# BCVEGPY2.0: A upgrade version of the generator BCVEGPY with an addendum about hadroproduction of the $P$-wave $B_{c}$ states 

Chao-Hsi Chang ${ }^{1,2}$ *, Jian-Xiong Wang ${ }^{3} \dagger$ and Xing-Gang Wu ${ }^{3 \ddagger}$<br>${ }^{1}$ CCAST (World Laboratory), P.O.Box 8730, Beijing 100080, P.R. China. ${ }^{\S}$<br>${ }^{2}$ Institute of Theoretical Physics, Chinese Academy of Sciences, P.O.Box 2735, Beijing 100080, P.R. China.<br>${ }^{3}$ Institute of High Energy Physics, Chinese Academy of Sciences, P.O.Box 918(4), Beijing 100049, P.R. China.


#### Abstract

The generator BCVEGPY is upgraded by adding the hadroproduction of the $P$-wave excited $B_{c}$ states (denoted by $B_{c J, L=1}^{*}$ or by $h_{B_{c}}$ and $\chi_{B_{c}}$ ) and by improving some features of the original one as well. We denote it as BCVEGPY2.0. The $P$-wave production is also calculated by taking only the dominant gluon-gluon fusion mechanism (with the subprocess $g g \rightarrow B_{c J, L=1}^{*}+\bar{c}+b$ being dominated) into account as that for $S$-wave. In order to make the addendum piece of the upgraded generator as compact as possible so as to increase its efficiency, we manipulate the amplitude as compact as possible with FDC (a software for generating Feynman diagrams and the algebra amplitudes, and for manipulating algebra formulae analytically etc) and certain simplification techniques. The correctness of the program is tested by checking the gauge invariance of the amplitude and by comparing the numerical results with the existent ones in the literature carefully.


PACS numbers: 13.85.Ni, 12.38.Bx, 14.40.Nd, 14.40.Lb.
Keywords: Event generator for hadronic production, $B_{c}$ meson, $P$-wave $B_{c}$ states, Upgraded version.

[^0]
## PROGRAM SUMMARY

Title of program : BCVEGPY

Version: 2.0 (December, 2004)

Program obtained from: CPC Program Library or Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing, P.R. China: www.itp.ac.cn/~zhangzx/BCVEGPY2.0/.

Reference to original program: BCVEGPY1.0

Reference in CPC : Comput. Phys. Commun. 159, 192(2004)

Does the new version supersede the old program: Yes

Computer: Any computer with FORTRAN 77 compiler. The program has been tested on HP-SC45 Sigma-X parallel computer, Linux PCs and Windows PCs with Virsual Fortran.

Operating systems : UNIX, Linux and Windows.

Programming language used: FORTRAN 77/90.

Memory required to execute with typical data: About 2.0 MB.

No. of bytes in distributed program, (including PYTHIA6.2) : About 1.1 MB.

Distribution format: Compressed tar file.

Keywords : Event generator for hadronic production, $B_{c}$ meson, $P$-wave $B_{c}$ states, Upgraded version

Nature of physical problem: Hadronic production of $B_{c}$ meson and its $P$-wave excited states.

Method of solution: The code with option can generate weighted and un-weighted events. For jet hadronization, an interface to PYTHIA is provided.

Restrictions on the complexity of the problem: The hadronic production of $c \bar{b}$-quarkonium in $S$-wave and $P$-wave states via the mechanism of gluon-gluon fusion are given by the so-called 'complete calculation' approach. The less important contributions from the other mechanisms have not been included.

Typical running time : It depends on which option one runs to match PYTHIA when generating the $B_{c}$ events. Typically, For the hadronic production of the $S$-wave $c \bar{b}$-quarkonium, if $\operatorname{IDWTUP}=1$, then it takes about 20 hour on a 1.8 GHz Intel P4-processor machine to generate 1000 events; however if IDWTUP $=3$, to generate $10^{6}$ events, it takes about 40 minutes only. For the hadronic production of the $P$-wave $c \bar{b}$-quarkonium, the time will be almost two times longer than the case of the $S$-wave quarkonium.

## LONG WRITE-UP

## I. INTRODUCTION

Recently, we have completed the hadronic production of the $P$-wave states of $B_{c}$ [1, 2], that has not been included in the existent version BCVEGPY1.0 3] yet, so we upgrade the generator here. The studies in Refs. 1, 2] involve not only the contributions from the colorsinglet components $\left|(c \bar{b})_{\mathbf{1}}\left({ }^{1} P_{1}\right)\right\rangle$ and $\left|(c \bar{b})_{\mathbf{1}}\left({ }^{3} P_{J}\right)\right\rangle(J=1,2,3)$ but also those from the coloroctet components $\left|(c \bar{b})_{\mathbf{8}}\left({ }^{1} S_{0}\right) g\right\rangle$ and $\left|(c \bar{b})_{\mathbf{8}}\left({ }^{3} S_{1}\right) g\right\rangle$ 4] to the $P$-wave production. Especially, it is found that the $P$-wave production at Tevatron and LHC can be so big even as that in the same order of the magnitude for the $S$-wave production in certain kinematics regions. Therefore, to compare with the original generator, in addition to some improvements on the program for the $S$-wave $B_{c}$ (and $B_{c}^{*}$ ) production, the main fresh feature of the upgraded version BCVEGPY2.0 is that it can generate the hadroproduction of the $P$-wave excited $B_{c}$ states (denoted by $B_{c J, L=1}^{*}$ or by $h_{B_{c}}, \chi_{B_{c}}^{0}, \chi_{B_{c}}^{1}$ and $\chi_{B_{c}}^{2}$ ) as well. At such high energies of Tevatron and LHC, the gluon-gluon fusion mechanism i.e. that via the subprocess $g g \rightarrow$
$B_{c J, L=1}^{*}+\bar{c}+b$, is also dominant for the $P$-wave production [1, 2], hence in the addendum we only take the gluon-gluon fusion mechanism into account.

Therefore, in fact, here we just need to rewrite the program used in Refs. 1, 2] into the format according to the environment PYTHIA 5] as the original version BCVEGPY1.0 properly.

Besides that the hadronic production of the $P$-wave $B_{c}$ states is added, for convenience and increasing efficiency, some improvements in BCVEGPY2.0 are made. They can be summarized as: i). an improved way to record the $B_{c}$ events is available, i.e., all the possible information, such as various distributions of the $B_{c}$ events with proper kinematic cuts, can be recorded just by running the program once; ii). an improved way is available to generate the useful sampling importance function just by running VEGAS program 6] carefully; iii). an 'extra switch' is added, i.e. whether one switches on or off to adopt the newly improved parton distribution functions (PDFs) (other than the inner PDFs provided by PYTHIA itself) to the simulation, so that one can apply the latest version of the PDFs provided by the several groups such as CTEQ [7], GRV [8], MRS [9] conveniently; iv). the color flow of the processes is re-written (as an important issue for the new version), so as to well-match that in PYTHIA.

