# GravitinoPack and decays of supersymmetric metastable particles 

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#### Abstract

We present the package GravitinoPack that calculates the two- and three-body decays of unstable supersymmetric particles involving the gravitino in the final or initial state. In a previous paper, we already showed results for the gravitino decays into two and three particles. In this paper, we incorporate the processes where an unstable neutralino, stau or stop decays into a gravitino and Standard Model particles. This is the case in gravitino dark matter supersymmetric models, where the gravitino is the lightest SUSY particle. We give instructions for the installation and the use of the package. In the numerical analysis, we discuss various MSSM scenarios. We show that the calculation of all the decay channels and the threebody decay branching ratios is essential for the accurate application of cosmological bounds on these models.


## 1 Introduction

In the past the gravitino dark matter (DM) scenario has been studied extensively [1]. In these scenarios assuming $R$-parity conservation, the gravitino, the supersymmetric partner of the graviton, is stable and can play the role of the DM particle. Other supersymmetric particles as neutralinos and sfermions (e.g. stops or staus) are unstable and can decay into a gravitino and other Standard Model (SM) particles. These decays produce electromagnetic energy and hadrons which affect the primordial Big-Bang Nucleosynthesis $(\mathrm{BBN})$ prediction for the abundances of the light nuclei, like $\mathrm{D},{ }^{4} \mathrm{He},{ }^{3} \mathrm{He}$ and ${ }^{7} \mathrm{Li}[2-4]$.

In a previous paper [5] we presented results for the complementary case where the gravitino is unstable, using the package GravitinoPack. In this paper we extend the scope of the package by calculating the decays widths and the branching ratios of decays of unstable supersymmetric particles into a gravitino.

The package GravitinoPack is a numerical tool developed with the help of the packages FeynArts (FA) and FormCalc (FC), [6-8]. It contains Fortran and Mathematica routines that calculate the decay rates for the main decay channels for the unstable supersymmetric particles, involving gravitinos. In [5] we studied in detail all dominant two-body channels $\widetilde{G} \rightarrow \widetilde{X} Y$, as well as the three-body channels $\widetilde{G} \rightarrow \tilde{\chi}_{1}^{0} X Y$, where $\tilde{X}$ is a sparticle, $\tilde{\chi}_{1}^{0}$ the lightest neutralino and $X, Y$ are SM particles. The two-body decays dominate the total gravitino width, and in particular the channel $\widetilde{G} \rightarrow \tilde{\chi}_{1}^{0} \gamma$, which is kinematically open in the whole region $m_{\widetilde{G}}>m_{\tilde{\chi}_{1}^{0}}$. On the other hand, also many three-body decay channels can be open, $\widetilde{G} \rightarrow \tilde{\chi}_{1}^{0} X Y$, even below thresholds of involved two-body decays, $m_{\widetilde{G}}<m_{\tilde{X}}+m_{Y}$.

In this paper we present results for the cases where the Next to the Lightest Supersymmetric Particle (NLSP) is the lightest neutralino $\left(\tilde{\chi}_{1}^{0}\right)$, or the lighter stau $\left(\tilde{\tau}_{1}\right)$, or the lighter stop $\left(\tilde{t}_{1}\right)$ decaying to a gravitino and Standard Model particles. The dominant two-body decay channels are $\tilde{\chi}_{1}^{0} \rightarrow \widetilde{G} \gamma(Z), \tilde{\tau}_{1} \rightarrow \widetilde{G} \tau$ and $\tilde{t}_{1} \rightarrow \widetilde{G} t$. In addition, we have included all possible three-body decays.

The amplitudes of the processes are generated using the package FA, that has been extended in order to deal with interactions with spin- $3 / 2$ particles. We have built a model file with all possible gravitino interactions with the particles of the Minimal Supersymmetric Standard Model (MSSM). The corresponding details are shown in Appendix A. FC has been extended so that it automatically generates a Fortran code for the numerical calculation of the squared amplitudes. As there are many gamma matrices involved we have done this within FC by using the Weyl-van-der-Waerden formalism [9] as imple-
mented into FC from [10]. There the complexity of the calculation only grows linearly with the number of Feynman graphs.

In our numerical study, we use a few benchmark points from supersymmetric models with different supersymmetry breaking patterns, like the phenomenological MSSM (pMSSM) [11], and the Constrained MSSM (CMSSM) [12, 13]. Actually, the cases we have selected in the pMSSM are mainly points where the neutralino carries significant Higgsino components as in the Non-Universal Higgs Model (NUHM) [14]. Therefore, we do not explicitly discuss the NUHM. These models are applied to the different NLSP cases. In our analysis, we have employed all phenomenological constraints from the LHC [15] experiments concerning the superpartner mass bounds, the Higgs boson mass [16] and the $\mathrm{LHCb}[17,18]$ data.

The paper is organized as follows: In Section 2 we present the decay channels into the LSP gravitino. In Section 3 we describe the package GravitinoPack and give details for its installation and use. We present in Section 4 a few representative numerical results in various MSSM models. In Section 5 we summarise our results. In Appendix A the detailed derivation of the gravitino couplings to the MSSM particles is given.

## 2 The Calculation

The main aspects were already presented in [5]. There an illustrative example is given how a specific gravitino-MSSM interaction is derived and implemented into FeynArts together with all possible Lorentz structures of gravitino interactions with MSSM particles. The detailed derivation of the total explicit gravitino-MSSM interaction Lagrangian with the 78 couplings can be found in Appendix A.

### 2.1 Two- and Three-body decays with Gravitino

In [5] we already discussed all two-body decays of the gravitino and all three-body of gravitino into a neutralino and a SM particle pair. We also present the general formulas for the decay widths. The analogous formulas for the decays into a gravitino can be derived from there.

### 2.1.1 NLSP neutralino decays

There are five two-body decays of $\tilde{\chi}_{1}^{0}$ possible:

$$
\begin{aligned}
& \widetilde{\chi}_{1}^{0} \rightarrow \tilde{G}\left(Z^{0}, \gamma\right), \\
& \widetilde{\chi}_{1}^{0} \rightarrow \tilde{G}\left(h^{0}, H^{0}, A^{0}\right) .
\end{aligned}
$$

The lightest neutralino decays into the gravitino and a pair of SM particles as

$$
\begin{aligned}
& \widetilde{\chi}_{1}^{0} \rightarrow \tilde{G} \tilde{f} f, \\
& \widetilde{\chi}_{1}^{0} \rightarrow \tilde{G} V V, \quad V V=Z^{0} Z^{0}, Z^{0} \gamma, W^{+} W^{-}, \\
& \widetilde{\chi}_{1}^{0} \rightarrow \tilde{G} V S, \quad V S=\left(Z^{0}, \gamma\right)\left(h^{0}, H^{0}, A^{0}\right), W^{+} H^{-}, W^{-} H^{+}, \\
& \widetilde{\chi}_{1}^{0} \rightarrow \tilde{G} S S, \quad S S=\left(h^{0}, H^{0}, A^{0}\right)\left(h^{0}, H^{0}, A^{0}\right), H^{+} H^{-},
\end{aligned}
$$

where $f=\nu_{e}, \nu_{\mu}, \nu_{\tau}, e^{-}, \mu^{-}, \tau^{-}, u, c, t, d, s, b$. These are 19 three-body decay channels. They are given in Table 1.

### 2.1.2 NLSP stau decays

There is only one two-body decay possible,

$$
\tilde{\tau}_{1}^{-} \rightarrow \tilde{G} \tau^{-}
$$

All possible three-body decays of $\widetilde{\tau}_{1}^{-}$are given in Table 2.

### 2.1.3 NLSP stop decays

There is only one two-body decay possible,

$$
\widetilde{t}_{1} \rightarrow \tilde{G} t
$$

All possible three-body decays of $\tilde{t}_{1}$ are given in Table 3. If $\tilde{t}_{1}$ is the NLSP only the top quark can be resonant in the channels with $W^{+}$or $H^{+}$.

## 3 GravitinoPack

GravitinoPack is a package for the evaluation of processes with gravitino interaction. The version GravitinoPack1.0 includes all two-body decays of $\tilde{G}$ and all three-body

| process <br> $\widetilde{\chi}_{1}^{0} \rightarrow \tilde{G} X Y$ | number <br> of graphs | first decay <br> $\widetilde{\chi}_{1}^{0} \rightarrow \tilde{X} Y$ | possible <br> resonances |
| :--- | :---: | :--- | :--- |
| $\tilde{G} f \bar{f}$ | 7 | $\tilde{G}\left(h^{0}, H^{0}, A^{0}, \gamma, Z^{0}\right), f \tilde{f}_{l}^{*}, \tilde{f}_{l} \bar{f}$ | $h^{0}, H^{0}, A^{0}, Z^{0}$ |
| $\tilde{G} Z^{0} Z^{0}$ | 4 | $\tilde{G}\left(h^{0}, H^{0}\right), Z^{0} \tilde{\chi}_{k}^{0}, \tilde{\chi}_{k}^{0} Z^{0}$ | $H^{0}$ |
| $\tilde{G} Z^{0} \gamma$ | 1 | $\tilde{\chi}_{k}^{0} Z^{0}$ | - |
| $\tilde{G} W^{+} W^{-}$ | $6+4 p t$ | $\tilde{G}\left(h^{0}, H^{0}, \gamma, Z^{0}\right), W^{+} \tilde{\chi}_{j}^{-}, \tilde{\chi}_{j}^{+} W^{-}$ | $H^{0}$ |
| $\tilde{G} Z^{0} h^{0}$ | $4+4 p t$ | $\tilde{G}\left(A^{0}, Z^{0}\right), Z^{0} \tilde{\chi}_{k}^{0}, \tilde{\chi}_{k}^{0} h^{0}$ | $A^{0}$ |
| $\tilde{G} Z^{0} H^{0}$ | $4+4 p t$ | $\tilde{G}\left(A^{0}, Z^{0}\right), Z^{0} \tilde{\chi}_{k}^{0}, \tilde{\chi}_{k}^{0} H^{0}$ | $A^{0}$ |
| $\tilde{G} Z^{0} A^{0}$ | $4+4 p t$ | $\tilde{G}\left(h^{0}, H^{0}\right), \tilde{\chi}_{k}^{0} Z^{0}, A^{0} \tilde{\chi}_{k}^{0}$ | $H^{0}$ |
| $\tilde{G} \gamma h^{0}$ | 1 | $\tilde{\chi}_{k}^{0} h^{0}$ | - |
| $\tilde{G} \gamma H^{0}$ | 1 | $\tilde{\chi}_{k}^{0} H^{0}$ | - |
| $\tilde{G} \gamma A^{0}$ | 1 | $\tilde{\chi}_{k}^{0} A^{0}$ | - |
| $\tilde{G} W^{+} H^{-}$ | $5+4 p t$ | $\tilde{G}\left(h^{0}, H^{0}, A^{0}\right), W^{+} \tilde{\chi}_{j}^{-}, \tilde{\chi}_{j}^{+} H^{-}$ | $H^{0}, A^{0}$ |
| $\tilde{G} W^{-} H^{+}$ | $5+4 p t$ | $\tilde{G}\left(h^{0}, H^{0}, A^{0}\right), W^{-} \tilde{\chi}_{j}^{+}, \tilde{\chi}_{j}^{-} H^{+}$ | $H^{0}, A^{0}$ |
| $\tilde{G} h^{0} h^{0}$ | 4 | $\tilde{G}\left(h^{0}, H^{0}\right), h^{0} \tilde{\chi}_{k}^{0}, \tilde{\chi}_{k}^{0} h^{0}$ | $H^{0}$ |
| $\tilde{G} H^{0} H^{0}$ | 4 | $\tilde{G}\left(h^{0}, H^{0}\right), H^{0} \tilde{\chi}_{k}^{0}, \tilde{\chi}_{k}^{0} H^{0}$ | - |
| $\tilde{G} h^{0} H^{0}$ | 4 | $\tilde{G}\left(h^{0}, H^{0}\right), h^{0} \tilde{\chi}_{k}^{0}, \tilde{\chi}_{k}^{0} H^{0}$ | - |
| $\tilde{G} A^{0} A^{0}$ | 4 | $\tilde{G}\left(h^{0}, H^{0}\right), A^{0} \tilde{\chi}_{k}^{0}, \tilde{\chi}_{k}^{0} A^{0}$ | $H^{0}$ |
| $\tilde{G} h^{0} A^{0}$ | 3 | $\tilde{G}\left(A^{0}, Z^{0}\right), h^{0} \tilde{\chi}_{k}^{0}$ | - |
| $\tilde{G} H^{0} A^{0}$ | 3 | $\tilde{G}\left(A^{0}, Z^{0}\right), H^{0} \tilde{\chi}_{k}^{0}$ | - |
| $\tilde{G} H^{+} H^{-}$ | 6 | $\tilde{G}\left(h^{0}, H^{0}, \gamma, Z^{0}\right), H^{+} \tilde{\chi}_{j}^{-}, \tilde{\chi}_{j}^{+} H^{-}$ | $H^{0}$ |

