ALOHA: Automatic Libraries Of Helicity Amplitudes for Feynman diagram computations

Priscila de Aquino^{a,b}, William Link^c, Fabio Maltoni^b, Olivier Mattelaer^b, Tim Stelzer^c

 ^aInstituut voor Theoretische Fysica, Katholieke Universiteit Leuven, Celestijnenlaan 200D, B-3001 Leuven, Belgium
 ^bCenter for Cosmology, Particle Physics and Phenomenology (CP3), Université Catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium
 ^cDepartment of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, IL 61801

Abstract

We present an application that automatically writes the Helas (Helicity Amplitude Subroutines) library corresponding to the Feynman rules of any in quantum field theory Lagrangian. The code is written in Python and takes the Universal FeynRules Output (Ufo) as an input. From this input it produces the complete set of routines, wave-functions and amplitudes, that are needed for the computation of Feynman diagrams at leading as well as at higher orders. The representation is language independent and currently it can output routines in Fortran, C++, and Python. A few sample applications implemented in the MADGRAPH 5 framework are presented.

PROGRAM SUMMARY

Manuscript Title: ALOHA: Automatic Libraries Of Helicity Amplitudes for Feynman diagram computations

Authors: Priscila de Aquino, William Link, Fabio Maltoni, Olivier Mattelaer, Tim Stelzer

Program Title: ALOHA

Journal Reference:

Catalogue identifier:

Licensing provisions: http://www.opensource.org/licenses/UoI-NCSA.php

Programming language: Python2.6

Computer: 32/64 bit

Operating system: Linux/Mac/Windows

RAM: 512 Mbytes

Keywords: Helicity Routine, Feynman Rules, Feynman diagram

Classification:

4.4 Feynman diagrams,

11.6 Phenomenological and Empirical Models and Theories

Nature of problem:

An efficient numerical evaluation of a squared matrix element can be done with the help of the helicity routines implemented in the HELAS library [1]. This static library contains a limited number of helicity functions and is therefore not always able to provide the needed routine in presence of an arbitrary interaction. This program provides a way to automatically create the corresponding routines for any given model.

Solution method:

ALOHA takes the Feynman rules associated to the vertex obtained from the model information (in the UFO format [2]), and multiply it by the different wavefunctions or propagators. As a result the analytical expression of the helicity routines is obtained. Subsequently, this expression is automatically written in the requested language (Python, Fortran or C++)

Restrictions:

The allowed fields are currently spin 0, 1/2, 1 and 2, and the propagators of these particles are canonical.

Unusual features:

None

Running time:

A few seconds for the SM and the MSSM, and up to few minutes for model with spin 2 particles.

References

- [1] Murayama, H. and Watanabe, I. and Hagiwara, K., HELAS: HELicity Amplitude Subroutines for Feynman diagram evaluations, KEK-91-11, (1992) http://www-lib.kek.jp/cgi-bin/img_index?199124011
- [2] C. Degrande, C. Duhr, B. Fuks, D. Grellscheid, O. Mattelaer, et al., UFO The Universal FeynRules Output, Comput. Phys. Commun. 183 (2012) 1201-1214. arXiv:1108.2040, doi:10.1016/j.cpc.2012.01.022.

1. Introduction and motivation

We have now entered the era of the Large Hadron Collider (LHC) and measurements based on the Standard Model (SM) predictions have already begun. As data continues to be analysed, deviations from the SM are likely to occur and the challenge will be understanding which Beyond the Standard Model theories (BSM) are responsible for such deviations. Extracting information from the LHC data requires the ability to accurately simulate events based on the theoretical models. While it is possible for researchers to manually create event simulators for the SM and a few popular BSM theories, such an approach would severely restrict the range of models that could be tested at the LHC.

To alleviate the potential bottleneck in simulating BSM theories, extensive efforts are being made to automate the process of going from the Lagrangian of a new theory, to simulating events that can be then compared with data [1, 2, 3, 4]. In this paper we present one important piece of that process, i.e., the ability to automatically create helicity amplitude routines necessary for calculating the matrix element associated with any new model. Such an ability will be directly useful to many groups writing custom generators, however, at present the greatest impact is on the very recently released MADGRAPH 5 suite [5]. Prior to this work, MADGRAPH was restricted to generating amplitudes for models that had the same Lorentz structures as the Standard Model. Adding new particles and new interactions, from the MSSM [6] to higher spin particles such as gravitons [7] or gravitinos [8], required identifying, writing and debugging new helicity amplitudes associated with the model. This severely restricted the range of models that could be studied, as the number of physicists able to implement new models is very limited. For example, technicolor, chiral perturbation theory, and chromomagnetic moments all require extensions to the HELAS library. The ability to dynamically generate the required helicity routines eliminates the bottleneck, giving all physicists the ability to easily simulate any new model within the MADGRAPH 5 framework. For instance, all models presently available in the FEYNRULES framework [9, 10] can be used via the UFO format [11] in MAD-Graph 5.

This paper is organized as follows. In Sec. 2 we give a simple example with the aim of explaining the idea of calculating Feynman diagrams routines for vertices

and propagators as implemented in Helas. This also allows us to establish a dictionary of terms that will be used in the following sections. In Sec. 3 we describe in more detail how the Aloha program works. In Sec. 4 we describe the validation procedure and collect our conclusions in Sec. 5.

2. Helicity amplitudes

Helicity amplitudes methods [12, 13, 14, 15, 16, 17, 18] are a convenient and effective way to evaluate the squared matrix element of any process. As the name implies, helicity amplitude methods work at the amplitude level, in contrast to the trace techniques based on completeness relations which work on squared amplitudes. This has two important advantages. First, the complexity in the calculation grows linearly with the number of diagrams instead of quadratically. Second, it is possible to "factorize" diagrams, such that if a particular substructure shows up in several diagrams, it only needs to be calculated once, significantly increasing the speed of the calculation (see for example [5]).

The Helas [19, 20] library has been a particularly successful implementation of an helicity amplitude method for tree-level processes. Amplitudes are generated by initializing a set of external wavefunctions using their helicity and momenta. These wavefunctions are combined based on the particle interactions in the Lagrangian to determine the wavefunctions of the internal lines (propagators). Once all of the wavefunctions are determined, they are combined to calculate the complex number corresponding to the amplitude for the diagram. These amplitudes can then be added and squared to give the required result. The set of routines needed to calculate amplitudes for the Standard Model at tree-level was released in the original Helas package [19] and successfully employed for many phenomenological studies thereafter.

In addition to having a complete and highly optimized set of routines, the structure of the Helas calls makes it particularly easy to write (and read) the code for any Feynman diagram: one line of code for each line of the associated Feynman diagram, and a final call that returns the amplitude. For example, the diagram for $W^+W^- \to t\bar{t}$ via a Z boson, shown in Fig. 1, is written in six lines. First, there are calls to functions associated with the external particles¹:

```
call VXXXXX(P_WM, W_MASS, WM_HEL, -1, WM)
call VXXXXX(P_WP, W_MASS, WP_HEL, -1, WP)
call OXXXXX(P_T, T_MASS, T_HEL, +1, F0)
call IXXXXX(P_TBn T_MASS, TB_HEL, -1, FI).
```

From the four resulting wavefunctions (WM, WP, FO, FI) one can combine either the two fermions or the two bosons in order to build the propagator for the Z boson. Choosing to connect the fermions, the code reads:

¹The notation follows the Helas convention [19].

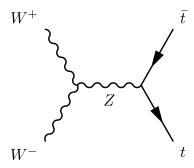


Figure 1: $W^+W^- \to t\bar{t}$ via Z exchange

call J3XXXX(FI, FO, GAU, GZU, Z_MASS, Z_WIDTH, J3).

Finally, the Z boson can be combined with the external bosons to evaluate the amplitude:

call VVVXXX(WP, WM, J3, GW, AMP).

The example above demonstrates that Helas consists of three categories of routines:

External particle routines: Each external leg of a Feynman diagram corresponds to a wavefunction computed for a given momentum and a given helicity. For example, the first four routines used in the $W^+W^- \to t\bar{t}$ calculation are of this class. The only input expected for this category of routines is the type of the particle (i.e. vector, scalar, tensor or incoming/outgoing fermion), is independent of the interactions in the model, and depends only on the Lorentz representation of the particles present in the theory.

