Tools for NLO automation: extension of the golem95C integral library

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Abstract

We present an extension of the program golem95C for the numerical evaluation of scalar integrals and tensor form factors entering the calculation of one-loop amplitudes, which supports tensor ranks exceeding the number of propagators. This extension allows various applications in Beyond the Standard Model physics and effective theories, for example higher ranks due to propagators of spin two particles, or due to effective vertices. Complex masses are also supported. The program is not restricted to the Feynman diagrammatic approach, as it also contains routines to interface to unitarity-inspired numerical reconstruction of the integrand at the tensorial level. Therefore it can serve as a general integral library in automated programs to calculate one-loop amplitudes.

PACS: 12.38.Bx

 $Key \ words:$ NLO computations, one-loop diagrams, tensor integrals, higher spin representations, effective theories

NEW VERSION PROGRAM SUMMARY

Manuscript Title: Tools for NLO automation: extension of the golem95C integral library Authors: J. Ph. Guillet, G. Heinrich, J. F. von Soden-Fraunhofen Program Title: golem95-1.3.0 Licensing provisions: none Programming language: Fortran95 Computer: Any computer with a Fortran95 compiler Operating system: Linux, Unix RAM: RAM used per integral/form factor is insignificant Keywords: NLO computations, one-loop diagrams, tensor integrals, higher spin representations, effective theories PACS: 12.38.Bx Classification: 4.4, 11.1 Externel meetings (libraries: some finite scalar integrals are called from Onel Opp [1, 2]

External routines/libraries: some finite scalar integrals are called from OneLOop [1,2], the option to call them from LoopTools [3,4] is also implemented.

Catalogue identifier of previous version: AEEO_v2_0

Journal reference of previous version: Comput. Phys. Commun. 182 (2011) 2276. Does the new version supersede the previous version?: yes

Nature of problem: Evaluation of one-loop multi-leg integrals occurring in the calculation of next-to-leading order corrections to scattering amplitudes in particle physics. In the presence of particles with spin two in the loop, or effective vertices, or certain gauges, tensor integrals where the rank exceeds the number of propagators N are required.

Solution method: Extension of the reduction algorithm to rank $r \leq 10$ for $N \leq 4$ and $r \leq N + 1$ for $N \geq 5$, which is sufficient for most applications in Beyond the Standard Model Physics.

Reasons for the new version: The previous version was restricted to tensor ranks less or equal to the number of propagators.

Summary of revisions: Tensor ranks r > N are supported, an alternative reduction method for the case of small Gram determinants is implemented, numerical stability for the case of small mass differences has been improved.

Running time: Depends on the nature of the problem. A single call to a rank 6 fivepoint form factor at a randomly chosen kinematic point, using real masses, takes 10^{-3} seconds on an Intel Core 4 i7-3770 CPU with a 3.4 GHz processor.

1 Introduction

Over the last years, enormous progress has been made to push the calculation of NLO corrections towards a higher number of particles in the final states, i.e. to "multi-leg" amplitudes, in QCD as well as in the electroweak sector.

Nowadays, the efforts are focused on the goals of *automating* multi-leg oneloop calculations and making them *publicly available*. Programs aiming at the complete automation of one-loop amplitude calculations, including the generation of the amplitude requested by the user "on the fly", are e.g. FEYN-ARTS/FORMCALC [3,5], HELAC-NLO [6], GOSAM [7], aMC@NLO [8], BlackHat [9], NJET [10], OpenLoops [11], Recola [12]. Public programs which contain a collection of pre-generated processes are e.g. MCFM [13], VBFNLO [14].

An important ingredient for such programs is an integral library containing the one-loop integrals which are the basic building blocks of any oneloop amplitude unless it is calculated purely numerically. Several libraries are publicly available to date: FF [15], Looptools [4], QCDLoop [16], OneLOop [2], golem95C [17,18], PJFRY [19]. A code for the calculation of one-loop four-point functions with complex masses (DOC) can be found in [20]. The latter has been integrated into the LoopTools library [4] where the complex versions of infrared finite integrals with less than four legs are already implemented. A complete set of scalar four-point integrals, both in dimensional and in mass regularisation and valid also for complex masses can be found in [21] in analytic form.

Public programs dedicated to the reduction of multi-leg one-loop amplitudes at integrand level are e.g. CutTools [22] and Samurai [23]. A novel reduction algorithm based on integrand reduction through Laurent series expansion also has been developed [24].

