# CLUMPY: a code for $\gamma$ -ray signals from dark matter structures

Aldée Charbonnier<sup>a</sup>, Céline Combet<sup>b,c</sup>, David Maurin<sup>a,b,c,d</sup>

Abstract

We present the first public code for semi-analytical calculation/decay in the Galaxy, including dark matter substructure the dark matter density squared (for annihilations) or densit to draw skymaps from the Galactic smooth component and/halo (that is not the Galactic halo, e.g. dwarf spheroidal galowed DM profiles for a given object. Extragalactic contribution will be included in a second release.

\*\*Reywords:\*\* Cosmic rays, Cosmology, Dark Matter, Indirection Program Title: CLUMPY

Program Title: CLUMPY

Programming language: C/C++

Computer: PC and Mac

Operating system: UNIX(Linux), MacOS X

RAM: depends on the requested size of skymaps (~40 Mb for 500 × 500 map)

\*\*Keywords:\*\* dark matter, indirect detection, gamma-rays

Classification: 1.1, 1.9

External routines/libraries: CERN ROOT¹, Doxygen² (optional)

Nature of problem: Calculation of y-ray signal from dark matter nihilation (resp. decay). This involves a particle physics term and astrophysical one. The focus here is on the latter.

Solution method: Integration of the DM density squared (resp. of sity) along a line of sight. The code is optimised to deal with the density peaks encountered along the line of sight (DM substructured A semi-analytical approach (calibrated on N-body simulations) is a strophysical order. The code is optimised to deal with the density peaks encountered along the line of sight (DM substructured in the Galaxy).

\*\*Restrictions:\*\* Some generic dark matter annihilation spectra are vided but are not included in the calculation so far as it is assumed the particle physics is independent of the astrophysics of the problem: this is highly dependent of the DM profiles conside the requested precision 6 and integration angle \$\text{cm}\$: The code is optimised to the profiles consider the requested precision 6 and integration angle \$\text{cm}\$: The code is optimised to the profiles consider the particle physics is independent of the astrophysics of the profiles. We present the first public code for semi-analytical calculation of the  $\gamma$ -ray flux astrophysical J-factor from dark matter annihilation/decay in the Galaxy, including dark matter substructures. The core of the code is the calculation of the line of sight integral of the dark matter density squared (for annihilations) or density (for decaying dark matter). The code can be used in three modes; i) to draw skymaps from the Galactic smooth component and/or the substructure contributions, ii) to calculate the flux from a specific halo (that is not the Galactic halo, e.g. dwarf spheroidal galaxies) or iii) to perform simple statistical operations from a list of allowed DM profiles for a given object. Extragalactic contributions and other tracers of DM annihilation (e.g. positrons, antiprotons)

Keywords: Cosmic rays, Cosmology, Dark Matter, Indirect detection, Gamma-rays

RAM: depends on the requested size of skymaps (~40 Mb for a

Nature of problem: Calculation of  $\gamma$ -ray signal from dark matter annihilation (resp. decay). This involves a particle physics term and an

Solution method: Integration of the DM density squared (resp. density) along a line of sight. The code is optimised to deal with the DM density peaks encountered along the line of sight (DM substructures). A semi-analytical approach (calibrated on N-body simulations) is used for the spatial and mass distributions of the dark matter substructures

Restrictions: Some generic dark matter annihilation spectra are provided but are not included in the calculation so far as it is assumed that the particle physics is independent of the astrophysics of the problem. Running time: this is highly dependent of the DM profiles considered, the requested precision  $\epsilon$  and integration angle  $\alpha_{\text{int}}$ :

- about 60 mn for a  $5^{\circ} \times 5^{\circ}$  map towards the Galactic centre, with  $\alpha_{\rm int} = 0.01^{\circ}$ , NFW dark matter profiles and  $\epsilon = 10^{-2}$ ;
- about 2h for the same set-up towards the anti-centre;
- 0.1 to 10 DM models per second, depending on integration angle and DM profile.

Email address: celine.combet@lpsc.in2p3.fr (Céline Combet)

#### 1. Introduction

Despite the several astrophysical evidences pointing at the existence of dark matter (flat rotation curves of galaxies, gravitational lensing, "bullet cluster", etc.), its nature still evades us. This search has become one of the major topic in both particle physics and astrophysics and is tackled using either direct or indirect detection methods. For the former, the hope is to directly witness the interaction of a dark matter particle with a detector. The indirect approaches aim at measuring the end products of dark matter annihilation/decay (e.g.,  $\gamma$ -rays, positrons, antiprotons). The detection of  $\gamma$ -rays was soon recognised to be a promising channel [1, 2]. If such a signal is yet to be measured by the existing  $\gamma$ -rays observatories (FERMI, HESS, MAGIC, VERITAS), the prospect may significantly improve with the forthcoming next-generation instruments, such as CTA [3].

Having a modelling tool that can compute the expected  $\gamma$ ray flux from dark matter annihilation/decay in a wide range of astrophysical configurations and making it available to the community should prove very useful. This is the motivation for developing the code presented in this paper. This paper highlights CLUMPY's main features but a more thorough description can be found in the documentation coming with the source code. The paper is organised as follows. Section 2 presents the formulation of the generic  $\gamma$ -ray signal calculation that is performed by CLUMPY, with special emphasis on the contribution from DM clumps. The code is described in sect. 3, which presents the structure, parameters and functions that are at the core of CLUMPY. Section 4 deals with two examples of the science that can be performed with CLUMPY. We conclude and discuss the future developments of the code in sect. 5.

<sup>&</sup>lt;sup>a</sup>Laboratoire de Physique Nucléaire et Hautes Energies, CNRS-IN2P3/Universités Paris VI et Paris VII, 4 place Jussieu, Tour 33, 75252 Paris Cedex 05, France  $^b$ Dept. of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH, UK

<sup>&</sup>lt;sup>c</sup>Laboratoire de Physique Subatomique et de Cosmologie, CNRS/IN2P3/INPG/Université Joseph Fourier Grenoble 1, 53 avenue des Martyrs, 38026 Grenoble, France

<sup>&</sup>lt;sup>d</sup>Institut d'Astrophysique de Paris, UMR7095 CNRS, Université Pierre et Marie Curie, 98 bis bd Arago, 75014 Paris, France

http://root.cern.ch

<sup>&</sup>lt;sup>2</sup>http://www.doxygen.org

# 2. Calculating the $\gamma$ -ray flux from dark matter annihilation/decay

The  $\gamma$ -ray flux  $d\Phi_{\gamma}/dE_{\gamma}$  from dark matter annihilating/decaying particles is expressed as the product of a particle physics term by an astrophysical contribution. For a given experiment with a spatial resolution  $\alpha_{\rm int}$  corresponding to an integration solid angle  $\Delta\Omega=2\pi\,(1-\cos\,\alpha_{\rm int})$ , at energy  $E_{\gamma}$  and pointing in the direction  $(\psi,\theta)$ , the flux is written as:

$$\frac{\mathrm{d}\Phi_{\gamma}}{\mathrm{d}E_{\gamma}}(E_{\gamma},\psi,\theta,\Delta\Omega) = \frac{\mathrm{d}\Phi_{\gamma}^{PP}}{\mathrm{d}E_{\gamma}}(E_{\gamma}) \times J(\psi,\theta,\Delta\Omega). \tag{1}$$

#### 2.1. Annihilation and decay: general statements

2.1.1. Annihilation: the particle physics term It can be generically expressed as

$$\frac{\mathrm{d}\Phi_{\gamma}^{PP}}{\mathrm{d}E_{\gamma}}(E_{\gamma}) \equiv \frac{1}{4\pi} \frac{\langle \sigma_{\mathrm{ann}} v \rangle}{2m_{\chi}^{2}} \cdot \sum_{f} \frac{dN_{\gamma}^{f}}{dE_{\gamma}} B_{f}, \qquad (2)$$

where  $m_\chi$  is the mass of the DM particle,  $\sigma_{\rm ann}$  is the annihilation cross section,  $\langle \sigma_{\rm ann} \nu \rangle$  the annihilation rate averaged over the DM velocity distribution,  $B_f$  is the branching ratio into the final state f and  $dN_\gamma^f/dE_\gamma$  is the photon yield per annihilation. A widely accepted value for  $\langle \sigma_{\rm ann} \nu \rangle$  estimated from present day thermal relic density is  $\langle \sigma_{\rm ann} \nu \rangle = 3 \cdot 10^{-26} \ {\rm cm}^3 {\rm s}^{-1}$  [4]<sup>3</sup>. The photon spectrum is dependent on the annihilation channels. We restrict ourselves to the popular *Minimal Supersymmetric Standard Model* (MSSM). In this framework, the neutralino is the lightest stable supersymmetric particle and is the favoured DM candidate.

Neutralino annihilation can produce  $\gamma$ -rays in three ways: i) DM annihilates directly into two photons or  $Z^0\gamma$ , giving rise to monochromatic lines, ii) or it can annihilate into primary products, the hadronization and decay of which produce a  $\gamma$ -ray continuum, iii) finally, if charged particles are produced, internal bremsstrahlung (IB) can also contribute to the continuum. The lines are very model-dependent and given the uncertainties on the particle physics models, we did not include any in clumpy. The user is however free to implement their own function if need be. There exists in the literature several parametrisations for the  $\gamma$ -ray continuum and we have implemented that of Bergström et al. [4], and Tasitsiomi and Olinto [5]. The IB contributions is also very model dependent, and we provide one benchmark model from Bringmann et al. [6]. We refer the reader to §3.3.7 (spectra.h) for more details on these spectra.