Wavefunction/Off-shell routines: When one leg of a vertex has no associated wavefunction, a routine is used to generate the internal particle (i.e., propagator) wavefunction. This is exemplified by the Z boson call above. These routines return both the internal wavefunction and the momentum of the propagator.

Amplitude/On-shell routines: When the wavefunctions for all the legs at a vertex are known, they can be used to compute the matrix-element of the diagram by the associated amplitude routines.

To simulate new models, relatively few external particles are required, as they depend only on the Lorentz representation of the particle. However, the wavefunction and amplitude routines depend explicitly on the interactions of the theory, making it impractical to anticipate and create a comprehensive library of routines for all interesting BSM theories.

3. Automation of the method

Originally, the Helas package contained external particle routines only for classes of particles present in the Standard Model (particles with spin 0, 1 and 1/2). Recently the library has been expanded to include spin-2 particles [20, 21], spin-3/2 particles [8, 22], as well as the Higgs effective theory [23]. This set of external particle routines is sufficient for calculating most renormalizable BSM theories of interest.

The challenge that remains is writing the required wavefunction and amplitude routines required for any BSM theory, renormalizable or not. Although the rules for writing such a routine are known, it can be very time consuming to verify that all of the sign conventions and normalizations have been properly implemented. This provides a severe limitation to Helas-based event generators. To fill this need, we develop Aloha: the Automatic Language-independent Output of Helicity Amplitudes, which is capable of automatically generating Helas routines in Fortran, C++ and Python.

3.1. Computation method

Aloha is written in Python, the same language used by the Ufo code [11] to ensure maximum flexibility and compatibility. Moreover, the input model file used by Aloha is the one provided by the Ufo.

In order to generate the necessary Helas routines, Aloha needs to perform specific analytic computations. A package to deal with these mandatory computations has been developed and its details can be found in Appendix B.

We begin by describing the relevant information existing in the UFO code, which is the starting point of Aloha. All information concerning the model, *i.e.* particles, interactions and parameters, is obtained from the UFO. Aloha connects particles, creating Lorentz structures to output the wavefunctions and amplitudes using the syntax defined by the original Helas package.

To illustrate how Aloha works, consider the $e^+e^-\gamma$ vertex produced by UFO:

The spins attribute contains the list of spins of each particle (present in the vertex) in the 2*S+1 scheme². The structure attribute collects the analytical expression for the Lorentz structure in the vertex. The arguments of the Lorentz object refer to the particle associated with each index of the object. The list of

²In the case of a fermion, the first particle is considered as an incoming fermion and the second as an outgoing fermion. For interactions containing more than two fermions, the convention is that the vertex should be defined for a given fermion flow. The outgoing fermion follows directly its associated incoming fermion.

Table 1: Elementary Lorentz structures			
Charge conjugation matrix: C_{ab} Epsilon matrix: $\epsilon^{\mu\nu\rho\sigma}$ Gamma matrices: $(\gamma^{\mu})_{ab}$ Gamma5 matrix: $(\gamma^{5})_{ab}$ (Spinorial) Kronecker delta: δ_{ab} Minkowski metric: $\eta_{\mu\nu}$ Momenta of particle N : p_{N}^{μ} Right chiral projector: $(\frac{1+\gamma 5}{2})_{ab}$ Left chiral projector $(\frac{1-\gamma 5}{2})_{ab}$ Sigma matrices: $(\sigma^{\mu\nu})_{ab}$ Scalar wavefunction: ϕ_{N} Spinor wavefunction: ψ_{N}^{a} Vector wavefunction: ϵ_{μ}^{N}	C(a,b) Epsilon(μ, ν, ρ, σ) Gamma(μ , a, b) Gamma5(a,b) Identity(a,b) Metric(μ, ν) P(μ ,N) ProjP(a,b) ProjM(a,b) Sigma(μ, ν ,a,b) Scalar(N) Spinor(a,N) Vector(μ ,N)		
Spin2 wavefunction: $T_N^{\mu,\nu}$	$\mathrm{Spin2}(\mu,\nu,\mathrm{N})$		

Table 1: The greek indices stands for lorentz indices while the lower case latin indices stands for spin indices. The indices N stands for the label of the particle.

available objects and the Python convention for different indices are explained in Table 1. The UFO convention requires:

- positive indices to be linked to the particle numbers;
- all Lorentz indices to be listed before spin indices;
- all repeated indices to be summed with the appropriate matrix;
- all momenta to be incoming. Note this is different for the Helas/Aloha convention.

From a given UFO model structure, Aloha creates the analytical expression linked to the associated Helas routine by contracting the expression with a set of wavefunctions. To illustrate how it works we use the structure shown in the FFV python object introduced above: Gamma(3,2,1).

The associated amplitude routine, which was previously named IOV by the Helas convention, is now:

$$Spinor(2) * Gamma(3,2,1) * Spinor(1) * Vector(3)$$

which correspond to the analytical expression:

$$FFV_{-}0(\psi_1, \psi_2, \epsilon_{3\mu}, g) = g * \bar{u}_2 \gamma^{\mu} v_1 \epsilon_{\mu}. \tag{1}$$

The analytical module of Aloha expands Eq. (1) component by component. The result is written to a C++, Python or Fortran file, language chosen by the user. For instance, the Fortran output would be:

```
С
     This File is Automatically generated by ALOHA
C
     The process calculated in this file is:
C
     Gamma(3,2,1)
C
     SUBROUTINE FFV1_0(F1,F2,V3,COUP,VERTEX)
     IMPLICIT NONE
     DOUBLE COMPLEX F1(*)
     DOUBLE COMPLEX F2(*)
     DOUBLE COMPLEX V3(*)
     DOUBLE COMPLEX COUP
     DOUBLE COMPLEX VERTEX
     VERTEX = COUP*((F2(1)*((F1(3)*((0, -1)*V3(1)+(0, 1)*V3(4))))
    +(F1(4)*((0, 1)*V3(2)+V3(3))))+((F2(2)*((F1(3)*((0, 1)
    *V3(2)-V3(3))+(F1(4)*((0,-1)*V3(1)+(0,-1)*V3(4))))
    +((F2(3)*((F1(1)*((0,-1)*V3(1)+(0,-1)*V3(4)))+(F1(2)
    *((0, -1)*V3(2)-V3(3))))+(F2(4)*((F1(1)*((0, -1)*V3(2))))
    +V3(3))+(F1(2)*((0,-1)*V3(1)+(0,1)*V3(4))))))
     END
```

Aloha's naming convention for generated routines is denoted by NAME_X, where NAME is the name provided by UFO and X is either 0 for an amplitude routine or the number corresponding to the off-shell particle for a wavefunction routine. The convention for the arguments is detailed in Appendix A.

To produce the necessary off-shell routines, Aloha loops over the list of particles and replaces the on-shell wavefunction with a propagator. For particle 1, the incoming spinor is replaced by a propagator, computing³:

which is analogous to:

$$FFV_{-1}(\psi_2, \epsilon_{3\mu}, g, m_1, \Gamma_1) = g * \bar{\psi}_2 \gamma^{\mu} \cdot i \frac{p + m_1}{p^2 - m_1^2 + i m_1 \Gamma_1} \epsilon_{3\mu}.$$
 (2)

FFV_1 is the wavefunction of the first spinor, and would be FOV in the original Helas language. The momentum for the propagator is computed by using conservation of momentum at the vertex.

³The propagator object is defined internally to Aloha.

Similarly, for the outgoing spinor, we have:

$$\text{FFV}_{-2}(\psi_1, \epsilon_{3\,\mu}, g, m_2, \Gamma_2) = i \, \frac{\not\!\! p + m_2}{p^2 - m_2^2 + i m_2 \Gamma_2} \cdot i \, \gamma^\mu \, \psi_2 \, \epsilon_{3\mu}. \tag{3}$$

The syntax used by Aloha reads:

which can be related to the FIV routine in the original Helas language.