In BCVEGPY1.0, the color flow is treated according to the usual fundamentalrepresentation decomposition [10]. Since the color flow for a given hard scattering subprocess in PYTHIA is treated in the way as that in Ref.[11], i.e., the color fundamentalrepresentation decomposition is replaced by the color-flow decomposition equivalently under the large $N_{c}$ limit. Such a color-flow decomposition has a more intuitive physical interpretation and practically it is easier to deal with than the usual fundamental-representation decomposition. Hence in order to well-match with PYTHIA and to be easy to deal with, especially, with the shower Monte Carlo for QCD jets, the replacement to treat the color flow as that in Ref. 11] for the relevant sub-processes is adopted in BCVEGPY2.0.

In order to increase the efficiency and to make the $P$-wave production amplitude as compact as possible, we manipulate the amplitude with the help of some techniques and the software FDC (a software for generating Feynman diagrams and the algebra amplitudes, and for manipulating algebra formulae analytically etc [12]). To guarantee the correctness of the program, as Refs. 1, 2], we have made numerical checks on the gauge invariance of the amplitude and numerical comparisons between the present obtained results and those
in the literature carefully.
The paper is organized: following Introduction, in Section II we outline the new features of BCVEGPY2.0 in detail, and explain how to use and test (check) the program. In Sec.III, we present some discussions on the generator and make a summary.

## II. THE PROGRAM: BCVEGPY2.0

Since in hadron collisions at high energies, the gluon-gluon fusion mechanism for the hadronic production of the $B_{c}$ meson is dominant over the other mechanisms, thus in BCVEGPY1.0 [3], the hadronic production of the $S$-wave $B_{c}$ and $B_{c}^{*}$ mesons via the gluongluon fusion mechanism has been accomplished. While the present target for the new version BCVEGPY2.0 now essentially is to add the hadronic production of the $P$-wave $B_{c}$ states to the generator BCVEGPY. In BCVEGPY2.0, for the production of the four $P$-wave $B_{c J, L=1}^{*}\left(h_{B_{c}}, \chi_{B_{c}}^{0}, \chi_{B_{c}}^{1}\right.$ and $\left.\chi_{B_{c}}^{2}\right)$ states, not only the contributions from color-singlet components $\left|(c \bar{b})_{\mathbf{1}}\left({ }^{1} P_{1}\right)\right\rangle$ and $\left|(c \bar{b})_{\mathbf{1}}\left({ }^{3} P_{J=0,1,2}\right)\right\rangle$ but also those from the color-octet components $\left|(c \bar{b})_{\mathbf{8}}\left({ }^{1} S_{0}\right) g\right\rangle$ and $\left|(c \bar{b})_{\mathbf{8}}\left({ }^{3} S_{1}\right) g\right\rangle$ are included. The formulation and additional techniques in calculating the hadronic production of $P$-wave states have been given in Refs. [1, 2] with explanations, hence for simplicity, we do not repeat them here. The readers may refer to Refs. 11, 2] for details if they are interested in them.

To supersede the old version BCVEGPY1.0 by the new version BCVEGPY2.0 conveniently, BCVEGPY2.0 is also written into a Fortran package, and it can generate the $S$-wave and the $P$-wave $B_{c}$ states accordingly with proper options. In BCVEGPY2.0, for comparison, the hadronic production of $S$-wave states $B_{c}\left({ }^{1} S_{0}\right)$ and $B_{c}^{*}\left({ }^{3} S_{1}\right)$ via the less important light quark-antiquark annihilation subprocess, $q+\bar{q} \rightarrow B_{c}\left(B_{c}^{*}\right)+\bar{c}+b$, (the annihilation mechanism) has also been included. Since BCVEGPY2.0 is also written in the format as PYTHIA (including common block variables), thus all the functions of PYTHIA can be utilized in connection with the use of BCVEGPY2.0.

## A. New features in BCVEGPY2.0

BCVEGPY2.0 is written in the same structure as that of BCVEGPY1.0, with some improvements in addition to the $P$-wave production being merged. It contains the following
new features:

- The amplitudes for the hadronic production of the color-singlet corresponding to the four $P$-wave states, $B_{c J, L=1}^{*}$ or ${ }^{1} P_{1}$ and ${ }^{3} P_{J}(\mathrm{~J}=0,1,2)$, are included. Four files to calculate the amplitudes for the $P$-wave states are added: p1p1amp.for, pj0amp.for, pj1amp.for and pj2amp.for, which are for the amplitudes of ${ }^{1} P_{1}$ and ${ }^{3} P_{J}(\mathrm{~J}=0,1,2)$ respectively. While the necessary subroutines and functions for calculating the square of the amplitudes for all the $P$-wave states are put into the files psamp.for and lorentz.for. In fact, the file lorentz.for includes basic subroutines, which are to calculate the polarization vectors and the polarization tensor of the heavy quarkonium, and to generate short notations for typical expressions of useful contracts of the vectors and/or tensors so as to make the program more compact. Three extra head files: inclamp.f, inclcon.f and inclppp.f are included in the package, which contain the frequently used common blocks for calculating each hard scattering amplitude. Note here, some new techniques to compute the $P$-wave amplitude [1], such as to expand the amplitude by 'independent and elementary fermion strings' analytically with the help of the FDC program [12] etc., are used instead of the original helicity ones that are used in computing the $S$-wave amplitude.
- To give full estimates of the $P$-wave production up to $v^{2}$, we also take into account the contributions to the hadronic $P$-wave production from the color-octet components $\left|(c \bar{b})_{\mathbf{8}}\left({ }^{1} S_{0}\right) g\right\rangle$ and $\left|(c \bar{b})_{\mathbf{8}}\left({ }^{3} S_{1}\right) g\right\rangle$. In the cases of the hadronic production $g g \rightarrow(c \bar{b})_{\mathbf{1}}+b+\bar{c}$ with $(c \bar{b})_{\mathbf{1}}$ in color-singlet, $\left|(c \bar{b})_{\mathbf{1}}\left({ }^{1} S_{0}\right)\right\rangle$ or $\left|(c \bar{b})_{\mathbf{1}}\left({ }^{3} S_{1}\right)\right\rangle$, there are only three independent color factors (i.e. three independent color flows [1, 13]). While in the cases of the $P$ wave hadronic production due to the color-octet components, such as $g q \rightarrow(c \bar{b})_{8}+b+\bar{c}$ with $(c \bar{b})_{8}$ in color-octet, there are totally ten independent color factors [2]. We shall show in more detail about how to deal with the color-flows in the next subsection. The hard scattering amplitudes corresponding to the color-octet components can be directly read from the Eqs.(4-8) in Ref. [3] but the color-singlet matrix elements (or the wave function at origin) are replaced by the color-octet matrix elements $\langle 0| \chi_{b}^{\dagger} T^{d} \psi_{c}\left(a_{H}^{\dagger} a_{H}\right) \psi_{c}^{\dagger} T^{d} \chi_{b}|0\rangle$ and $\langle 0| \chi_{b}^{\dagger} \sigma^{i} T^{d} \psi_{c}\left(a_{H}^{\dagger} a_{H}\right) \psi_{c}^{\dagger} \sigma^{i} T^{d} \chi_{b}|0\rangle$ accordingly. We can approximately relate the color-octet matrix elements to the color-singlet one by NRQCD scaling rules [4] as the method described in Ref. 14]. Applying the heavy-quark spin symmetry, the above two production
matrix elements are related by [4]