The calculation of scalar one-loop integrals has a long tradition of pioneering work, see e.g. [25,26,27,28,29,30,31]. For processes involving unstable particles, these integrals are also required for complex internal masses, in order to be able to work within the so-called "complex-mass scheme" developed in Refs. [32,33]. An extension of the golem95 library to complex masses has been presented in Ref. [18], called golem95C. The strategy in golem95C to avoid numerical instabilities due to small Gram determinants is to avoid the reduction to scalar basis integrals in such cases, in favour of turning to the numerical evaluation of a convenient one-dimensional integral representation of the respective tensor integral. Recent new developments in this direction can be found in Ref. [34].

In this article, we present an extension of the golem95C library to integrals with tensor ranks r exceeding the number of propagators N. Such integrals occur for example in effective theories (a prominent example is the effective coupling of gluons to the Higgs boson), or in calculations within theories containing spin two particles beyond the leading order. Even though most of the modern methods to reduce one-loop amplitudes do not entirely rely on Feynman diagrams and tensor reduction anymore, the latter is still important in a number of cases. In particular, methods based on the reconstruction of tensor coefficients at integrand level [35,36] have proven very efficient recently [11,12]. Therefore, a tensor and scalar integral library represents an important tool for calculations within and beyond the Standard Model.

Further, tensor reduction, or tensorial reconstruction [36], can be an important "rescue system" for phase space points where cut-based techniques do not provide the desired accuracy. As such, the golem95C library is an important ingredient for the automated one-loop program GOSAM [7]. The extension of cut based reduction methods at integrand level to higher ranks also has been tackled already [24]. The higher rank integrals (closely related to integrals in more than $D = 4 - 2\epsilon$ space-time dimensions) can be written in terms of some "basis integrals" which can be called from the golem95C library presented here. However, the library contains the higher rank form factors in full generality. Therefore it can be used in various applications where the number of loop momenta in the integrand is large, in combination with integrand reduction methods.

This article is organized as follows. In Section 2, we review the theoretical background, while in Section 3 we focus on the new extension of the library to integrals of higher ranks. Section 4 contains installation instructions, while in Section 5 the user can find simple examples how to run the program. Section 6 contains our conclusions.

2 Theoretical background

The program is an update of the tensor and scalar integral library described in more detail in Ref. [17], based on the formalism developed in Refs. [37,38] to reduce tensor integrals to a convenient set of basis integrals. Similar reduction schemes can be found e.g. in Refs. [30,31,39,40,41,42,43,44,45]. Here we will describe the theoretical framework only briefly and focus on the new features of the program.

2.1 Form Factors

Tensor integrals can be divided into a part containing the Lorentz structure and a part consisting of scalar quantities, which we call *form factors*. We define an N-point tensor integral of rank r in $D = 4 - 2\epsilon$ dimensions as

$$I_N^{D,\,\mu_1\dots\mu_r}(a_1,\dots,a_r) = \int \frac{d^D q}{i\,\pi^{D/2}} \,\frac{q_{a_1}^{\mu_1}\dots q_{a_r}^{\mu_r}}{(q_1^2 - m_1^2 + i\delta)\dots(q_N^2 - m_N^2 + i\delta)} \tag{1}$$

where $q_a = q + r_a$, q is the loop momentum, and r_a is a combination of external momenta. Using the shift invariant vectors

$$\Delta^{\mu}_{ij} = r^{\mu}_i - r^{\mu}_j , \qquad (2)$$

we can write down a general form factor decomposition of an arbitrary tensor integral

