Domination versus edge domination

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Abstract

We propose the conjecture that the domination number $\gamma(G)$ of a Δ-regular graph G with $\Delta \geq 1$ is always at most its edge domination number $\gamma_e(G)$, which coincides with the domination number of its line graph. We prove that $\gamma(G) \leq \left(1 + \frac{2(\Delta-1)}{\Delta 2^{\Delta}}\right)\gamma_e(G)$ for general $\Delta \geq 1$, and $\gamma(G) \leq \left(\frac{7}{6} - \frac{1}{204}\right)\gamma_e(G)$ for $\Delta = 3$. Furthermore, we verify our conjecture for cubic claw-free graphs.

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1 Introduction

We consider finite, simple, and undirected graphs, and use standard terminology. Let G be a graph. A set D of vertices of G is a *dominating set* in G if every vertex in $V(G) \setminus D$ has a neighbor in D, and the *domination number* $\gamma(G)$ of G is the minimum cardinality of a dominating set in G. For a set M of edges of G, let $V(M)$ denote the set of vertices of G that are incident with an edge in M. The set M is a *matching* in G if the edges in M are pairwise disjoint, that is, $|V(M)| = 2|M|$. A matching M in G is *maximal* if it is maximal with respect to inclusion, that is, the set $V(G) \setminus V(M)$ is independent. Let the *edge domination number* $\gamma_e(G)$ of G be the minimum size of a maximal matching in G. A maximal matching in G of size $\gamma_e(G)$ is a *minimum maximal matching*.

A natural connection between the domination number and the edge domination number of a graph G becomes apparent when considering the line graph $L(G)$ of G. Since a maximal matching M in G is a maximal independent set in $L(G)$, the edge domination number $\gamma_e(G)$ of G equals the independent domination number $i(L(G))$ of $L(G)$. Since $L(G)$ is always claw-free, and since the independent domination number equals the domination number in claw-free graphs [1], $\gamma_e(G)$ actually equals the domination number $\gamma(L(G))$ of $L(G)$. While the domination number [7] and the edge domination number [11], especially with respect to computational hardness and algorithmic approximability [3–6, 8, 10], have been studied extensively for a long time, little seems to be known about their relation. For regular graphs, we conjecture the following:

Conjecture 1. *If G is a Δ-regular graph with $\Delta \geq 1$, then $\gamma(G) \leq \gamma_e(G)$.*

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The conjecture is trivial for $\Delta \leq 2$, and fails for non-regular graphs, see Figure 1. As pointed out by Felix Joos [9], for $\Delta \geq 13$, Conjecture 1 follows by combining the known results $\gamma(G) \leq \frac{(1+\ln(\Delta+1))n}{\Delta+1}$ (cf. [2]) and $\gamma_e(G) \geq \frac{\Delta n}{4\Delta-2}$ (cf. (1) below), that is, it is interesting for small values of Δ only. Furthermore, he observed that the union of two triangles plus a perfect matching shows that Conjecture 1 is tight for $\Delta = 3$.

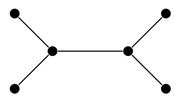

Figure 1: A non-regular graph G with $\gamma(G) = 2 > 1 = \gamma_e(G)$.

Our contributions are three results related to Conjecture 1. A simple probabilistic argument implies a weak version of Conjecture 1, which, for $\Delta \leq 12$, is better than the above-mentioned consequence of [2] and (1).

Theorem 1. *If G is a Δ-regular graph with $\Delta \geq 1$, then $\gamma(G) \leq \left(1 + \frac{2(\Delta-1)}{\Delta 2^\Delta}\right)\gamma_e(G)$.*

For cubic graphs, Theorem 1 implies $\gamma(G) \leq \frac{7}{6}\gamma_e(G)$, which we improve with our next result. Even though the improvement is rather small, we believe that it is interesting especially because of the approach used in its proof.

Theorem 2. *If G is a cubic graph, then $\gamma(G) \leq \left(\frac{7}{6} - \frac{1}{204}\right)\gamma_e(G)$.*

Finally, we show Conjecture 1 for cubic claw-free graphs.

Theorem 3. *If G is a cubic claw-free graph, then $\gamma(G) \leq \gamma_e(G)$.*

All proofs are given in the following section.