$$
\begin{equation*}
\langle 0| \chi_{b}^{\dagger} \sigma^{i} T^{d} \psi_{c}\left(a_{H}^{\dagger} a_{H}\right) \psi_{c}^{\dagger} \sigma^{i} T^{d} \chi_{b}|0\rangle=(2 J+1) \cdot\langle 0| \chi_{b}^{\dagger} T^{d} \psi_{c}\left(a_{H}^{\dagger} a_{H}\right) \psi_{c}^{\dagger} T^{d} \chi_{b}|0\rangle\left[1+\mathcal{O}\left(v^{2}\right)\right] \tag{1}
\end{equation*}
$$

where $J$ is the total angular momentum of the hadron state. More specifically, based on the velocity scale rule [4], we estimate

$$
\begin{align*}
\langle 0| \chi_{b}^{\dagger} T^{d} \psi_{c}\left(a_{H}^{\dagger} a_{H}\right) \psi_{c}^{\dagger} T^{d} \chi_{b}|0\rangle & \simeq \Delta_{S}(v)^{2} \cdot\langle 0| \chi_{b}^{\dagger} \psi_{c}\left(a_{H}^{\dagger} a_{H}\right) \psi_{c}^{\dagger} \chi_{b}|0\rangle \\
& \left.\simeq \Delta_{S}(v)^{2} \cdot\left|\langle 0| \chi_{b}^{\dagger} \psi_{c}\right| B_{c}\left({ }^{1} S_{0}\right)\right\rangle\left.\right|^{2} \cdot\left[1+\mathcal{O}\left(v^{4}\right)\right] \tag{2}
\end{align*}
$$

where the second equation comes from the vacuum-saturation approximation. $v$ is the relative velocity between the constitute quarks in the bound states. $\Delta_{S}(v)$ is of the order $v^{2}$ or so, and one can take it to be within the region of 0.10 to 0.30 , which is in consistent with the identification: $\Delta_{S}(v) \sim \alpha_{s}(M v)$.

- The $S$-wave $\left({ }^{1} S_{0}\right.$ and $\left.{ }^{3} S_{1}\right)$ hadronic production via the less important mechanism, i.e. the light quark-antiquark annihilation mechanism with the subprocess $q+\bar{q} \rightarrow B_{c}\left(B_{c}^{*}\right)+\bar{c}+b$, is also included in BCVEGPY2.0. In the light quark-antiquark annihilation mechanism, the technique as that in the gluon-gluon fusion one [3] is applied, but it is much more simple (only seven Feynman diagrams are needed to be considered), so we do not present any detail for the calculation here. Numerical results of the calculation show that its contributions to the production are smaller than those from the gluon-gluon fusion $(\sim 1 \%)[15,16]$.
- For convenience, in order to record more data at one run which may interest us, we add 24 extra files to record the information of the generated events, and then all these output files are put in a subdirectory named data. For example, the generated files, pt005y.dat, pt020y.dat, pt035y.dat, pt050y.dat and pt100y.dat are used to record the rapidity distributions with different $p_{t}$ cuts, i.e. $p_{\text {tcut }}=5,20,35,50,100 \mathrm{GeV}$, respectively. The user may record other interested information in a similar way.
- With the option IVEGASOPEN=1, one can use the VEGAS program [6] to achieve an important sampling-function, which may be used to increase the efficiency of the Monte Carlo simulation [5]. Otherwise one may set IVEGASOPEN=0. Furthermore, in order to improve the precision of the existed important sampling-function obtained by previous VEGAS run, we add one more input parameter IVEGGRADE, i.e., with IVEGGRADE $=1$, one can use the existed sampling important function recorded in the file grade.dat to generate a more precise important sampling-function and record it in a new file newgrade.dat. Otherwise one should set IVEGGRADE $=0$.
- PYTHIA itself provides sixteen parton distribution functions (PDFs). It is known that, in fact, the PDFs are obtained through global fitting of the experimental data and then involuted to the requested characteristic scale $Q^{2}$ by a standard way of perturbative Quantum Chromodynamics (pQCD). Several groups, CTEQ [7], GRV [8] and MRS 9] etc, devote themselves to offer accurate PDFs to the world and upgrade them continually with fresh experimental data being available. BCVEGPY2.0 provides a switch (IOUTPDF) to determine which type of PDF one uses: if (IOUTPDF=1), then one can use one of the latest version of PDFs from the above three groups with a proper value for the parameter (IPDFNUM) to do the hadronic production; otherwise, with (IOUTPDF $=0$ ), one can directly use a certain existent PDF (by setting the value of the PYTHIA parameter MSTP(51)) provided in PYTHIA to do the hadronic production. For convenience, we have downloaded all the source files for the latest version of the leading order (LO) PDFs of those three groups, i.e. CTEQ6L, GRV98L and MRST2001L, and saved them into a single fortran file outerpdf.for. The three corresponding data files (cteq6l.tbl, grv98lo.grid and lo2002.dat), which are necessary when running the file outerpdf.for for these three PDFs, have been also added in the package. In fact, it is shown by numerical calculations that the uncertainties for the estimates due to the different PDFs can reach up to the level $\sim 10 \%$ in both cases for the $S$-wave and $P$-wave hadronic production [1, 15].
- For convenience, we have written an additional file parameter.for to set the initial values of the parameters shown in TAB.I. One may fix the value of 'IREADDATA' in parameter.for as zero ('IREADDATA $=\mathbf{0}$ ') and then one returns back the original way (that in version 1.0) to set the input parameters by reading the parameter values from the date file: totput.dat. In practice, if one want to generate a large number of results with various input parameters, we would like to suggest him/her to adopt the way of reading the input data directly from the date files accordingly. In this way, only the input files need to be changed and all the source files (*.for) need not to be changed, so we can save a lot of time on making the source files and furthermore, all the works can be written in a single executable file and be executed automatically.

In summary, in the new package BCVEGPY2.0, there are twenty one files totally, which includes thirteen main source files: bcvegpy.for, foursets.for, genevnt.for, lorentz.for, outerpdf.for, p1p1amp.for, parameter.for, pj0amp.for, pj1amp.for, pj2amp.for, psamp.for, py6208.for (PYTHIA version 6.208), ssamp.for; three date files for the corresponding three

TABLE I: The parameter values in the sequential order in the totput.dat file.