Finally, Aloha also creates the routine associated to the off-shell vector. This routine returns a vector wavefunction from a pair of incoming and outgoing fermion wavefunctions, and it is written by Aloha as:

analogous to the analytic expression FFV_3, which would be JIO in the original Helas language:

$$FFV_{-3}(\psi_1, \psi_2, g, m_3, \Gamma_3) = \bar{\psi}_2 i \gamma_\mu \psi_1 \cdot -i \frac{\eta^{\mu\nu} - \frac{p^\mu p^\nu}{m_3^2}}{p^2 - m_3^2 + i m_3 \Gamma_3}.$$
 (4)

Following the original Helas convention, Aloha uses the unitary gauge for massive particles and the light cone gauge for massless particles. Notice that Aloha is not designed to create external particles routines, e.g., IXXXXX and OXXXXX for incoming and outgoing fermions, VXXXXX for vectors, SXXXXX for scalars and TXXXXX for spin-2 particles. There is a relatively small number of routines needed, and it is therefore reasonable to use a static library⁴.

In order to cover the full range of routines required for any type of interaction (and model), Aloha still needs two additional types of special routines:

• Conjugate Routines: The presence of majorana fermions and of fermion flow violation, requires for some processes the presence of the conjugated version of the routines [24] not present in the Standard Model. In the case of the previous example, the Lorentz expression would have to be replaced by:

$$C(-2,2) * Spinor(2) * Gamma(3,2,1) * Spinor(1) * Vector(3) * C(3,-3)$$

In order to distinguish these conjugate routines from the standard ones, Aloha adds to their name the letter C followed by the number of the

⁴These external particle routines are available in Python, C++ and Fortran languages in the Aloha package.

fermion pair that is conjugated. In this case the name of the conjugate routine would be FFVC1_X with X being either 0, 1, 2 or 3 depending on the number corresponding to the amplitude or off-shell particle. Note that _1 (_2) returns the wavefunctions associated to the incoming (outcoming) fermion.

• Multiple Couplings: In the UFO scheme each Lorentz structure is associated to a single coupling. In practice, this means that some interactions are linked to more than one Lorentz structure (for example, this is the case for the Z boson in the Standard Model). However, it is convenient for the readability and the compactness of the code to have one single routine per interaction. Therefore, Aloha defines a series of wrapper functions which call different routines associated by their couplings. The name of these wrapper routines are the concatenation of the name of the associated routines combined by an underscore. If all of the associated routines start with the same prefix (with letter S, F, V, T) then this prefix is only kept the first time. For example, the wrapper function of FFV1_X and FFV2_X will be called FFV1_2_X.

In some cases, this method provides redundancy. Consider a four-Higgs-interaction. Due to the symmetry of the Lorentz structure, the four wavefunction routines will each have the same content. This type of symmetry is detected automatically by Aloha and the computation is done only once. However, in order to simplify the work of the matrix element generator, all four routines are explicitly defined and have their own names. The first one is computed and the remaining three are written as an alias of the first one.

3.2. Resulting output

The major advantage of Aloha comes from using it in combination with a Monte Carlo (MC) event generator. To this aim, Aloha offers two options. The first is to use Aloha as a standalone program. The user can call Aloha from the shell:

- ./bin/aloha UFO_path [-f Fortran | Python | CPP] [-o output_dir] where UFO_path is the path to the UFO model directory, and the two possible arguments are
 - the language in which the routine should be written (default is Fortran).
 - the path for the output directory (which is by default UFO_path/LANGUAGE, where LANGUAGE can be either Fortran, Python or C++).

The second possibility is to internally link the Aloha code to a MC event generator. This option is only possible for a generator written in Python, and is currently used by Madgraph version 5. For each process Madgraph requests the required set of helicity routines, which is produced by Aloha. This first implementation of Aloha inside a matrix element generator played an essential role in fully validating the package.

4. Tests and validation

The validity and precision of the original Helas routines has been known for many years. A natural choice for testing Aloha routines is therefore to compare them with the original Helas routines. Since the format of the Aloha is different from that of Helas, a line-by-line comparison would not work as a general validation test. Instead, validation is provided by the calculation of amplitudes of a particular process using the two sets of routines and comparing results. Below is a full description for the validation tests, as well as an example of the power and utility provided by such tests.

4.1. The validation process

MADGRAPH version 4 uses Helas routines to compute values for the square of the matrix element for a specific phase-space point of a known process. New routines can be validated by computing these values using both the original Helas and the new Aloha routines.

The idea is to use two different versions of Madgraph to compute these values. We employ Madgraph version 4.4.44, which uses the original Helas routines, and compare the results with those from Madgraph version 5.1.1.0, which uses the new Aloha routines.

The comparison is made by specifying a point of phase space (randomly generated by RAMBO [25]) and a center-of-mass energy, after fixing properly the set of external parameters. The values for each matrix element squared are computed, and the difference is given by ΔD :

$$\Delta D = 2 \times \frac{|\mathcal{M}(\text{ALOHA})|^2 - |\mathcal{M}(\text{HELAS})|^2}{|\mathcal{M}(\text{ALOHA})|^2 + |\mathcal{M}(\text{HELAS})|^2}.$$
 (5)

Due to effects of numerical precision, one can not expect exact agreement between the two codes. Instead, the test is considered to pass if values of the squared matrix element computed by Aloha and Helas agree to at least five orders of magnitude, i.e., $\Delta D \leq 1 \times 10^{-5}$.

These tests are carried out for different processes based on the Standard Model (SM), the Minimal Supersymmetric Model (MSSM) and the Randall-Sundrum type I (RS-I) model, the later including spin-2 particles. Additional tests are made by checking gauge and Lorentz invariance of each process. We observe a great agreement for all processes, as illustrated in Tables 2 - 7 and explained below.

4.2. Standard Model processes

For the Standard Model, we test a very large sample of processes. Namely around 800 2 \rightarrow 2, 600 2 \rightarrow 3 and 50 2 \rightarrow 4 processes. These are chosen to ensure that all helicity routines have been covered.

Some examples of these results can be found in Tables 2 and 3. In both tables, the first column shows the process chosen for the comparison. The second column shows the difference ΔD given by Eq. (5) for a specific point of phase space.

4.3. MSSM processes

One common approach to extend the Standard Model is the 4-dimensional Supersymmetric theory, based on the popular field description delineated by Wess and Zumino [26]. Within this category of Beyond Standard Model theories is the Minimal Supersymmetric Standard Model, which relates bosons to fermions by unifying symmetries in the most straightforward way [27, 28, 29, 30]. It addresses the Hierarchy Problem by adding new states whose contribution effectively remove the quadratic divergences to the Higgs mass. It also provides a natural candidate for dark matter if the presence of an additional symmetry, R-parity, is assumed. On account of being a popular extension to the Standard Model, the MSSM has been extensively searched for at Hadron Colliders and several model implementations are available. We can therefore use these implementations to test Aloha within a theory which uses the full set of implemented subroutines. Testing Aloha for the MSSM is similar to testing it for the SM, since the Lorentz structures are for the most identical. However, the presence of majorana fermions and of fermion flow violations, require the presence conjugated versions of the routines [24] not present in the Standard Model. The same validation tests used in Sec. 4.2 for the Standard Model are therefore performed for the MSSM. Approximately 1000 2 \rightarrow 2 processes and 7000 2 \rightarrow 3 processes have been tested. The results are conclusive and 100% agreement is obtained. A sample of processes checked for the MSSM is shown in Tables 4 and 5.

4.4. RS-I processes

Several models with extra dimensions have recently been proposed. Examples of such theories include the Large Extra Dimensional model (or ADD) [31, 32] and Randall-Sundrum (RS) models [33, 34]. These models contain spin-2 particles allowed to propagate through higher dimensional space-time that couple to the SM particles in a very particular way. To complement the implementation of the ALOHA helicity routines, it is interesting to test and validate also spin-2 routines [20].