#### 2.1.2. Annihilation: the astrophysical contribution

The  $\gamma$ -rays being produced from the annihilation of pairs of dark matter particles, the  $\gamma$ -ray flux is proportional to the DM density squared. The second term of Eq. (1), termed *astrophysical* contribution, corresponds to the integration of the density

squared along the line of sight  $(l,\psi,\theta)$ , in the observational cone of solid angle  $\Delta\Omega$  (see Appendix A and geometry.h for definitions and the geometry used)

$$J(\psi, \theta, \Delta\Omega) = \int_0^{\Delta\Omega} \int_{l, 0.5} \rho^2(l(\psi, \theta)) \, dl \, d\Omega , \qquad (3)$$

where  $\rho(l(\psi, \theta))$  is the local DM density at distance l from Earth in the direction  $\psi$ ,  $\theta$  (longitude and latitude in Galactic coordinates).

This integral is the core of the CLUMPY code and can be rather complicated to evaluate. The difficulty comes partially from the steepness of some DM profiles considered but also, and mainly, from the existence of DM clumps, the contribution of which must be added to the smooth Galactic DM background. Both components are dealt with hereafter.

#### 2.1.3. Decaying dark matter

 $\gamma$ -rays can also be produced from the decay of DM. Formally, the flux is still given by Eq. (1) but the astrophysical and particle physics terms take different forms:

- The particle physics term now corresponds to the decay spectrum and no parametrisation of that term has been included in the code so far. The parametrisation from Bertone et al. [7] could be used.
- Because we are now considering a decay reaction, the astrophysics term is the integration of the density (rather that density squared as in Eq. (3)) along the line of sight, namely

$$J(\psi, \theta, \Delta\Omega) = \int_{0}^{\Delta\Omega} \int_{0}^{\Omega} \rho(l(\psi, \theta)) dl d\Omega . \tag{4}$$

This quantity can be straightforwardly computed in CLUMPY by setting the flag parameter gDM\_IS\_ANNIHIL\_OR\_DECAY to false (see table 3).

#### 2.2. Dark matter distribution

To date, there is no consensus to what the Galactic DM profile,  $\rho(r)$ , should be. There is some dynamical evidence that the Galactic DM halo might be triaxial [8] and numerical simulations, such as the Aquarius [9] or the Via Lactea runs [10] also find non-spherical halos. When included, the baryons tend however to make the halos more spherical [11]. In this first version of CLUMPY we simplify the problem (as often done) using spherically symmetric DM halos, but allowing for tri-axiality is being considered for future releases. The total averaged density profiles of the DM halos measured in these simulations are generally fitted by a form of the Zhao profile [12]:

$$\rho_{\text{tot}}(r) = \frac{\rho_s}{(r/r_s)^{\gamma} \left[1 + (r/r_s)^{\alpha}\right]^{(\beta - \gamma)/\alpha}},$$
(5)

where  $\gamma$ ,  $\alpha$  and  $\beta$  are respectively the inner, transition and outer slope of the profile,  $\rho_s$  and  $r_s$  a scale density and radius. The Navarro et al. [13], Moore et al. [14], and Diemand et al. [15] profiles fall in this category. Some other profiles having

<sup>&</sup>lt;sup>3</sup>Note that a velocity dependent cross section should be included in a future release of the code in order to account for the Sommerfeld effect in the calculation. This effect results in much larger cross sections for some resonant neutralino masses and therefore boost the signal for these DM candidates.

a log-varying inner slope [16, 17] are also included in CLUMPY. The reader is referred to §3.3.4 (profiles.h) for a detailed description.

Some of these DM profiles are steep enough in the inner regions to lead to a singularity of the  $\gamma$ -ray luminosity at the centre of the halo. However, a cut-off radius  $r_{\rm cut}$  naturally appears, within which the DM density saturates due to the balance between the annihilation rate and the gravitational infalling rate of DM particles. The saturation density reads [18]

$$\rho_{\rm sat} = 3.10^{18} \left( \frac{m_{\chi}}{100 \text{ GeV}} \right) \times \left( \frac{10^{-26} \text{cm}^3 \text{ s}^{-1}}{\langle \sigma v \rangle} \right) \text{M}_{\odot} \text{ kpc}^{-3}. (6)$$

Plugged in any profile parametrisation, this saturation density defines the cut-off radius below which the annihilation rate is constant. This is a very crude description, but this is not important as this cut-off matters only for the steepest profiles  $(\gamma \geq 1.5)$  which are disfavoured by current numerical simulations.

The hierarchical scenario of galaxy formation in a  $\Lambda$ -CDM cosmology is characterised by a high degree of clumpiness of the DM distribution (supported by N-body simulations) so that the total averaged density computed from these simulations corresponds to

$$\rho_{\text{tot}}(r) = \rho_{\text{sm}}(r) + \langle \rho_{\text{subs}}(r) \rangle \tag{7}$$

where  $\rho_{\rm sm}(r)$  is the "true" smooth component and  $\langle \rho_{\rm sub}(r) \rangle$  represents the average density from the substructures. These two components are discussed below.

#### 2.2.1. The clumps

Whether substructures in subhalos are scaled-down versions of substructure in main halos remains an open question [9]. In this release of CLUMPY, we only consider one level of substructure within the halo under scrutiny (Galactic halo or individual halo such as a dwarf spheroidal galaxy), and the properties of these sub-halos can be independently chosen from that of the parent halo (see below).

Clump individual properties. In the code, all clumps have the same inner DM distribution, which can be any of the profiles discussed in §3.3.4. The mass of the clump generally suffices to determine all its properties, i.e., its size  $R_{\rm vir}^{\rm cl}$ , and once an inner density profile is chosen, its scale radius  $r_s$  and scale density  $\rho_s$ . This is done using the so-called *concentration parameter*  $c_{\rm vir}$ . Parametrisations of the mass-concentration relation have been established from numerical simulations for isolated field halos [19, 20] but the concentration generally presents a significant scatter [21]. These relations have been implemented in the code, regardless of that scatter or of the environment and formation history of the halos that can also affect the value of the concentration. We refer the reader to §3.3.6 (clumps.h) and the Doxygen documentation for more details.

Spatial and mass distribution of the clumps. Following Lavalle et al. [22], spherical symmetry is assumed and the two distri-

butions are assumed independent<sup>4</sup>. If  $N_{\text{tot}}$  is the total number of clumps in a DM host halo, then the overall distribution of clumps is written as:

$$\frac{\mathrm{d}^2 N}{\mathrm{d}V \mathrm{d}M} = N_{\text{tot}} \frac{\mathrm{d}\mathcal{P}_V(r)}{\mathrm{d}V} \frac{\mathrm{d}\mathcal{P}_M(M)}{\mathrm{d}M} , \qquad (8)$$

where the spatial and mass distribution are probabilities, respectively normalised as:

$$\int_{V} \frac{d\mathcal{P}_{V}(r)}{dV} dV = 1 \quad \text{and} \quad \int_{M_{min}}^{M_{max}} \frac{d\mathcal{P}_{M}(M)}{dM} dM = 1.(9)$$

Analysis of N-body simulations have shown the mass distribution to vary as a simple power law

$$\frac{\mathrm{d}\mathcal{P}_M(M)}{\mathrm{d}M} \propto M^{-\alpha_M} \text{ with } \alpha_M \approx 1.9. \tag{10}$$

Given the mass and spatial distribution, the total number of clumps  $N_{\text{tot}}$  can be determined in two ways. If the mass fraction f of clumps and the mass of the host  $M_{\text{tot}}$  halo are known,

$$N_{\rm tot} = \frac{f M_{\rm tot}}{\langle M_{\rm 1cl} \rangle} \,,$$

with  $\langle M_{1\text{cl}} \rangle$  the average mass of one clump. An other approach is to know the number of clumps  $N_0$  in a given mass interval  $[M_1, M_2]$  and to calculate the total number of clumps as

$$N_{\text{tot}} = \frac{N_0}{\int_{M_1}^{M_2} \frac{\mathrm{d}\mathcal{P}_M(M)}{\mathrm{d}M}} \ .$$

Clumpy uses the first approach when dealing with substructures in individual host halos (that are not the Milky Way) and the second one for the Galactic halo clumps (numerical simulations have found about 100 clumps with masses above  $10^8~M_{\odot}$  in Milky Way-like halos). clumpy allows to select any of the profiles given in §3.3.4 (profiles.h) for the clump spatial distribution, independently of the choice made for the total density profile.

#### 2.2.2. The smooth component

From the spatial distribution  $d\mathcal{P}_V(r)/dV$  one can define the average clump density  $\langle \rho_{\rm cl} \rangle$  used in Eq. (7) as:

$$\langle \rho_{\text{subs}}(r) \rangle = f M_{\text{tot}} \frac{\mathrm{d} \mathcal{P}_V(r)}{\mathrm{d} V} \ . \tag{11}$$

For a given total density profile and clump spatial distribution, Eq. (7) then gives the smooth density profile as

$$\rho_{\rm sm}(r) = \rho_{\rm tot}(r) - \langle \rho_{\rm subs}(r) \rangle . \tag{12}$$

Using this approach, rather than providing independently a smooth and a clump profile, ensures (by construction) that the total density profiles follows the one measured in N-body simulations. This has been discussed in Pieri et al. [23].