PDFs: lo2002.dat, cteq6l.tbl, grv98lo.grid; three head files: inclamp.f, inclcon.f, inclppp.f; one input data file: totput.dat and finally a simple help file: readme.dat.

With all these new features in mind, the sequential order and the format of the parameters in the input data file, totput.dat, now changes to Table.I. Some new parameters specified in the input file are:

- $\mathbf{F B C}=$ : is different from the meaning in BCVEGPY1.0, and now it's value is the radial wave function $R(0)$ for $S$-wave production or the first derivative of the radial wave function $R^{\prime}(0)$ for $P$-wave production. In our calculations, we take $\mathrm{FBC}=1.24 \mathrm{GeV}^{3 / 2}$, which corresponds to $\psi(0)=1 / \sqrt{4 \pi} R(0)=0.35 \mathrm{GeV}^{3 / 2}$ for $S$-wave, and $\mathrm{FBC}=0.448 \mathrm{GeV}^{5 / 2}$, which corresponds to $\psi^{\prime}(0)=\sqrt{3 / 4 \pi} R^{\prime}(0)=0.219 G e V^{5 / 2}$ for $P$-wave.
- IBCSTATE=: indicates the $S$ - and $P$-states of $B_{c}$ meson: IBCSTATE $=1$, for $B_{c}\left[{ }^{1} S_{0}\right]$; IBCSTATE $=2$, for $B_{c}^{*}\left[{ }^{3} S_{1}\right]$; IBCSTATE $=3,4, \cdots, 6$, for the four $P$-wave states $B_{c}^{*}\left[{ }^{1} P_{1}\right]$, $B_{c}^{*}\left[{ }^{3} P_{0}\right], B_{c}^{*}\left[{ }^{3} P_{1}\right]$ and $B_{c}^{*}\left[{ }^{3} P_{2}\right]$ respectively.
- INUMEVNT $=$ : indicates whether to switch on or off to keep the event number distributions other than the differential distributions of $p_{T}, y$ and etc.. INUMEVNT=1 means to switch on and INUMEVNT $=0$ means to switch off.
- IVEGGRADE $=$ : indicates whether to switch on or off to use the existed grade (importance sampling function) before running VEGAS. IVEGGRADE $=1$ means to switch on and IVEGGRADE=0 means to switch off. With this parameter, one can improve the efficiency of the importance sampling function.
- IQQBAR=: indicates whether to switch on or off to use the less important light quark-
antiquark annihilation subprocess $q+\bar{q} \rightarrow(c \bar{b})+\bar{c}+b$ to generate the ( $c \bar{b}$ )-quarkonium events, where $(c \bar{b})$-quarkonium is in ${ }^{1} S_{0}$ or ${ }^{3} S_{1}$ state. IQQBAR $=1$ means to switch on and IQQBAR $=0$ means to switch off. When IQQBAR $=0$, it will use the default gluon-gluon fusion subprocess to generate the $(c \bar{b})$-quarkonium events.
- IQCODE =: indicates the species of the light quark for the light quark-antiquark annihilation mechanism. Under the option IQQBAR $=1, \mathrm{IQCODE}=1$ means $u$ and $\bar{u}, \mathrm{IQCODE}=2$ means $d$ and $\bar{d}$, while IQCODE $=3$ means $s$ and $\bar{s}$ used in the quark-antiquark annihilation subprocess.
- IOUTPDF =: indicates whether to switch on or off to use the three latest version of PDFs: CTEQ6L, GRV98L and MRST2001L offered in the program for the hadronic production. IOUTPDF $=1$ means to switch on, while IOUTPDF $=0$ means to switch off and then the inner PDFs of PYTHIA is used to generate the events.
- IPDFNUM=: indicates which one of the three latest version of PDFs: CTEQ6L, GRV98L and MRST2001L is used for the hadronic production. It comes into operation only under the option IOUTPDF=1. IPDFNUM=100 means to use GRV98L; IPDFNUM=200 means to use MRST2001L; IPDFNUM=300 means to use CTEQ6L.
- IOCTET $=$ : indicates whether to switch on or off to use the subprocess $g g \rightarrow(c \bar{b})_{\mathbf{8}}+b+\bar{c}$ to generate the color-octet components $\left|(c \bar{b})_{\mathbf{8}}\left({ }^{1} S_{0}\right) g\right\rangle$ and $\left|(c \bar{b})_{\mathbf{8}}\left({ }^{3} S_{1}\right) g\right\rangle$. IOCTET $=1$ means to switch on and IOCTET $=0$ means to switch off.
- $\mathbf{C O E O C T}=$ : the value for $\Delta_{S}(v)$, where $\Delta_{S}(v)$ is defined in Eq.(2).


## B. On the color flow for the hardronic production of $B_{c}$ meson

A string-based fragmentation scheme for the cross sections, such as that in the Lund model and used in PYTHIA, is needed for different color flows. In order to merge well with the shower Monte Carlo used in PYTHIA, in BCVEGPY2.0 we adopt the color-flow decomposition that has been introduced in Ref. [11] to analyze the production. In Ref. [11], it is demonstrated that the color-ordered amplitudes (i.e. amplitudes pertain to the corresponding color-flows), that appear in the color-flow decomposition, are identical to those appear in the usual fundamental-representation decomposition under the large $N_{c}$ limit. So practically, one can first use the color-flow decomposition to analyze how many independent color-flows we need for a concerned subprocess and then transform the results into the usual


FIG. 1: Color flow diagrams for the color-singlet production based on the color-flow decomposition 11]. Each pair of indices $i_{k}$ and $j_{k}$ corresponds to an external gluon, i.e. $k=1$ is for gluon-1 and $k=2$ for gluon-2. $i$ and $j$ are the decomposed color indices for the outgoing $\bar{c}$ and $b$ respectively.
fundamental-representation decomposition according to the color-flow Feynman rules. In this way, we can read out the color-ordered amplitudes easily from our original results in Refs. 11, 2, 3].

Since the less important light quark-antiquark annihilation mechanics with the subprocess $q+\bar{q} \rightarrow B_{c}\left(B_{c}^{*}\right)+\bar{c}+b$ has only two independent orthogonal color-flows, so we do not repeat the results here. We show the color-flows for the dominant gluon-gluon fusion subprocess in detail as the follows. In BCVEGPY2.0, we need to deal with the color-flows for the dominant gluon-gluon fusion subprocess $g g \rightarrow(c \bar{b})+b+\bar{c}$ in two cases: one is for $(c \bar{b})$-quarkonium in a color-singlet state (i.e. $B_{c}, B_{c}^{*}$ and $B_{c J, L=1}^{*}$ ) and the other is for $(c \bar{b})$-quarkonium in a color-octet state (i.e. $\left|(c \bar{b})_{\mathbf{8}}\left({ }^{1} S_{0}\right) g\right\rangle$ and $\left.\left|(c \bar{b})_{\mathbf{8}}\left({ }^{3} S_{1}\right) g\right\rangle\right)$.