To this aim, a simplified version of the Randall-Sundrum type I model is used. Consider a warped 5-dimensional model in which the extra dimension is spatial and compactified in a line segment using the S_1/Z_2 symmetry. The propagation on the extra dimension is restricted to spin-2 particles, which will be perceived in 4-dimensions as a tower of excited Kaluza-Klein modes of the same particle with

Table 2: SM $2 \rightarrow 2$ results		Table 3: SM $2 \rightarrow 3$ results		
Process	ΔD	Process	ΔD	
$e^{+}e^{-} \rightarrow \gamma \gamma$ $e^{+}e^{-} \rightarrow W^{+}W^{-}$ $e^{-}\bar{\nu}_{e} \rightarrow \gamma W^{-}$ $\gamma e^{-} \rightarrow \nu_{e}W^{-}$ $b e^{-} \rightarrow b e^{-}$ $e^{-}W^{-} \rightarrow e^{-}W^{-}$ $e^{+}e^{+} \rightarrow e^{+}e^{+}$ $e^{+}\bar{\nu}_{e} \rightarrow e^{+}\bar{\nu}_{e}$	1.512×10^{-7} 3.376×10^{-8} 4.541×10^{-8} 4.545×10^{-8} 7.310×10^{-8} 5.557×10^{-8} 9.974×10^{-8} 3.911×10^{-8}	$\begin{array}{c} e^{+}e^{-} \rightarrow H H Z \\ e^{-} g \rightarrow e^{-} u \bar{u} \\ d e^{-} \rightarrow d e^{-} H \\ e^{-} t \rightarrow t \nu_{e} W^{-} \\ e^{-} W^{-} \rightarrow e^{-} W^{-} Z \\ e^{+} \nu_{e} \rightarrow u \bar{u} W^{+} \\ \gamma e^{+} \rightarrow \bar{\nu}_{e} W^{+} Z \\ \nu_{e} Z \rightarrow \gamma e^{-} W^{+} \end{array}$	2.029×10^{-8} 4.934×10^{-8} 1.712×10^{-7} 6.658×10^{-8} 9.767×10^{-8} 8.593×10^{-8} 7.611×10^{-9} 2.692×10^{-8}	
$e^{+} {t} {\rightarrow} e^{+} {t}$ $e^{+} {W}^{+} \rightarrow e^{+} {W}^{+}$ $e^{+} {Z} \rightarrow \gamma e^{+}$ $\nu_{e} {\nu_{e}} \rightarrow e^{+} e^{-}$ $\nu_{e} {\nu_{e}} \rightarrow W^{+} {W}^{-}$ $t \nu_{e} \rightarrow t \nu_{e}$ $\nu_{e} {W}^{-} \rightarrow \gamma e^{-}$ $\nu_{e} {Z} \rightarrow \nu_{e} {Z}$ $t {\nu_{e}} \rightarrow t {\nu_{e}}$ ${\nu_{e}} {W}^{+} \rightarrow e^{+} {Z}$ ${\nu_{e}} {Z} \rightarrow {\nu_{e}} {Z}$	1.317×10^{-7} 5.557×10^{-8} 5.458×10^{-8} 7.164×10^{-8} 7.477×10^{-8} 1.068×10^{-7} 4.545×10^{-8} $< 1.0 \times 10^{-10}$ 8.411×10^{-8} 5.586×10^{-8} $< 1.0 \times 10^{-10}$	$\begin{array}{c} \nu_e Z \to \gamma e^- w \\ u \bar{\nu}_e \to \gamma u \bar{\nu}_e \\ t \bar{\nu}_e \to e^+ t W^- \\ \bar{\nu}_e W^+ \to b b e^+ \\ g g \to \gamma d \bar{d} \\ b g \to \gamma b g \\ \gamma u \to e^+ e^- u \\ \gamma d \to d e^+ e^- \\ \gamma H \to e^+ e^- Z \\ \gamma t \to t t t \\ u \bar{u} \to e^+ e^- H \\ u \bar{u} \to u \bar{u} Z \end{array}$	1.179×10^{-7} 8.095×10^{-8} 8.430×10^{-8} 7.526×10^{-8} 7.415×10^{-8} 1.662×10^{-7} 6.045×10^{-8} 5.547×10^{-8} 7.878×10^{-8} 1.487×10^{-7} 9.593×10^{-8}	
$\begin{array}{c} \gamma g \rightarrow t t \\ g t \rightarrow t Z \\ b g \rightarrow b g \\ b g \rightarrow \gamma b \\ \gamma \gamma \rightarrow e^+ e^- \\ \gamma t \rightarrow g t \\ \gamma t \rightarrow \gamma t \\ \gamma b \rightarrow b Z \\ \gamma Z \rightarrow b b \\ t t \rightarrow \nu_e \bar{\nu}_e \\ t t \rightarrow \gamma \gamma \\ t t \rightarrow W^+ W^- \\ b Z \rightarrow \gamma b \\ b W^- \rightarrow b W^- \\ W^+ W^- \rightarrow \gamma Z \\ W^+ W^- \rightarrow Z Z \\ Z Z \rightarrow e^+ e^- \end{array}$	7.528×10^{-8} 3.173×10^{-8} $< 1.0 \times 10^{-10}$ 7.665×10^{-8} 1.512×10^{-7} 7.611×10^{-8} 1.524×10^{-7} 4.564×10^{-8} 4.554×10^{-8} 1.530×10^{-7} 1.833×10^{-8} 4.564×10^{-8} 6.291×10^{-9}	$H u \rightarrow H u Z$ $u W^{+} \rightarrow e^{+} u \nu_{e}$ $\bar{u} \bar{u} \rightarrow H \bar{u} \bar{u}$ $t \bar{u} \rightarrow H t \bar{u}$ $\bar{u} Z \rightarrow e^{+} e^{-} \bar{u}$ $d \bar{d} \rightarrow H \nu_{e} \bar{\nu}_{e}$ $H Z \rightarrow g t t$ $t t \rightarrow \gamma \nu_{e} \bar{\nu}_{e}$ $t t \rightarrow u \bar{u} Z$ $t W^{-} \rightarrow e^{-} t \bar{\nu}_{e}$ $b t \rightarrow \gamma b t$ $t Z \rightarrow d \bar{d} t$ $b b \rightarrow g H H$ $b b \rightarrow Z Z Z$ $b b \rightarrow \gamma b b$ $b Z \rightarrow b Z Z$ $W^{+} W^{-} \rightarrow d \bar{d} H$	4.478×10^{-8} 8.576×10^{-8} 1.145×10^{-7} $< 1.0 \times 10^{-10}$ 3.816×10^{-8} 2.566×10^{-8} 2.457×10^{-8} 2.065×10^{-8} 3.962×10^{-8} 4.744×10^{-8} 7.409×10^{-8} $< 1.0 \times 10^{-10}$ 9.055×10^{-8} 7.375×10^{-8} 8.196×10^{-8}	