<sup>&</sup>lt;sup>4</sup>Keep in mind that such an assumption fails near the Galactic centre, where small clumps are expected to be disrupted by tidal forces. In this direction, the signal from the smooth contribution dominates anyway.

#### 2.3. Calculating the annihilation J-factor

The astrophysical contribution to the annihilation flux is formally given by Eq. (3). The J-factor is more explicitly written as:

$$J = \int_0^{\Delta\Omega} \int_{l_{\min}}^{l_{\max}} \frac{1}{l^2} \left( \rho_{\text{sm}} + \sum_i \rho_{\text{cl}}^i \right)^2 l^2 dl d\Omega . \tag{13}$$

where  $\rho_{\rm sm}$  is given by Eq. (12) and the second term corresponds to the sum of the inner densities squared of all clumps i contained with the volume element. The latter should not be confused with  $\langle \rho_{\rm subs}(r) \rangle$ . Three terms arise from this equation:

$$J_{\rm sm} \equiv \int_0^{\Delta\Omega} \int_{l_{\rm min}}^{l_{\rm max}} \rho_{\rm sm}^2 dl d\Omega , \qquad (14)$$

$$J_{\text{subs}} \equiv \int_{0}^{\Delta\Omega} \int_{l_{\text{min}}}^{l_{\text{max}}} \left( \sum_{i} \rho_{\text{cl}}^{i} \right)^{2} dl d\Omega , \qquad (15)$$

and the cross product

$$J_{\rm cross-prod} \equiv 2 \int_0^{\Delta\Omega} \int_{l_{\rm min}}^{l_{\rm max}} \rho_{\rm sm} \sum_i \rho_{\rm cl}^i dl d\Omega . \tag{16}$$

#### 2.4. Statistical considerations

Taking  $\alpha_M = 1.9$  as a canonical value for the slope of the mass distribution, with 100 clumps above  $10^8 \text{ M}_{\odot}$  as determined from N-body simulations, and a minimal theoretical mass of  $10^{-6}\ M_{\odot}$  for the smallest ones, the total number of clumps in the Milky Way halo (i.e., for  $\Delta\Omega = 4\pi$ ) is estimated to be  $\sim 10^{14}$ , which is computationally prohibitive. Even if we are interested in a 5°  $\times$  5° skymap only (i.e.,  $\Delta\Omega \sim 6 \times 10^{-3}$ ), the number of clumps is  $\sim 5 \times 10^{10}$ . This is too large a number to realise a statistical drawing according to the  $dP_V/dV$  distribution and then numerically integrate the J component for each of these clumps. However, their high number means that a statistical average can be safely performed, as long as the variance is small. The CLUMPY code computes the  $\gamma$ -ray flux from DM clumps by either randomly drawing clumps given user-defined spatial and mass distributions and inner DM profiles, or by calculating the mean contribution whenever the variance of this contribution is smaller than some user-defined value.

#### 2.4.1. Averaged quantities from the continuum limit

Given the clump mass and spatial distributions given above, it is possible to define average values for the primary random variables of the problem (a clump mass and position) and the one deriving from them. The mean mass and the mean luminosity are respectively defined by

$$\langle M \rangle = \int_{M_{\text{min}}}^{M_{\text{max}}} M \frac{\mathrm{d}\mathcal{P}_M}{\mathrm{d}M} \mathrm{d}M ,$$
 (17)

$$\langle \mathcal{L} \rangle = \int_{M_{\min}}^{M_{\max}} \mathcal{L}(M) \frac{\mathrm{d}\mathcal{P}_M}{\mathrm{d}M} \mathrm{d}M \ .$$
 (18)

where  $\mathcal{L}(M)$  is the intrinsic luminosity of a clump given by

$$\mathcal{L}(M) \equiv \int_{V_{cl}} (\rho_{cl})^2 \, dV \,. \tag{19}$$

The average of the distance to any power n is written as

$$\langle l^n \rangle = \int_0^{\Delta\Omega} \int_{l_{\min}}^{l_{\max}} l^n \frac{\mathrm{d}\mathcal{P}_V}{\mathrm{d}V} l^2 dl d\Omega \ . \tag{20}$$

Combining these, it can be shown that the average contribution from the clumps is expressed as

$$\langle J_{\text{subs}} \rangle = N_{\text{tot}} \int_{0}^{\Delta\Omega} \int_{l_{\text{min}}}^{l_{\text{max}}} \frac{\mathrm{d}\mathcal{P}_{V}}{\mathrm{d}V} dl d\Omega \int_{M_{\text{min}}}^{M_{\text{max}}} \mathcal{L}(M) \frac{\mathrm{d}\mathcal{P}_{M}}{\mathrm{d}M} \mathrm{d}M$$
, (21)

In the same spirit, in the continuum limit the cross-product becomes

$$\langle J_{\text{cross-prod}} \rangle = 2 \int_0^{\Delta\Omega} \int_{l_{\text{min}}}^{l_{\text{max}}} \rho_{\text{sm}} \langle \rho_{\text{subs}} \rangle dl d\Omega .$$
 (22)

The importance of the cross product can easily be identified when considering that the clump spatial distribution follows the total average density profile. In that case,  $\langle \rho_{\text{subs}}(r) \rangle = f \rho_{\text{tot}}(r)$  and  $\rho_{\text{sm}}(r) = (1-f)\rho_{\text{tot}}(r)$ , leading directly to  $J_{\text{cross-prod}}/J_{\text{tot}} = 2f(1-f)$ . For a typical clump mass fraction f=10%, this cross product amounts to 18% of the reference J-factor  $J_{\text{tot}} \equiv \int \rho_{\text{tot}}^2 dl d\Omega$ . This shows that when an averaged clump description is used to integrate the signal along the line of sight, this cross product is not negligible and should be taken into account. In Appendix B, it is however shown that on an individual basis, the cross product of one clump in an underlying smooth Galactic component can be safely discarded. This will apply to the clumps that need to be statistical drawn in our model (see below).

# 2.4.2. Variance of the clump contribution $\sigma_{\text{clumps}}^2$

Along a given line of sight and for a given integration angle  $\alpha_{\rm int}$ , the average flux between  $l_{\rm min}$  and  $l_{\rm max}$  given by Eq. (21) can also be expressed as

$$\langle J_{\text{subs}} \rangle = \langle N_{\text{cl}} \rangle \langle J_{1 \, \text{cl}} \rangle ,$$
 (23)

where  $\langle N_{\rm cl} \rangle$  is the average number of clumps in this volume (given the distance interval and mass range considered), and  $\langle J_{1\,{\rm cl}} \rangle$  the average flux of one clump (see also [24]). Using the point-like approximation, a clump of mass M at a distance l has a flux

$$J_{1 \text{ cl}} = \frac{\mathcal{L}(M)}{l^2} , \qquad (24)$$

so that, recalling that the mass of the clump and its location are independent variables,

$$\langle J_{1 \text{ cl}} \rangle = \langle \mathcal{L} \rangle \left( \frac{1}{l^2} \right).$$
 (25)

The variance on the flux of a single clump is then defined as

$$\sigma_{1 \text{ cl}}^2 = \langle J_{1 \text{ cl}}^2 \rangle - \langle J_{1 \text{ cl}} \rangle^2 = \langle \mathcal{L}^2 \rangle \left\langle \frac{1}{l^4} \right\rangle - \langle J_{1 \text{ cl}} \rangle^2 . \tag{26}$$

The variance of a population of  $\langle N_{\rm cl} \rangle$  clumps follows as

$$\sigma_{\rm cl}^2 = \langle N_{\rm cl} \rangle \, \sigma_{\rm 1\,cl}^2 \,. \tag{27}$$

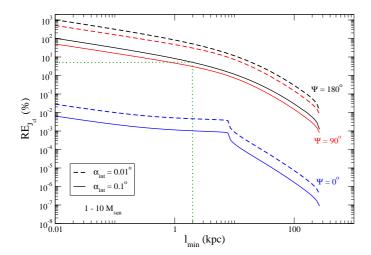


Figure 1: Relative error between the *average* and *true J* value from clumps when the signal is integrated between  $l_{\rm min}$  and  $l_{\rm max}$ , according to Eq. (28), for  $M_{\rm cl} \in [1,10]~{\rm M}_{\odot}$ . The signal is integrated over  $\alpha_{\rm int} = 0.1^{\circ}$  (solid line) or  $\alpha_{\rm int} = 0.01^{\circ}$  (dashed line). Three directions in the Galactic plane are considered: towards the Galactic centre (blue), towards the anti-centre (black) and towards the Galactic East (red). For the latter the green dotted line illustrates how to get the critical distance (here 2 kpc) below which clumps (in this mass range) must be drawn if the user requires an accuracy of 5%. All distributions (smooth, clump distribution and inner DM clump profile) are taken as NFW profiles.

#### 2.4.3. Criterion for using the averaged description

For a given integration domain, there is a threshold mass above which the clumps are not numerous enough to be described by an averaged description. Conversely, for a given mass decade there is a critical distance below which the averaged description fails and where clumps should be statistically drawn.

The averaged description can be safely used, as long as the relative error *RE* made on the clump flux with respect to the *total* flux (i.e. including the smooth contribution)

$$RE_{J_{\text{clumps}}} = \frac{\sqrt{N_{\text{cl}}} \,\sigma_{1\,\text{cl}}}{N_{\text{cl}}\langle J_{1\,\text{cl}}\rangle + J_{\text{smooth}}}$$
(28)

is smaller than some user-defined prescription.