- The first case: $(c \bar{b})$-quarkonium in a color-singlet state. In this case, we does not need to consider the color structure of the $(c \bar{b})$-quarkonium and one may find that there are only three types of independent color flows as shown in Fig 1 :

$$
\begin{equation*}
c_{1}=\left(\delta_{i_{2}}^{j} \delta_{i_{1}}^{j_{2}} \delta_{i}^{j_{1}}\right), c_{2}=\left(\delta_{i_{1}}^{j} \delta_{i_{2}}^{j_{1}} \delta_{i}^{j_{2}}\right), c_{3}=\left(\delta_{i}^{j} \delta_{i_{1}}^{j_{2}} \delta_{i_{2}}^{j_{1}}\right) \tag{3}
\end{equation*}
$$

where $\left(i_{n}, j_{n}\right)(\mathrm{n}=1,2)$ are the color indices for the gluon- 1 and gluon- 2 respectively, $i$ and $j$ are color indices of the outgoing $\bar{c}$ and $b$. In the conventions, the lower indices $i$ and $i_{n}$ $(\mathrm{n}=1,2)$ transform under the anti fundamental-representation of $S U\left(N_{c}\right)$ and upper indices


FIG. 2: Color flow diagrams for the color-octet production based on the color-flow decomposition [11]. Each pair of indices $i_{k}$ and $j_{k}$ corresponds to an external gluon, i.e. $k=1$ is for gluon- $1, k=2$ for gluon- 2 and $k=3$ for the color-octet $(c \bar{b})$-quarkonium. $i$ and $j$ are the decomposed color indices for the outgoing $\bar{c}$ and $b$ respectively. The diagrams for $c_{n+1}(n=1,2, \cdots, 5)$ can be directly obtained by gluon exchange.
$j$ and $j_{n}(\mathrm{n}=1,2)$ under the fundamental-representation. Note here that the color bases are not normalized as done in Ref. [11], because we only need the relative probabilities among each color flow. According to the color-flow Feynman rules 11], the color-flow decomposition can be related to the fundamental-representation through the following transformation:

$$
\begin{equation*}
c_{1} \rightarrow\left(T^{a} T^{b}\right)_{i j}, \quad c_{2} \rightarrow\left(T^{b} T^{a}\right)_{i j}, \quad c_{3} \rightarrow\left(\delta_{i j} \operatorname{Tr}\left[T^{a} T^{b}\right]\right) \tag{4}
\end{equation*}
$$

Equivalently, the total amplitude of the process can be written in the fundamentalrepresentation decomposition in the following way:

$$
\begin{equation*}
M=\left(T^{a} T^{b}\right)_{i j} M_{1}+\left(T^{b} T^{a}\right)_{i j} M_{2}+\left(\delta_{i j} \operatorname{Tr}\left[T^{a} T^{b}\right]\right) M_{3} \tag{5}
\end{equation*}
$$

where $T^{a}$ and $T^{b}$ are the color matrices of the gluon-1 and gluon- $2 ; i, j=1,2,3$ are color indices of the outgoing quarks $\bar{c}$ and $b$ respectively. $M_{i}$ are the color-ordered amplitudes respectively, which can be directly read out from Refs. 11, 3].

- The second case: $(c \bar{b})$-quarkonium in the color-octet state. In this case, we need to consider the color structure of the $(c \bar{b})$-quarkonium itself, and one may find that there are ten types of independent color flows totally as shown in Fig2

$$
c_{1}=\left(\delta_{i}^{j_{2}} \delta_{i_{2}}^{j_{1}} \delta_{i_{1}}^{j_{3}} \delta_{i_{3}}^{j}\right), \quad c_{2}=\left(\delta_{i}^{j_{1}} \delta_{i_{1}}^{j_{2}} j_{i_{2}}^{j_{3}} \delta_{i_{3}}^{j}\right), \quad c_{3}=\left(\delta_{i_{1}}^{j_{3}} \delta_{i_{3}}^{j_{1}} \delta_{i}^{j_{2}} \delta_{i_{2}}^{j}\right),
$$

$$
\begin{gather*}
c_{4}=\left(\delta_{i_{2}}^{j_{3}} \delta_{i_{3}}^{j_{2}} \delta_{i}^{j_{1}} \delta_{i_{1}}^{j}\right), \quad c_{5}=\left(\delta_{i}^{j} \delta_{i_{2}}^{j_{1}} \delta_{i_{1}}^{j_{3}} \delta_{i_{3}}^{j_{2}}\right), \quad c_{6}=\left(\delta_{i}^{j} \delta_{i_{1}}^{j_{2}} \delta_{i_{2}}^{j_{3}} \delta_{i_{3}}^{j_{1}}\right), \quad c_{7}=\left(\delta_{i}^{j_{3}} \delta_{i_{3}}^{j_{2}} \delta_{i_{2}}^{j_{1}} \delta_{i_{1}}^{j}\right), \\
c_{8}=\left(\delta_{i}^{j_{3}} \delta_{i_{3}}^{j_{1}} \delta_{i_{1}}^{j_{2}} \delta_{i_{2}}^{j}\right), \quad c_{9}=\left(\delta_{i}^{j_{1}} \delta_{i_{1}}^{j_{3}} \delta_{i_{3}}^{j_{2}} \delta_{i_{2}}^{j}\right), \quad c_{10}=\left(\delta_{i}^{j_{2}} \delta_{i_{2}}^{j_{3}} \delta_{i_{3}}^{j_{1}} \delta_{i_{1}}^{j}\right), \tag{6}
\end{gather*}
$$

where $i_{3}$ and $j_{3}$ are the decomposed color indices of $c$ and $\bar{b}$ in the $(c \bar{b})$-quarkonium. According to the color-flow Feynman rules [11], the color-flow decomposition can be related to the fundamental-representation through the following 'correspondences':

$$
\begin{gather*}
c_{1} \rightarrow\left(T^{b} T^{a} T^{d}\right)_{i j}, \quad c_{2} \rightarrow\left(T^{a} T^{b} T^{d}\right)_{i j}, \quad c_{3} \rightarrow\left(T_{i j}^{b} \operatorname{Tr}\left[T^{a} T^{d}\right]\right), \\
c_{4} \rightarrow\left(T_{i j}^{a} \operatorname{Tr}\left[T^{b} T^{d}\right]\right), \quad c_{5} \rightarrow\left(\delta_{i j} \operatorname{Tr}\left[T^{b} T^{a} T^{d}\right]\right), \quad c_{6} \rightarrow\left(\delta_{i j} \operatorname{Tr}\left[T^{a} T^{b} T^{d}\right]\right), \quad c_{7} \rightarrow\left(T^{d} T^{b} T^{a}\right)_{i j}, \\
c_{8} \rightarrow\left(T^{d} T^{a} T^{b}\right)_{i j}, \quad c_{9} \rightarrow\left(T^{a} T^{d} T^{b}\right)_{i j}, \quad c_{10} \rightarrow\left(T^{b} T^{d} T^{a}\right)_{i j}, \tag{7}
\end{gather*}
$$