Table 1: All possible three-body decays channels of the NLSP neutralino $\widetilde{\chi}_{1}^{0}$ decaying into the LSP gravitino $\tilde{G}$ and a pair of SM particles; $4 p t$ denotes a Feynman graph with four-point interaction. The indices are $i=1, \ldots, 4 ; k=2,3,4 ; j, l=1,2$, and $f=\nu_{e}, \nu_{\mu}, \nu_{\tau}, e^{-}, \mu^{-}, \tau^{-}, u, c, t, d, s, b$.
decays of $\tilde{G}$ to a neutralino and a pair of two particles. In the case the gravitino is the LSP all two- and three-body decays of the $\tilde{\chi}_{1}^{0}, \tilde{\tau}_{1}$ or $\tilde{t}_{1}$ NLSP are included. GravitinoPack works at the Fortran level and has a Mathematica interface. All two-body decay widths with a gravitino were cross-checked with the analytic results of the package FeynRules [19], version 2.0.23. We have found agreement in all channels. In GravitinoPack the input can also be given in SUSY Les Houches Accord convention [20, 21]. For convenience we have directly included the code of the

| process <br> $\tilde{\tau}_{1} \rightarrow \tilde{G} X Y$ | number <br> of graphs | first decay <br> $\tilde{\tau}_{1} \rightarrow \tilde{X} Y$ |
| :--- | :---: | :--- |
| $\tilde{G} Z^{0} \tau$ | $3+4 p t$ | $\tilde{G} \tau, \tilde{\tau}_{i} Z^{0}, \tilde{\chi}_{k}^{0} \tau$ |
| $\tilde{G} W^{-} \nu_{\tau}$ | $3+4 p t$ | $\tilde{G} \tau, \tilde{\nu}_{\tau} W^{-}, \tilde{\chi}_{j}^{-} \nu_{\tau}$ |
| $\tilde{G} h^{0} \tau$ | 3 | $\tilde{G} \tau, \tilde{\tau}_{i} h^{0}, \tilde{\chi}_{k}^{0} \tau$ |
| $\tilde{G} H^{0} \tau$ | 3 | $\tilde{G} \tau, \tilde{\tau}_{i} H^{0}, \tilde{\chi}_{k}^{0} \tau$ |
| $\tilde{G} A^{0} \tau$ | 3 | $\tilde{G} \tau, \tilde{\tau}_{i} A^{0}, \tilde{\chi}_{k}^{0} \tau$ |
| $\tilde{G} H^{-} \nu_{\tau}$ | 3 | $\tilde{G} \tau, \tilde{\nu}_{\tau} H^{-}, \tilde{\chi}_{j}^{-} \nu_{\tau}$ |

Table 2: All possible three-body decays channels of the NLSP stau $\tilde{\tau}_{1}$ decaying into the LSP gravitino $\tilde{G}$ and a pair of SM particles; $4 p t$ denotes a Feynman graph with four-point interaction. The indices are $i, j=1,2$ and $k=1,2,3,4$. There are no resonances possible.

| process <br> $\tilde{t}_{1} \rightarrow \tilde{G} X Y$ | number <br> of graphs | first decay <br> $\tilde{t}_{1} \rightarrow \tilde{X} Y$ | possible <br> resonances |
| :--- | :---: | :--- | :--- |
| $\tilde{G} Z^{0} t$ | $3+4 p t$ | $\tilde{G} t, \tilde{t}_{i} Z^{0}, \tilde{\chi}_{k}^{0} t$ | $\tilde{\chi}_{k}^{0}$ |
| $\tilde{G} W^{+} b$ | $3+4 p t$ | $\tilde{G} t, \tilde{b}_{i} W^{+}, \tilde{\chi}_{j}^{+} b$ | $t, \tilde{b}_{i}, \tilde{\chi}_{j}^{+}$ |
| $\tilde{G} h^{0} t$ | 3 | $\tilde{G} t, \tilde{t}_{i} h^{0}, \tilde{\chi}_{k}^{0} t$ | $\tilde{\chi}_{k}^{0}$ |
| $\tilde{G} H^{0} t$ | 3 | $\tilde{G} t, \tilde{t}_{i} H^{0}, \tilde{\chi}_{k}^{0} t$ | $\tilde{\chi}_{k}^{0}$ |
| $\tilde{G} A^{0} t$ | 3 | $\tilde{G} t, \tilde{t}_{i} A^{0}, \tilde{\chi}_{k}^{0} t$ | $\tilde{\chi}_{k}^{0}$ |
| $\tilde{G} H^{+} b$ | 3 | $\tilde{G} t, \tilde{b}_{i} H^{+}, \tilde{\chi}_{j}^{+} b$ | $t, \tilde{b}_{i}, \tilde{\chi}_{j}^{+}$ |

Table 3: All possible three-body decays channels of the stop $\tilde{t}_{1}$ decaying into the LSP gravitino $\tilde{G}$ and a pair of SM particles; $4 p t$ denotes a Feynman graph with four-point interaction. The indices are $i, j=1,2$ and $k=1,2,3,4$.
libraries SLHALib-2.2 [22], and Cuba-3.3 [23] for the three-body phase space integrations.

### 3.1 Installation

To compile the package, a Fortran 77 compiler and the GNU C compiler (gcc) are required.

1. Download the file GravitinoPack1.0.tar.gz at
http://www.hephy.at/susytools
2. Unpack the archive by
```
ar -xvzf GravitinoPack1.0.tar.gz
```

3. Go to the GravitinoPack1.0 folder and write
./configure
which creates the makefile
4. Fortran programs with the main fortran file example1.F can be compiled by make example1
5. That will generate an executable called example1. To run it type ./example1
6. The Mathematica link program Mgravitinopack is compiled by make Mgravitinopack

### 3.2 Use of GravitinoPack

In the main directory of the package there are five examples in order to explain the functionality at the Fortran level, see the main files example1.F, example1slha.F, example2slha.F, example3slha.F, and example4slha.F. Furthermore, the file GravitinoPack.nb shows the usage at the Mathematica level, working with the MathLink executable Mgravitinopack. Executing the command make creates all six executables at once.

GravitinoPack works at tree-level with possible complex flavor conserving parameters. The input can be set locally or read in from a SLHA file. Then decay widths and branching ratios can be calculated.

First of all, one has to define a scenario. We set the SM parameters given in setSMparameters.F:

```
call setSMparameters() (Fortran)
SetSMparameters[] (Mathematica)
```

If a SLHA file is read in, this call becomes redundant.

Then the input parameters vector with 43 entries must be set, see also the code in example1.F,
input $=\{$ MAO, TB, absM1, phiM1, M2, absMUE, phiMUE, MSQ[1], MSQ[2], $\operatorname{MSQ}[3], \operatorname{MSU}[1], \operatorname{MSU}[2], \operatorname{MSU}[3], \operatorname{MSD}[1], \operatorname{MSD}[2], \operatorname{MSD}[3], \operatorname{MSL}[1], \operatorname{MSL}[2]$, MSL[3], MSE[1], MSE[2], MSE[3], absAe[1], phiAe[1], absAe[2], phiAe[2], absAe[3], phiAe[3], absAu[1], phiAu[1], absAu[2], phiAu[2], absAu[3], phiAu[3], absAd[1], phiAd[1], absAd[2], phiAd[2], absAd[3], phiAd[3], absM3, phiM3, MGr\}

Using a SLHA file, one reads in the input parameters vector by

```
call getslhapara(slhainfile, input) (Fortran)
input = GetSLHAParameters[slhainfile] (Mathematica)
```

The gravitino mass $m_{\tilde{G}}$ must be set in addition,

```
input(43) = m
input[[43]] = m
```

All parameters including SUSY masses, mixing angles and total widths of possible resonant propagators in three-body decays are calculated and some default values are set by

```
call setMSSMparameters(input, flag) (Fortran)
SetMSSMparameters[input, flag] (Mathematica)
```

with flag $=\{f l a g 1, f l a g 2\}$ where
flag1 $=1 / 0$ : print information on/off,
flag2 $=1 / 0$ : gauge unification on/off.

Working with a SLHA input file, flag2 becomes redundant, and one further can use

```
call setMSSMOSparameters() (Fortran)
SetMSSMOSparameters[] (Mathematica)
```

which reads from the slhainfile file all on-shell SUSY and Higgs masses, the rotation matrices of charginos, neutralinos, stops, sbottoms and staus, the $h^{0}-H^{0}$ mixing angle $\alpha$, and all the widths for possibly resonant propagators.

```
call setMSSMOSmasses() (Fortran)
SetMSSMOSmasses [] (Mathematica)
```

takes from the SLHA file all on-shell SUSY and Higgs masses.

The Mathematica variable SLHAfile shows the currently read LesHouches file. Further useful functions are GetSMparameters [] and GetMSSMmasses [], see the description for them directly in examples.nb.

Now we come to the functions which calculate decay widths given in GeV and branching ratios (BRs). Note, in the following A denotes the photon and HH stands for the heavy CP even Higgs $H^{0}$. All the other particle names in the code should be self-explanatory.