$$I_{N}^{D,\mu_{1}...\mu_{r}}(a_{1},...,a_{r};S) = \sum_{j_{1},...,j_{r}\in S} \left[\Delta_{j_{1}\cdot}^{\cdot}\cdots\Delta_{j_{r}\cdot}^{\cdot} \right]_{\{a_{1}...a_{r}\}}^{\{\mu_{1}...\mu_{r}\}} A_{j_{1}...j_{r}}^{N,r}(S)$$
(3)
+
$$\sum_{j_{1},...,j_{r-2}\in S} \left[g^{\cdot}\Delta_{j_{1}\cdot}^{\cdot}\cdots\Delta_{j_{r-2}\cdot}^{\cdot} \right]_{\{a_{1}...a_{r}\}}^{\{\mu_{1}...\mu_{r}\}} B_{j_{1}...j_{r-2}}^{N,r}(S)$$
+
$$\sum_{j_{1},...,j_{r-4}\in S} \left[g^{\cdot}g^{\cdot}\Delta_{j_{1}\cdot}^{\cdot}\cdots\Delta_{j_{r-4}\cdot}^{\cdot} \right]_{\{a_{1}...a_{r}\}}^{\{\mu_{1}...\mu_{r}\}} C_{j_{1}...j_{r-4}}^{N,r}(S)$$
+
$$\sum_{j_{1},...,j_{r-4}\in S} \left[g^{\cdot}g^{\cdot}g^{\cdot}\Delta_{j_{1}\cdot}^{\cdot}\cdots\Delta_{j_{r-6}\cdot}^{\cdot} \right]_{\{a_{1}...a_{r}\}}^{\{\mu_{1}...\mu_{r}\}} D_{j_{1}...j_{r-6}}^{N,r}(S)$$
+
$$\sum_{i_{1},...,j_{r-4}\in S} \left[g^{\cdot}g^{\cdot}g^{\cdot}\Delta_{j_{1}\cdot}^{\cdot}\cdots\Delta_{j_{r-6}\cdot}^{\cdot} \right]_{\{a_{1}...a_{r}\}}^{\{\mu_{1}...\mu_{r}\}} D_{j_{1}...j_{r-6}}^{N,r}(S)$$
+
$$\sum_{i_{1},...,i_{r-4}\in S} \left[g^{\cdot}g^{\cdot}g^{\cdot}\Delta_{j_{1}\cdot}^{\cdot}\cdots\Delta_{j_{r-6}\cdot}^{\cdot} \right]_{\{a_{1}...a_{r}\}}^{\{\mu_{1}...\mu_{r}\}} D_{j_{1}...j_{r-6}}^{N,r}(S)$$

The notation $[\cdots]_{\{a_1\cdots a_r\}}^{\{\mu_1\cdots \mu_r\}}$ stands for the distribution of the *r* Lorentz indices μ_i , and the momentum labels a_i , to the vectors $\Delta_{ja_i}^{\mu_i}$ and metric tensors in all distinguishable ways. Note that the choice $r_N = 0$, $a_i = N \forall i$ leads to the well known representation in terms of external momenta where the labels a_i are not necessary, but we prefer a completely shift invariant notation here.

S denotes an ordered set of propagator labels, corresponding to the momenta forming the kinematic matrix \mathcal{S} , defined by

$$S_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2 , \quad i, j \in \{1, \dots, N\} .$$
(4)

The kinematic matrix S is related to the Gram matrix G_{ij} (i, j = 1, ..., N-1for $r_N = 0$ by

det
$$G = (-1)^{N+1} B \det \mathcal{S}$$
, $B = \sum_{i,j=1}^{N} \mathcal{S}_{ij}^{-1}$. (5)

We should point out that the form factors of type $D_{j_1...j_{r-6}}^{N,r}$ and beyond, i.e. form factors associated with three or more metric tensors, are not needed for integrals where the rank r does not exceed the number N of propagators, no

matter what the value of N is. This is because integrals with $N \ge 6$ can be reduced algebraically to pentagons, without generating higher dimensional remainder terms, using the following formula recursively [37]

$$I_N^{D,\mu_1\dots\mu_r}(a_1,\dots,a_r;S) = -\sum_{j\in S} \mathcal{C}_{ja_r}^{\mu_r} I_{N-1}^{D,\mu_1\dots\mu_{r-1}}(a_1,\dots,a_{r-1};S\setminus\{j\}) \quad (N\geq 6) ,$$

where C_{ja}^{μ} is the solution of the equation

$$\sum_{j\in S} \mathcal{S}_{ij} \, \mathcal{C}^{\mu}_{j\,a} = \Delta^{\mu}_{i\,a} \,, \quad a \in S \;. \tag{6}$$

If $N \geq 7$, or in the case of exceptional kinematics, S is not invertible, so eq. (6) does not have a unique solution. However, an explicit solution can be constructed as shown in [37,46]. In this sense the tensor reduction of N-point integrals with $N \geq 6$ is trivial: integrals with $N \geq 6$ can be reduced iteratively to 5-point integrals. Therefore form factors for $N \geq 6$ are never needed. If the rank r of an N-point integral does not exceed N, Lorentz structures carrying at most two factors of $g^{\mu\nu}$ are sufficient, as the maximal form factor needed is $C_j^{5,5}$, corresponding to a rank five 5-point integral. However, for r > N, we can also have e.g. rank six 5-point integrals, which contain Lorentz structures involving three factors of $g^{\mu\nu}$, and therefore form factors which go beyond the types $A^{N,r}, B^{N,r}, C^{N,r}$ are needed.

The form factors are linear combinations of algebraic reduction coefficients, derived from the matrix S, and N-point integrals with $N \leq 4$. Explicit expressions for $r \leq N$ are given in [37].