where $T^{d}$ stands for the color matrix of the color-octet $(c \bar{b})$-quarkonium. Equivalently, the total amplitude of the process can be written in the fundamental-representation decomposition in the following,

$$
\begin{align*}
M & =\left(T^{b} T^{a} T^{d}\right)_{i j} M_{1}+\left(T^{a} T^{b} T^{d}\right)_{i j} M_{2}+\left(T_{i j}^{b} \operatorname{Tr}\left[T^{a} T^{d}\right]\right) M_{3}+\left(T_{i j}^{a} \operatorname{Tr}\left[T^{b} T^{d}\right]\right) M_{4} \\
& +\left(\delta_{i j} \operatorname{Tr}\left[T^{b} T^{a} T^{d}\right]\right) M_{5}+\left(\delta_{i j} \operatorname{Tr}\left[T^{a} T^{b} T^{d}\right]\right) M_{6}+\left(T^{d} T^{b} T^{a}\right)_{i j} M_{7}+\left(T^{d} T^{a} T^{b}\right)_{i j} M_{8} \\
& +\left(T^{a} T^{d} T^{b}\right)_{i j} M_{9}+\left(T^{b} T^{d} T^{a}\right)_{i j} M_{10} . \tag{8}
\end{align*}
$$

$M_{i}$ are the corresponding color-ordered amplitudes, which can be directly read out from the results in Refs. 2, 3].

- In both cases, when squaring the amplitude and summing over colors, the leading terms in $1 / N_{c}$ obtained by the square of each color flow. The cross terms between different color flows are suppressed by a certain power of $N_{c}$ comparing with the leading terms and under the large $N_{c}$ limit they can be safely neglected. Therefore, we can obtain the probability for each color flow, which are just $\left|M_{i}\right|^{2}$ accordingly, with $\mathrm{i}=(1,2,3)$ in color-singlet case and $\mathrm{i}=(1,2, \cdots, 10)$ in color-octet case.


## C. Checks for the generator

The amplitude for $S$-wave production has been checked well in the original version already, thus the check for the whole Fortran package BCVEGPY2.0 essentially is on the new contents in comparison to the old version. As for BCVEGPY2.0, it is checked by examining the gauge invariance of the amplitude for each production, $S$-wave and $P$-wave of the $B_{c}$ meson,

TABLE II: The total cross section for the hard subprocess $g+g \rightarrow B_{c J, L=1}^{*}+b+\bar{c}$ (gluon fusion into $P$-wave excited states $\left.B_{c J, L=1}^{*}\right)$ at different C.M. energies. The input parameters are taken as those used in Ref. [17]: $m_{b}=5.0 \mathrm{GeV}, m_{c}=1.7 \mathrm{GeV}, M=6.7 \mathrm{GeV}$ and the running $\alpha_{s}$ is fixed to 0.2 etc.

| C.M. energy (GeV) | 20 GeV | 40 GeV | 60 GeV | 80 GeV | 100 GeV | 200 GeV |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\sigma\left({ }^{1} P_{1}\right)(p b)$ | 0.367 | 0.743 | 0.657 | 0.538 | 0.439 | 0.195 |
| $\sigma\left({ }^{3} P_{0}\right)(p b)$ | 0.184 | 0.207 | 0.175 | 0.141 | 0.114 | 0.0496 |
| $\sigma\left({ }^{3} P_{1}\right)(p b)$ | 0.346 | 0.598 | 0.503 | 0.402 | 0.324 | 0.139 |
| $\sigma\left({ }^{3} P_{2}\right)(p b)$ | 0.721 | 1.49 | 1.31 | 1.06 | 0.862 | 0.374 |



FIG. 3: $p_{t}$ and $y$ differential distributions for the subprocess, where $c \bar{b}$ is in $P$-wave states. The dotted line, the dash-dot line, the dash line and the solid line are for ${ }^{1} P_{1},{ }^{3} P_{0},{ }^{3} P_{1}$ and ${ }^{3} P_{2}$, respectively.
individually, i.e. the amplitude vanishes when the polarization vector of an initial gluon is substituted by the momentum vector of this gluon. Numerically we find that the gauge invariance is guaranteed at the computer ability (double precision).

For further checks, we have performed several test runs. With the option ISUBONLY=1, we obtained the transverse momentum $p_{\mathrm{t}}$ and rapidity $y$ distributions and the total crosssections for the sub-process $g g \rightarrow B_{c J, L=1}^{*}+\bar{c}+b$. The results by taking the same parameters

TABLE III: Generation parameters used in the sample generation.

```
............. INITIAL PARAMETERS
GET THE RESULT FOR Bc IN '3}\mp@subsup{P}{0}{
Bc IN COLOR-SINGLET STATE
GENERATE EVNTS 30000000 FOR TEVA ENERGY(GEV) 0.196E + 4
* USING SUBPROCESS: }g+g->Bc+b+\overline{c}\quad
****************************************
M_{Bc}=6.700 M_{B}=5.000 M_{C}=1.700 f_ {Bc}=0.2879
Q2 TYPE= 3 ALPHAS ORDER= LO
THE NEW VERSION OF OUTER PDFs WHICH ARE FROM WWW
PDF: CTEQ6L; ALPHA IN LO
USING PYTHIA IDWTUP= 3
PTCUT =0.000 GeV
NO RAPIDITY CUT
USING VEGAS: NUMBER IN EACH ITERATION= 200000 ITERATION= 30
............. END OF INITIALIZATION
```

$\qquad$
as those used in Refs. 17, 18] are shown in the TABLE II and in FIG[3. One may see that the results for the total cross-section of the hard subprocess $g+g \rightarrow B_{c J, L=1}^{*}+\bar{c}+b$ with various C.M. energies are agree with those in Ref. 17] (TABLE.I there) within the MC error, and the $p_{t}$ and $y$ distributions are consistent well with the ones obtained in Refs. [17, 18]. Since our present method is quite different from that in Refs. 17, 18] the comparison is really a good check for the generator and also for the results in Refs. 17, 18]. In fact, in Refs. 17, 18], the derivative over the relative momentum $q$ of the constitute quarks in the $B_{c}$ meson, which is necessary for $P$-wave production, is done numerically; while in our case, it is done analytically with the help of the FDC program [12].