For the decays there are five couples of miscellaneous functions given, which all work basically on the same principle. By calling SetMSSMparameters the default values of the corresponding flags are set, see Table 4.

| flag | default value |
| :---: | :---: |
| itwobody | 1 |
| iCMS | 1 |
| ithreebody | 1 |
| ionlygamma | 0 |
| igrhel | 0 |

Table 4: SetMSSMparameters sets the default values of five flags.

```
call seti2body[i]
(Fortran)
Seti2body[i]
(Mathematica)
```

sets the flag itwobody $=1$.
itwobody = 0: analytic formulas are used,
itwobody = 1: FormCalc results are used.

```
getitwobody() (Fortran)
Geti2body[]
(Mathematica)
```

returns the actual value of itwobody.

```
call setiCMS(i)
    (Fortran)
SetiCMS[i]
(Mathematica)
```

sets the flag iCMS = i for the frame used for the three-body center-of-mass system, iCMS can be 1,2 , or 3 .

```
getiCMS() (Fortran)
GetiCMS[] (Mathematica)
```

returns the actual value of iCMS.

```
call seti3body[i]
(Fortran)
Seti3body[i] (Mathematica)
```

sets the flag ithreebody = i.
ithreebody $=0$ : full calculation, also in case of resonances
ithreebody = 1: NWA for resonances and non-resonant part
ithreebody $=2$ : narrow width approx. for resonances only
ithreebody = 3: non-resonant part only.

| getithreebody() | (Fortran) |
| :--- | :--- |
| Geti3body[] | (Mathematica) |

returns the actual value of ithreebody.

```
call setionlygamma(i) (Fortran)
Setionlygamma[i]
(Mathematica)
```

sets the flag ionlygamma = i.
ionlygamma = 0: all possible Feynman graphs are taken in decays with a $W^{+} W^{-}$pair.
ionlygamma = 1: only the Feynman graph with photon line and four-point interaction are taken in decays with a $W^{+} W^{-}$pair.

```
getionlygamma()
(Fortran)
Getionlygamma[]
(Mathematica)
```

returns the actual value of ionlygamma.

```
call setiGrHel[i] (Fortran)
SetiGrHel[i] (Mathematica)
```

sets igrhel $=i$ it affects only decays of $\tilde{G}$ and works only for itwobody $=1$. igrhel = 0: unpolarized gravitino decay
igrhel = 1: only spin $1 / 2+\operatorname{spin}-1 / 2$ contributions are taken
igrhel = 3 : only spin $3 / 2+$ spin $-3 / 2$ contributions are taken

```
getigrhel()
    (Fortran)
GetiGrHel[]
(Mathematica)
```

returns the actual value of igrhel.

Furthermore, the functions

```
IsGrtheLSP()
(Fortran)
IsGrtheLSP[]
(Mathematica)
IsNeu1theNLSP() (Fortran)
IsNeu1theNLSP[]
    (Mathematica)
IsStau1theNLSP()
(Fortran)
IsStau1theNLSP []
(Mathematica)
```

```
IsStop1theNLSP()
IsStop1theNLSP []
(Fortran)
(Mathematica)
```

are useful for the decays of $\tilde{\chi}_{1}^{0}, \tilde{\tau}_{1}$, and $\tilde{t}_{1}$ into $\tilde{G}$. If the return value is 1 , the condition is fulfilled, otherwise not.

```
call gravwidth2body(args) (Fortran)
GravWidth2body[] (Mathematica)
```

returns the branching ratios (BRs) and the total width of the gravitino decaying into all possible two-body final states, $\{\operatorname{args}\}=\{$ BR_NeuA [4], BR_NeuZ[4], BR_GluinoG, BR_ChaW[2], BR_FSF[24], BR_NeuHn[12], BR_ChaH[2], gamma2tot $\}$,
$A$ denotes the photon. The Fortran BR of the gravitino to fermion sfermion, BR_FSF(i(2),type(4), gen(3)) is mapped into BR_FSF[1, 1,1$]$, BR_FSF[2, 1,1$]$, BR_FSF[1,2,1], BR_FSF[2,2,1], ..., BR_FSF[2,4,3], type $=\{1,2,3,4\}==$ \{sneutrino, slepton, sup, sdown\} type. Similar is the mapping of BR_NeuH0: BR_Neuh0 $==$ elements $\{1,2,3,4\}$, BR_NeuHO $==$ elements $\{5,6,7,8\}$, and BR_NeuAO $==$ elements $\{9,10,11,12\}$ of BR_NeuHn. Note that for the decays into a neutralino and a pair of charged particles the charged conjugated channel is already included in the BR, e.g. BR_ChaW $==$ BR_ChapWm + BR_ChamWp.

```
GravWidth3body(idecnumber, ineu, igen) (Fortran)
GravWidth3body[idecnumber, ineu, igen] (Mathematica)
```

returns a certain gravitino three-body decay width to a neutralino and a pair of non-SUSY particles. ineu $=1,2,3,4$; for the decay to neutralino and a fermion pair also igen $=1,2,3$ must be set, otherwise it is a dummy argument. idecnumber stands for the gravitino decay to $\{\{$ NeuNNb, 1\}, $\{$ NeuLLb, 2\}, \{NeuUUb, 3\}, \{NeuDDb, 4\}, \{NeuWmWp, 5\}, $\{$ NeuWpWm, 6\}, \{NeuHpHm, 26\}, \{NeuHmHp, 27\}, \{NeuZZ, 7\}, \{NeuAZ, 8\}, \{NeuZA, 9\}, $\{$ Neuh0A, 10\}, \{NeuAh0, 11\}, \{NeuHHA, 12\}, \{NeuA0A, 13\}, \{Neuh0Z, 14\}, $\{$ NeuZh0, 15\}, \{NeuHHZ, 16\}, \{NeuAOZ, 17\}, \{NeuHpWm, 18\}, \{NeuWpHm, 19\}, \{Neuh0h0, 20\}, \{NeuHHHH, 21\}, \{NeuAOAO, 22\}, \{Neuh0HH, 23\}, \{Neuh0A0, 24\}, \{NeuHHAO, 25\}\}.

```
call gravtotalwidth(k, gam2tot, gam3tot)
GravTotalWidth[k]
(Fortran)
(Mathematica)
```

calculates the total two-body decay width gam2tot and the non-resonant three-body decay width into SM particle pairs and $\tilde{\chi}_{k}^{0}$, gam3tot, with $k=1, \ldots, 4$. For $k=0$ all four contributions are summed up in the total non-resonant three-body decay width. The Mathematica result is given as \{gam2tot, gam3tot $\}$.

```
call neu1toGrwidth2body(args) (Fortran)
Neu1toGrWidth2body[]
(Mathematica)
```

returns all possible two-body BRs and the total two-body width of $\tilde{\chi}_{1}^{0}$ decaying into $\tilde{G}$ and a SM particle, assuming that $\tilde{\chi}_{1}^{0}$ is the NLSP and Gr the LSP, $\{\operatorname{args}\}=\left\{B R \_G r A\right.$, BR_GrZ, BR_Grh0, BR_GrHO, BR_GrAO, gamma2tot $\}$.

```
Neu1toGrWidth3body(idecnumber, igen) (Fortran)
Neu1toGrWidth3body[idecnumber, igen] (Mathematica)
```

returns the width of a certain $\tilde{\chi}_{1}^{0}$ three-body decay to $\tilde{G}$ and a pair of non-SUSY particles, assuming that $\tilde{\chi}_{1}^{0}$ is the NLSP and Gr the LSP. For the decay to $\tilde{G}$ and a fermion pair igen $=1,2,3$ must be set, otherwise it is a dummy argument. idecnumber stands for the $\tilde{\chi}_{1}^{0}$ decay to $\{\{N e u 2 G r N N b, 1\},\{N e u 2 G r L L b, 2\},\{N e u 2 G r U U b, 3\},\{N e u 2 G r D D b$, 4\}, \{Neu2GrWpWm, 6\}, \{Neu2GrZZ, 7\}, \{Neu2GrZA, 9\}, \{Neu2GrZh0, 15\}, $\{$ Neu2GrZHH, 16\}, \{Neu2GrZAO, 17\}, \{Neu2GrAh0, 10\}, \{Neu2GrAHH, 12\}, $\{$ Neu2GrAAO, 13\}, \{Neu2GrWpHm, 19\}, \{Neu2Grh0h0, 20\}, \{Neu2GrHHHH, 21\}, \{Neu2GrA0A0, 22\}, \{Neu2Grh0HH, 23\}, \{Neu2Grh0A0, 24\}, \{Neu2GrHHAO, 25\}, \{Neu2GrHpHm, 26\}\}.

```
call neu1toGrtotalwidth(gam2tot, gam3tot) (Fortran)
Neu1toGrTotalWidth[] (Mathematica)
```

calculates the total two-body decay width gam2tot and the non-resonant three-body decay width into SM particle pairs of $\tilde{\chi}_{1}^{0}$, assuming that $\tilde{\chi}_{1}^{0}$ is the NLSP and Gr the LSP. The Mathematica result is given as $\{$ gam2tot, gam3tot $\}$.

```
call stau1toGrwidth2body(gamma_Grtau)
Stau1toGrTauWidth[]
```

(Fortran)
(Mathematica)
calculates the decay width of $\tilde{\tau}_{1}$ decaying into $\tilde{G} \tau$, named gamma_Grtau. Assuming that $\tilde{\tau}_{1}$ is the NLSP and $\tilde{G}$ the LSP, this is the only possible two-body decay of $\tilde{\tau}_{1}$.

```
Stau1toGrWidth3body(idecnumber) (Fortran)
Stau1toGrWidth3body[idecnumber] (Mathematica)
```

returns a certain $\tilde{\tau}_{1}$ three-body decay to $\tilde{G}$ and a pair of non-SUSY particles, assuming that $\tilde{\tau}_{1}$ is the NLSP and $\tilde{G}$ the LSP. decnumber stands for $\{\{$ Stau2GrZTau, 1\}, $\{$ Stau2GrWmNutau, 2\}, \{Stau2Grh0Tau, 3\}, \{Stau2GrHHTau, 4\}, \{Stau2GrA0Tau, 5\}, \{Stau2GrHmNutau, 6\}\}.

```
call stau1toGrtotalwidth(gam2tot, gam3tot) (Fortran)
Stau1toGrTotalWidth[] (Mathematica)
```

calculates the total two-body decay width and the non-resonant three-body decay width into SM particle pairs of $\tilde{\tau}_{1}$, assuming that $\tilde{\tau}_{1}$ is the NLSP and Gr the LSP. Note, that in this case all possible $\tilde{\tau}_{1}$ three-body decay widths are non-resonant. The Mathematica result is given as $\{$ gam2tot, gam3tot $\}$.

```
call stop1toGrwidth2body(gamma_Grtop) (Fortran)
Stop1toGrTopWidth[]
(Mathematica)
```

calculates the decay width of $\tilde{t}_{1}$ decaying into $\tilde{G} t$, named gamma_Grtop. Assuming that $\tilde{t}_{1}$ is the NLSP and $\tilde{G}$ the LSP, this is the only possible two-body decay of $\tilde{t}_{1}$.

```
Stop1toGrWidth3body(idecnumber)
Stop1toGrWidth3body[idecnumber]
(Fortran)
(Mathematica)
```

returns a certain $\tilde{t}_{1}$ three-body decay to $\tilde{G}$ and a pair of non-SUSY particles, assuming that $\tilde{t}_{1}$ is the NLSP and $\tilde{G}$ the LSP. decnumber stands for $\{\{$ Stop2GrZTop, 1\}, \{Stop2GrWpBottom, 2\}, \{Stop2Grh0Top, 3\}, \{Stop2GrHHTop, 4\}, \{Stop2GrAOTop, 5\}, \{Stop2GrHpBottom, 6\}\}.

```
call stop1toGrtotalwidth(gam2tot, gam3tot) (Fortran)
Stop1toGrTotalWidth[]
(Mathematica)
```

calculates the total two-body decay width and the non-resonant three-body decay width into SM particle pairs of $\tilde{t}_{1}$, assuming that $\tilde{t}_{1}$ is the NLSP and Gr the LSP. Note, that in this case phenomenologically only the top propagator in the channels with $W^{+}$or $H^{+}$ can become resonant. The Mathematica result is given as $\{$ gam2tot, gam3tot $\}$.