Table 4: MSSM $2 \rightarrow 2$ results		Table 5: MSSM $2 \rightarrow 3$ results		
Process	ΔD	Process	ΔD	
$e^-W^+ \to e^-W^+$	9.451×10^{-10}	$e_L^-W^+ \rightarrow \gamma e^- x_1^+$	6.045×10^{-7}	
$e_R^+W^+ \rightarrow e_R^+W^+$	5.544×10^{-8}	$u_R W^- \to g u_R W^-$	5.029×10^{-9}	
$H_2W^+ \to H_3W^+$	1.108×10^{-8}	$s\nu_e W^- \rightarrow \gamma \nu_e x_1^-$	2.792×10^{-7}	
$H^-W^+ o H_2H_3$	9.787×10^{-7}	$n_1 Z \to u_R \bar{u} Z$	8.805×10^{-10}	
$e_L^+W^- \rightarrow e_L^+H^-$	5.285×10^{-9}	$u_L Z \rightarrow e^+ e_L^- u$	6.262×10^{-8}	
$W^- x_1^+ \to H^+ x_1^-$	2.493×10^{-6}	$e^+ Z \rightarrow e^+ e_L^- e_R^+$	$< 1.0 \times 10^{-10}$	
$H^+W^- \rightarrow H^+W^-$	2.055×10^{-7}	$s\nu_e Z \rightarrow e_L^- n_1 x_1^+$	9.288×10^{-9}	
$\nu_e \bar{\nu}_e o H^+ H^-$	6.214×10^{-9}	$\gamma g \to u_L \bar{u}_L Z$	4.678×10^{-8}	
$s\bar{\nu}_e\nu_e \to e^+e_L^-$	1.038×10^{-7}	$\gamma \bar{u}_L \to e^+ e_R^- \bar{u}$	5.273×10^{-10}	
$H_3 \nu_e \rightarrow e_L^- x_1^{\mp}$	1.075×10^{-7}	$\gamma e_L^- \to e^- s \nu_e \bar{\nu}_e$	4.001×10^{-9}	
$e_L^+ \bar{\nu}_e \rightarrow e_L^{\bar{+}} \bar{\nu}_e$	5.987×10^{-8}	$x_1^+ \bar{x}_1^- \to e^- \bar{\nu}_e W^+$	2.483×10^{-7}	
$H_1 \bar{\nu}_e \to e^+ H^-$	8.885×10^{-7}	$e^- x_1^+ \to \nu_e W^+ x_1^-$	3.355×10^{-7}	
$e^+ e^- \to s \nu_e s \bar{\nu}_e$	8.665×10^{-8}	$s\bar{\nu}_e x_1^+ \to e^+ \nu_e \bar{\nu}_e$	6.548×10^{-8}	
$e^- s \bar{\nu}_e \rightarrow H_3 x_1^-$	1.613×10^{-7}	$e^+ x_1^- \to s \bar{\nu}_e u_R \bar{u}_R$	1.116×10^{-10}	
$e^- H_1 \rightarrow e^- H_3$	8.890×10^{-7}	$n_1 n_1 \rightarrow n_1 u_R \bar{u}$	5.398×10^{-9}	
$e^+ e_L^- \to s \nu_e \bar{\nu}_e$	1.792×10^{-9}	$n_1 \bar{u}_L \to go \bar{u}_L Z$	3.231×10^{-8}	
$e^+ e_R^- \to H^- x_1^+$	6.679×10^{-9}	$e_L^- n_1 \to \gamma e^- Z$	1.842×10^{-9}	
$e^+ H^+ \to e_L^+ x_1^+$	6.977×10^{-9}	$n_1 s \nu_e \to \nu_e \nu_e \bar{\nu}_e$	2.176×10^{-9}	
$H_1 \mu^+ \to H_3 \mu^+$	8.890×10^{-7}	$g u_L \to e^- e_R^+ u$	3.677×10^{-10}	
$e_L^+ e_L^- \to \mu^+ \mu^-$	1.984×10^{-9}	$go go \to u_R \bar{u}_R Z$	1.089×10^{-7}	
$e_L^- x_1^+ \to H_2 \nu_e$	3.028×10^{-8}	$gou_R \to u Z Z$	1.526×10^{-10}	
$e_L^- H^+ \to H_1 s \nu_e$	8.264×10^{-7}	$u\bar{u}\to\gammau_L\bar{u}_L$	1.541×10^{-7}	
$e_L^+ x_1^- \to H_3 \bar{\nu}_e$	1.075×10^{-7}	$u\bar{u}_R \to e^+\nu_ex_1^-$	2.903×10^{-10}	
$e_L^+ H^- \to H_2 s \bar{\nu}_e$	1.871×10^{-9}	$u\bar{\nu}_e \to n_1u_L\bar{\nu}_e$	9.610×10^{-10}	
$s\nu_e x_1^- \to \nu_e W^-$	3.714×10^{-8}	$u_R \bar{u} \to \gamma s \bar{\nu}_e \nu_e$	6.985×10^{-10}	
$H^- s \nu_e \rightarrow \nu_e x_1^-$	1.039×10^{-8}	$\bar{u}\nu_e \to gs\nu_e\bar{u}_L$	1.233×10^{-8}	
$H_2 s \bar{\nu}_e \rightarrow e_L^+ H^-$	1.871×10^{-9}	$e^+ u_L \rightarrow e^+ u_L Z$	1.542×10^{-8}	
$e_R^+ e_R^- \to s \nu_e s \bar{\nu}_e$	4.856×10^{-9}	$e_L^+ \bar{u}_L \to s\bar{\nu}_e \bar{u}_L W^+$	8.574×10^{-7}	
$e_R^+ H^- \rightarrow e_R^+ W^-$	7.992×10^{-9}	$e^+ u_R \to e_R^+ u Z$	$< 1.0 \times 10^{-10}$	
$x_1^+ x_1^+ \to x_1^+ x_1^+$	2.863×10^{-8}	$e_R^- \bar{u}_R \to e^- g \bar{u}$	2.204×10^{-7}	
$H_2 x_1^+ \to H_3 x_1^+$	2.019×10^{-8}	$e^-e_L^+ \rightarrow n_1 u \bar{u}$	2.122×10^{-8}	
$H_2 x_1^- \rightarrow e^- s \bar{\nu}_e$	1.527×10^{-7}	$e^+e^+ \rightarrow \gamma e_L^+ e_R^+$	1.285×10^{-9}	
$H_1 H_1 \to H_1 H_1$	1.139×10^{-8}	$e^+ s \nu_e \rightarrow \gamma n_1 W^+$	7.022×10^{-9}	
$H_1 H_3 \rightarrow H_2 H_3$	8.991×10^{-7}	$e_R^- \nu_e \rightarrow e^- e_L^- W^+$	$< 1.0 \times 10^{-10}$	
$H_2 H_2 \rightarrow H_3 H_3$	1.021×10^{-8}	$\bar{\nu}_e \bar{\nu}_e \to e_L^+ \bar{\nu}_e x_1^-$	4.661×10^{-8}	

increasing mass. We implement a reduced version of the RS-I model: it contains only the first mode of the spin-2 particle explicitly defined, which couples to SM particles. Validation tests, similar to the ones presented above for the SM and MSSM, are performed. Approximately $500\ 2 \rightarrow 2$ and $2000\ 2 \rightarrow 3$ processes are tested, and some examples can be found in Tables 6 and 7. Notice the first mode of the spin-2 particle is defined as y. The results are again in full agreement.

4.5. Beyond the HELAS library

An important achievement of Aloha is the ability to create an helicity routine for any type of interaction, even ones not found in the Standard Model. In this context, the Aloha package, via MadGraph 5, has already been used for some analyses.

As an example we cite the Strongly Interacting Light Higgs model (SILH). Within this theory, the Higgs is the Goldstone Boson linked to a new strongly interacting sector. This model has been implemented into UFO and presented in [11]. The authors provide a validation example where the partial decay width of the Higgs into two W bosons is computed. This decay is modified by a Non-Standard Model interaction corresponding to the following Lorentz structure:

This structure corresponds to a dimension 6 operator (and should therefore be proportional to the vacuum expectation value).

The value for the partial decay width is computed by Madgraph version 5, which uses Aloha, and compared to results previously known. This example not only validates the Ufo model but also the routines automatically generated by Aloha.

A second example can be seen in the recent study on effective operators that modify the top production and decay [35, 36]. These analyses are motivated by the precise result on the top sector provided by both the Tevatron and the LHC, and which allow one to constrain the higher dimensional operators. For instance, one of these operators is known as the chromo-magnetic operator [35]:

$$\mathcal{L} = \frac{(H\bar{Q})\sigma^{\mu\nu}T^AtG^A_{\mu\nu}}{\Lambda^2} + h.c.,$$

where Λ represents the cutoff of the effective theory. Adding such a term to the Lagrangian of the Standard Model leads to additional interactions shown by Fig. 2. These additional interactions are not covered by the traditional Helas library. In collaboration with the authors of [35], we check the Aloha results