2.2 Integrals

The golem95C program uses the fact that tensor integrals are related to Feynman parameter integrals with Feynman parameters in the numerator. A scalar integral, after Feynman parametrisation, can be written as

$$I_{N}^{D}(S) = (-1)^{N} \Gamma(N - \frac{D}{2}) \int \prod_{i=1}^{N} dz_{i} \,\delta(1 - \sum_{l=1}^{N} z_{l}) \,\left(R^{2}\right)^{\frac{D}{2} - N} R^{2} = -\frac{1}{2} \sum_{i,j=1}^{N} z_{i} \,\mathcal{S}_{ij} z_{j} - i\delta \,.$$
(7)

The general relation between tensor integrals and parameter integrals with Feynman parameters in the numerator is well known [47,30,38]:

$$I_{N}^{D,\mu_{1}\dots\mu_{r}}(a_{1},\dots,a_{r};S) = (-1)^{r} \sum_{m=0}^{[r/2]} \left(-\frac{1}{2}\right)^{m}$$
$$\sum_{j_{1}\dots j_{r-2m}=1}^{N} \left[(g^{\cdot})^{\otimes m} \Delta_{j_{1}\cdot}^{\cdot} \dots \Delta_{j_{r}\cdot}^{\cdot} \right]_{\{a_{1}\dots a_{r}\}}^{\{\mu_{1}\dots\mu_{r}\}} I_{N}^{D+2m}(j_{1}\dots,j_{r-2m};S) , \qquad (8)$$

where $I_N^{D+2m}(j_1 \ldots, j_{r-2m}; S)$ is an integral with Feynman parameters in the numerator. [r/2] stands for the nearest integer less or equal to r/2 and the symbol $\otimes m$ indicates that m powers of the metric tensor are present. Feynman parameter integrals corresponding to diagrams where propagators l_1, \ldots, l_m are omitted with respect to the "maximal" topology can be defined as

$$I_{N}^{D}(j_{1},\ldots,j_{r};S \setminus \{l_{1},\ldots,l_{m}\}) = (-1)^{N}\Gamma(N-\frac{D}{2})$$
$$\int \prod_{i=1}^{N} dz_{i} \,\delta(1-\sum_{k=1}^{N} z_{k}) \,\delta(z_{l_{1}})\ldots\delta(z_{l_{m}}) z_{j_{1}}\ldots z_{j_{r}} \left(R^{2}\right)^{D/2-N} \,. \tag{9}$$

The program golem95C reduces the integrals internally to a set of basis integrals, i.e. the endpoints of the reduction (they do not form a basis in the mathematical sense, as some of them are not independent). The choice of the basis integrals can have important effects on the numerical stability in certain kinematic regions. Our reduction endpoints are 4-point functions in 6 dimensions I_4^6 , which are IR and UV finite, 4-point functions in D + 4 dimensions, and various 2-point and 3-point functions, some of the latter with Feynman parameters in the numerator. This provides us with a convenient separation of IR and UV divergences, as the IR poles are exclusively contained in the triangle functions.

Note that I_3^{D+2} and I_4^{D+4} are UV divergent, while I_3^D can be IR divergent. In the code, the integrals are represented as arrays containing the coefficients of their Laurent expansion in $\epsilon = (4 - D)/2$.

If the endpoints of the reduction contain finite scalar four-point integrals, the latter have not all been coded explicitly, but some are automatically called from the library OneLOop [2].

3 Extension of the program to higher rank

We focus here on the new features, for more details on the software components which are the same as in version 1.2, we refer to [17,18] and to the documentation contained in the program.

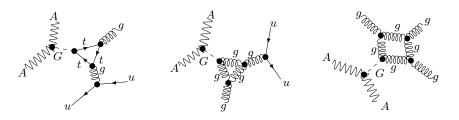


Fig. 1. Examples of diagrams with graviton exchange, where rank four 3-point and rank five 4-point integrals are needed. The graviton is denoted by G.

The call syntax for the higher rank form factors is analogous to the one for $r \leq N$, where the form factors defined in eq. (3) have been extended to include higher ranks. The code is built such that the reduction procedure is valid for arbitrary ranks. Form factors up to rank ten for $N \leq 4$ and up to rank six for N = 5 have been hardcoded, additional ones can be easily generated using the python script gen_form_factors.py in the subdirectory tool/highrank. Explicit examples how to call the form factor for a rank four triangle, a rank five box and a rank six pentagon are given in section 5.

4 Installation

The program can be downloaded as golem95-1.3.0.tar.gz from the following URL: http://golem.hepforge.org/95/. The installation instructions given below also can be found in the README file coming with the code.