## 4 Numerical results

The unstable gravitino case, using GravitinoPack, has been discussed in [5]. On the other hand, if the gravitino $\widetilde{G}$ is the stable LSP it can play the role of the DM particle. This scenario is called gravitino DM model (GDM). In this case other sparticles that play the role of the NLSP as the neutralino, stau and stop can decay to $\widetilde{G}$ and Standard Model particles. The details have been already discussed in the introduction. As basis for the numerical analysis we will use the CMSSM and pMSSM supersymmetric models, assuming that the gravitino is stable.

Usually in the GDM based on CMSSM (GDM/CMSSM) models the NLSP is either the lightest neutralino or the stau. There is also a narrow part of the parameter space, especially for large value of the trilinear couplings $A_{0}$ favoured by the Higgs mass, where the NLSP can be the lightest stop. These cases are provided as options in the GravitionoPack 1.0. In the general pMSSM there are many more possibilities for NLSP. In the present analysis we use stau and stop as representative examples for a slepton and squark, in particular including non-trivial mixing effects in the mass eigenstates.

We start discussing the neutralino NLSP case in the context of the GDM/CMSSM. For this particular case we have chosen a benchmark point with the CMSSM parameters $m_{0}=$ $1600, M_{1 / 2}=5000, A_{0}=-4000 \mathrm{GeV}$, and $\tan \beta=10$. The mass of $\tilde{\chi}_{1}^{0}$ is 2282 GeV . The benchmark points we study in this section are compatible with the cosmological- [24-26] and LHC constraints (Higgs mass $\simeq 126 \mathrm{GeV}$, LHCb bounds for rare decays etc.) [27,28]. It is worth mentioning that the gravitino DM relic density and the NLSP relic density are related by

$$
\begin{equation*}
\frac{\Omega_{\mathrm{NLSP}}}{\Omega_{\widetilde{G}}}=\frac{m_{\mathrm{NLSP}}}{m_{\widetilde{G}}}>1 . \tag{1}
\end{equation*}
$$

The cosmological bound for the gravitino relic density can be understood as upper bound $\Omega_{\widetilde{G}} h^{2} \leq 0.12$. Therefore, one can have in addition gravitino production during reheating


Figure 1: The three-body decay widths of the neutralino NLSP decaying into the gravitino $\widetilde{G}$ and other particles, in the GDM/CMSSM scenario. The dominant channels $q \bar{q}, l \bar{l}$, $W^{+} W^{-}$, and $Z Z$ are marked in the figure; $q \bar{q}$ stands for the sum over all six quark flavors and $l \bar{l}$ for the sum over the three charged lepton and three neutrino flavors. The red dotted lines denote the two-body decay $\tilde{\chi}_{1}^{0} \rightarrow \widetilde{G} \gamma$. In the right figure we display the corresponding branching ratios for the decay channels plotted in the left figure.
after inflation, if the reheating temperature is relatively large, of the order of $\sim 10^{10} \mathrm{GeV}$.


Figure 2: The three-body decay widths of the stau NLSP decaying into the gravitino $\widetilde{G}$ and other SM particles, in the GDM/CMSSM scenario. We present the dominant two body channel $\widetilde{G} \tau$ and the three-body channels $\widetilde{G} Z \tau, \widetilde{G} W^{-} \nu_{\tau}$ and $\widetilde{G} h \tau$. In the right figure we display the corresponding branching ratios for the decay channels plotted in the left figure except $\tilde{\tau}_{1} \rightarrow \widetilde{G} \tau$.

In Figure 1 we present the corresponding decay widths (left figure) and the branching ratios (right figure) for the neutralino decays into $\widetilde{G}$ and other particles. The dominant channels $q \bar{q}, l \bar{l}, W^{+} W^{-}$, and $Z Z$ are marked in the figure; $q \bar{q}$ stands for the sum over all six quark flavors and $l \bar{l}$ for the sum over the three charged lepton and three neutrino flavors. The red dotted lines denote the two-body decay $\tilde{\chi}_{1}^{0} \rightarrow \widetilde{G} \gamma$, that dominates the
neutralino decay amplitude, as can be seen in the left panel that illustrates the branching ratios. On the other hand, the three-body decay channels $q \bar{q}$ and $l \bar{l}$ are of the order of $10 \%$, while $W^{+} W^{-}$, and $Z Z$ channels are much smaller. For this particular CMSSM point the other decay channels are even smaller. This happens because the neutralino is predominantly bino, at this particular point of the parameter space. Later we will present cases where the higgsino components of the neutralino NLSP will boost other channels.

In Figure 2 we present the decays widths (left figure) and the branching ratios (right figure) for the stau NLSP decays into the gravitino and other particles, in the GDM scenario. The CMSSM parameters are $m_{0}=1000, M_{1 / 2}=4200, A_{0}=-2500 \mathrm{GeV}$, and $\tan \beta=10$. The mass of $\tilde{\tau}_{1}$ is 1795 GeV . The dominant decay channel is the two body decay $\tilde{\tau} \rightarrow \widetilde{G} \tau$. In addition we plot the three-body channels $\widetilde{G} Z^{0} \tau, \widetilde{G} W^{-} \nu_{\tau}$, and $\widetilde{G} h^{0} \tau$. The widths of the channels involving heavier Higgs bosons in the final state, are much smaller or even zero.

Similarly in Figure 3 we present the decays widths (left figure) and the branching ratios (right figure) for the stop NLSP decays into the gravitino $\widetilde{G}$ other particles. The CMSSM parameters are $m_{0}=3000, M_{1 / 2}=1090, A_{0}=-7500 \mathrm{GeV}$ and $\tan \beta=30$. The mass of the NLSP $\tilde{t}_{1}$ is 501 GeV . As in the case of the $\tilde{\tau}$ decays we present also the three-body channels $\widetilde{G} Z^{0} t, \widetilde{G} W^{-} b$ and $\widetilde{G} h^{0} t$. Again the channels involving the heavier Higgs bosons are negligible. The dominant decay channel is the two body decay $\tilde{t}_{1} \rightarrow \widetilde{G} t$, up to the kinematical threshold $m_{\widetilde{G}}=m_{\tilde{t}_{1}}-m_{t} \sim 325 \mathrm{GeV}$. As can been seen in both plots in Figure 3, beyond this point the 2-body channel is closed and it dominates the 3 -body channel $\widetilde{G} W^{-} b$. This is clearer visible in the right plot, where for $m_{\widetilde{G}}>m_{\tilde{t}}-m_{t}$ the $\widetilde{G} W^{-} b$ decay channel grows after this point and eventually reach the maximum value one outside of the displayed region.

In addition, we will study three representative points from the pMSSM [11] supersymmetric scenario, each for the neutralino, stau and stop NLSP case as before. In the pMSSM model we have relaxed the unification conditions for the soft parameters at the GUT scale and we used the values given in Table 5.

In Fig. 4 we present the neutralino NLSP point, in the pMSSM. We have chosen the soft SUSY parameters for this particular point in such a way that the NLSP is predominantly higgsino, that is $\tilde{\chi}_{1}^{0}=0.263 \tilde{B}-0.210 \tilde{W}^{3}+0.673 \tilde{H}_{1}^{0}-0.659 \tilde{H}_{2}^{0}$. For this reason we see that the 3 -body channels $q \bar{q}$ and $l \bar{l}$ that are driven by the higgsino dominant $\widetilde{G} \tilde{\chi}_{1}^{0} Z^{0}$ coupling to predominate the neutralino decay width, up to $m_{\widetilde{G}} \simeq 950 \mathrm{GeV}$ where the 2-body channel $\tilde{\chi}_{1}^{0} \rightarrow \widetilde{G} \gamma$ becomes kinematically accessible. In addition, we also present the $W^{+} W^{-}$and $Z^{0} Z^{0}$ channels that are significantly enhanced in comparison to the


Figure 3: The three-body decay widths of the stop NLSP decaying into the gravitino $\widetilde{G}$ and other SM particles, in the GDM/CMSSM scenario. We present the dominant two body channel $\widetilde{G} \tau$ and the three-body channels $\widetilde{G} Z t, \widetilde{G} W^{-} b$ and $\widetilde{G} h^{0} b$. In the right figure we display the corresponding branching ratios for the decay channels plotted in the left figure except $\tilde{t}_{1} \rightarrow \widetilde{G} t$.


Figure 4: The three-body decay widths of the neutralino decaying into the gravitino $\widetilde{G}$ and other particles, in the pMSSM scenario. The dominant channels $q \bar{q}, l \bar{l}, W$-pairs, and $Z$-pairs are marked in the figure; $q \bar{q}$ stands for the sum over all six quark flavors and $l \bar{l}$ for the sum over the three charged lepton and three neutrino flavors. The red dotted lines denote the two-body decay $\tilde{\chi}_{1}^{0} \rightarrow \widetilde{G} \gamma$. In the right figure we display the corresponding branching ratios for the decay channels plotted in the left figure.
corresponding CMSSM scenario presented in Figure 1, for the same reason.
On the other hand Figure 5 is similar to the corresponding one of the CMSSM case presented in Figure 2. The only difference is that in the pMSSM case the $\widetilde{G} W^{-} \nu_{\tau}$ decay dominates, while the $\widetilde{G} Z^{0} \tau$ decay is by far the most dominant three-body decay in the CMSSM stau NLSP point. This is because in the pMSSM point the $\widetilde{G} W^{-} \nu_{\tau}$ channel is enlarged due to the sizeable $\tilde{\tau}_{1} \tilde{\chi}_{1}^{+} \nu_{\tau}$ coupling.