Critical distance  $l_{\rm crit}$  for the clumps in the Galaxy. The RE is plotted in Fig. 1 as a function of the lower limit of the integration  $l_{\rm min}$ . As can be seen, for any required precision (e.g., 5% in Fig. 1, tagged by the dotted green line), one can infer the corresponding distance  $l_{\rm crit}$  below which the relative error becomes larger than the request.

In practise, it is convenient to perform the calculation in terms of mass decades: for each decade, we find (by dichotomy) the critical distance below which clumps must be drawn<sup>5</sup>. A lower limit of  $10^{-3}$  kpc is set on  $l_{\rm crit}$  to avoid divergences for a clump sitting at the observer's location. An example for the final number of clumps to draw below  $l_{\rm crit}$  is provided in Tab. 1. The flowchart given in Fig. 2 summarises the various steps implemented in CLUMPY to obtain the skymap (see, e.g., Fig. 3).

Table 1: Example of  $l_{\rm crit}$  and the associate number of clumps to draw toward the anti-centre (criterion  $RE_{J_{\rm clumps}}$  5%) for a 2°×2° skymap with an integration angle (instrument) of  $\alpha_{\rm int}$ =0.1°. Actually, a given  $l_{\rm crit}$  gives  $< n_{\rm cl} >$  number of clumps:  $n_{\rm cl}$  to draw is obtained from a Poisson distribution of mean value  $< n_{\rm cl} >$ .

Mass decade	# clumps	l <sub>crit</sub> (kpc)	<# clumps>	# to draw
$\log_{10}\left(\frac{M_{\rm cl}}{M_{\odot}}\right)$	(Galaxy)	kpc	$(5^{\circ} \times 5^{\circ})$	$(5^{\circ} \times 5^{\circ})$
-6:-5	$3.5 \times 10^{14}$	$1.0 \times 10^{-3}$	$1.5 \times 10^{-3}$	0
-5:-4	$4.4 \times 10^{13}$	$1.0 \times 10^{-3}$	$1.8 \times 10^{-4}$	0
-4:-3	$5.6 \times 10^{12}$	$1.0 \times 10^{-3}$	$2.4 \times 10^{-5}$	0
-3:-2	$7.0 \times 10^{11}$	$7.3 \times 10^{-3}$	$1.2 \times 10^{-3}$	0
-2:-1	$8.8 \times 10^{10}$	$5.8 \times 10^{-2}$	$7.6 \times 10^{-2}$	0
-1:0	$1.1 \times 10^{10}$	$4.0 \times 10^{-1}$	2.8	3
0:1	$1.4 \times 10^{9}$	2.0	$3.7 \times 10^{+1}$	45
1:2	$1.7 \times 10^{8}$	6.7	$1.0 \times 10^{+2}$	117
2:3	$2.2 \times 10^{7}$	$1.6 \times 10^{1}$	$8.7 \times 10^{+1}$	99
3:4	$2.8 \times 10^{6}$	$3.3 \times 10^{1}$	$3.6 \times 10^{+1}$	44
4:5	$3.5 \times 10^{5}$	$5.8 \times 10^{1}$	$1.0 \times 10^{+1}$	7
5:6	$4.4 \times 10^{4}$	$9.5 \times 10^{1}$	2.2	1
6:7	$5.6 \times 10^{3}$	$1.5 \times 10^{2}$	$4.1 \times 10^{-1}$	0
7:8	$7.0 \times 10^{2}$	$2.5 \times 10^{2}$	$6.6 \times 10^{-2}$	0
8:9	$8.8 \times 10^{1}$	$2.9 \times 10^{2}$	$9.5 \times 10^{-3}$	0
9:10	$1.1 \times 10^1$	$2.9 \times 10^{2}$	$1.3 \times 10^{-3}$	0

Threshold mass  $m_{\rm thresh}$  for the sub-clumps in an "far-away" halo. The second case where the criterion is used is for the sub-clumps in a halo located away from us, e.g., a dSph galaxy. In that case, the distance d to the clumps is known as they are all distributed within their parent halo's radius and we can determine in a similar fashion the threshold mass above which clumps must be drawn.

### 3. Description of the code

CLUMPY is written in C/C++ and relies of the CERN ROOT library that is publicly available from http://root.cern.ch. The main features of the code are described hereafter along with brief descriptions of its most important functions. The code's structure is standard, with separate directories for the various pieces of code: declarations are in include/\*.h, sources in src/\*.cc, compiled libraries, objects and executables are respectively in the lib/, obj/, and bin/ directories.

#### 3.1. Code executables

The code has one executable that can take three main options depending whether the user i) is interested in performing a generic analysis on the Galactic halo, ii) is focusing on a list of specific halos (e.g. dSph galaxies), or iii) wishes to perform a statistical analysis on a single object. These main three usages of the code all accommodate several cases, the details of which are given below.

# 3.1.1. Galactic mode: ./bin/clumpy -g[option]

This calculates the *J*-factor for the Galactic halo, including both the smooth and the clump component. For the clump component, the mean flux or a stochastic realisation can be chosen. Note that a list of specific halos can be added to the calculation. Several options allow to select the quantity to be calculated/plotted from a text interface. A mandatory input is the

<sup>&</sup>lt;sup>5</sup>This mass decade corresponds to a typical clump radius  $R_{\rm cl}$  and we also check that the point-like hypothesis used to derived the continuum limit is appropriate ( $l_{\rm crit} \gg R_{\rm cl}$ ).

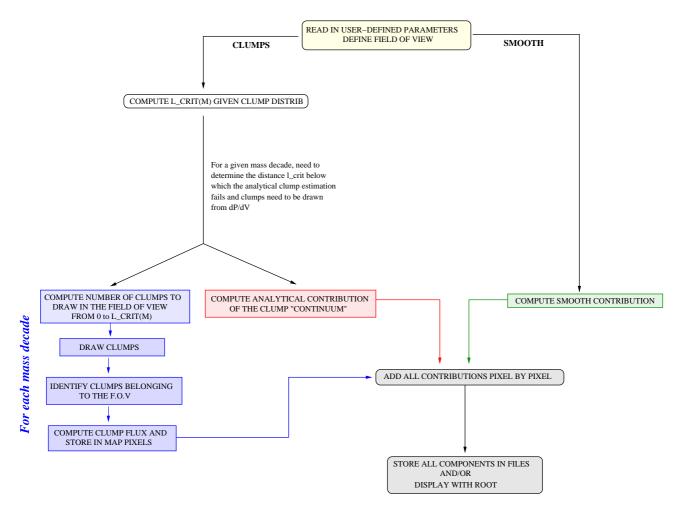


Figure 2: Flow chart of the different steps leading to the creation of a  $\gamma$ -ray skymap.

Table 2: Format of the ASCII file to describe halos processed by ./bin/clumpy\_dsph -h[option]. 'Type' should take one of the values of gENUM\_TYPEHALOES, namely DSPH, GALAXY or CLUSTER. 'long','lat' and 'd' are the galactic longitude, latitude, and distance to the halo, z its redshift and  $R_{\text{vir}}$  its virial radius. The profile parameters are  $\rho_s$ ,  $r_s$ , 'prof.' (see profile.h) and the shape parameters. In particular, if 'prof'=kZHAO, see §3.2), the profile corresponds to the  $(\alpha, \beta, \gamma)$  profile (in which case the next-to-last three columns are used). If an kEINASTO is used instead, only the first shape parameter is considered.

Name	Type	long.	lat.	d	Z	$R_{ m vir}$	$\rho_s$	$r_s$	prof.	α	β	γ
-	-	deg	deg	kpc	-	kpc	$M_{\odot}{\rm kpc^{-3}}$	kpc	-	-	-	-

parameter file clumpy\_params.txt described in §3.2 and table 3. The options are the following:

- 'option = -g1' plots the total, smooth and averaged clump density in 1D, given the user-defined total density and the number of clumps in a given mass range.
- 'option = -g2' plots  $J_{\rm sm} + \langle J_{\rm subs} \rangle + \langle J_{\rm cross-prod} \rangle^6$ , respectively defined by Eq. (14), Eq. (21) and Eq. (22) as a function of the integration angle  $\alpha_{\rm int}$ .
- 'option = -g3' is the same as '-g2' but as a function of the angular distance to the Galactic centre, for a given integration angle.

- 'option = -g4' is the same as '-g3' but including a list of specific objects, such as dSph galaxies.
- 'option = -g5' plots a 2D skymap of the *J*-factor, using the averaged clump description. where
- 'option = -g6' is similar '-g5', but adds specific halos to the plots (e.g. dSph), to identify their contrast to the background.
- 'option = -g7' and -g8 are respectively the same as '-g5' and '-g6' but with the clumps randomly drawn from their user-defined mass and spatial distributions.

## 3.1.2. Halo list mode: ./bin/clumpy -h[option]

This usage of CLUMPY performs operations that are basically the identical to those performed in the 'Galactic' mode (see

<sup>&</sup>lt;sup>6</sup>Does not exist for decaying DM.