Finally, we show a test run for the hadronic production of $\left({ }^{3} P_{0}\right) c \bar{b}$-quarkonium at the TEVATRON. When running the program, the initialization is shown as a screen snap-shot in TABLE III Here the value of $f_{B c}=0.2879$ corresponds to $R^{\prime}(0)=0.448 \mathrm{GeV}^{5 / 2}$. The


FIG. 4: Test run results of the hadronic production of ${ }^{3} P_{0}$ state at TEVATRON. The transverse momentum $p_{\mathrm{t}}$ and rapidity $y$ distributions for the CTEQ6L by using $\alpha_{s}$ in the leading order (LO) and adopting the characteristic energy scale squared $Q^{2}=p_{t}^{2}+M_{(c \bar{b})}^{2}$.


FIG. 5: Flow chart for using VEGAS. The meaning of the symbols can be found in the program. results are shown in FIG.4.

## D. About VEGAS

This subsection is devoted to give a supplementary explanation on VEGAS.

By using VEGAS, one can obtain an important sampling-function (or simply called grade function) into the file grade.dat, and as it is well known, by using such function to do the event simulation, the efficiency can be greatly increased [5]. A modification to the original version in Ref. [6] is done so as to record the obtained grade function into the data file grade.dat. Such small trick is useful, i.e. when one wants to generate the events with the same input parameters, then one can directly apply the grade function obtained by the previous running and does not need to run VEGAS again. The flow chart for this part is presented in FIG5, where as mentioned above, one can use the existent grade function to do the initialization further and then to obtain a more precise grade function.

In calling VEGAS, we need to input two parameters, i.e. NUMBER (maximum total number of the times to call the integrand in each iteration) and ITMX (maximum number of allowed iterations). These two parameters should be adjusted in a suitable way so as to obtain a grade function in good precision and also to save the CPU as much as possible. One subtle point is that, in VEGAS, the default number of the column bin is 50 (i.e. the region of $(0,1)$ is divided into 50 pieces); by taking a bigger proper value for the column bins, one can improve the production efficiency but need more iterations (or numbers) to obtain a stable result. In practice, the values for NUMBER, ITMX and the column bins should be carefully chosen to obtain a best important sampling-function within the least of time. In our program, we keep the column bins to be 50 (according to our experience, it is enough); however the user who wants to do some very precise studies, a proper variation of column bins might be needed.

## III. CONCLUSIONS

An upgrade version of $B_{c}$ meson generator BCVEGPY2.0 is given. In this new version, the hadronic production of $P$-wave states up to $v^{2}$, that include both the color-singlet components $B_{c J, L=1}^{*}$ and the color-octet components $\left|(c \bar{b})_{\mathbf{8}}\left({ }^{1} S_{0}\right) g\right\rangle$ and $\left|(c \bar{b})_{\mathbf{8}}\left({ }^{3} S_{1}\right) g\right\rangle$, is available. Since almost all of these $P$-wave excited states decay to the ground state $B_{c}$ meson through the electromagnetic transition (if the phase space for strong interaction transitions is not allowed) finally, thus in fact the $P$-wave production provides additional contributions to the hadronic production of the $B_{c}$ meson. In the new version, the hadronic production of $S$-wave $c \bar{b}$-quarkonium from the quark-antiquark annihilation subprocess is also included for
comparison, although the contributions from the quark-antiquark annihilation mechanism to the $B_{c}$ hadronic production, in comparison with those from gluon-gluon fusion mechanism, can be ignored safely at LHC/TEVATRON energies in the allowed kinematic region.

To achieve a compact program, we apply the FDC program to manipulate the amplitude of the dominant gluon-gluon fusion subprocess for the production corresponding to the four color-singlet $P$-wave states [1]. The correctness of the program is guaranteed by various checks. The generator is interfaced with PYTHIA, which takes care of producing the full event and filling the standard PYTHIA event common block. In view of the prospects for $B_{c}$ physics at Tevatron and at LHC, the generator offers a platform for further experimental studies.

Acknowledgement: The work was supported in part by Nature Science Foundation of China (NSFC). The authors would like to thank G.M. Chen, S.H. Zhang, A. Belkov, S. Shulga, L. Han, Y. Jiang and Y.N. Gao for helpful discussions and suggestions. One of the author (X.G. Wu) would like to thank A.V. Berezhnoy for discussions on the $P$-wave production.

## APPENDIX A: ADDITIONAL ROUTINES AND FUNCTIONS FOR CALCULATING THE HADRONIC PRODUCTION OF $B_{c J, L=1}^{*}$

In this Appendix, additional subroutines and functions for calculating the amplitudes for the gluon-gluon fusion subprocess, $g g \rightarrow B_{c J, L=1}^{*}+b+\bar{c}$, are explained.

## SUBROUTINE ampI_1p1(cc)

Purpose: with $\mathrm{I}=1,2, \cdots, 36$ to compute the amplitude of the 36 Feynman diagrams of the gluon-gluon fusion subprocess correspondingly, with the $(c \bar{b})$-quarkonium in ${ }^{1} P_{1}$ state.
Real*8 cc=: $c c= \pm \sqrt{\frac{9 R^{\prime}(0)^{2}}{2 \pi} \frac{1}{72 M^{3}}}$, where $R^{\prime}(0)^{2}$ is the square of the first derivative of the radial wave function at the origin, and $M$ is the mass of the $(c \bar{b})$-quarkonium. Whether it takes the plus or the minus sign is our inner convention for the 36 Feynman diagrams and it depends on how to decompose the Feynman diagrams.

## SUBROUTINE ampI_3p0(cc)

Purpose: with $\mathrm{I}=1,2, \cdots, 36$, to compute the amplitude of the 36 Feynman diagrams of the gluon-gluon fusion subprocess correspondingly, with the $(c \bar{b})$-quarkonium in ${ }^{3} P_{0}$ state.

Real* $8 \mathrm{cc}=: c c= \pm \sqrt{\frac{9 R^{\prime}(0)^{2}}{2 \pi} \frac{1}{216 M^{3}}}$, where $R^{\prime}(0)^{2}$ is the square of the first derivative of the radial wave function at the origin and $M$ is the mass of the $(c \bar{b})$-quarkonium. Whether it takes the plus or the minus sign is our inner convention for the 36 Feynman diagrams and it depends on how to decompose the Feynman diagrams.

## SUBROUTINE ampI_3p1(cc)

Purpose: with $\mathrm{I}=1,2, \cdots, 36$, to compute the amplitude of the 36 Feynman diagrams of the gluon-gluon fusion subprocess correspondingly, with $(c \bar{b})$-quarkonium in ${ }^{3} P_{1}$ state.
Real ${ }^{*} 8 \mathrm{cc}=: c c= \pm \sqrt{\frac{9 R^{\prime}(0)^{2}}{2 \pi} \frac{1}{144 M^{3}}}$, where $R^{\prime}(0)^{2}$ is the square of the first derivative of the radial wave function at the origin and $M$ is the mass of the $(c \bar{b})$-quarkonium. Whether it takes the plus or the minus sign is our inner convention for the 36 Feynman diagrams and it depends on how to decompose the Feynman diagrams.

