Time-dependent magnetotransport in semiconductor nanostructures via the generalized master equation

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Abstract

Transport of electrons through two-dimensional semiconductor structures on the nanoscale in the presence of perpendicular magnetic field depends on the interplay of geometry of the system, the leads, and the magnetic length. We use a generalized master equation (GME) formalism to describe the transport through the system without resorting to the Markov approximation. Coupling to the leads results in elastic and inelastic processes in the system that are described to a high order by the integro-differential equation of the GME formalism. Geometrical details of systems and leads leave their fingerprints on the transport of electrons through them. The GME formalism can be used to describe both the initial transient regime immediately after the coupling of the leads to the system and the steady state achieved after a longer time.

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Commonly, various transport formalisms have been tried on very simple "model systems". Here, we show that the generalized master equation can be used to investigate magnetotransport properties of a twodimensional electron system with nontrivial geometry. Furthermore, we find that the geometrical shape of the leads coupled to the system strongly influence the transport. The GME formalism has been used by several groups to study transport [1-4]. To derive the non-Markovian GME we project the Liouville-von Neumann equation for the density operator of the leads and the system on the system by tracing out all operators pertaining to the leads, obtaining the reduced density operator (RDO) $\rho_{\rm S}(t)$ describing the evolution of the system under the influence of the leads (the reservoirs). The derivation had its origin in quantum optics [5, 6]. For technical details see [7] for a lattice model and [8] for a continuous model. The semi-infinite leads are coupled to the finite system at t = 0. Here, we will as-

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sume no Coulomb interaction between the electrons in the system, but we have added the Coulomb interaction via "exact diagonalization" in a different communication [9]. An external magnetic field with strength Bis perpendicular to the 2D electron system in the leads and the system. The derivation of the GME is carried out with the assumption of weak tunneling coupling between the system and the leads. In the continuous model the coupling is described by a nonlocal overlap integral between states in the system and the wire in the contact area marked green in Fig. 1, (see [8]). This gives us a complex coupling scheme between the leads and system that depends on the geometry of the subsystems instead of a single coupling constant often used.

The GME-formalism is a many-electron formalism and thus we select as relevant for the transport singleelectron states (SESs) in and around the bias-window shown in Fig. 2 to build the necessary many-electron states (MESs). In the calculation here we use 10 SESs, and since it is carried out at magnetic field B = 1.0 T we have to remember that some states have the character of

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Figure 1: Schematic of the system, the leads, and the coupling between them represented by the tunneling Hamiltonian $H_T^{L,R}(t)$, (see [8])

an edge state while others are bulk states. The coupling to the states will vary with their character, and the magnetic field influences this coupling to the largest extent.



Figure 2: Schematic of the relevant single electron states (SESs) in and around the bias window included in the transport calculation.

We assume that the system is a finite parabolically confined quantum wire of length 300 nm, and use GaAs parameters, $m^* = 0.067m_e$. The characteristic confinement energy is $\hbar\Omega_0 = 1.0$ meV. The ends of the system have a hard wall for t < 0 and are for later times tunnel coupled to the semi-infinite parabolic leads that will either be broad, with confinement energy scale E_0 = $\hbar \Omega_0^{L,R} = 1.0$ meV, or more narrow with $E_0 = 2.0$ meV. The energy spectrum of the lowest MESs of the system is seen in Fig. 3 together with the continuous SESs spectra for the broad and the more narrow leads. The chemical potential of the right lead is fixed at $\mu_R = 1.4$ meV and the bias $\Delta \mu$ is varied by changing μ_L . The total time-dependent occupation of the system with electrons is displayed in Fig. 4 for the two types of leads and different values of the bias. The largest amount of charge is accumulated in the system for the higher bias and the broad leads, and for the broad leads and the lower bias the system is very close to a steady state regime after the transient charging. Furthermore, we see in Fig. 5 how the charging for the broad leads occurs through both leads in the transient regime, and how the steady state value of the current through the system depends on the bias.

Interesting is to see how the different MESs contribute to the transport in Fig. 6. Not only more MESs participate in the case of the broad leads, but more in-



Figure 3: The discrete spectra of the MESs of the system, the continuous SESs spectra for a broad or narrow semi-infinite lead, and the chemical potential in the right lead μ_R .



Figure 4: The total charge in the system as a function of time for different values of the bias $\Delta \mu$ and confinement of the leads $E_0 = \hbar \Omega_0^{R,L}$.

teresting is to see their location with respect to μ_R . If we analyze the structure of the MESs using the Fock space of SESs $|\mu\rangle = |i_1^{\mu}, i_2^{\mu}, i_3^{\mu}, \cdots, i_{N_{\text{SES}}}^{\mu}\rangle$ with $i_a^{\mu} \in \{0, 1\}$ we find that in case of the broad leads a large contribution comes from |01.00000000, |01.10000000, and |01.01000000>, where we have used a dot to indicate the location of μ_R . In the case of the more narrow leads we find large contributions from $|00.1000000\rangle$, $|00.11000000\rangle$, and $|00.10100000\rangle$. In both cases the lowest SES is not occupied since it is a bulk state with low coupling to the contact region. For the broad wire we see the state just under μ_R is occupied, and states in the bias window. For the narrow leads this state is not occupied, but rather the next state just above μ_R . In Fig. 3 we see that this can be explained by the lack of low enough energy states in the narrow leads. In the case of the broad leads we notice that initially one-



Figure 5: Total current in the right and left leads for a narrow and a broad confinement for $\mu_R = 0.1$ meV (upper), and $\mu_R = 1.1$ meV (lower). A negative current in the right leads is a current into the system from the right lead.

electron states are occupied, but for later times twoelectron states are favored.

We see thus that the energy spectrum of the leads is very important when considering which states in the system will contribute to the transport, both in the transient and the steady state regime. In addition, the character of the state in the system both with respect to energy and geometry is essential as the coupling depends on all these details. The coupling to the leads enforces certain correlation on the electrons in the system. To describe these correlation effects in the weak coupling limit for a low density of electrons we needed the GME many-electron formalism. Elsewhere, we have shown that the Coulomb interaction between the electrons influences the charging of the system by changing the energy scales, the spectra, and the correlation between the electron states [9].

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Figure 6: Partial time-dependent charge in some MESs states for $E_0 = \hbar \Omega_0^{R,L} = 1.0 \text{ meV}$ (upper), and $E_0 = 2.0 \text{ meV}$ (lower). $\Delta \mu = 1.1 \text{ meV}$.

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