§3.1.1), but for a list of halos that are not the Galactic halo. This mode was initially implemented for the study of dwarf spheroidal galaxies [25], but can be extended to any DM halo (e.g., galaxy clusters). If only the smooth total density profile is considered, the halo profiles are read from a simple ASCII list of profiles along with their parameters, as exemplified in Tab. 2. If sub-clumps are considered, the CLUMPY parameter file clumpy\_params.txt must also be provided (see table 3).

Again, further options are accessed for this mode by means of a simple text-user interface:

- '-h1' to '-h3' are identical to '-g1' to '-g3' but transposed to any halo that is not the Galactic DM halo;
- '-h4' to '-h5' create 2D skymaps of the halo under scrutiny using respectively an averaged description or a statistical realisation for the sub-clumps within this halo;

#### 3.1.3. 'Statistical' mode: ./bin/clumpy -s[option]

Confidence levels on the astrophysical factor J, for e.g. dSphs, can also be estimated from an ASCII statistical file. The format for the latter is quite similar to the halo list file (see documentation) as it requires the profile parameters, but it differs in the sense that for a single halo, thousands of profiles 'selected' by a statistical analysis must be provided (their respective  $\chi^2$  value is necessary, although not used for all CL calculations). Such a statistical analysis can be the Markov Chain Monte Carlo (MCMC), as discussed in Charbonnier et al. [25]; Walker et al. [26]. The most important options in the statistical mode are:

- '-s2' plots the probability density functions (PDF) and correlations of the parameters given in the statistical file;
- '-s4' locates the x% confidence intervals on the PDF of each parameters;
- '-s5' computes, for each radius from the dSph centre, the median and confidence levels of the DM density  $\rho(r)$ ;
- '-s6' is the same as '-s5' but for  $J(\alpha_{\rm int})$ , i.e. as a function of the integration angle;
- '-s7' is same as '-s5' but for  $J(\theta)$ , i.e. as a function of the angle to the centre of the halo.

#### 3.2. User-defined parameters

For most of the options used above, a user-define parameter file is read. Table 3 gives the list of the main parameters. Each parameter is a single word formatted as g (for global parameter) + physics keyword + parameter name. For instance, COSMO refers to cosmology-related parameters, TYPE defines the type of halo under scrutiny (dSPh galaxy, galaxy or galaxy clusters) and GAL refers to the Milky Way halo parameters. Note also that the parameters ending in FLAG\_CVIRMVIR, FLAG\_SPECTRUM, and FLAG\_PROFILE should take values from the following enumerations:

• enum gENUM\_CVIRMVIR {kB01, kENS01, kJS00} for the clump concentration;

- enum gENUM\_GAMMASPECT {kSUSY\_BUB98, kSUSY\_T002, kSUSY\_BBE08} for the γ-ray spectrum;
- enum gENUM\_PROFILE {kZHAO, kEINASTO, kEINASTO\_N} for the DM spatial distributions (smooth and clumpy components). The corresponding shape parameters (e.g.  $(\alpha, \beta, \gamma)$  for kZHAO,  $\alpha$  for kEINASTO and n for kEINASTO\_N) should be provided by the user in the input parameter file (see below).

This enumerations allow the user to use their explicit denomination kXXX, rather than the corresponding integer number. These parameters are gathered in the ASCII file clumpy\_params.txt which is formatted as "Parameter name", "Units", "Value". The function load\_parameters(file\_name) defined in params.h reads in the input file and assigns the appropriate values to the parameters. The user can change any "Value" in this file<sup>7</sup>.

#### 3.3. CLUMPY libraries

The functions of CLUMPY are located in eleven "library" files, accordingly to their field of action. We briefly describe hereafter the most important functions of these libraries. We are omitting in this document the more minor functions, the description of which can be found in the documentation provided with the code. Also, we do not detail here what arguments are passed in these functions as it can be easily retrieved from the documentation.

#### 3.3.1. integr.h

This library contains standard integration algorithms. The Simpson integrator with doubling step is the most widely used throughout the code to ensure that the user-defined precision is reached. Depending on the function to integrate, either linear or logarithmic stepping can be used.

# 3.3.2. geometry.h

In geometry.h are gathered functions performing geometrical tasks (change of coordinates, etc.). They are not essential for the presentation of the code and generally self-explanatory. We refer the reader to the extensive Doxygen documentation and to Appendix A for more details.

#### 3.3.3. inlines.h and misc.h

These contain some definitions (unit conversion and inline functions) and miscellaneous formatting functions respectively.

#### 3.3.4. profiles.h

This library contains all the density profiles available in the code and operations on those. Among the functions in the "profiles" section of the code, some act in a very straightforward way (e.g. squaring the density) and are not included in the present document. The reader is, once again, referred to the Doxygen documentation for a full description.

<sup>&</sup>lt;sup>7</sup>Some of these parameters are optional and used only for specific modes[options]

Table 3: clumpy user-defined input parameter file (clumpy\_params.txt).

Name	Definition				
gCOSMO_RHOO_C	Critical density of the universe $[M_{\odot} \text{ kpc}^{-3}]$				
gCOSMO_OMEGAO_M	Present-day pressure-less matter density				
gCOSMO_OMEGAO_LAMBDA	Present-day dark energy density				
gDM_GAMMARAY_FLAG_SPECTRUM	$\gamma$ -ray spectrum flag [gENUM_GAMMASPECT]				
gDM_MMIN_SUBS	Min. mass of a DM (sub-)clump $[M_{\odot}]$				
gDM_MMAXFACTOR_SUBS	Defines max. mass of clump in host halo as $M_{\text{max}} = \text{Factor} \times M_{\text{host}}$				
gDM_RHOSAT	Saturation density $[M_{\odot} \text{ kpc}^{-3}]$				
gTYPE_CLUMPS_FLAG_CVIRMVIR <sup>†</sup>	Clump concentration flag in the parent TYPE halo[gENUM_CVIRMVIR]				
gTYPE_CLUMPS_FLAG_PROFILE	Clump inner profile flag [gENUM_PROFILE]				
gTYPE_CLUMPS_SHAPE_PARAMS[0-2]	Shape parameters for sub-clumps inner profile				
gTYPE_DPDM_SLOPE	Slope subclump mass function				
gTYPE_DPDV_FLAG_PROFILE	Spatial distribution subclumps [gENUM_PROFILE]				
gTYPE_DPDV_RSCALE	Scale radius for the subclump distribution in the parent halo [kpc]				
gTYPE_DPDV_SHAPE_PARAMS[0-2]	Shape parameters for the clump spatial distribution				
gTYPE_SUBS_MASSFRACTION	Mass fraction of the parent halo in sub-clumps				
gGAL_CLUMPS_FLAG_CVIRMVIR	Concentration flag [gENUM_CVIRMVIR]				
gGAL_CLUMPS_FLAG_PROFILE	Clump inner profile flag [gENUM_PROFILE]				
gGAL_CLUMPS_SHAPE_PARAMS[0-2]	Shape parameter for the Galactic clumps inner profile				
gGAL_DPDM_SLOPE	Slope the clump mass function				
gGAL_DPDV_FLAG_PROFILE	Clump number distribution flag				
gGAL_DPDV_RS	Scale radius for clumps [kpc]				
gGAL_DPDV_SHAPE_PARAMS[0-2]	Three shape parameters for the Galactic clump spatial distribution				
gGAL_SUBS_M1	Reference mass for gGAL_SUBS_N_INM1M2 $[M_{\odot}]$				
gGAL_SUBS_M2	Reference mass for gGAL_SUBS_N_INM1M2 $[M_{\odot}]$				
gGAL_SUBS_N_INM1M2	# of clumps in $[M_1, M_2]$				
gGAL_RHOSOL	Local DM density [GeV cm <sup>-3</sup> ]				
gGAL_RSOL	Distance Sun – Galactic centre [kpc]				
gGAL_RVIR	Virial radius of the Galaxy [kpc]				
gGAL_TOT_FLAG_PROFILE	Total DM profile of the Milky Way [gENUM_PROFILE]				
gGAL_TOT_RSCALE	Scale radius for DM halo of the Milky Way [kpc]				
gGAL_TOT_SHAPE_PARAMS[0-2]	Shape parameter for the Galactic total density profile				
gSIMU_ALPHAINT_DEG	Integration angle [deg]				
gSIMU_EPS	Default precision for numerical integrations				
gSIMU_IS_ANNIHIL_OR_DECAY	For annihilating or decaying DM				
gSIMU_SEED	Seed of random number generator				

<sup>†</sup> TYPE corresponds to the type of halo under scrutiny (not the Galactic halo): DSPH, GALAXY or CLUSTER are the values allowed by gENUM\_TYPEHALOES in this version.

Table 4: Standard DM density profiles parameters, following Eq. (29).

	α	β	γ	Reference
ISO	2	2	0	-
NFW97	1	3	1	Navarro et al. [13]
M98	1.5	3	1.5	Moore et al. [14]
DMS04	1	3	1.2	Diemand et al. [15]

 rho\_ZHAO returns the DM density of a user-defined profile having the generic functional form given in Zhao [12]

$$\rho(r)^{\text{ZHAO}} = \frac{\rho_s}{(r/r_s)^{\gamma} \left[1 + (r/r_s)^{\alpha}\right]^{(\beta - \gamma)/\alpha}}.$$
 (29)

The isothermal, Navarro et al. [13], Moore et al. [14], and Diemand et al. [15] profiles are all from the Zhao family. They are defined using Eq. (29) with the set of parameters of table 4. The user is required to specify these parameters in the ASCII input file.

rho\_EINASTO and rho\_EINASTO\_N: it has been argued that
a profile with a logarithmic inner slope gives a better fit to
the simulated data. This so-called Einasto is generically
written as

$$\rho(r) \propto \exp(-Ar^{\alpha})$$
.

