente, J.M.S., 2010. Beam search heuristics for quadratic earliness and tardiness scheduling. J. Oper. Res. Soc. 61 (4), 620–631. doi:10.1057/jors.2008.191.

Valente, J.M.S., Alves, R.A.F.S., 2005. Improved heuristics for the early/tardy scheduling problem with no idle time. Comput. Oper. Res. 32 (3), 557–569. doi:10.1016/j.cor.2003.08.003.

Valente, J.M.S., Alves, R.A.F.S., 2008. Beam search algorithms for the single machine total weighted tardiness scheduling problem with sequence-dependent setups. Comput. Oper. Res. 35 (7), 2388–2405. doi:10.1016/j.cor.2006.11.004.

Valente, J.M.S., Schaller, J.E., 2012. Dispatching heuristics for the single machine weighted quadratic tardiness scheduling problem. Comput. Oper. Res. 39 (9), 2223–2231. doi:10.1016/j.cor.2011.11.005.

van Wassenhove, L.N., Gelders, L.F., 1978. Four solution techniques for a general one machine scheduling problem. Eur. J. Oper. Res. 2 (4), 281–290. doi:10.1016/0377-2217(78)90043-7.

Vepsalainen, A.P.J., Morton, T.E., 1987. Priority rules for job shops with weighted tardiness costs. Manag. Sci. 33 (8), 1035–1047. doi:10.1287/mnsc.33.8.1035.

Vila, M., Pereira, J., 2013. Exact and heuristic procedures for single machine scheduling with quadratic earliness and tardiness penalties. Comput. Oper. Res. 40 (7), 1819–1828. doi:10.1016/j.cor.2013.01.019.

Wemmerlöv, U., Hyer, N.L., 1989. Cellular manufacturing in the U.S. industry: a survey of users. Int. J. Prod. Res. 27 (9), 1511–1530. doi:10.1080/00207548908942637.

Wilkerson, L.J., Irwin, J.D., 1971. An improved method for scheduling independent tasks: A I I E transactions. AIIE Trans. 3 (3), 239–245. doi:10.1080/05695557108974812.