The installation setup is based on autotools [48]. To install the golem95C library, type the following commands:

./configure [--prefix=mypath] [--precision=quadruple] [FC=compiler]
make

make install

The --prefix option denotes the installation prefix, under which the directories lib/ and include/ are generated. If no option is given, on a Linux system the configure script would choose prefix=/usr/local. The argument --precision selects double or quadruple precision to be used in the library; it should be noted that quadruple precision is not supported by all Fortran compilers and that precision=double is the default value. If the variable FC is not set the first Fortran compiler which is automatically detected will be used. Another variable commonly used is FCFLAGS which allows one to pass compiler flags to the Fortran compiler.

As an alternative to the call of finite box integrals from OneLOop [2], it is possible to call finite scalar box and triangle integrals with internal masses from LoopTools [3]. To use this option, the user should (a) install LoopTools, and

(b) use the option --with-looptools=path_to_libooptools.a for the configure script, i.e. type the following commands:

```
./configure [--prefix=mypath] [--with-looptools=path_to_libooptools.a]
[--precision=quadruple] [FC=compiler] [F77= fortran77compiler]
make
```

make install.

5 Usage and examples

Examples for the usage of the program can be found in the subdirectory **demos**. The examples for the higher rank form factors are also described below.

The basic structure of a program using the calculation of N-point scalar integrals or form factors by golem95C is:

```
call initgolem95(N)
```

 \dots fill kinematic matrix \mathcal{S} \dots

```
call preparesmatrix()
```

... evaluate integrals/form factors ...

```
call exitgolem95()
```

A simple example for the calculation of a scalar three-point integral, which is evaluated by calling the form factor A30, is given in Figure 1.

Listing 1. Example of a program calling a scalar three-point integral.

```
use precision_golem ! to get the type ki (for real and complex)
                      ! needed for initgolem95, s_mat, etc.
! contains useful constants
use matrice_s
{\bf use} \ {\rm constante}
use constants
use form_factor_type
use parametre ! contains default parameter settings
use form_factor_3p ! module containing the three-point form factors
implicit none
{\bf type}({\rm form\_factor}) \,:: \,\, {\rm res}
real(ki) :: s1, s2, s3, m1sq, m2sq, m3sq
s1 = 1.ki
m1sq = 2.ki
m2sq = 0._ki
m3sq = 0._ki
call initgolem95(3) ! initialisation of caching system and 3x3 matrix S
! definition of the kinematic matrix S
      s_mat(1,:) = (/ -m1sq*2..ki , s2-m1sq-m2sq, s1-m1sq-m3sq /) \\       s_mat(2,:) = (/ s2-m1sq-m2sq, -m2sq*2..ki , s3-m2sq-m3sq /) \\       s_mat(3,:) = (/ s1-m1sq-m3sq, s3-m2sq-m3sq, -m3sq*2..ki /) 
call preparesmatrix()
! call the scalar triangle
```

 $\rm res~=a30(s_null)$

```
write (6,*) 'result='
write (6,*) 'result='
write (6,*(" 1/epsilon^2 * (",e16.10,1x,"+ I*",1x,e16.10,")")') real(res%a,ki),aimag(res%a)
write (6,*("+1/epsilon * (",e16.10,1x,"+ I*",1x,e16.10,")")') real(res%b,ki),aimag(res%b)
write (6,*("+1 * (",e16.10,1x,"+ I*",1x,e16.10,")")') real(res%c,ki),aimag(res%c)
call exitgolem95()
end program main
```

5.1 Three-point rank 4 example

The executable for this example is created by make demo_3point. The program demos/demo_three_point.f90 contains as an option the call to a rank four three-point form factor, A34(1, 1, 2, 3, S). The integers in the argument list denote the labels of the corresponding momenta in the form factor representation, i.e. $A_{1123}^{3,4}$ in the notation of eq. (3), denoting the coefficient of the term $\Delta_{1a_1}^{\mu_1} \Delta_{1a_2}^{\mu_2} \Delta_{2a_3}^{\mu_3} \Delta_{3a_4}^{\mu_4}$ in the form factor decomposition of the Lorentz structure. Note that using $a_j = 3$ and $r_3 = 0$, Δ_{ia_j} is simply replaced by r_i . The call syntax is the same as for the lower rank form factors. The results are written to the file test3point.txt. As a check for the user, the results to be obtained when calling A34(1, 1, 2, 3, S) are listed in table_of_results_3point_option_n.txt. The option n in table_of_results_3point_option_n.txt denotes different choices for the kinematics of the three-point function. Running the executable demo_3point, the user is prompted to give (a) the option for the kinematics and (b) the option for the rank (or other features). The higher rank three-point example corresponds to option (b) number 7.