In Navarro et al. [16] and Springel et al. [9] the Einasto profile is defined as

$$\rho^{\text{EINASTO}}(r) = \rho_{-2} \exp\left(-\frac{2}{\alpha} \left[ \left(\frac{r}{r_{-2}}\right)^{\alpha} - 1 \right] \right), \quad (30)$$

where  $r_{-2}$  is the radius at which the profile's slope  $d \ln \rho / d \ln r = -2$ , and  $\rho_{-2}$  the corresponding density. Both Navarro et al. [16] and Springel et al. [9] find  $\alpha \sim 0.17$  to be a good fit to the smooth component of simulated halos. Springel et al. [9] further notes that  $\alpha \sim 0.68$  allows a good description of the spatial distribution of substructures dP/dV.

Merritt et al. [17] use an Einasto  $r^{1/n}$  profile, slightly different from the above and given by

$$\rho^{\text{EINASTO\_N}}(r) = \rho_e \exp\left(-d_n \cdot \left[ \left(\frac{r}{r_e}\right)^{1/n} - 1 \right] \right). \quad (31)$$

where n = 6 and  $d_n = 3n - 1/3 + 0.0079/n$ . The scale radius  $r_e$  in this relation correspond to the radius within which half of the halo mass is enclosed.

• The normalisation density of all these profiles  $(\rho_s, \rho_{-2} \text{ or } \rho_e)$  can be done by i) either requesting a given value for the density at a given point or ii) by requesting a given mass within a given radius. The functions set\_par0\_given\_rhoref and set\_par0\_given\_mref achieve the normalisation using both approaches.

- rho simply returns the value of the density by calling any
  of the functions above, depending on the value of the profile flag given in clumpy\_params.txt.
- For two density profiles  $\rho_1$  and  $\rho_2$ , rho\_mix returns either the difference  $\rho_1 \rho_2$  or the product  $\rho_1 \rho_2$ . This is needed when computing Eq. (12) for the smooth density profile or Eq. (22) for the cross product.

#### 3.3.5. integr\_los.h

This is the core of the CLUMPY code as it contains the routines performing the integration along the line of sight direction  $(\theta, \psi)$  and over the chosen angular resolution  $\alpha_{\text{int}}$ . Given a function  $g(l, \alpha, \beta; \theta, \psi)$  where  $(l, \alpha, \beta)$  are spherical coordinates in the  $(\theta, \psi)$  direction (see Fig. 6), the integration along the observation cone, from  $l_1$  to  $l_2$  is generically written as:

$$I(\theta, \psi) = \int_0^{2\pi} d\beta \int_0^{\alpha_{\text{int}}} \sin\alpha \, d\alpha \int_{l_1}^{l_2} g(l, \alpha, \beta; \theta, \psi) \, l^2 dl . (32)$$

- integrand\_1: returns the value of the function  $f(l,\alpha,\beta;\theta,\psi) \equiv g(l,\alpha,\beta;\theta,\psi) \, l^2$ . A switch is included to integrate different functions of the density profile  $\rho$ . Note that here  $\rho$  can refer to any profile or distribution, be it the density or the spatial distribution of clumps  $dP_V/dV$  depending on the normalisation used. The functions that can be integrated along the line of sight are:
  - $f(l, \alpha, \beta; \theta, \psi) = \rho$  when calculating the clumps averaged contribution  $\langle J_{\text{subs}} \rangle$  of Eq. (21).
  - $-f(l,\alpha,\beta;\theta,\psi) = \rho^2$  in order to evaluate J such as in Eq. (14).
  - $f(l, \alpha, \beta; \theta, \psi) = l^2 \rho$  is used to calculate the mass fraction or number fraction of clumps in the integration volume;
  - $-f(l,\alpha,\beta;\theta,\psi) = l^3\rho$  is needed when estimating the mean distance of the clumps as in Eq. (20).
  - $f(l, \alpha, \beta; \theta, \psi) = l^4 \rho$  is needed when estimating the variance of distance of the clumps.
  - $-f(l,\alpha,\beta;\theta,\psi) = l^{-2}\rho$  must be evaluated to evaluate the variance on J as in Eq. (26).

Note that if decaying rather than annihilating DM is being considered, i.e. gDM\_IS\_ANNIHIL\_OR\_DECAY is false instead of true, any  $\rho^2$  is replaced by  $\rho$  in the above functions<sup>8</sup>.

A variation of this function, called integrand\_lrel, is used whenever the steepness of the profile requires it (see Doxygen documentation for more details).

 fn\_beta\_alpha: computes the third integrand in Eq. (32), namely the integration over l. The functions to integrate roughly behave as power laws, with a varying slope,

<sup>&</sup>lt;sup>8</sup>This makes some of these functions redundant for decay, but they are still required for annihilation.

and log-step Simpson integration schemes are used (see integr.h). Some profiles can be very steep and several tests and tricks are needed to ensure the numerical stability/optimisation of the integration. The reader is again referred to the documentation for an explicit description of the integration strategy.

- fn\_beta: computes the integration over the angle α in Eq. (32). A simple linear Simpson integration scheme is used.
- los\_integral: it is the final part of the triple integration, i.e. the integration over the angle  $\beta$ . This is again performed quite easily with a linear Simpson scheme. This function is called whenever a single density profile is to be dealt with.
- los\_integral\_mix: as seen previously, it may be necessary to integrate a difference or a product of two densities along the line of sight. A flag parameter switch\_rho is introduced to tell rho\_mix to return either ρ<sub>1</sub> ρ<sub>2</sub> (switch\_rho = 0) or ρ<sub>1</sub>ρ<sub>2</sub> (switch\_rho = 1) (see profiles.h). This function then behaves identically to los\_integral and uses the same flags with respect to the quantity to integrate.

### 3.3.6. clumps.h

All the functions related to the clumps mass and spatial distributions are gathered in this file. Here, we only give an overview of the most important ones.

- add\_halo\_in\_map: when using the code to generate a skymap where clumps are randomly drawn from their mass and spatial distribution, this adds the J contribution of the drawn clump to the appropriate pixels. The clump is only added to a pixel if  $J_{\rm cl}({\rm pixel}) > \epsilon \times J_{\rm sm}({\rm pix}_0)$  where pix $_0$  is the pixel containing the centre of the clump and  $\epsilon$  is the requested integration precision gSIMU\_EPS. This avoids the clump from being integrated "too far" which would only be time consuming without changing the result.
- dpdv\_XXX and set\_normprob\_dpdv: the former function returns the spatial distribution of the clumps as described in  $\S 2.2.1$  and the second calculates the appropriate normalisation to make it a probability according to Eq. (9). The function dpdv\_XXX is written in several versions depending of the reference frame and coordinate system in which the clumps are drawn (cartesian XXX=xyz, spherical centred on the galactic centre XXX=rthetapsi, spherical centred on Earth XXX=lthetapsi, or centred on Earth, in the cone  $\Delta\Omega$ , in the direction  $(\psi, \theta)$  XXX=lalphabeta).
- dpdm and set\_normprob\_dpdm: this is the equivalent of the above but for the mass distribution of the clumps as given by Eq. (10).
- frac\_nsubs\_in\_foi returns the number fraction of clumps in a given field of integration w.r.t. the total number of clumps in the host halo

- frac\_nsubs\_in\_m1m2 returns the fraction of clumps with a mass in the range  $[m_1, m_2]$ , given the total number of clumps.
- jcrossprod\_continuum returns the value of the crossproduct integrated along the l.o.s as defined by Eq. (22). This is a simple call to los\_integral\_mix.
- jcrossprod\_interpolation interpolates the value of *J* in the pixels of a skymap, provided that *J* has previously been estimated in some locations of the map. This makes use of ROOT interpolation function based on Delaunay's triangles and allows to speed-up the calculation significantly when the skymap has a (too) large number of pixels.
- j\_smooth returns the *J*-factor when a single density profile is used (e.g.  $\rho_{\text{tot}}$ ). This is a simple call to los\_integral.
- j\_smooth\_mix is the same as above but deals with a combination of densities, e.g. when computing Eq. (14) where  $\rho_{sm} = \rho_{tot} \langle \rho_{cl} \rangle$ . This is a simple call to los\_integral\_mix.
- j\_sub\_continuum computes and returns the averaged J
  generated by the sub-clumps in a halo, in a given line of
  sight.
- lum\_singlehalo returns the total intrinsic luminosity of a DM halo, given its inner density profile as in Eq. (19).
- mass\_singlehalo returns the total mass of a DM halo, given its inner density profile.