| Parameters | $\tilde{\chi}_{1}^{0}$ decay | $\tilde{\tau}_{1}$ decay | $\tilde{t}_{1}$ decay |
| :---: | :---: | :---: | :---: |
| $\tan \beta=\left\langle H_{2}^{0}\right\rangle /\left\langle H_{1}^{0}\right\rangle$ | 40 | 20 | 30 |
| $\mu$, higgsino mixing parameter | 1 | 1.5 | 2 |
| $M_{A}, A^{0}$ Higgs boson mass | 2.2 | 1.5 | 2 |
| $\left(M_{1}, M_{2}, M_{3}\right)$, gauginos masses | $(1.1,1.2,2.8)$ | $(2,3,7)$ | $(1,2,2.5)$ |
| $A_{t}$, top trilinear coupling | -4.3 | -3 | -4.4 |
| $A_{b}$, bottom trilinear coupling | -6.3 | -3 | -8 |
| $A_{\tau}$, tau trilinear coupling | -2.8 | -3 | -6.7 |
| $m_{\tilde{q}_{L}}, 1^{\text {st }} / 2^{\text {nd }}$ family $Q_{L}$ squark mass | 2 | 3 | 3.6 |
| $m_{\widetilde{u}_{R}}, 1^{\text {st }} / 2^{\text {nd }}$ family $U_{R}$ squark | 4 | 3 | 3.6 |
| $m_{\widetilde{d}_{R}}, 1^{\text {st }} / 2^{\text {nd }}$ family $D_{R}$ squark | 4 | 3 | 3.6 |
| $m_{\tilde{\ell}_{L}}, 1^{\text {st }} / 2^{\text {nd }}$ family $L_{L}$ slepton | 2 | 2 | 3 |
| $m_{\widetilde{e}_{R}}, 1^{\text {st }} / 2^{\text {nd }}$ family $E_{R}$ slepton | 4 | 2 | 3 |
| $m_{\widetilde{Q}_{3 L}}, 3^{\text {rd }}$ family $Q_{L}$ squark | 3.5 | 7 | 2.3 |
| $m_{\tilde{t}_{R}}, 3^{\text {rd }}$ family $U_{R}$ squark | 3.5 | 7 | 1 |
| $m_{\tilde{b}_{R}}, 3^{\text {rd }}$ family $D_{R}$ squark | 3.5 | 7 | 3 |
| $m_{\widetilde{L}_{3 L}}, 3^{\text {rd }}$ family $L_{L}$ slepton | 1.25 | 1.2 | 2.7 |
| $m_{\tilde{\tau}_{R}}, 3^{\text {rd }}$ family $E_{R}$ slepton | 3.5 | 1.2 | 2.2 |

Table 5: The pMSSM parameters used as input for the three scenarios in our analysis. All values but $\tan \beta$ are given in TeV .

Finally, in Figure 6 we present the stop NLSP point in the pMSSM case. For this point the $\tilde{t}_{1}$ NLSP mass is about 830 GeV . As in the corresponding CMSSM case the 2-body channel $\tilde{t} \rightarrow \widetilde{G} t$ dominates up to the kinematical threshold $m_{\widetilde{G}}=m_{\tilde{t}}-m_{t}$. After this value of the gravitino mass the 2-body channel is closed and the 3 -body $\widetilde{G} W^{-} b$ channels takes over, as has happened in the CMSSM case.

To summarise the representative cases both in CMSSM and pMSSM, we can see that the full knowledge of all the two- and three-body decay channels of the NLSP unstable particle is essential for the precise calculation of the decay width and the various branching ratios. This is actually the big advantage of using the GravitinoPack, since it gives all computed results both in Fortran and within the Mathematica environment. It also supports SLHA input format. These results enable the user of the package to apply precisely the relevant cosmological constraints, especially those related to the BBN


Figure 5: The three-body decay widths of the stau NLSP decaying into the gravitino $\widetilde{G}$ and other SM particles, in the pMSSM scenario. We present the dominant two body channel $\widetilde{G} \tau$ and the three-body channels $\widetilde{G} Z^{0} \tau, \widetilde{G} W^{-} \nu_{\tau}$ and $\widetilde{G} h^{0} \tau$. In the right figure we display the corresponding branching ratios for the decay channels plotted in the left figure except $\tilde{\tau}_{1} \rightarrow \widetilde{G} \tau$.


Figure 6: The three-body decay widths of the stop NLSP decaying into the gravitino $\widetilde{G}$ and other SM particles, in the pMSSM scenario. We present the dominant two body channel $\widetilde{G} \tau$ and the three-body channels $\widetilde{G} Z t, \widetilde{G} W^{-} b$ and $\widetilde{G} h^{0} b$. In the right figure we display the corresponding branching ratios for the decay channels plotted in the left figure except $\tilde{t}_{1} \rightarrow \widetilde{G} t$.
predictions, to various supersymmetric models.

## 5 Summary

We have studied supersymmetric models, where the gravitino is the lightest supersymmetric particle (LSP). In this case the gravitino can play the role of the dark matter particle. The Next to Lightest Supersymmetric Particle (NLSP) can be either the lightest
neutralino $\tilde{\chi}_{1}^{0}$ or a sfermion, as stau $\tilde{\tau}_{1}$ or stop $\tilde{t}_{1}$. These three cases have been discussed in this work using GravitinoPack. Although these cases can usually be found in the CMSSM parameter space, they are also representative for a slepton, squark and a gaugino NLSP in a more general supersymmetric scenario like in the pMSSM.

We have calculated all two- and three-body decays of the NLSP neutralino, stau, and stop to the gravitino LSP and one or two SM particles. The products of these decays carry electromagnetic energy and can build hadrons that influence the predictions of BBN, since the gravitational nature of these decays place them in this time scale. Therefore, the detailed knowledge of the relevant branching ratios and decay widths is important to study and constrain various supergravity models.

To facilitate the application of BBN constraints, we have developed the public available computer tool GravitinoPack. This numerical package based on an autogenerated Fortran 77 code, calculates the branching ratios and decay widths for the NSLP decays, if the gravitino is stable and the DM particle. On the other hand we have already presented in [5] the complementary case, where the gravitino is unstable and can decay into a neutralino and SM particles. Moreover, we have provided all relevant technical details for its use. GravitinoPack can be used directly at the Fortran level or more conveniently with Mathematica via MathLink functions.

As in the case of the decays of the unstable gravitino, the three-body decays can be important in the case of the unstable neutralino, stau or stop NLSP. In Section 4 we have seen this feature especially in the region below the kinematical threshold of the subleading two-body decays. Thus, GravitinoPack provides important results on the decays of unstable NLSP's or gravitino, making the application of the BBN data more predictive.

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## A Gravitino Interactions with the MSSM

We start with the relevant supergravity Lagrangian [29].

$$
\begin{align*}
\mathcal{L}_{\widetilde{G}, \text { int }}^{(\alpha)} & =-\frac{i}{\sqrt{2} M_{\mathrm{P}}}\left[\mathcal{D}_{\mu}^{(\alpha)} \phi^{* i} \overline{\widetilde{G}}_{\nu} \gamma^{\mu} \gamma^{\nu} \chi_{L}^{i}-\mathcal{D}_{\mu}^{(\alpha)} \phi^{i} \bar{\chi}_{L}^{i} \gamma^{\nu} \gamma^{\mu} \widetilde{G}_{\nu}\right] \\
& -\frac{i}{8 M_{\mathrm{P}}} \overline{\widetilde{G}}_{\mu}\left[\gamma^{\rho}, \gamma^{\sigma}\right] \gamma^{\mu} \lambda^{(\alpha) a} F_{\rho \sigma}^{(\alpha) a}, \tag{2}
\end{align*}
$$

with the covariant derivative given by

$$
\begin{equation*}
\mathcal{D}_{\mu}^{(\alpha)} \phi^{i}=\partial_{\mu} \phi^{i}+i g_{\alpha} A_{\mu}^{(\alpha) a} T_{a, i j}^{(\alpha)} \phi^{j}, \tag{3}
\end{equation*}
$$

and the field strength tensor $F_{\mu \nu}^{(\alpha) a}$ reads

$$
\begin{equation*}
F_{\mu \nu}^{(\alpha) a}=\partial_{\mu} A_{\nu}^{(\alpha) a}-\partial_{\nu} A_{\mu}^{(\alpha) a}-g_{\alpha} f^{(\alpha) a b c} A_{\mu}^{(\alpha) b} A_{\nu}^{(\alpha) c} . \tag{4}
\end{equation*}
$$

The index $\alpha$ corresponds to the three groups $\mathrm{U}(1)_{Y}, \mathrm{SU}(2)_{I}$, and $\mathrm{SU}(3)_{c}$ with $a=1,3,8$ and $i=1,2,3$, respectively.

In detail, we get for the three covariant derivatives

$$
\begin{align*}
\mathcal{D}_{\mu}^{(1)} \phi & =\partial_{\mu} \phi+i \frac{g_{1}}{2} B_{\mu} Y(\phi) \phi \\
\mathcal{D}_{\mu}^{(2)}\binom{\phi^{1}}{\phi^{2}} & =\left(\left(\begin{array}{cc}
\partial_{\mu} & 0 \\
0 & \partial_{\mu}
\end{array}\right)+i \frac{g_{2}}{2}\left(\begin{array}{cc}
s_{W} A_{\mu}+c_{W} Z_{\mu} & \sqrt{2} W_{\mu}^{+} \\
\sqrt{2} W_{\mu}^{-} & -s_{W} A_{\mu}-c_{W} Z_{\mu}
\end{array}\right)\right)\binom{\phi^{1}}{\phi^{2}}, \\
\mathcal{D}_{\mu}^{(3)} \phi^{r} & =\partial_{\mu} \phi^{r}+i g_{s} G_{\mu}^{a} T_{r s}^{a} \phi^{s} \tag{5}
\end{align*}
$$

with $g_{s}$ the strong coupling, $r, s$ are color indices, and $T_{r s}^{a}=\lambda_{r s}^{a} / 2, \lambda_{r s}^{a}$ are the $8(3 \times 3)$ Gell-Mann matrices. We already substituted $B_{\mu}=c_{W} A_{\mu}-s_{W} Z_{\mu}, W_{\mu}^{3}=s_{W} A_{\mu}+c_{W} Z_{\mu}$, $W_{\mu}^{1}=\left(W_{\mu}^{+}+W_{\mu}^{-}\right) / \sqrt{2}, W_{\mu}^{2}=i\left(W_{\mu}^{+}-W_{\mu}^{-}\right) / \sqrt{2}$.
The three field strength tensors read

$$
\begin{align*}
F_{\mu \nu}^{(1)} & =\partial_{\mu} B_{\nu}-\partial_{\nu} B_{\mu},  \tag{6}\\
F_{\mu \nu}^{(2) a} & =\partial_{\mu} W_{\nu}^{a}-\partial_{\nu} W_{\mu}^{a}-g_{2} \epsilon^{a b c} W_{\mu}^{b} W_{\nu}^{c},  \tag{7}\\
F_{\mu \nu}^{(3) a} & =\partial_{\mu} G_{\nu}^{a}-\partial_{\nu} G_{\mu}^{a}-g_{s} f^{a b c} G_{\mu}^{b} G_{\nu}^{c}, \tag{8}
\end{align*}
$$

We fix $\epsilon^{123}=1, f^{a b c}$ is the structure constant of $\mathrm{SU}(3)$.
We have to fill in all the gauge fields, see Table 6, and matter fields, see Table 7, of the MSSM into (2).