2 Proofs

We begin with the simple probabilistic proof of Theorem 1, which is also the basis for the proof of Theorem 2.

Proof of Theorem 1. Let M be a minimum maximal matching in G. Since every vertex in $V(G) \setminus V(M)$ has Δ neighbors in $V(M)$, and every vertex in $V(M)$ has at most $\Delta - 1$ neighbors in $V(G) \setminus V(M)$, we have

$$\Delta(n - 2\gamma_e(G)) \leq 2(\Delta - 1)\gamma_e(G), \tag{1}$$

where n is the order of G.

Let the set D arise by selecting, for every edge in M, one of the two incident vertices independently at random with probability $1/2$. Clearly, $|D| = \gamma_e(G)$. If u is a vertex in $V(G) \setminus V(M)$, then u has no neighbor in D with probability at most $1/2^\Delta$. Note that u might be adjacent to both endpoints of some edge in M in which case it always has a neighbor in D. If B is the set of vertices in $V(G) \setminus V(M)$ with no neighbor in D, then linearity of expectation implies

$$\mathbb{E}[|B|] = \sum_{u \in V(G)\setminus V(M)} \mathbb{P}[u \in B] \leq \frac{|V(G) \setminus V(M)|}{2^\Delta} = \frac{n - 2\gamma_e(G)}{2^\Delta}.$$

Since $D \cup B$ is a dominating set in G, the first moment method implies

$$\gamma(G) \leq |D| + \mathbb{E}[|B|] = \gamma_e(G) + \frac{n - 2\gamma_e(G)}{2^\Delta} \stackrel{(1)}{\leq} \gamma_e(G) + \frac{2(\Delta-1)\gamma_e(G)}{\Delta 2^\Delta},$$

which completes the proof. □

The next proof arises by modifying the previous proof.

Proof of Theorem 2. Clearly, we may assume that G is connected. Let M be a minimum maximal matching in G. Let R_0 be the set of vertices from $V(G) \setminus V(M)$ that are adjacent to both endpoints of some edge in M, and let R be $(V(G) \setminus V(M)) \setminus R_0$. Also in this proof, we construct a random set D containing exactly one vertex from every edge in M. Note that every vertex from R_0 will always have a neighbor in D. Again, let B be the set of vertices in R with no neighbor in D. As before, we will use the estimate

$$\gamma(G) \leq \gamma_e(G) + \mathbb{E}[|B|] = \gamma_e(G) + \sum_{u \in R} \mathbb{P}[u \in B].$$

Initially, we choose D exactly as in the proof of Theorem 1, which implies

$$\mathbb{E}[|B|] = \frac{|R|}{8}.$$

In order to obtain an improvement, we iteratively modify the random choice of D in such a way that $\mathbb{E}[|B|]$ becomes smaller. We do this using two operations. Each individual operation leads to some reduction of $\mathbb{E}[|B|]$, and we ensure that all these reductions combine additively. While the first operation leads to a reduction of $\mathbb{E}[|B|]$ regardless of additional structural properties of G, our argument that the second operation leads to a reduction is based on the assumption that the first operation has been applied as often as possible.

The first operation is as follows.

- If there are two edges uv and $u'v'$ in M such that the set X of vertices x in R with

  $$N_G(x) \cap \{u, v, u', v'\} \in \big\{\{u, u'\}, \{v, v'\}\big\}$$

  is larger than the set Y of vertices y in R with

  $$N_G(y) \cap \{u, v, u', v'\} \in \big\{\{u, v'\}, \{v, u'\}\big\},$$

  see Figure 2, then we *couple* the random choices for the pair $\{uv, u'v'\}$ in such a way that D contains $\{u, v'\}$ with probability $1/2$ and $\{u', v\}$ with probability $1/2$.

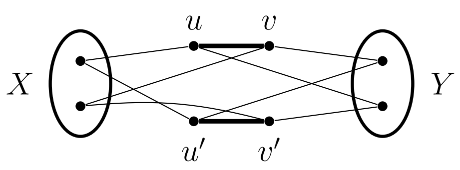

Figure 2: The edges uv, $u'v'$ and the sets X and Y.