Table 6: RS $2 \rightarrow 2$ results		Table 7: RS $2 \rightarrow 3$ results		
Process	ΔD	Process	ΔD	
$u\bar{u} \to \gamma\gamma$	$< 1.0 \times 10^{-10}$	$\gamma g \rightarrow \gamma u \bar{u}$	4.431×10^{-8}	
$b \ u \rightarrow b \ u$	1.293×10^{-8}	$g g \rightarrow d \bar{d} g$	1.412×10^{-9}	
$u\bar{\nu}_e \to u\bar{\nu}_e$	9.699×10^{-9}	$\gamma \gamma \rightarrow g g g$	8.610×10^{-10}	
$\bar{u}\bar{u} o \bar{u}\bar{u}$	7.021×10^{-9}	$\gamma g \rightarrow \gamma b b$	4.426×10^{-8}	
$\gamma \bar{u} \to \gamma \bar{u}$	5.389×10^{-8}	$\gamma t \rightarrow \gamma g t$	2.749×10^{-8}	
$\bar{u} W^+ \to \bar{u} W^+$	1.836×10^{-8}	$g g \rightarrow g g g$	3.175×10^{-8}	
bb o gg	$< 1.0 \times 10^{-10}$	$g g \rightarrow e^+ e^- g$	9.659×10^{-10}	
$b t \to b t$	9.656×10^{-9}	$b g \rightarrow \gamma \gamma b$	1.449×10^{-8}	
$b u_{\mu} o b u_{\mu}$	1.293×10^{-8}	$b g \rightarrow b e^+ e^-$	9.753×10^{-8}	
$b \ t \rightarrow b \ t$	9.656×10^{-9}	$g t \to t \nu_e \bar{\nu}_e$	4.294×10^{-9}	
$b \nu_{\mu} \rightarrow b \nu_{\mu}$	1.293×10^{-8}	$gt \rightarrow ggt$	2.231×10^{-8}	
$t t \rightarrow u \bar{u}$	$< 1.0 \times 10^{-10}$	$g \nu_e \rightarrow b b \nu_e$	9.876×10^{-8}	
$t t \to \nu_{\mu} \bar{\nu}_{\mu}$	$< 1.0 \times 10^{-10}$	$g\bar{\nu}_e \to tt\bar{\nu}_e$	1.019×10^{-7}	
$t\bar{\nu}_e \to t\bar{\nu}_e$	9.656×10^{-9}	$e^+ g \rightarrow e^+ g g$	1.251×10^{-8}	
$t \ t \rightarrow t \ t$	7.071×10^{-9}	$g W^+ \rightarrow g g W^+$	4.333×10^{-9}	
$t \ \bar{\nu}_{\mu} \rightarrow t \ \bar{\nu}_{\mu}$	9.656×10^{-9}	$b b \rightarrow \gamma \gamma g$	6.926×10^{-9}	
$g g \to t t$	$< 1.0 \times 10^{-10}$	$b b \rightarrow g \nu_e \bar{\nu}_e$	2.382×10^{-7}	
$g g \to W^+ W^-$	$< 1.0 \times 10^{-10}$	$b \nu_e \to b g \nu_e$	3.671×10^{-8}	
$g Z \to g Z$	6.757×10^{-8}	$b \ b \rightarrow b \ b \ g$	2.067×10^{-7}	
$\gamma \gamma \to g g$	$< 1.0 \times 10^{-10}$	$b e^+ \to b e^+ g$	3.529×10^{-9}	
$\gamma \nu_e \to \gamma \nu_e$	5.389×10^{-8}	$tt \rightarrow bb g$	3.815×10^{-10}	
$\gamma W^- \to \gamma W^-$	6.240×10^{-8}	$t t \rightarrow g W^+ W^-$	4.773×10^{-7}	
$\nu_e \bar{\nu}_e \to \gamma \gamma$	$< 1.0 \times 10^{-10}$	$e^+ t \rightarrow e^+ g t$	3.378×10^{-9}	
$\nu_e \bar{\nu}_\mu \to \nu_e \bar{\nu}_\mu$	9.699×10^{-9}	$e^+ t \rightarrow e^+ g t$	3.378×10^{-9}	
$\bar{\nu}_e \bar{\nu}_\mu \to \bar{\nu}_e \bar{\nu}_\mu$	9.699×10^{-9}	$e^+ e^- \rightarrow g g g$	1.903×10^{-10}	
$ u_{\mu} \bar{\nu}_{\mu} \to b b$	$< 1.0 \times 10^{-10}$	$ZZ \rightarrow bb g$	4.314×10^{-9}	
$ u_{\mu}\bar{\nu}_{\mu} o ZZ$	4.294×10^{-9}	$W^+W^ \rightarrow gtt$	1.595×10^{-9}	
$\bar{\nu}_{\mu} Z \to \bar{\nu}_{\mu} Z$	3.671×10^{-8}	$du \rightarrow dgu$	8.234×10^{-9}	
$ZZ \rightarrow gg$	1.067×10^{-8}	$g u \to d d u$	1.012×10^{-8}	
$W^+Z \to W^+Z$	7.339×10^{-8}	$\bar{u}\bar{u} \to g\bar{u}\bar{u}$	1.421×10^{-8}	
$W^+ W^- \to g g$	2.677×10^{-9}	$\gamma \bar{u} \rightarrow \gamma g \bar{u}$	2.774×10^{-8}	
$g u \rightarrow u y$	$< 1.0 \times 10^{-10}$	$\bar{t} y \to g g \bar{t}$	9.766×10^{-10}	
$b y \rightarrow b g$	$< 1.0 \times 10^{-10}$	$g g \rightarrow g g y$	3.979×10^{-10}	
$\bar{t}y \to g\bar{t}$	$< 1.0 \times 10^{-10}$	$b g \rightarrow b g y$	8.009×10^{-10}	
$g y \to t \bar{t}$	$< 1.0 \times 10^{-10}$	$b \xrightarrow{b} \xrightarrow{b} \xrightarrow{b} y$	1.864×10^{-9}	
$g t \to t y$	$< 1.0 \times 10^{-10}$	$b \overline{t} \rightarrow b \overline{t} y$	7.411×10^{-10}	
$g g \to g y$	$< 1.0 \times 10^{-10}$	$\int t \bar{t} \to b b y$	1.928×10^{-10}	

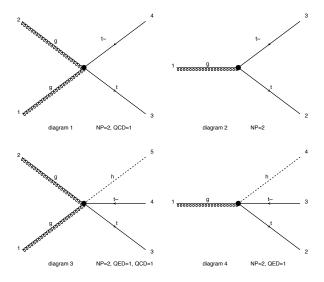


Figure 2: Interactions induced by the chromomagnetic operator.

against i) their analytical computation, and ii) their private implementation of the Helas routines inside the MadGraph version 4 framework. A very good agreement is again obtained.

The implementation of Aloha inside Madgraph version 5, combined with the ability to automatically generate the Ufo model from a general Lagrangian via Feynrules, Aloha has been used and tested on a wide class of different models. The list of currently available models is presented on the Feynrules web-page. These models include, for example, the Higss Effective Theory, the MUED model, the NMSSM and the RMSSM.

4.6. Speed comparison

In addition to the validity checks carried out to test the accuracy of the Aloha routines, one extra comparison is performed to check the speed of the new routines. Table 8 shows the time required to compute the squared matrix element using the original optimized Helas routines and these generated by Aloha. The third column shows the time ratio (Aloha/Helas) revealing that the automatically generated routines are typically within a factor of two in speed of the optimized routines.

5. Summary and perspectives

The automation of cross section calculations and event generation at leading order (LO), as well as at higher orders, from the Lagrangian level requires several non-trivial steps. In this paper, we have presented a code that automatically generates the helicity routines required by a MC event generator in several widely used formats (Fortran, C++ and Python). This is an important step in achieving a fully automatized chain from the Feynman rules, or the Lagrangian level, up to the event generation level.

Aloha represents a non-trivial extension of the current Helas library which is limited to renormalizable interactions and fixed Lorentz structures. The code

Table 8: Computational time (s) to produce values for the $ \mathcal{M} ^2$ using either Aloha or Helas				
Process	ALOHA	HELAS	ALOHA/HELAS	
$\begin{array}{c} u\bar{u} \rightarrow u\bar{u}u\bar{u} \\ u\bar{u} \rightarrow e^-e^+gg \\ e^-u \rightarrow e^-ggu \\ gu \rightarrow guu\bar{u} \\ \bar{u}\bar{u} \rightarrow e^+e^-\bar{u}\bar{u} \\ gg \rightarrow ggg \end{array}$	0.2492 0.1922 0.1871 0.2286 0.2026 0.1966	0.1169 0.107 0.1057 0.1105 0.1062 0.1035	2.1 1.8 1.8 2.1 1.9	

is extremely flexible and easily extendible. A first simple example is the implementation of the complex-mass scheme [37] for the gauge-invariant computation of amplitudes with unstable intermediate particles (such as top, W and Z). As a further example, the current implementation of the Lorentz algebra assumes 4 space-time dimensions, but it could be easily generalized to any other number of even dimensions, as the algebra is symbolic and its actual representation enters only at the final stage of the output writing.

In addition, the implementation of the so-called R_2 Feynman rules, which are needed for the automatic loop computation within the Cuttools method [38], can be easily obtained. The necessary modifications for the routines to run on GPU's can be included, and the corresponding HEGET [39, 40] library can be automatically generated for any BSM model. Finally, extensions to matrix element generator other than MADGRAPH, such as that used in HERWIG++, is straightforward. Work in these directions is on-going.