5.2 Four-point rank 5 example

The executable for this example is created by make demo_4point. The program demos/demo_four_point.f90 contains as an option the call to a rank five four-point form factor, A45(1, 2, 2, 3, 3, S). The integers in the argument list denote the labels of the corresponding momenta in the form factor representation, i.e. $A_{12233}^{4,5}$ in the notation of eq. (3), denoting the coefficient of the term $\Delta_{1a_1}^{\mu_1} \Delta_{2a_2}^{\mu_2} \Delta_{2a_3}^{\mu_3} \Delta_{3a_4}^{\mu_4} \Delta_{3a_5}^{\mu_5}$ in the form factor decomposition of the Lorentz structure. The call syntax is the same as for the lower rank form factors. The results are written to the file test4point.txt. As a check for the user, the results to be obtained when calling A45(1, 2, 2, 3, 3, S) are listed in table_of_results_4point_option_n.txt, where again n denotes different choices for the kinematics. Once the kinematics is chosen, the rank five 4-point example corresponds to option (b) number 9.

5.3 Five-point rank 6 example

The executable for this example is created by make demo_5point. The program demos/demo_five_point.f90 contains as an option the call to a rank four three-point form factor, A56(1, 1, 2, 3, 4, 5, S). The integers in the argument list denote the labels of the corresponding momenta in the form factor representation, i.e. $A_{112345}^{5,6}$ in the notation of eq. (3), denoting the coefficient of the term $\Delta_{1a_1}^{\mu_1} \Delta_{1a_2}^{\mu_2} \Delta_{2a_3}^{\mu_3} \Delta_{3a_4}^{\mu_5} \Delta_{5a_6}^{\mu_6}$ in the form factor decomposition of the Lorentz structure. The call syntax is the same as for the lower rank form factors. The results are written to the file test5point.txt. As a check for the user, the results to be obtained when calling A56(1, 1, 2, 3, 4, 5, S) (option 5 in demo_five_point.f90) are listed in table_of_results_5point.txt.

6 Conclusions

We have presented an extension of the program golem95C which provides a library of scalar and tensor integrals where the tensor rank r, denoting the number of loop momenta in the numerator, can exceed the number of propagators N. We have implemented ranks up to r = 10 for $N \leq 4$, and 5-point and 6-point functions up to rank r = N + 1. The use of both real or complex masses is possible within the same setup. The program, which is an extension of on an earlier version of the golem95C library, now also can be used in the presence of effective vertices and in models where the presence of spin two particles can require integrals with higher ranks than usually needed in renormalizable theories. Due to an appropriate interface, the program can be used both within a traditional tensor reduction approach as well as within unitarity-inspired numerical reconstruction of the integrand at the tensorial level. The program is publicly available at http://golem.hepforge.org/95/.

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A Appendix: \tilde{k} - integrals

Here we give a list of integrals which are needed if the momenta in the numerator are split into a 4-dimensional and a (D-4)-dimensional part, according to $k_{(D)}^{\mu} = \hat{k}_{(4)}^{\mu} + \tilde{k}_{(-2\epsilon)}^{\mu}, k_{(D)}^{2} = \hat{k}^{2} + \tilde{k}^{2}$. Using $\hat{q}_{a} = \hat{k} + r_{a}$, where \hat{k} is the loop momentum in 4 dimensions, and r_{a} is a combination of external momenta, we define

$$I_N^{D,\alpha;\mu_1\dots\mu_r}(a_1,\dots,a_r;S) \equiv \int \frac{\mathrm{d}^D k}{i\pi^{D/2}} \frac{\left(\hat{k}^2\right)^{\alpha} \hat{q}_{a_1}^{\mu_1}\cdots\hat{q}_{a_r}^{\mu_r}}{\prod_{j=1}^N (q_j^2 - m_j^2 + i\delta)}.$$
 (A.1)