#### 3.3.7. spectra.h

This library contains the different parametrisations of the average  $\gamma$ -ray spectra,

$$\frac{\mathrm{d}N_{\gamma}}{\mathrm{d}E_{\gamma}} \equiv \sum_{f} \frac{\mathrm{d}N^{f}}{\mathrm{d}E_{\gamma}} B_{f} \tag{33}$$

in the particle physics term of Eq. (2). As mentioned in §2.1.1, we have included the Bergström et al.[4] and Tasitsiomi & Olinto [5] parametrisations for the  $\gamma$ -ray continuum and the Bringmann et al. [6] expression of the internal bremsstrahlung. Defining  $x \equiv E_{\gamma}/m_{\chi}$ , the three parametrisations are given by their corresponding functions below.

dNdE\_BERGSTROM returns

$$\frac{\mathrm{d}N_{\gamma}}{\mathrm{d}E_{\gamma}} = 0.73 \cdot \frac{1}{m_{\gamma}} \cdot \frac{\exp\left(-7.8x\right)}{x^{1.5}} \ .$$

ullet dNdE\_TASITSIOMI computes the  $\gamma$ -ray spectrum as

$$\frac{\mathrm{d}N_{\gamma}}{\mathrm{d}E_{\gamma}} = \frac{1}{m_{\gamma}} \cdot \left( \frac{10}{3} - \frac{5}{4} x^{0.5} - \frac{5}{2} x^{-0.5} + \frac{5}{12} x^{-1.5} \right) \, .$$

• dNdE\_BRINGMANN estimates the internal bremsstrahlung contribution with

$$\frac{\mathrm{d}N_{\gamma}}{\mathrm{d}E_{\gamma}} = \frac{1}{m_{\chi}} \cdot \frac{\alpha}{\pi} \cdot \frac{4\left(x^2 - x + 1\right)^2}{x\left(1 - x + \epsilon/2\right)} \cdot \left[\ln\left(2 \cdot \frac{1 - x + \epsilon/2}{\epsilon}\right) - \frac{x^3 + x - \frac{1}{2}}{4}\right].$$
(density, J-factor)
$$\frac{\mathrm{d}N_{\gamma}}{\mathrm{d}E_{\gamma}} = \frac{1}{m_{\chi}} \cdot \frac{\alpha}{\pi} \cdot \frac{4\left(x^2 - x + 1\right)^2}{x\left(1 - x + \epsilon/2\right)} \cdot \left[\ln\left(2 \cdot \frac{1 - x + \epsilon/2}{\epsilon}\right) - \frac{x^3 + x - \frac{1}{2}}{4}\right].$$

where  $\alpha = 1/137$  is the fine structure constant and  $\epsilon$  is the ratio of the mass of the W boson to the neutralino mass  $\epsilon = m_W/m_\chi$ .

#### 3.3.8. stat.h

The functions included in this library perform simple statistical operations on the random variables of the problem (mass and distance of the clumps) and on quantities deriving from them (luminosity, J-factor). Ultimately, these are mainly used to determine the critical distance (or threshold mass) below (above) which clumps need to be drawn (see §2.4.3 for the adopted procedure). Given these dark matter spatial  $(d\mathcal{P}_V/dV)$ and mass  $(d\mathcal{P}_M/dM)$  distributions defined in §2.2.1):

- for a given mass and/or distance range, mean1cl\_mass, mean1cl, mean1cl\_lum2, mean1cl\_l, and mean1cl\_j return respectively the average mass, luminosity, luminosity squared, distance and J-factor of one clump;
- var1cl\_mass, var1cl\_lum, var1cl\_l, and var1cl\_j returns the variance of the corresponding random variable;
- finally, knowing the variance of the J-factor of one clump, find\_lcrit\_los and find\_mthresh\_los find the critical distance and threshold mass for drawing the clumps. They both rely on a simple dichotomy to bracket the desired quantity, as illustrated on Fig.1.

#### 3.3.9. janalysis.h

This is the top-level library. It contains the main functions called when invoking any of the three options of clumpy (galactic halo, halo list or statistics, see §3.1). Note that the first word 'xxx' of the function's name refers to which option they belong to (xxx=gal, xxx=halo, or xxx=stat).

- xxx\_j1D handles all the 1D calculations of the Galactic or halo options. It computes  $\rho(r)$ ,  $J(\alpha_{int})$ , and  $J(\theta)$ , where  $\theta$ is the angle measured w.r.t. the Galactic centre.
- xxx\_j2D generates the 2D skymaps for the smooth, averaged and statistically drawn clump contributions. It also include the cross product term and It applies to both the Galactic and halo options.
- xxx\_load\_list loads the list of halo under scrutiny either for the halo mode or the statistical option. The list should be provided as an ASCII file with the format given in table 2.
- xxx\_set\_par\* are a set of functions filling the parameters arrays to be passed to the above functions with the proper quantities (e.g., profile flag and shape parameters, concentration flag, etc.). It applies to the Galactic and halo options.

• stat\_CLs is the most important function in the statistics option and computes the confidence levels of a quantity (density, J-factor) from its PDF.

$$-x^3 + x - \frac{1}{2}$$
.
**4. Run examples**

This section provides a few plots obtained by using the clumpy executable. Many more examples and illustrations are provided along with the Doxygen documentation.

#### 4.1. Sky maps

Figure 3 shows two examples of a  $5^{\circ} \times 5^{\circ}$  skymap of the J-factor (option '-g7' of clumpy\_gal), when looking 10° off the Galactic centre (left) or towards the anti-centre (right). The angular resolution is 0.01° and the Galactic smooth DM density, clump distribution and clump inner profile are all assumed to follow an NFW parametrisation.

Similarly, skymaps of individual objects can be performed (option '-h5' of clumpy\_dsph). In Fig. 4, a "generic" dwarf galaxy, including a population of subclumps, is integrated over  $0.1^{\circ}$  (left),  $0.05^{\circ}$  (middle), and  $0.01^{\circ}$  (right). As the resolution increases, more substructures become apparent but the J-factor is strongly reduced.

#### 4.2. dSphs and statistical tools

The three panels in Fig 5 illustrate CLUMPY capabilities on the dSphs. The top left and bottom figures are taken from Charbonnier et al. [25], and the top right one is a direct result of the analysis performed in the above paper. Running CLUMPY on the default parameters will not give these plots, but similar ones in spirit.

#### 5. Conclusions

The CLUMPY code is optimised to calculate the line-of-sight integration of the density and/or the density square of the DM distributions (smooth and clump distributions). It can be used by (i) experimentalists looking for realistic  $\gamma$ -ray skymaps to calculate their new instrument sensitivity, or simply to use them in model/template analyses for the DM diffuse emission near the Galactic centre (or for dSphs), (ii) for astrophysicists working on the reconstruction of the DM content of dSphs who wish to calculate the J factor as a by-product of their study, (iii) theoreticians who want to plug their preferred particle physics model and see what is the corresponding  $\gamma$ -ray flux in the Galaxy/dSph. The code is fully documented and provides many functions that makes it versatile and flexible.

Many improvements are possible for a future release. The simplest and more straightforward ones to consider are:

- Extra-galactic signal from the local structures (e.g. local clusters, Nezri et al. in prep.), but also from the highredshift isotropic contribution.
- The possibility to have the so-called Sommerfeld-like enhancements. This requires to couple the calculation of the spatial density of DM to the position-dependant velocity of the DM particles.

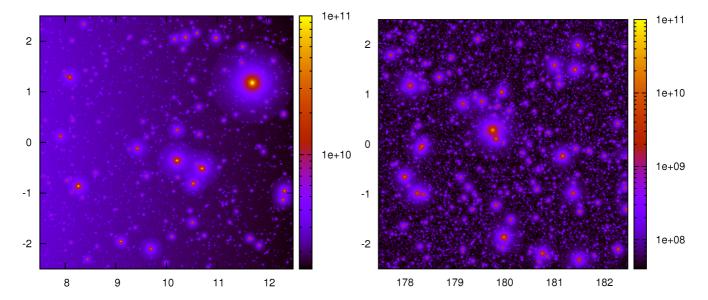


Figure 3: Example of two  $5^{\circ} \times 5$  skymaps, at  $10^{\circ}$  from the Galactic centre (left) and towards the anti-centre(right), for a  $0.01^{\circ}$  angular resolution, obtained from the skymap mode of CLUMPY. The colour scale gives the J values. These maps are obtained with the following command option: ./bin/clumpy -g7. Clumpy will display the results automatically but these images have been produced with Gnuplot for aesthetic reasons.

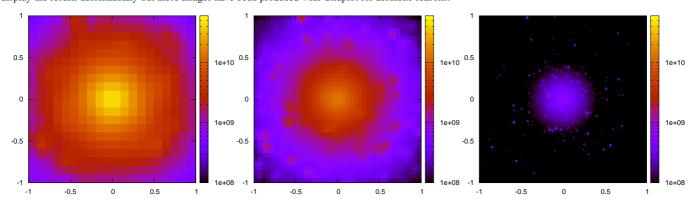


Figure 4: Skymaps of a fake dSph galaxy at three different resolutions: 0.1° (left), 0.05° (middle), and 0.01° (right). Note that ROOT will automatically display the image, but these ones have been plotted using the output files with Gnuplot. These maps are obtained with the following command option: ./bin/clumpy -h5

• Neutrino annihilation spectra and fluxes.

Some other improvements are intrinsically more difficult to add, as they depend on astrophysical inputs that are not well determined. Indeed, in the absence of a smoking gun signature for indirect detection (e.g. a line detection), multi-wavelength and multi-messenger analyses seem to be a compulsory approach. To do so, one needs to take into account

- the anti-proton and anti-deuteron signal;
- electron and positron production;
- secondary multi-wavelength emissions from leptons.

Finally, the CLUMPY code could be coupled to some particle physics code to get more specific annihilation spectra (e.g., darkSUSY, Gondolo et al. [27]; MicrOMEGAs, Belanger et al. [28]), or to some cosmic-ray propagation code (e.g. GAL-PROP<sup>9</sup>, DRAGON<sup>10</sup> or USINE, Maurin et al., in preparation)

to compare with the Galactic astrophysical backgrounds. This will be considered for a second release of the code.