The choice for the *coupled pair* $\{uv, u'v'\}$ will remain independent of all other random choices involved in the construction of D. Furthermore, the two edges in a coupled pair will not be involved in any other operation modifying the choice of D.

Let π be a coupled pair $\{uv, u'v'\}$. By construction, we obtain $\mathbb{P}[x \in B] = 0$ for every vertex x in X. Now, consider a vertex y in Y. The two neighbors of y in the two coupled edges are either both in D or both outside of D, each with probability exactly $1/2$. We will ensure that the third neighbor of y, which is necessarily in a third edge from M, will belong to D still with probability exactly $1/2$. By the independence mentioned above, we have $\mathbb{P}[y \in B] = 1/4$. Recall that, for the choice of D as in the proof of Theorem 1, each vertex from $X \cup Y$ belongs to B with probability exactly $1/8$. Hence, by coupling the pair π, the expected cardinality $\mathbb{E}[|B|]$ of B is reduced by $(|X| - |Y|)/8$, which is at least $1/8$.

The second operation is as follows.

- We select a suitable vertex z from R such that it has no neighbor in any of the coupled edges. If the edges u_1v_1, u_2v_2, and u_3v_3 from M are such that u_1, u_2, and u_3 are the three neighbors of z, then we *derandomize* the selection for these three edges, and D will always contain u_1, u_2, and u_3. We call $\{u_1v_1, u_2v_2, u_3v_3\}$ a *derandomized triple with center* z.

We will first couple a maximal number of pairs, and then derandomize triples one after the other as long as possible.

Let $\tau = \{u_1v_1, u_2v_2, u_3v_3\}$ be the next triple to be derandomized at some point. Let $S(\tau)$ be the set of all vertices that are incident with an edge e from $M \setminus \tau$ such that some vertex in R has a neighbor in $V(\tau)$ as well as in e, see Figure 3.

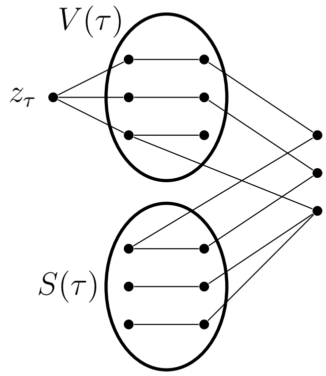

Figure 3: The set $S(\tau)$.

During all changes of the initial random choice of D performed so far, we ensure that the following property holds just before we derandomize the triple τ:

$$\text{\textit{For every vertex u in R that has a neighbor in $V(\tau) \cup S(\tau)$, the three neighbors of u in $V(M)$ belong to D independently with probability} } 1/2. \tag{2}$$

All coupled pairs and derandomized triples will be disjoint.

For every edge in M that does not belong to any coupled pair or derandomized triple, we select the endpoint that is added to D exactly as in the proof of Theorem 1, that is, with probability $1/2$ independently of all other random choices involved in the construction of D.

We fix a maximal collection $\mathcal{P}$ of pairwise disjoint coupled pairs $\pi_1, \ldots, \pi_p$.

Let $S_{\rm paired}$ be the set of the $4p$ vertices from $V(M)$ that are incident with some of the $2p$ paired edges. Let R_1 be the set of vertices in R with exactly one neighbor in $S_{\rm paired}$, and let R_2 be the set of vertices in R with at least two neighbors in $S_{\rm paired}$. Note that the sets R_0, R_1, and R_2 are disjoint by definition. Let S'_{paired} be the set of vertices from $V(M)\setminus S_{\text{paired}}$ that are incident with an edge in M that contains a neighbor of some vertex in $R_0 \cup R_2$. Let R_3 be the set of vertices from $R \setminus (R_0 \cup R_1 \cup R_2)$ that have a neighbor in S'_{paired}. All sets are illustrated in Figure 4. Let

$$R^{(1)} = R \setminus (R_0 \cup R_1 \cup R_2 \cup R_3),$$

$r = |R|$, $r^{(1)} = |R^{(1)}|$, and $r_i = |R_i|$ for $i \in \{0, 1, 2, 3\}$.