The integrals with additional powers of $(\tilde{k}^2)^{\alpha}$ in the numerator are related to integrals in higher dimensions by

$$I_{N}^{D,\alpha;\mu_{1}...\mu_{r}}(a_{1},...,a_{r};S) = (-1)^{r+\alpha} \frac{\Gamma(\alpha+D/2-2)}{\Gamma(D/2-2)} \sum_{m=0}^{\lfloor r/2 \rfloor} \left(-\frac{1}{2}\right)^{m} \times \sum_{j_{1},...,j_{r-2m}=1}^{N} \underbrace{\left[\hat{g}^{\bullet} \cdots \hat{g}^{\bullet}\right]_{m}^{\bullet}}_{m} \Delta_{j_{1}}^{\bullet} \cdots \Delta_{j_{r-2m}}^{\bullet} \cdot \left[\hat{g}_{1}^{\mu_{1}...\mu_{r}}I_{N}^{D+2\alpha+2m}(j_{1},...,j_{r-2m};S)\right]_{m} (A.2)$$

where $\Delta_{ij}^{\mu} = r_i^{\mu} - r_j^{\mu}$. Note that for $D = 4 - 2\epsilon$, the prefactor in eq. (A.2) reads

$$\frac{\Gamma(\alpha + D/2 - 2)}{\Gamma(D/2 - 2)} = \frac{\Gamma(\alpha - \epsilon)}{\Gamma(-\epsilon)} = -\epsilon \left(\alpha - 1\right)! + \mathcal{O}(\epsilon^2) .$$
(A.3)

Therefore, integrals with $\alpha > 0$ will only contribute if the integrals $I_N^{D+2\alpha+2m}$ are UV divergent, since we can drop terms of $\mathcal{O}(\epsilon)$ for one-loop applications. The coefficient of the UV pole of these integrals, which is projected out in this way, will contribute to the so-called "rational part" of an amplitude. For $D = 4 - 2\epsilon$, the integral $I_N^{D+2\alpha+2m}$ will be proportional to $\Gamma(\epsilon - \eta)$, with $\eta = 2 - N + \alpha + m$. Therefore it will contain an UV divergence if $\eta \geq 0$.

The results for those integrals which are relevant for the rational part can be given in a general form [49].

$$\epsilon I_N^{D+2\alpha+2m}(l_1,\dots,l_r;S) = \frac{(-1)^N}{2^\eta \eta!} \sum_{j_1,\dots,j_{2\eta}=1}^N \mathcal{S}_{j_1j_2} \cdots \mathcal{S}_{j_{2\eta-1},j_{2\eta}} P_N(l_1,\dots,l_r,j_1,\dots,j_{2\eta})$$

$$\eta = 2 - N + \alpha + m, \tag{A.4}$$

where we define

$$P_{t_1, t_2, \dots, t_N} = \frac{\prod_{j=1}^N (t_j!)}{(N - 1 + \sum_{i=1}^N t_i)!}$$
(A.5)

to arrive at $P_N(j_1, \ldots, j_s)$, which counts the indices in an expression:

$$P_N(j_1, \dots, j_s) = P_{\left(\sum_{i=1}^s \delta_{1, j_i}\right), \dots, \left(\sum_{i=1}^s \delta_{N, j_i}\right)}.$$
 (A.6)

Further, $S_{j_1j_2}$ denotes an element of the kinematic matrix S.

The integrals $I_N^{D+2\alpha+2m}$ will be UV divergent for $2\alpha + 2m \ge 2N - 4$. Below give the relevant explicit expressions, derived from eq. (A.4). Terms which will be of order $\mathcal{O}(\epsilon)$ are dropped.

$$\epsilon I_N^{D-4+2N}(S) = \frac{(-1)^N}{(N-1)!}$$
(A.7a)

$$\epsilon I_N^{D-4+2N}(l_1;S) = \frac{(-1)^N}{N!}$$
(A.7b)

$$\epsilon I_N^{D-4+2N}(l_1, l_2; S) = \frac{(-1)^N}{(N+1)!} (1 + \delta_{l_1 l_2})$$
(A.7c)

$$\epsilon I_N^{D-4+2N}(l_1, l_2, l_3; S) = \frac{(-1)^N}{(N+2)!}$$
(A.7d)
 $\times (1 + \delta_{N-1} + \delta_{N-1} + 2\delta_{N-1} + \delta_{N-1})$

$$\times (1 + o_{l_1 l_2} + o_{l_1 l_3} + o_{l_2 l_3} + 2o_{l_1 l_2} o_{l_2 l_3})$$

$$\epsilon I_N^{D-4+2N}(l_1, l_2, l_3, l_4; S) = \frac{(-1)^N}{(N+3)!}$$
(A.7e)