## SUBROUTINE ampI_3p2(cc)

Purpose: with $\mathrm{I}=1,2, \cdots, 36$, to compute the amplitude of the 36 Feynman diagrams of the gluon-gluon fusion subprocess correspondingly, with $(c \bar{b})$-quarkonium in ${ }^{3} P_{2}$ state.
Real $* 8 \mathrm{cc}=: c c= \pm \sqrt{\frac{9 R^{\prime}(0)^{2}}{2 \pi} \frac{1}{72 M^{3}}}$, where $R^{\prime}(0)^{2}$ is the square of the first derivative of the radial wave function at the origin and $M$ is the mass of the $(c \bar{b})$-quarkonium. Whether it takes the plus or the minus sign is our inner convention for the 36 Feynman diagrams and it depends on how to decompose the Feynman diagrams.

## FUNCTION amps20_1p1()

Purpose: to compute the square of the whole amplitude for a particular combination of two gluons' polarization states and the polarization vector of ${ }^{1} P_{1} c \bar{b}$-quarkonium state.

## FUNCTION amps2_1p1()

Purpose: to compute the square of the whole amplitude with the help of the FUNCTION amps20_1p1(), where the summation over the gluons' polarization states and the polarization vector of ${ }^{1} P_{1} c \bar{b}$-quarkonium state is done.

## FUNCTION amps20_3p0()

Purpose: to compute the square of the whole amplitude for the production of ${ }^{3} P_{0} c \bar{b}$ quarkonium and for a particular combination of the two gluons' polarization states.

## FUNCTION amps2_3p0()

Purpose: to compute the square of the whole amplitude with the help of the FUNCTION amps20_3p0(), where the summation over the gluons' polarization states is done.

FUNCTION amps20_3p1()

Purpose: to compute the square of the whole amplitude for a particular combination of two gluons' polarization states and the polarization vector of ${ }^{3} P_{1} c \bar{b}$-quarkonium state.

## FUNCTION amps2_3p1()

Purpose: to compute the square of the whole amplitude with the help of the FUNCTION amps20_3p1(), where the summation over the gluons' polarization states and the polarization vector of ${ }^{3} P_{1} c \bar{b}$-quarkonium state is done.

## FUNCTION amps20_3p2()

Purpose: to compute the square of the whole amplitude for a particular combination of two gluons' polarization states and the polarization tensor of ${ }^{3} P_{2} c \bar{b}$-quarkonium state.

## FUNCTION amps2_3p2()

Purpose: to compute the square of the whole amplitude with the help of the FUNCTION amps20_3p2(), where the summation over the gluons' polarization states and the polarization vector of ${ }^{3} P_{2} c \bar{b}$-quarkonium state is done.

## SUBROUTINE genpolar3(p, ep, pm)

Purpose: to generate the polarization vectors (ep) of a vector meson with mass pm and momentum p.
real*8 $\mathrm{p}(4), \mathrm{ep}(4,2: 4): \mathrm{p}(4)$ is the momentum for the vector meson, $p(1)=E, p(2)=p_{x}$, $p(3)=p_{y}$ and $p(4)=p_{z} . \operatorname{ep}(4,2: 4)$ records three polarization vectors of the vector meson, each of which has four components. The explicit formulas of ep can be found in appendix A of Ref. 1].

## SUBROUTINE genpolar(p, ep)

Purpose: to generate the polarization vectors (ep) of a gluon with momentum p.
real* $8 \mathrm{p}(4), \mathrm{ep}(4,2: 3): \mathrm{ep}(4,2: 3)$ records two transverse polarization vectors of the gluon.

## SUBROUTINE gentensor ( $\mathrm{p}, \mathrm{ep}, \mathrm{pm}$ )

Purpose: to generate the polarization tensor (ep) for the meson in ${ }^{3} P_{2}$ state with mass pm and momentum p .
real*8 $\mathrm{p}(4), \mathrm{ep}(4,2: 6): \mathrm{ep}(4,2: 6)$ records five polarization tensors of the meson in ${ }^{3} P_{2}$ state. The explicit formulas of $e p$ can be found in appendix A of Ref.[1].

There are also some accessory subroutines and functions that are used to simplify the amplitude and to make the amplitude more compact and not difficult to be understood, such as the SUBROUTINE genppp which is used to generate short notations for all the typical linear combination of dotted products of two momenta, thus for shortening the paper
we do not explain them precisely here.
[1] Chao-Hsi Chang, Jian-Xiong Wang and Xing-Gang Wu, Phys. Rev. D70, 114019(2004).
[2] Chao-Hsi Chang, Cong-Feng Qiao, Jian-Xiong Wang and Xing-Gang Wu, hep-ph/0502155, accepted by Phys. Rev. D.
[3] Chao-Hsi Chang, Chafik Driouich, Paula Eerola and Xing-Gang Wu, Comput. Phys. Commun. 159, 192 (2004).
[4] Geoffrey T. Bodwin, Eric Braaten and G. Peter Lepage, Phys. Rev. D 51, 1125 (1995); Erratum-ibid. D 55, 5853 (1997).
[5] T. Sjostrand, Comput. Phys. Commun. 82, 74 (1994).
[6] G.P. Lepage, J. Comp. Phys 27, 192 (1978).
[7] H.L. Lai, et al., hep-ph/0201195, JHEP 0207, 012 (2002)
[8] M. Glueck, E. Reya, A. Vogt, Eur. Phys. J. C 5, 461 (1998).
[9] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, Eur. Phys. J. C 23, 73 (2002).
[10] M.L. Mangano, S. Parke and Z. Xu, Nucl.Phys. B298, 653 (1988).
[11] F. Maltoni, K. Paul, T. Stelzer and S. Willenbrock, Phys.Rev. D67, 014026(2003).
[12] Jian-Xiong Wang, Nucl. Instrum. Methods Phys. Res., Sect. A 534, 241 (2004).
[13] A.V. Berezhnoy, hep-ph/0407315.
[14] Xing-Gang Wu, Chao-Hsi Chang, Yu-Qi Chen and Zheng-Yun Fang, Phys. Rev. D 67, 094001 (2003).
[15] Chao-Hsi Chang and Xing-Gang Wu, Eur. Phys. J. C 38, 267 (2004).
[16] Chao-Hsi Chang and Yu-Qi Chen, Phys. Rev. D 48, 4086 (1993).
[17] A.V. Berezhnoy, V.V. Kiselev and A.K. Likhoded, Phys. Atom. Nucl. 60, 100 (1997); Yad. Fiz. 60, 108 (1997).
[18] A.V. Berezhnoy, V.V. Kiselev and A.K. Likhoded, Z. Phys. A 356, 79 (1996).


[^0]:    * email: zhangzx@itp.ac.cn
    ${ }^{\dagger}$ email:jxwang@mail.ihep.ac.cn
    $\ddagger$ email: wuxg@mail.ihep.ac.cn
    ${ }^{\S}$ Not correspondence address.