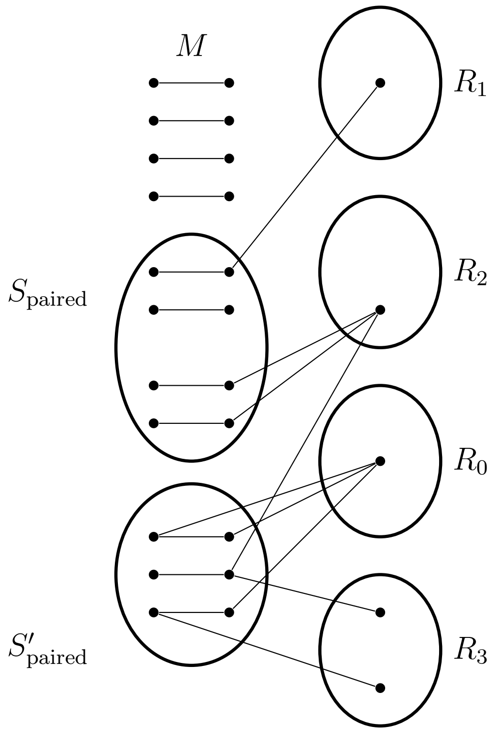

Figure 4: The sets S_{paired}, S'_{paired}, R_0, R_1, R_2, and R_3

Since G has at most $8p$ edges leaving S_{paired}, we have $2r_2 + r_1 \leq 8p$, which implies $r_1 + 7r_2 \leq 28p$. By definition, we obtain $|S'_{\text{paired}}| \leq 4r_0 + 2r_2$. Considering the number of edges leaving S'_{paired}, we obtain $r_3 \leq 3|S'_{\text{paired}}| \leq 12r_0 + 6r_2$. Therefore,

$$\begin{aligned} r^{(1)} &= r - r_0 - r_1 - r_2 - r_3 \\ &\geq r - 13r_0 - r_1 - 7r_2 \\ &\geq r - 13r_0 - 28p. \end{aligned} \tag{3}$$

Note that, only coupling the pairs $\pi_1, \dots, \pi_p$ and not derandomizing any triple, we have

$$\mathbb{E}[|B|] \leq \frac{|R|}{8} - \frac{p}{8} = \frac{1}{8}(n - 2\gamma_e(G) - r_0) - \frac{p}{8}. \tag{4}$$

If r_0+p is large enough, then this already yields the desired improvement. Since we cannot guarantee this, we now form derandomized triples one by one with centers from $R^{(1)}$. For every selected triple to be derandomized, we remove suitable vertices from $R^{(1)}$ in order to ensure (2). Suppose that we have already formed $t-1$ such derandomized triples with centers $z_1, \dots, z_{t-1}$, then the center z_t for the triple τ_t will be selected from $R^{(t)}$, where t is initially 1, and $R^{(t+1)}$ is obtained from $R^{(t)}$ by removing every vertex from $R^{(t)}$ that has a neighbor in $V(\tau_t) \cup S(\tau_t)$. This ensures that all coupled pairs and derandomized triples are disjoint as well as (2).

Now, we analyze the reduction of $\mathbb{E}[|B|]$, or rather the reduction of the upper bound on $\mathbb{E}[|B|]$ given in (4), incurred by some derandomized triple τ_t with center z_t. Let e_1, e_2, and e_3 in M be such that $e_i = u_i v_i$ for $i \in [3]$ and z_t is adjacent to u_1, u_2, and u_3, that is, $\tau_t = \{u_1v_1, u_2v_2, u_3v_3\}$.

We consider two cases.

Case 1 *Some vertex z in R distinct from z_t has three neighbors in $V(\tau_t)$.*

First, suppose that z is adjacent to u_1 and u_2. In this case, the pair e_1 and e_2 could be coupled and added to $\mathcal{P}$, contradicting the choice of $\mathcal{P}$. Next, suppose that z is adjacent to v_1, v_2, and v_3. Since the pair e_1 and e_2 cannot be coupled and added to $\mathcal{P}$, there are two vertices z' and z'' in R such that z' is adjacent to u_1 and v_2, and z'' is adjacent to u_2 and v_1. Since the pair e_2 and e_3 cannot be coupled and added to $\mathcal{P}$, the vertex z' is adjacent to u_3, which implies the contradiction that the pair e_1 and e_3 could be coupled and added to $\mathcal{P}$.