#### Acknowledgements

We thank E. Nezri and J.-C. Lambert for their useful comments on the code. We also thank the anonymous referee the many constructive comments that improved both the code and the paper.

#### Appendix A. Geometry and change of coordinates

We define two frames of reference: one attached to the observer the origin of which is the Earth, and the other linked to the Galactic Centre (see Fig. 6). The frame related to the Earth  $(E, l, \psi, \theta)$  is spherical, and is identified to the Galactic coordinate system. l is the distance to the Earth, the Galactic longitude  $\psi$  is measured in the plane of the Galaxy using an axis pointing from the Earth to the Galactic Centre, the Galactic latitude is measured from the plane of the Galaxy to the object.

<sup>9</sup>http://galprop.stanford.edu/

<sup>10</sup>http://www.desy.de/ maccione/DRAGON/

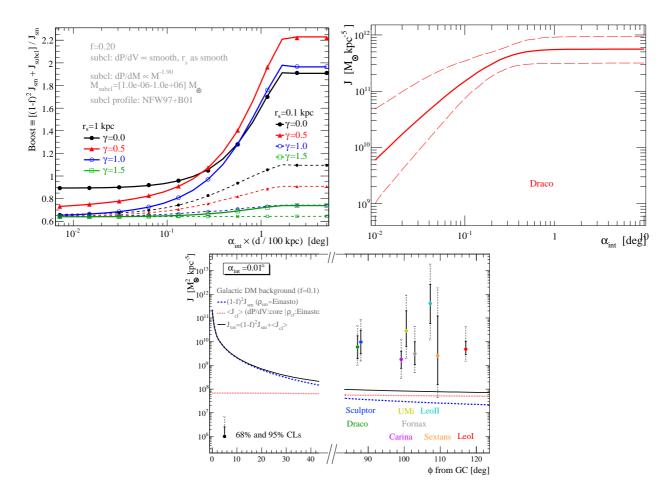


Figure 5: **Top left:** boost factor as a function of the integration angle for 'several' generic dSphs (./bin/clumpy -h2). **Top right:** median (solid line) and 95% confidence levels (dashed lines) of the *J*-factor as a function of the integration angle for the Draco dSph (./bin/clumpy -s6). **Bottom panel:** the *J*-factor for the dSph can be over-plotted on the galactic smooth and mean-clump *J* factors (./bin/clumpy -g4).

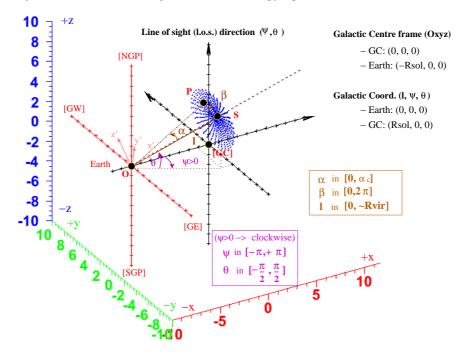


Figure 6: The geometry as defined in the CLUMPY simulation. Two frames are used: the Galactic Centre frame and the Galactic coordinates. For a given direction of observation  $(\psi, \theta)$  (l.o.s. for line of sight), we will run over all the  $\alpha$  and  $\beta$  directions within the solid angle  $\Delta\Omega$ .

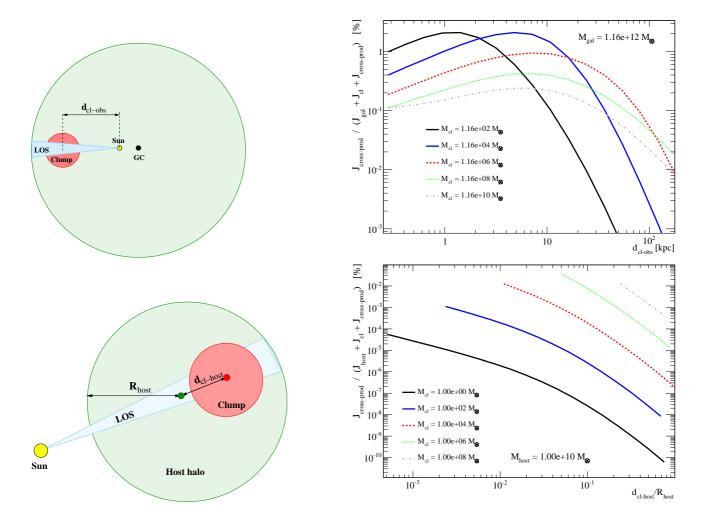


Figure 7: Relative importance of the cross-product w.r.t to the total J calculation for i) a clump embedded in the Galactic halo in the anti-centre direction (top panels) and ii) a sub-clump in DM halo located 100 kpc away (bottom panels). Several masses for the clump have been considered and the integration angle is  $\alpha_{\text{int}} = 0.1^{\circ}$ .

The Galactic Centre frame (GC, x, y, z) is a cartesian frame, where the (GC, x) and (GC, y) axes are in the Galactic plane, and the (GC, z) axis is pointing to the North Galactic pole. In this frame, the position of the Earth is  $(-R_{\odot}, 0, 0)$ , where  $R_{\odot}$  is the distance between the Earth and the Galactic Centre.

The change of coordinates formulae are given in the documentation (geometry.h).

#### Appendix B. Cross-product of individual clumps

In §2.3 it has been shown that the cross-product between the underlying smooth DM density and that of all the clumps in the line of sight (when using an average description) can significantly contribute to the value of J in the line of sight. It is interesting to check what contribution it brings when considering one clump only (rather that the entire distribution). We use the same equations as in §2.3 but written for one clump only and numerically integrates J. Two configurations are explored: i) a clump embedded in the Galactic halo (fig. 7, top panels) and ii) a clump embedded in a host halo located away from the sun (fig. 7, bottom panels).

In fig. 7, the relative error made when neglecting the cross-product is plotted against i) the distance to the clump for the first

configuration (top), ii) the distance of the clump to the centre of its host halo (bottom). In both cases, these curves show that, as far as one individual clump is concerned, the cross-product is completely negligible, whatever situation occurs: it amounts at most to  $\sim 1\%$ .

#### References

- Gunn, J. E., Lee, B. W., Lerche, I., Schramm, D. N., and Steigman, G., ApJ, 223 (1978) 1015.
- [2] Stecker, F. W., ApJ, 223 (1978) 1032.
- [3] CTA Consortium, arXiv:1008.3703, 2011.
- [4] Bergström, L., Ullio, P., and Buckley, J. H., Astroparticle Physics, 9 (1998) 137.
- [5] Tasitsiomi, A. and Olinto, A. V., Phys. Rev. D, 66 (2002) 083006.
- [6] Bringmann, T., Bergström, L., and Edsjö, J., Journal of High Energy Physics, 1 (2008) 49.
- [7] Bertone, G., Buchmüller, W., Covi, L., and Ibarra, A., J. Cosmology Astropart. Phys., 11 (2007) 3.
- [8] Law, D. R., Majewski, S. R., and Johnston, K. V., ApJ, 703 (2009) L67.
- [9] Springel, V. et al., MNRAS, **391** (2008) 1685.
- [10] Diemand, J. et al., Nature, **454** (2008) 735.
- [11] Kazantzidis, S. et al., ApJ, **611** (2004) L73.
- [12] Zhao, H., MNRAS, 278 (1996) 488.
  [13] Navarro, J. F., Frenk, C. S., and White, S. D. M., ApJ, 490 (1997) 493.
- [14] Moore, B., Governato, F., Quinn, T., Stadel, J., and Lake, G., ApJ, 499 (1998) L5.

- [15] Diemand, J., Moore, B., and Stadel, J., MNRAS, 353 (2004) 624.
- [16] Navarro, J. F. et al., MNRAS, 349 (2004) 1039.
- [17] Merritt, D., Graham, A. W., Moore, B., Diemand, J., and Terzić, B., AJ, 132 (2006) 2685.
- [18] Berezinsky, V. S., Gurevich, A. V., and Zybin, K. P., Physics Letters B, 294 (1992) 221.
- [19] Bullock, J. S. et al., MNRAS, 321 (2001) 559.
- [20] Eke, V. R., Navarro, J. F., and Steinmetz, M., ApJ, 554 (2001) 114.
- [21] Wechsler, R. H., Bullock, J. S., Primack, J. R., Kravtsov, A. V., and Dekel, A., ApJ, 568 (2002) 52.
- [22] Lavalle, J., Yuan, Q., Maurin, D., and Bi, X., A&A, 479 (2008) 427.
- [23] Pieri, L., Lavalle, J., Bertone, G., and Branchini, E., Phys. Rev. D, 83 (2011) 023518.
- [24] Lavalle, J., Pochon, J., Salati, P., and Taillet, R., A&A, 462 (2007) 827.
- [25] Charbonnier, A. et al., MNRAS 418 (2011) 1526.
- [26] Walker, M. G., Combet, C., Hinton, J. A., Maurin, D., and Wilkinson, M. I., ApJ, 733 (2011) L46.
- [27] Gondolo, P. et al., J. Cosmology Astropart. Phys., 7 (2004) 8.
- [28] Belanger, G. et al., Comput. Phys. Commun. 182 (2011) 842.