| Name | Gauge bosons $A_{\mu}^{(\alpha) a}$ | Gauginos $\lambda^{(\alpha) a}$ | $\left(\mathrm{SU}(3)_{\mathrm{c}}, \mathrm{SU}(2)_{\mathrm{L}}\right)_{Y}$ |
| :--- | :---: | :---: | :---: |
| B-boson, bino | $A_{\mu}^{(1) a}=B_{\mu} \delta^{a 1}$ | $\lambda^{(1) a}=\widetilde{B} \delta^{a 1}$ | $(\mathbf{1}, \mathbf{1})_{0}$ |
| W-bosons, winos | $A_{\mu}^{(2) a}=W_{\mu}^{a}$ | $\lambda^{(2) a}=\widetilde{W}^{a}$ | $(\mathbf{1}, \mathbf{3})_{0}$ |
| gluon, gluino | $A_{\mu}^{(3) a}=G_{\mu}^{a}$ | $\lambda^{(3) a}=\widetilde{g}^{a}$ | $(\mathbf{8}, \mathbf{1})_{0}$ |

Table 6: Gauge fields of the MSSM

We start with the gravitino interaction with the Higgs bosons and higgsinos together with the $\mathrm{SU}(2)$ and $\mathrm{U}(1)$ gauginos. First of all, the Lagrangian (2) is hermitian. This means, $\left(i \mathcal{D}_{\mu}^{(\alpha)} \phi^{i} \bar{\chi}_{L}^{i} \gamma^{\nu} \gamma^{\mu} \widetilde{G}_{\nu}\right)^{\dagger}=-i \mathcal{D}_{\mu}^{(\alpha)} \phi^{* i} \overline{\widetilde{G}}_{\nu} \gamma^{\mu} \gamma^{\nu} \chi_{L}^{i}$, and the second line in (2) is a real quantity. This gives $\mathcal{D}_{\mu}^{(\alpha)} \phi^{* i}=\left(\mathcal{D}_{\mu}^{(\alpha)} \phi^{i}\right)^{\dagger}$, which is true because $\mathcal{D}_{\mu}^{(\alpha)}$ is an operator which acts on $\phi^{i}$ e.g. under $\mathrm{SU}(2)$, but it acts on $\phi^{* i}$ under $\overline{\mathrm{SU}(2)}$. So we can calculate first the second term of (2) and by hermitian conjugation we get the first one. We need all the insertions for $\phi^{i}, \chi_{L}^{i}$, and $\lambda^{\prime}$ 's. The Higgs fields are $\left(H_{d}^{0}, H_{d}^{-}\right)$, and $\left(H_{u}^{+}, H_{u}^{0}\right)$ with the transformations to physical states,

$$
\begin{align*}
H_{1}^{1} \equiv H_{d}^{0} & =\frac{1}{\sqrt{2}}\left(v_{1}+c_{\alpha} H^{0}-s_{\alpha} h^{0}+i\left(-c_{\beta} G^{0}+s_{\beta} A^{0}\right)\right.  \tag{9}\\
H_{1}^{2} \equiv H_{d}^{-} & =-c_{\beta} G^{-}+s_{\beta} H^{-}  \tag{10}\\
H_{2}^{1} \equiv H_{u}^{+} & =s_{\beta} G^{+}+c_{\beta} H^{+},  \tag{11}\\
H_{2}^{2} \equiv H_{u}^{0} & =\frac{1}{\sqrt{2}}\left(v_{2}+s_{\alpha} H^{0}+c_{\alpha} h^{0}+i\left(s_{\beta} G^{0}+c_{\beta} A^{0}\right),\right. \tag{12}
\end{align*}
$$

$v_{1}=v c_{\beta}$ and $v_{2}=v s_{\beta}$ using the SM convention $v=2 m_{W} / g_{2}$. The Higgs superpartners, called higgsinos are left-handed by definition and transform to charginos and neutralinos as

$$
\begin{align*}
\tilde{H}_{d}^{0} & =Z_{k, 3}^{*} P_{L} \tilde{\chi}_{k}^{0} \\
\tilde{H}_{d}^{-} & =U_{j, 2}^{*} P_{L} \tilde{\chi}_{j}^{-}, \\
\tilde{H}_{u}^{+} & =V_{j, 2}^{*} P_{L} \tilde{\chi}_{j}^{+} \\
\tilde{H}_{u}^{0} & =Z_{k, 4}^{*} P_{L} \tilde{\chi}_{k}^{0}, \tag{13}
\end{align*}
$$

| Name | Bosons $\phi^{i}$ | Fermions $\chi_{L}^{i}$ | $\left(\mathrm{SU}(3)_{\mathrm{c}}, \mathrm{SU}(2)_{\mathrm{L}}\right)_{Y}$ |
| :--- | :--- | :--- | :--- |
| Sleptons, leptons <br> $I=1,2,3$ | $\widetilde{L}^{I}=\binom{\widetilde{\nu}_{L}^{I}}{\widetilde{e}_{L}^{-I}}$ | $L^{I}=\binom{\nu_{L}^{I}}{e_{L}^{-I}}$ | $(\mathbf{1}, \mathbf{2})_{-1}$ |
|  | $\widetilde{E}^{* I}=\widetilde{e}_{R}^{-* I}$ | $E^{c I}=e_{R}^{-c I}$ | $(\mathbf{1}, \mathbf{1})_{+2}$ |
| Squarks, quarks <br> $I=1,2,3$ <br> $(\times 3$ colors $)$ | $\widetilde{Q}^{I}=\binom{\widetilde{u}_{L}^{I}}{\widetilde{d}_{L}^{I}}$ | $\widetilde{U}^{I}=\binom{u_{L}^{I}}{d_{L}^{I}}$ | $(\mathbf{3}, \mathbf{2})_{+\frac{1}{3}}$ |
|  | $\widetilde{D}^{* I}=\widetilde{u}_{R}^{* I}=\widetilde{d}_{R}^{* I}$ | $U^{c I}=u_{R}^{c I}$ | $\left(\overline{\mathbf{3}, \mathbf{1})_{-\frac{4}{3}}}\right.$ |
| Higgs, higgsinos | $H_{d}=\binom{H_{d}^{0}}{H_{d}^{-}}$ | $\widetilde{H}_{d}=\binom{\widetilde{H}_{d}^{0}}{\widetilde{H}_{d}^{-}}$ | $(\overline{\mathbf{3}}, \mathbf{1})_{+\frac{2}{3}}$ |
|  | $H_{u}=\binom{H_{u}^{+}}{H_{u}^{0}}$ | $\widetilde{H}_{u}=\binom{\widetilde{H}_{u}^{+}}{\widetilde{H}_{u}^{0}}$ | $(\mathbf{1}, \mathbf{2})_{+1}$ |

Table 7: Matter fields of the MSSM
and the $\mathrm{U}(1)$ and $\mathrm{SU}(2)$ gauginos, which have left and right spin components, follow

$$
\begin{align*}
\lambda^{B} & =Z_{k, 1}^{*} P_{L} \tilde{\chi}_{k}^{0}+Z_{k, 1} P_{R} \tilde{\chi}_{k}^{0} \\
\lambda^{+} & =V_{j, 1}^{*} P_{L} \tilde{\chi}_{j}^{+}+U_{j, 1} P_{R} \tilde{\chi}_{j}^{+} \\
\lambda^{-} & =U_{j, 1}^{*} P_{L} \tilde{\chi}_{j}^{-}+V_{j, 1} P_{R} \tilde{\chi}_{j}^{-} \\
\lambda^{3} & =Z_{k, 2}^{*} P_{L} \tilde{\chi}_{k}^{0}+Z_{k, 2} P_{R} \tilde{\chi}_{k}^{0} \tag{14}
\end{align*}
$$

For the second term of (2) we need $\bar{\chi}_{L}$. For next steps we need the formulas

$$
\begin{array}{cll}
\gamma^{0} \gamma^{\mu \dagger} \gamma^{0}=\gamma^{\mu} & , & \gamma^{0} P_{L, R}^{\dagger} \gamma^{0}=P_{R, L} \\
C \gamma^{\mu T} C^{-1}=-\gamma^{\mu} & , \quad C P_{L, R}^{T} C^{-1}=P_{L, R} \tag{16}
\end{array}
$$

with $\gamma^{0} \gamma^{0}=1$, and for the charge conjugate matrix it holds $C^{-1}=-C=C^{T}$. Applying (15) we derive $\chi_{L}=P_{L} \chi, \overline{\chi_{L}}=\left(P_{L} \chi\right)^{\dagger} \gamma^{0}=\chi^{\dagger} P_{L}^{\dagger} \gamma^{0}=\chi^{\dagger} \gamma^{0} \gamma^{0} P_{L}^{\dagger} \gamma^{0}=\bar{\chi} P_{R}$. We get from
(13) and (14)

$$
\begin{array}{ll}
\overline{\tilde{H}_{d}^{0}}=Z_{k, 3} \overline{\tilde{\chi}_{k}^{0}} P_{R}, & \overline{\lambda^{B}}=Z_{k, 1} \overline{\tilde{\chi}_{k}^{0}} P_{R}+Z_{k, 1}^{*} \overline{\tilde{\chi}_{k}^{0}} P_{L}, \\
\overline{\tilde{H}_{d}^{-}}=U_{j, 2} \overline{\tilde{\chi}_{j}^{-}} P_{R}, & \overline{\lambda^{-}}=U_{j, 1} \overline{\tilde{\chi}_{j}^{-}} P_{R}+V_{j, 1}^{*} \tilde{\chi}_{j}^{-} P_{L}, \\
\overline{\tilde{H}_{u}^{+}}=V_{j, 2} \overline{\tilde{\chi}_{j}^{+}} P_{R}, & \overline{\lambda^{+}}=V_{j, 1} \overline{\tilde{\chi}_{j}^{+}} P_{R}+U_{j, 1}^{*} \overline{\tilde{\chi}_{j}^{+}} P_{L}, \\
\overline{\tilde{H}_{u}^{0}}=Z_{k, 4} \overline{\tilde{\chi}_{k}^{0}} P_{R}, & \overline{\lambda^{3}}=Z_{k, 2} \overline{\tilde{\chi}_{k}^{0}} P_{R}+Z_{k, 2}^{*} \overline{\tilde{\chi}_{k}^{0}} P_{L} \tag{17}
\end{array}
$$

For the second term for the doublet $\left(H_{d}^{0}, H_{d}^{-}\right)$we have

$$
\begin{align*}
\mathcal{L}_{2} \sim \frac{1}{\sqrt{2} M_{\mathrm{P}}} & {\left[\left(i \partial_{\mu} H_{d}^{0}-\frac{1}{2}\left(g_{1} Y\left(H_{d}\right) B_{\mu}+g_{2} W_{\mu}^{3}\right) H_{d}^{0}-\frac{1}{\sqrt{2}} g_{2} W_{\mu}^{+} H_{d}^{-}\right) \overline{H_{d}^{0}} \gamma^{\nu} \gamma^{\mu} \widetilde{G}_{\nu}+\right.} \\
& \left.\left(i \partial_{\mu} H_{d}^{-}-\frac{1}{2}\left(g_{1} Y\left(H_{d}\right) B_{\mu}-g_{2} W_{\mu}^{3}\right) H_{d}^{-}-\frac{1}{\sqrt{2}} g_{2} W_{\mu}^{-} H_{d}^{0}\right) \overline{H_{d}^{-}} \gamma^{\nu} \gamma^{\mu} \widetilde{G}_{\nu}\right] \tag{18}
\end{align*}
$$