Hence, by symmetry, we may assume that z is adjacent to u_1, v_2, and v_3. Since the pair e_2 and e_3 cannot be coupled and added to $\mathcal{P}$, there are two vertices z' and z'' in R such that z' is adjacent to u_2 and v_3, and z'' is adjacent to u_3 and v_2. If z'' is adjacent to v_1, then, considering the pair e_1 and e_2, it follows that z' must be adjacent to v_1. In this case, the connected graph G has order 10, and $\{u_1, u_2, u_3\}$ is a dominating set, which implies the statement. Hence, we may assume that z'' is not adjacent to v_1. A symmetric argument implies that z' is not adjacent to v_1. See Figure 5 for an illustration.

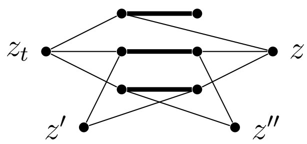

Figure 5: The edges in τ and the vertices z_t, z, z', and z''.

Our derandomized choice of adding always u_1, u_2, and u_3 to D yields

$$\mathbb{P}[z_t \in B] = \mathbb{P}[z \in B] = \mathbb{P}[z' \in B] = \mathbb{P}[z'' \in B] = 0.$$

Furthermore, property (2) implies $\mathbb{P}[w \in B] = 1/4$ for every neighbor w of v_1 in R. Since v_1 has at most two such neighbors, derandomizing the triple τ_t additionally reduces the upper bound on $\mathbb{E}[|B|]$ given in (4) by at least $\frac{4}{8} - \frac{2}{8} = \frac{1}{4}$. Since z' and z'' both have at most one neighbor not in $V(\tau_t)$, and at most two neighbors of v_1 in R both have at most two neighbors not in $V(\tau_t)$, we obtain $|S(\tau_t)| \leq 12$, and

$$\begin{aligned}
|R^{(t+1)}| &= |R^{(t)}| - \left|\left\{v \in R^{(t)} : v \text{ has a neighbor in } V(\tau_t) \cup S(\tau_t)\right\}\right| \\
&= |R^{(t)}| - \left|\left\{v \in R^{(t)} : v \text{ has a neighbor in } V(\tau_t)\right\}\right| \\
&\quad - \left|\left\{v \in R^{(t)} : v \text{ has a neighbor in } S(\tau_t) \text{ but no neighbor in } V(\tau_t)\right\}\right| \\
&\geq |R^{(t)}| - 6 - 3 \cdot 6 \\
&= |R^{(t)}| - 24.
\end{aligned}$$

Case 2 *z_t is the only vertex in R that has three neighbors in $V(\tau_t)$.*

Since the pair e_1 and e_2 cannot be coupled and added to $\mathcal{P}$, we may assume, by symmetry, that there is a vertex z in R that is adjacent to u_1 and v_2. Since the pair e_2 and e_3 cannot be coupled and added to $\mathcal{P}$, we may assume that there is a vertex z' in R such that either z' is adjacent to u_3 and v_2 or z' is adjacent to u_2 and v_3.

If z' is adjacent to u_3 and v_2, then the assumption of Case 2 implies the contradiction that the pair e_1 and e_3 can be coupled and added to $\mathcal{P}$. Hence, we may assume that z' is adjacent to u_2 and v_3. Since the pair e_1 and e_3 cannot be coupled and added to $\mathcal{P}$, there is a vertex z'' in R adjacent to u_3 and v_1. See Figure 6 for an illustration.

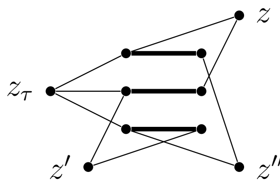

Figure 6: The edges in τ and the vertices z_t, z, z', and z''.