$$\times (\delta - (6\delta - \delta - + 2\delta - + 2\delta - + \delta -)$$

$$\times \left(\delta_{l_{1}l_{2}}(0\delta_{l_{1}l_{3}}\delta_{l_{2}l_{4}} + 2\delta_{l_{1}l_{3}} + 2\delta_{l_{2}l_{4}} + \delta_{l_{3}l_{4}} \right) + 2\delta_{l_{3}l_{4}}(\delta_{l_{1}l_{3}} + \delta_{l_{2}l_{4}}) + \delta_{l_{1}l_{3}}\delta_{l_{2}l_{4}} + \delta_{l_{1}l_{4}}\delta_{l_{2}l_{3}} + \delta_{l_{1}l_{2}} + \delta_{l_{1}l_{3}} + \delta_{l_{1}l_{4}} + \delta_{l_{2}l_{3}} + \delta_{l_{2}l_{4}} + \delta_{l_{3}l_{4}} + 1 \right) \epsilon I_{N}^{D-4+2(N+1)}(S) = \frac{(-1)^{N}}{2(N+1)!} \left(\sum_{j_{1},j_{2}=1}^{N} S_{j_{1}j_{2}} + tr\{S\} \right)$$
(A.7f)

$$\epsilon I_N^{D-4+2(N+1)}(l_1; S) = \frac{(-1)^N}{2(N+2)!} \sum_{j_1, j_2=1}^N \mathcal{S}_{j_1 j_2} \left(1 + \delta_{j_1 j_2}\right)$$
(A.7g)

$$\times (1 + \delta_{l_1 j_1} + \delta_{l_1 j_2})$$

$$\epsilon I_N^{D-4+2(N+1)}(l_1, l_2; S) = \frac{(-1)^N}{2(N+3)!} \sum_{j_1, j_2=1}^N S_{j_1 j_2}$$

$$\times (\delta_{j_1 j_2}(6\delta_{j_1 l_1}\delta_{j_2 l_2} + 2\delta_{j_1 l_1} + 2\delta_{j_2 l_2} + \delta_{l_1 l_2})$$

$$+ 2\delta_{l_1 l_2}(\delta_{j_1 l_1} + \delta_{j_2 l_2}) + \delta_{j_1 l_1}\delta_{j_2 l_2} + \delta_{j_1 l_2}\delta_{j_2 l_1}$$

$$+ \delta_{j_1 j_2} + \delta_{j_1 l_1} + \delta_{j_1 l_2} + \delta_{j_2 l_1} + \delta_{j_2 l_2} + \delta_{l_1 l_2} + 1) .$$

$$(A.7h)$$

We also give here the explicit formulas for rank 6 pentagon integrals involving \tilde{k}^2 terms, because the latter are special to higher rank extensions. Again, terms of $\mathcal{O}(\epsilon)$ are dropped.

$$I_5^{D,3}(S) = \int \frac{\mathrm{d}^D k}{i\pi^{D/2}} \frac{\left(\tilde{k}^2\right)^3}{\prod_{j=1}^5 (q_j^2 - m_j^2 + i\delta)} = -\frac{1}{12} \tag{A.8}$$

$$I_5^{D,2;\mu_1\mu_2}(a_1, a_2; S) = \int \frac{\mathrm{d}^D k}{i\pi^{D/2}} \frac{\left(\tilde{k}^2\right)^2 \hat{q}_{a_1}^{\mu_1} \hat{q}_{a_2}^{\mu_2}}{\prod_{j=1}^5 (q_j^2 - m_j^2 + i\delta)} = -\frac{1}{48} g^{\mu_1\mu_2} \tag{A.9}$$

$$I_{5}^{D,1;\mu_{1}\cdots\mu_{4}}(a_{1},\ldots,a_{4};S) = \int \frac{\mathrm{d}^{2}k}{i\pi^{D/2}} \frac{k^{2} q_{a_{1}}^{\mu_{1}} \dots q_{a_{4}}^{\mu_{4}}}{\prod_{j=1}^{5}(q_{j}^{2}-m_{j}^{2}+i\delta)}$$
$$= -\frac{1}{96} \left[g^{\mu_{1}\mu_{2}} g^{\mu_{3}\mu_{4}} + g^{\mu_{1}\mu_{3}} g^{\mu_{2}\mu_{4}} + g^{\mu_{1}\mu_{4}} g^{\mu_{2}\mu_{3}} \right].$$
(A.10)

As another example, we give the expressions for the rational part of box integrals in D + 6 dimensions, needed for rank six four-point functions, which can be obtained from (A.7f):

$$\epsilon I_4^{D+6}(S) = \frac{1}{240} \left(\sum_{i,j=1}^4 (\Delta_{ij}^2 - m_i^2 - m_j^2) - 2\sum_{i=1}^4 m_i^2 \right) + \mathcal{O}(\epsilon) .$$
 (A.11)

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