As already mentioned, the first term we get by hermitian conjugation,

$$
\begin{align*}
\mathcal{L}_{1} \sim \frac{1}{\sqrt{2} M_{\mathrm{P}}} & {\left[\left(-i \partial_{\mu} H_{d}^{0 *}-\frac{1}{2}\left(g_{1} Y\left(H_{d}\right) B_{\mu}+g_{2} W_{\mu}^{3}\right) H_{d}^{0 *}-\frac{1}{\sqrt{2}} g_{2} W_{\mu}^{-} H_{d}^{+}\right) \overline{\widetilde{G}}_{\nu} \gamma^{\mu} \gamma^{\nu} \tilde{H}_{d}^{0}+\right.} \\
& \left.\left(-i \partial_{\mu} H_{d}^{+}-\frac{1}{2}\left(g_{1} Y\left(H_{d}\right) B_{\mu}-g_{2} W_{\mu}^{3}\right) H_{d}^{+}-\frac{1}{\sqrt{2}} g_{2} W_{\mu}^{+} H_{d}^{0 *}\right) \overline{\widetilde{G}}_{\nu} \gamma^{\mu} \gamma^{\nu} \tilde{H}_{d}^{-}\right] \tag{19}
\end{align*}
$$

The terms for the doublet $\left(H_{u}^{+}, H_{u}^{0}\right)$ we simply get by $H_{d}^{0} \rightarrow H_{u}^{+}, H_{d}^{-} \rightarrow H_{u}^{0}$ and $Y\left(H_{d}\right) \rightarrow$ $Y\left(H_{u}\right)$.
The third term is a product of two antisymmetric tensors in two Lorentz indices. Thus, we can simplify them,

$$
\begin{align*}
{\left[\gamma^{\rho}, \gamma^{\sigma}\right] F_{\rho \sigma}^{(1)} } & =2\left[\gamma^{\rho}, \gamma^{\sigma}\right] \partial_{\rho} B_{\sigma}  \tag{20}\\
{\left[\gamma^{\rho}, \gamma^{\sigma}\right] F_{\rho \sigma}^{(2) 1} } & =2\left[\gamma^{\rho}, \gamma^{\sigma}\right]\left(\partial_{\rho} W_{\sigma}^{1}-g_{2} W_{\rho}^{2} W_{\sigma}^{3}\right) \\
{\left[\gamma^{\rho}, \gamma^{\sigma}\right] F_{\rho \sigma}^{(2) 2} } & =2\left[\gamma^{\rho}, \gamma^{\sigma}\right]\left(\partial_{\rho} W_{\sigma}^{2}+g_{2} W_{\rho}^{1} W_{\sigma}^{3}\right) \\
{\left[\gamma^{\rho}, \gamma^{\sigma}\right] F_{\rho \sigma}^{(2) 3} } & =2\left[\gamma^{\rho}, \gamma^{\sigma}\right]\left(\partial_{\rho} W_{\sigma}^{3}-g_{2} W_{\rho}^{1} W_{\sigma}^{2}\right) \tag{21}
\end{align*}
$$

The gaugino superpartner transform analogously to the vector fields. We need $\lambda^{1}=$ $\left(\lambda^{+}+\lambda^{-}\right) / \sqrt{2}, \lambda^{2}=i\left(\lambda^{+}-\lambda^{-}\right) / \sqrt{2}$. Having inserted the rules for $W^{1,2}$ and $\lambda^{1,2}$, the third term reads

$$
\begin{align*}
\mathcal{L}_{3} \sim-\frac{i}{4 M_{\mathrm{P}}} \overline{\widetilde{G}}_{\mu}\left[\gamma^{\rho}, \gamma^{\sigma}\right] \gamma^{\mu}[ & \lambda^{B} \partial_{\rho} B_{\sigma}+\lambda^{+} \partial_{\rho} W_{\sigma}^{-}+\lambda^{-} \partial_{\rho} W_{\sigma}^{+}+\lambda^{3} \partial_{\rho} W_{\sigma}^{3} \\
& \left.-i g_{2} \lambda^{+} W_{\rho}^{3} W_{\sigma}^{-}+i g_{2} \lambda^{-} W_{\rho}^{3} W_{\sigma}^{+}-i g_{2} \lambda^{3} W_{\rho}^{-} W_{\sigma}^{+}\right] \tag{22}
\end{align*}
$$

For the Feynman rules one also needs the third term in a different form. We know that $F_{\rho \sigma}^{(\alpha) a}$ is real. Thus, it holds: $i \widetilde{\widetilde{G}}_{\mu}\left[\gamma^{\rho}, \gamma^{\sigma}\right] \gamma^{\mu} \lambda=\left(i \overline{\widetilde{G}}_{\mu}\left[\gamma^{\rho}, \gamma^{\sigma}\right] \gamma^{\mu} \lambda\right)^{\dagger}=-i \lambda^{\dagger} \gamma^{\mu \dagger}\left[\gamma^{\rho}, \gamma^{\sigma}\right]^{\dagger} \overline{\widetilde{G}}_{\mu}^{\dagger}=$ $i \bar{\lambda} \gamma^{\mu}\left[\gamma^{\rho}, \gamma^{\sigma}\right] \widetilde{G}_{\mu}$ and we get the second form

$$
\begin{align*}
\mathcal{L}_{3} \sim-\frac{i}{4 M_{\mathrm{P}}}[ & \partial_{\rho} B_{\sigma} \bar{\lambda}^{B}+\partial_{\rho} W_{\sigma}^{+} \bar{\lambda}^{+}+\partial_{\rho} W_{\sigma}^{-} \bar{\lambda}^{-}+\partial_{\rho} W_{\sigma}^{3} \bar{\lambda}^{3} \\
& \left.+i g_{2} W_{\rho}^{3} W_{\sigma}^{+} \bar{\lambda}^{+}-i g_{2} W_{\rho}^{3} W_{\sigma}^{-} \bar{\lambda}^{-}+i g_{2} W_{\rho}^{+} W_{\sigma}^{-} \bar{\lambda}^{3}\right] \gamma^{\mu}\left[\gamma^{\rho}, \gamma^{\sigma}\right] \widetilde{G}_{\mu} \tag{23}
\end{align*}
$$

Now all formulas are given to calculate the total Lagrangian without fermion and gluino interactions.

Next we focus on the electroweak interaction with fermions and sfermions. First of all, we need the transformations from the interaction to the physical field states,

$$
\begin{equation*}
\tilde{f}_{L}=R_{i 1}^{\tilde{f} *} \tilde{f}_{i} \quad, \quad \tilde{f}_{R}=R_{i 2}^{\tilde{f} *} \tilde{f}_{i} . \tag{24}
\end{equation*}
$$

Only the first line of (2) is relevant. In the following the quark or lepton flavour indices $I(=1,2,3)$ will be suppressed. We start with the left handed doublets $\left(\tilde{u}_{L}, \tilde{d}_{L}\right)$ and their SM partners $\left(u_{L}, d_{L}\right)$. Recall, that $\psi_{L}=P_{L} \psi$ and thus $\overline{\psi_{L}}=\bar{\psi} P_{R}$. We get from (18) and (19) by appropriate substitutions

$$
\begin{align*}
\mathcal{L}_{2}=\frac{1}{\sqrt{2} M_{\mathrm{P}}} & {\left[\left(i \partial_{\mu} \tilde{u}_{L}-\frac{1}{2}\left(g_{1} Y(Q) B_{\mu}+g_{2} W_{\mu}^{3}\right) \tilde{u}_{L}-\frac{1}{\sqrt{2}} g_{2} W_{\mu}^{+} \tilde{d}_{L}\right) \bar{u} \gamma^{\nu} \gamma^{\mu} P_{R} \widetilde{G}_{\nu}+\right.} \\
& \left.\left(i \partial_{\mu} \tilde{d}_{L}-\frac{1}{2}\left(g_{1} Y(Q) B_{\mu}-g_{2} W_{\mu}^{3}\right) \tilde{d}_{L}-\frac{1}{\sqrt{2}} g_{2} W_{\mu}^{-} \tilde{u}_{L}\right) \bar{d} \gamma^{\nu} \gamma^{\mu} P_{R} \widetilde{G}_{\nu}\right], \tag{25}
\end{align*}
$$

and

$$
\begin{align*}
\mathcal{L}_{1}=\frac{1}{\sqrt{2} M_{\mathrm{P}}} & {\left[\left(-i \partial_{\mu} \tilde{u}_{L}^{*}-\frac{1}{2}\left(g_{1} Y(Q) B_{\mu}+g_{2} W_{\mu}^{3}\right) \tilde{u}_{L}^{*}-\frac{1}{\sqrt{2}} g_{2} W_{\mu}^{-} \tilde{d}_{L}^{*}\right) \overline{\widetilde{G}}_{\nu} \gamma^{\mu} \gamma^{\nu} P_{L} u+\right.} \\
& \left.\left(-i \partial_{\mu} \tilde{d}_{L}^{*}-\frac{1}{2}\left(g_{1} Y(Q) B_{\mu}-g_{2} W_{\mu}^{3}\right) \tilde{d}_{L}^{*}-\frac{1}{\sqrt{2}} g_{2} W_{\mu}^{+} \tilde{u}_{L}^{*}\right) \widetilde{\widetilde{G}}_{\nu} \gamma^{\mu} \gamma^{\nu} P_{L} d\right] .(26 \tag{26}
\end{align*}
$$

For the right handed quarks the situation is more complex, because one has to work with left handed fermion fields. We need some identities for charge conjugated spinor fields. Most important is

$$
\begin{equation*}
\psi_{R}^{c} \equiv\left(\psi_{R}\right)^{c}=\left(\psi^{c}\right)_{L}=P_{L} \psi^{c} \tag{27}
\end{equation*}
$$