6. Acknowledgments

The authors would like to thank Johan Alwall and Celine Degrande for their help and support during all phases of this project. We would also like to thank Kaoru Hagiwara, Kentarou Mawatari, Qiang Li and Michel Herquet for their important support. A big thanks to the FeynRules, Madgraph and Ufo teams without whom this work would not be possible, and to Benjamin Fuks and Claude Duhr for their overall assistance within this project. Will Link was supported in part by NSF grant PHY-0705682. OM is Chercheur scientifique logistique postdoctoral F.R.S-FNRS, Belgium. This work is supported in part by the FWO - Vlaanderen, Project number G.0651.11, and in part by the Federal Office for Scientific, Technical and Cultural Affairs through the 'Interuniversity Attraction

Table A.9: Argument format in different output languages				
argument	symbol	Python	Fortran	C++
mass	M	float	real	complex
width	W	float	real	complex
coupling	Coup	complex	complex	complex
scalar wavefunctions	S	list (3)	complex(3)	complex[3]
fermion wavefunctions	F	list (6)	complex(6)	complex[6]
vector wavefunctions	V	list (6)	complex(6)	complex[6]
tensor wavefunctions	Τ	list (18)	complex(18)	complex[18]
Amplitude	A	complex	complex	complex

All Fortran, C++ arguments are double precision. The arguments in Python are only informative, since Python is dynamically typed. However, the list of wavefunctions should be initialized with the correct number of elements.

Poles Programme' Belgian Science Policy P6/11-P and by the IISN MadGraph convention 4.4511.10.

Appendix A. Argument convention

Since the Fortran, Python and C++ languages are distinct languages, there are small differences in the procedure used by each language to call the Aloha functions. Table A.9 presents the different types of arguments, and Table A.10 presents various examples on how the routines should be called in each language and what is the equivalent analytical formula.

Appendix B. Structure of the computation

In order to perform the necessary analytical computations in Python, we design a dedicated analytical package. The fundamental object of this package is the Variable object, which contains three different pieces of information:

 ${\bf prefactor} \ \ {\bf The\ numerical\ value\ by\ which\ the\ variable\ is\ multiplied.} \ \ {\tt [default=1]}$

power The power of the variable. [default=1]

name A string representing the variable.

Table A.10: Syntax Examples

$VVSS_0$

analytical: $Coup * V_1^{\mu} \Gamma_{\mu\nu} V_2^{\nu} S_3 S_4$

 $\label{eq:composition} \begin{array}{ll} \text{Python} & A = VVSS_0(V1,\,V2,\,S3,\,S4,\,Coup) \\ \text{Fortran/C++} & VVSS_0(V1,\,V2,\,S3,\,S4,\,Coup,\,A) \\ \end{array}$

FFV_0

analytical: $Coup * \bar{F}_2\Gamma_{\mu}F_1V_3^{\mu}$

Python $A = FFV_0(F1, F2, V3, Coup)$ Fortran/ C++ $FFV_0(F1, F2, V3, Coup, A)$

FFV_{-1}

analytical: $Coup * \bar{F}_2\Gamma_{\mu}P_1^fV_3^{\mu}$

 $\begin{array}{lll} {\tt Python} & {\tt F1} = {\tt FFV_1}({\tt F2},\,{\tt V3},\,{\tt Coup},\,{\tt M1},{\tt W1}) \\ {\tt Fortran/}\ {\tt C++} & {\tt FFV_1}({\tt F2},\,{\tt V3},\,{\tt Coup},\,{\tt M1},\,{\tt W1},\,{\tt F1}) \\ \end{array}$

FFVC1_0

analytical: $Coup * \bar{F}_1 C \Gamma_u^T C^{\dagger} F_2 V_3^{\mu}$

 $\begin{array}{ll} \mbox{Python} & A = \mbox{FFVC1_0}(F2,\,F1,\,V3,\,Coup) \\ \mbox{Fortran/C++} & \mbox{FFVC1_0}(F2,\,F1,\,V3,\,Coup,\,A) \end{array}$

$FFVC1_{-1}$

analytical: $Coup * \bar{F}_1 C \Gamma_u^T C^{\dagger} P_2^f V_3^{\mu}$

 $\begin{array}{lll} \mbox{Python} & \mbox{F2} = \mbox{FFVC1_0}(\mbox{ F1, V3, Coup, M2, W2)} \\ \mbox{Fortran/ C++} & \mbox{FFVC1_0}(\mbox{F1, V3, Coup, M2, W2, F2)} \\ \end{array}$

$VVV1_2_0$

 $\begin{array}{ll} \textbf{analytical:} & V_1^\mu V_2^\nu V_3^\rho (Coup_1\Gamma_{\mu\nu\rho}^1 + Coup_2 * \Gamma_{\mu\nu\rho}^2) \\ \textbf{Python} & \mathbf{A} = \mathbf{VVV4_5_0} \big(\ \mathbf{V1,V2}, \ \mathbf{V3,Coup1}, \ \mathbf{Coup2} \big) \end{array}$

Python $A = VVV4_5_0(V1,V2, V3,Coup1, Coup2)$ Fortran/ C++ $VVV4_5_0(V1,V2, V3, Coup1, Coup2, A)$

C stands for the conjugate matrix, Γ stands for the Lorentz structures associated to the vertex, P_i^f is the (fermion) propagator associated to the particle i. Other symbols are defined in Table A.9. Note in particular that the order of the arguments in the case of conjugate routines (FFVC1) follow the incoming/outgoing convention (and not the particle order). The last routine (VVV1_2_0) is an example of routines with multiple couplings, corresponding to two Lorentz structures.

For instance, $3x^2$ is Variable(name='x', prefactor=3, power=2). Variable objects with identical names are assumed to be referring to the same mathematical variable. Therefore, addition and multiplication are performed as usual. For example:

In the case of a sum (product) between different variables, the sum (product) returns an object which is essentially the list of the summed (multiplied) variables.

With such a structure it is possible to represent any type of scalar expression.

Non-scalar objects (like p^{μ} , γ^{μ} , ...) have basically the same structure, but slightly extended. There are two additional properties: lorentz_ind and spin_ind, which are the list of Lorentz and spinor indices linked to the object. Each object is related to a representation, *i.e.*, each component should be linked to a scalar expression which can be a number, a variable or even an expression.

Aloha inputs are based on Lorentz objects, and the calculations are performed in seven steps:

- 1. Change of momentum convention in order to pass from the UFO into the Helas convention.
- 2. Multiplication of the Lorentz expression (from UFO) to the wavefunction or propagator as required.
- 3. Evaluation of the expression at the abstract level and simplification where appropriated (to adjust prefactors, etc.).
- 4. Use of the representation of each object in order to create a representation of the result.
- 5. Simplification of each component.
- 6. Factorization of each component to have an efficient way to evaluate the result.
- 7. Writing the routine.

Once the final expression is obtained, the result is passed to a specific routine which writes out the corresponding Helas routines.

Aloha primarily uses 4 files that correspond to the different steps of the computation:

create_aloha.py: contains the driver of the code and the part needed to build the different routines associated with each lorentz structure.

aloha_lib.py: contains all of the code needed for the analytical computations and for the evaluation of the representation of the object.

- aloha_object.py: contains the definition of all basic objects. The current list of basic objects are displayed in Table 1. Adding a new object is straightforward since it is enough to specify the index of the objects and the representation.
- aloha_writers.py: contains the code that writes all Aloha functions in different languages (currently Python, Fortran and C++).

References

- [1] A. Semenov, LanHEP a package for automatic generation of Feynman rules from the Lagrangian. Updated version 3.1 arXiv:1005.1909.
- [2] A. Pukhov, CalcHEP 2.3: MSSM, structure functions, event generation, batchs, and generation of matrix elements for other packagesarXiv:hep-ph/0412191.
- [3] E. Boos, V. Bunichev, M. Dubinin, L. Dudko, V. Edneral, et al., CompHEP 4.5 Status Report, PoS ACAT08 (2008) 008. arXiv:0901.4757.
- [4] G. Brooijmans, B. Gripaios, F. Moortgat, J. Santiago, P. Skands, et al., Les Houches 2011: Physics at TeV Colliders New Physics Working Group ReportarXiv:1203.1488.
- [5] J. Alwall, M. Herquet, F. Maltoni, O. Mattelaer, T. Stelzer, MadGraph
 5: Going Beyond, JHEP 06 (2011) 128. arXiv:1106.0522, doi:10.1007/ JHEP06(2011)128.
- [6] G. C. Cho, et al., Weak boson fusion production of supersymmetric particles at the LHC, Phys. Rev. D73 (2006) 054002. arXiv:hep-ph/0601063, doi: 10.1103/PhysRevD.73.054002.
- [7] K. Hagiwara, J. Kanzaki, Q. Li, K. Mawatari, HELAS and Mad-Graph/MadEvent with spin-2 particles, Eur. Phys. J. C56 (2008) 435–447. arXiv:0805.2554, doi:10.1140/epjc/s10052-008-0663-x.
- [8] K. Hagiwara, K. Mawatari, Y. Takaesu, HELAS and MadGraph with spin-3/2 particles, Eur. Phys. J. C71 (2011) 1529. arXiv:1010.4255, doi:10. 1140/epjc/s10052-010-1529-6.
- [9] N. D. Christensen, C. Duhr, FeynRules Feynman rules made easy, Comput. Phys. Commun. 180 (2009) 1614–1641. arXiv:0806.4194, doi:10.1016/j.cpc.2009.02.018.
- [10] C. Duhr, B. Fuks, A superspace module for the FeynRules package, Comput. Phys. Commun. 182 (2011) 2404-2426. arXiv:1102.4191, doi:10.1016/j.cpc.2011.06.009.

- [11] C. Degrande, C. Duhr, B. Fuks, D. Grellscheid, O. Mattelaer, et al., UFO The Universal FeynRules Output, Comput.Phys.Commun. 183 (2012) 1201–1214. arXiv:1108.2040, doi:10.1016/j.cpc.2012.01.022.
- [12] F. A. Berends, R. Kleiss, P. De Causmaecker, R. Gastmans, T. T. Wu, Single Bremsstrahlung Processes in Gauge Theories, Phys.Lett. B103 (1981) 124. doi:10.1016/0370-2693(81)90685-7.
- [13] P. De Causmaecker, R. Gastmans, W. Troost, T. T. Wu, Multiple Bremsstrahlung in Gauge Theories at High-Energies. 1. General Formalism for Quantum Electrodynamics, Nucl. Phys. B206 (1982) 53. doi: 10.1016/0550-3213(82)90488-6.
- [14] R. Kleiss, W. Stirling, Spinor Techniques for Calculating p anti-p \rightarrow W+- / Z0 + Jets, Nucl.Phys. B262 (1985) 235–262. doi:10.1016/0550-3213(85) 90285-8.
- [15] R. Gastmans, T. Wu, The Ubiquitous photon: Helicity method for QED and QCD.
- [16] Z. Xu, D.-H. Zhang, L. Chang, Helicity Amplitudes for Multiple Bremsstrahlung in Massless Nonabelian Gauge Theories, Nucl. Phys. B291 (1987) 392. doi:10.1016/0550-3213(87)90479-2.
- [17] J. Gunion, Z. Kunszt, Improved Analytic Techniques for Tree Graph Calculations and the G g q anti-q Lepton anti-Lepton Subprocess, Phys.Lett. B161 (1985) 333. doi:10.1016/0370-2693(85)90774-9.
- [18] K. Hagiwara, D. Zeppenfeld, Helicity Amplitudes for Heavy Lepton Production in e+ e- Annihilation, Nucl. Phys. B274 (1986) 1. doi:10.1016/0550-3213(86)90615-2.
- [19] H. Murayama, I. Watanabe, K. Hagiwara, HELAS: HELicity Amplitude Subroutines for Feynman diagram evaluations. KEK-91-11 (1992) http://www-lib.kek.jp/cgi-bin/img_index?199124011.
- [20] K. Hagiwara, J. Kanzaki, Q. Li, K. Mawatari, HELAS and Mad-Graph/MadEvent with spin-2 particles, Eur. Phys. J. C56 (2008) 435-447. arXiv:0805.2554, doi:10.1140/epjc/s10052-008-0663-x.
- [21] P. de Aquino, K. Hagiwara, Q. Li, F. Maltoni, Simulating graviton production at hadron colliders, JHEP 06 (2011) 132. arXiv:1101.5499, doi:10.1007/JHEP06(2011)132.
- [22] K. Mawatari, Y. Takaesu, HELAS and MadGraph with goldstinos, Eur. Phys. J. C71 (2011) 1640. arXiv:1101.1289, doi:10.1140/epjc/ s10052-011-1640-3.

- [23] J. Alwall, et al., MadGraph/MadEvent v4: The New Web Generation, JHEP 09 (2007) 028. arXiv:0706.2334.
- [24] A. Denner, H. Eck, O. Hahn, J. Kublbeck, Feynman rules for fermion number violating interactions, Nucl. Phys. B387 (1992) 467–484. doi: 10.1016/0550-3213(92)90169-C.
- [25] R. Kleiss, W. Stirling, S. Ellis, A new monte carlo treatment of multiparticle phase space at high-energies, Comput.Phys.Commun. 40 (1986) 359. doi: 10.1016/0010-4655(86)90119-0.
- [26] J. Wess, B. Zumino, Supergauge Transformations in Four-Dimensions, Nucl. Phys. B70 (1974) 39–50. doi:10.1016/0550-3213(74)90355-1.
- [27] H. E. Haber, G. L. Kane, The Search for Supersymmetry: Probing Physics Beyond the Standard Model, Phys. Rept. 117 (1985) 75–263. doi:10.1016/ 0370-1573(85)90051-1.
- [28] H. P. Nilles, Supersymmetry, Supergravity and Particle Physics, Phys. Rept. 110 (1984) 1–162. doi:10.1016/0370-1573(84)90008-5.
- [29] J. Rosiek, Complete Set of Feynman Rules for the Minimal Supersymmetric Extension of the Standard Model, Phys. Rev. D41 (1990) 3464. doi:10. 1103/PhysRevD.41.3464.
- [30] S. P. Martin, A supersymmetry primer arXiv:hep-ph/9709356.
- [31] N. Arkani-Hamed, S. Dimopoulos, G. R. Dvali, The hierarchy problem and new dimensions at a millimeter, Phys. Lett. B429 (1998) 263–272. arXiv: hep-ph/9803315, doi:10.1016/S0370-2693(98)00466-3.
- [32] N. Arkani-Hamed, S. Dimopoulos, G. R. Dvali, Phenomenology, astrophysics, and cosmology of theories with submillimeter dimensions and TeV scale quantum gravity, Phys. Rev. D59 (1999) 086004. arXiv:hep-ph/9807344, doi:10.1103/PhysRevD.59.086004.
- [33] L. Randall, R. Sundrum, A large mass hierarchy from a small extra dimension, Phys. Rev. Lett. 83 (1999) 3370–3373. arXiv:hep-ph/9905221.
- [34] L. Randall, R. Sundrum, An alternative to compactification, Phys. Rev. Lett. 83 (1999) 4690–4693. arXiv:hep-th/9906064, doi:10.1103/PhysRevLett. 83.4690.
- [35] C. Degrande, J.-M. Gerard, C. Grojean, F. Maltoni, G. Servant, Non-resonant New Physics in Top Pair Production at Hadron Colliders, JHEP 03 (2011) 125. arXiv:1010.6304, doi:10.1007/JHEP03(2011)125.

- [36] C. Degrande, J.-M. Gerard, C. Grojean, F. Maltoni, G. Servant, An Effective approach to same sign top pair production at the LHC and the forward-backward asymmetry at the Tevatron, Phys.Lett. B703 (2011) 306–309. arXiv:1104.1798, doi:10.1016/j.physletb.2011.08.003.
- [37] A. Denner, S. Dittmaier, The Complex-mass scheme for perturbative calculations with unstable particles, Nucl. Phys. Proc. Suppl. 160 (2006) 22–26. arXiv:hep-ph/0605312, doi:10.1016/j.nuclphysbps.2006.09.025.
- [38] G. Ossola, C. G. Papadopoulos, R. Pittau, CutTools: A Program implementing the OPP reduction method to compute one-loop amplitudes, JHEP 0803 (2008) 042. arXiv:0711.3596, doi:10.1088/1126-6708/2008/03/042.
- [39] K. Hagiwara, J. Kanzaki, N. Okamura, D. Rainwater, T. Stelzer, Fast calculation of HELAS amplitudes using graphics processing unit (GPU), Eur.Phys.J. C66 (2010) 477–492. arXiv:0908.4403, doi:10.1140/epjc/s10052-010-1276-8.
- [40] K. Hagiwara, J. Kanzaki, N. Okamura, D. Rainwater, T. Stelzer, Calculation of HELAS amplitudes for QCD processes using graphics processing unit (GPU), Eur.Phys.J. C70 (2010) 513–524. arXiv:0909.5257, doi:10.1140/epjc/s10052-010-1465-5.