With $C P_{L}^{T}=P_{L} C$ the proof is $\left(\psi_{R}\right)^{c}=C{\overline{\psi_{R}}}^{T}=C\left(\bar{\psi} P_{L}\right)^{T}=C P_{L}^{T} \bar{\psi}^{T}=P_{L} C \bar{\psi}^{T}=P_{L} \psi^{c}$. The gravitino is a Majorana particle, $\widetilde{G}_{\nu}^{c}=\widetilde{G}_{\nu}$. Thus, $\widetilde{\widetilde{G}}_{\nu} \gamma^{\mu} \gamma^{\nu} f_{R}^{c}=\bar{f} \gamma^{\nu} \gamma^{\mu} P_{L} \widetilde{G}_{\nu}$ and
$\overline{f_{R}^{c}} \gamma^{\nu} \gamma^{\mu} \widetilde{G}_{\nu}=\overline{\widetilde{G}}_{\nu} \gamma^{\mu} \gamma^{\nu} P_{R} f$. As $u_{R}$ and $d_{R}$ are isospin singlett fields we get

$$
\begin{gather*}
\mathcal{L}_{2}=\frac{1}{\sqrt{2} M_{\mathrm{P}}}\left[\left(i \partial_{\mu} \tilde{u}_{R}^{*}-\frac{1}{2} g_{1} Y\left(\tilde{u}_{R}^{*}\right) B_{\mu} \tilde{u}_{R}^{*}\right) \overline{\widetilde{G}}_{\nu} \gamma^{\mu} \gamma^{\nu} P_{R} u+\right. \\
\left.\left(i \partial_{\mu} \tilde{d}_{R}^{*}-\frac{1}{2} g_{1} Y\left(\tilde{d}_{R}^{*}\right) B_{\mu} \tilde{d}_{R}^{*}\right) \overline{\widetilde{G}}_{\nu} \gamma^{\mu} \gamma^{\nu} P_{R} d\right], \tag{28}
\end{gather*}
$$

and

$$
\begin{align*}
\mathcal{L}_{1}=\frac{1}{\sqrt{2} M_{\mathrm{P}}} & {\left[\left(-i \partial_{\mu} \tilde{u}_{R}-\frac{1}{2} g_{1} Y\left(\tilde{u}_{R}^{*}\right) B_{\mu} \tilde{u}_{R}\right) \bar{u} \gamma^{\nu} \gamma^{\mu} P_{L} \widetilde{G}_{\nu}+\right.} \\
& \left.\left(-i \partial_{\mu} \tilde{d}_{R}-\frac{1}{2} g_{1} Y\left(\tilde{d}_{R}^{*}\right) B_{\mu} \tilde{d}_{R}\right) \bar{d} \gamma^{\nu} \gamma^{\mu} P_{L} \widetilde{G}_{\nu}\right] . \tag{29}
\end{align*}
$$

The analogous results for the leptons we get by substitution of $u \rightarrow \nu$ and $d \rightarrow e$ in (25), (26), (28), and (29). Note, as there is no $\nu_{R}$ included, $\tilde{u}_{R} \rightarrow \tilde{\nu}_{R} \rightarrow 0$. This means, only the second lines of (28) and (29) contribute in the leptonic case.

Only the interactions with gluons and gluinos are still missing. Using (5) we get for the part with the left handed squarks $\tilde{q}_{L}$

$$
\begin{equation*}
\mathcal{L}_{2}=\frac{1}{\sqrt{2} M_{\mathrm{P}}}\left(i \partial_{\mu} \tilde{q}_{L}^{r}-g_{s} G_{\mu}^{a} T_{r s}^{a} \tilde{\tilde{L}}_{\mathcal{s}}^{s}\right) \delta_{r t} \bar{q}^{t} \gamma^{\nu} \gamma^{\mu} P_{R} \widetilde{G}_{\nu} \tag{30}
\end{equation*}
$$

The matrices $T^{a}$ are hermitian, $T_{r s}^{a *}=T_{s r}^{a}$. We get

$$
\begin{equation*}
\mathcal{L}_{1}=\frac{1}{\sqrt{2} M_{\mathrm{P}}}\left(-i \partial_{\mu} \tilde{q}_{L}^{*}-g_{s} G_{\mu}^{a} T_{s q}^{a} \tilde{G}_{L}^{* *}\right) \delta_{r t} \overline{\widetilde{G}}_{\nu} \gamma^{\mu} \gamma^{\nu} P_{L} q^{t} \tag{31}
\end{equation*}
$$

For the right handed quarks we have to be careful again, because we work with the left handed antiparticle, which transforms under $\overline{\mathrm{SU}(3)}$ with $-T_{r s}^{a *}=-T_{s r}^{a}$. We get

$$
\begin{equation*}
\mathcal{L}_{2}=\frac{1}{\sqrt{2} M_{\mathrm{P}}}\left(i \partial_{\mu} \tilde{q}_{R}^{r *}+g_{s} G_{\mu}^{a} T_{s r}^{a} \tilde{\tilde{q}}_{R}^{* *}\right) \delta_{r t} \overline{\widetilde{G}}_{\nu} \gamma^{\mu} \gamma^{\nu} P_{R} q^{t}, \tag{32}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathcal{L}_{1}=\frac{1}{\sqrt{2} M_{\mathrm{P}}}\left(-i \partial_{\mu} \tilde{q}_{R}^{r}+g_{s} G_{\mu}^{a} T_{r s}^{a} \tilde{q}_{R}^{s}\right) \delta_{r t} q^{\bar{t}} \gamma^{\nu} \gamma^{\mu} P_{L} \widetilde{G}_{\nu} \tag{33}
\end{equation*}
$$

Similar to (21-21) we write

$$
\begin{equation*}
\left[\gamma^{\rho}, \gamma^{\sigma}\right] F_{\rho \sigma}^{(3) a}=\left[\gamma^{\rho}, \gamma^{\sigma}\right]\left(2 \partial_{\rho} G_{\sigma}^{a}-g_{s} f^{a b c} G_{\rho}^{b} G_{\sigma}^{c}\right) . \tag{34}
\end{equation*}
$$

We get

$$
\begin{equation*}
\mathcal{L}_{3}=-\frac{i}{8 M_{\mathrm{P}}}\left(2 \partial_{\rho} G_{\sigma}^{a}-g_{s} f^{a b c} G_{\rho}^{b} G_{\sigma}^{c}\right) \overline{\widetilde{G}}_{\mu}\left[\gamma^{\rho}, \gamma^{\sigma}\right] \gamma^{\mu} \tilde{g}^{a}, \tag{35}
\end{equation*}
$$

or in a second form,

$$
\begin{equation*}
\mathcal{L}_{3}=-\frac{i}{8 M_{\mathrm{P}}}\left(2 \partial_{\rho} G_{\sigma}^{a}-g_{s} f^{a b c} G_{\rho}^{b} G_{\sigma}^{c}\right) \bar{g}^{a} \gamma^{\mu}\left[\gamma^{\rho}, \gamma^{\sigma}\right] \widetilde{G}_{\mu} \tag{36}
\end{equation*}
$$

In Fig. 7 all possible structures are depicted for the gravitino interactions to MSSM particles in eq. (2). We extended the FA generic Lorentz file with these structures and the 78 couplings given by the coupling (or coefficient) vectors $C[\ldots]$ we appended to the FA MSSM model file.

For creating the coupling vectors of the FA model file the following relations are helpful:

$$
\begin{align*}
\left(c \bar{f} \gamma^{\mu}\left[\gamma^{\rho} \gamma^{\sigma}\right] P_{L, R} \widetilde{G}_{\mu}\right)^{\dagger} & =-c^{*} \overline{\widetilde{G}}_{\mu}\left[\gamma^{\rho} \gamma^{\sigma}\right] \gamma^{\mu} P_{L, R} f  \tag{37}\\
\left(c \bar{f} \gamma^{\mu} \gamma^{\nu} P_{L, R} \widetilde{G}_{\mu}\right)^{\dagger} & =-c^{*} \overline{\widetilde{G}}_{\mu} \gamma^{\nu} \gamma^{\mu} P_{R, L} f . \tag{38}
\end{align*}
$$

and (recall that $\widetilde{G}_{\mu}^{c}=\widetilde{G}_{\mu}$ )

$$
\begin{align*}
\bar{f}^{c} \gamma^{\mu}\left[\gamma^{\rho} \gamma^{\sigma}\right] P_{L, R} \widetilde{G}_{\mu} & =\overline{\widetilde{G}}_{\mu}\left[\gamma^{\rho} \gamma^{\sigma}\right] \gamma^{\mu} P_{R, L} f,  \tag{39}\\
\bar{f}^{c} \gamma^{\mu} \gamma^{\nu} P_{L, R} \widetilde{G}_{\mu} & =\widetilde{G}_{\mu} \gamma^{\nu} \gamma^{\mu} P_{L, R} f . \tag{40}
\end{align*}
$$

As an illustrative example we showed in [5] how to get the explicit $\tilde{\chi}^{0} \widetilde{G}_{\mu} W^{+} W^{-}$ interaction Lagrangian. Here we show that for the $\widetilde{G} \tilde{\tau}_{i} \tau$ interaction. From eqs. (25) and (29) we get

$$
\begin{equation*}
\mathcal{L}=\frac{i}{\sqrt{2} M_{\mathrm{P}}}\left[\left(\partial_{\mu} \tilde{\tau}_{L}\right) \bar{\tau} \gamma^{\nu} \gamma^{\mu} P_{R} \widetilde{G}_{\nu}-\left(\partial_{\mu} \tilde{\tau}_{R}\right) \bar{\tau} \gamma^{\nu} \gamma^{\mu} P_{L} \widetilde{G}_{\nu}\right]+\text { h.c. } \tag{41}
\end{equation*}
$$

Writing the fields $\tilde{\tau}_{L, R}$ in terms of physical states by using eq. (24) and further using eq. (38) gives

$$
\begin{equation*}
\mathcal{L}=\frac{i}{\sqrt{2} M_{\mathrm{P}}}\left[\left(\partial_{\nu} \tilde{\tau}_{i}\right) \bar{\tau} \gamma^{\mu} \gamma^{\nu}\left(R_{i 1}^{\tilde{\tau} *} P_{R}-R_{i 2}^{\tilde{\tau} *} P_{L}\right) \widetilde{G}_{\mu}+\left(\partial_{\nu} \tilde{\tau}_{i}^{*}\right) \overline{\widetilde{G}}_{\mu} \gamma^{\nu} \gamma^{\mu}\left(R_{i 2}^{\tilde{\tau}} P_{R}-R_{i 1}^{\tilde{\tau}} P_{L}\right) \tau\right] . \tag{42}
\end{equation*}
$$

Comparing this result with struc3 defined in Fig. 7 we get the coupling vectors

$$
C\left[\bar{\tau}, \tilde{G}^{\mu}, \tau_{i}\right]=\frac{i}{\sqrt{2} M_{\mathrm{P}}}\left(\begin{array}{c}
-R_{i 2}^{\tilde{\tau} *}  \tag{43}\\
R_{i 1}^{\tilde{\tau} *} \\
0 \\
0
\end{array}\right) \quad \text { and } \quad C\left[\tau, \tilde{G}^{\mu}, \tau_{i}^{*}\right]=\frac{i}{\sqrt{2} M_{\mathrm{P}}}\left(\begin{array}{c}
0 \\
0 \\
-R_{i 1}^{\tilde{\tau}} \\
R_{i 2}^{\tilde{\tau}}
\end{array}\right)
$$

The Feynman rule for the decay $\tau^{-} \rightarrow \widetilde{G} \tilde{\tau}_{i}^{-}$is shown in Fig. 8 .


Figure 7: Possible structures of gravitino interactions with MSSM particles, detailed explanation is given in the text. As all momenta are defined incoming in FA, $\partial_{\mu} \rightarrow-i p_{\mu}$.


Figure 8: Feynman rule for $\tau^{-} \rightarrow \widetilde{G} \tilde{\tau}_{i}^{-}$derived from the Lagrangian in Eq. (42).

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