The choice of $\mathcal{P}$ implies that no vertex from $R^{(t)}$ distinct from z_t, z, z', and z'' has two neighbors in $V(\tau_t)$. Arguing as above, we obtain that derandomizing the triple τ_t additionally reduces the upper bound on $\mathbb{E}[|B|]$ given in (4) by at least $\frac{4}{8} - \frac{3}{8} = \frac{1}{8}$. Similarly as in Case 1, it follows that $|S(\tau_t)| \leq 18$, and that

$$|R^{(t+1)}| = |R^{(t)} \setminus \{v \in R : \ v \text{ has a neighbor in } V(\tau_t) \cup S(\tau_t)\}| \geq |R^{(t)}| - 7 - 3 \cdot 9 = |R^{(t)}| - 34.$$

Since we derandomize as many triples as possible, it follows that the number t of derandomized triples satisfies

$$t \geq \frac{r^{(1)}}{34} \overset{(3)}{\geq} \frac{r - 13r_0 - 28p}{34},$$

and that the joint reduction of the upper bound on $\mathbb{E}[|B|]$ given in (4) is at least

$$\frac{t}{8} \geq \frac{r - 13r_0 - 28p}{272}.$$

Altogether, coupling all p pairs in $\mathcal{P}$, and derandomizing the t triples, we obtain

$$\begin{aligned}
\mathbb{E}[|B|] &\leq \frac{1}{8}\left(n - 2\gamma_e(G) - r_0\right) - \frac{p}{8} - \frac{t}{8} \\
&\leq \frac{1}{8}\left(n - 2\gamma_e(G) - r_0 - p\right) - \frac{r - 13r_0 - 28p}{272} \\
&= \frac{1}{8}\left(n - 2\gamma_e(G) - r_0 - p\right) - \frac{n - 2\gamma_e(G) - r_0 - 13r_0 - 28p}{272} \\
&= \frac{33}{272}(n - 2\gamma_e(G)) - \frac{5}{68}r_0 - \frac{3}{136}p \\
&\leq \frac{33}{272}(n - 2\gamma_e(G)) \\
&\overset{(1)}{\leq} \frac{11}{68}\gamma_e(G).
\end{aligned}$$

Therefore,

$$\gamma(G) \leq \gamma_e(G) + \mathbb{E}[|B|] \leq \frac{79}{68}\gamma_e(G) = \left(\frac{7}{6} - \frac{1}{204}\right)\gamma_e(G),$$

which completes the proof. $\square$

We proceed to the final proof.

Proof of Theorem 3. Let M be a minimum maximal matching in G. Let the set D of $|M|$ vertices intersecting each edge in M be chosen such that the set $B = \{u \in V(G) \setminus V(M) : |N_G(u) \cap D| = 0\}$ is smallest possible. For a contradiction, we may suppose that B is non-empty. Let $C = \{u \in V(G) \setminus V(M) : |N_G(u) \cap D| = 1\}$. Let b be a vertex in B. Let $u_{-1}v_{-1}$, u_0v_0, and u_1v_1 in M be such that $N_G(b) = \{v_{-1}, v_0, v_1\}$. Since D intersects each edge in M, we have $u_{-1}, u_0, u_1 \in D$. Since G is claw-free, we may assume, by symmetry, that v_0 and v_1 are adjacent, which implies that v_{-1} is not adjacent to v_0 or v_1. Let x be the neighbor of v_{-1} distinct from u_{-1} and b. Since G is claw-free, the vertex x is adjacent to u_{-1}. If $x = u_0$, then u_0 has no neighbor in C, and exchanging u_0 and v_0 within D reduces $|B|$, which is a contradiction. Hence, by symmetry between u_0 and u_1, the vertex x is distinct from u_0 and u_1. Since exchanging u_1 and v_1 within D does not reduce $|B|$, the vertex u_1 has a neighbor c_1 in C, which is necessarily distinct from x.

Now, let $\sigma : v_1, u_1, c_1, v_2, u_2, c_2, \ldots, v_k, u_k, c_k$ be a maximal sequence of distinct vertices from $V(G) \setminus \{u_{-1}, u_0, v_{-1}, v_0, b, x\}$ such that $u_iv_i \in M$, $u_i \in D$, $c_i \in C$, u_i is adjacent to c_i for every $i \in [k]$, and v_{i+1} is adjacent to u_i for every $i \in [k-1]$. Let $X = \{u_{-1}, u_0, v_{-1}, v_0, b, x\} \cup \{v_1, u_1, c_1, v_2, u_2, c_2, \ldots, v_k, u_k, c_k\}$, and see Figure 7 for an illustration.

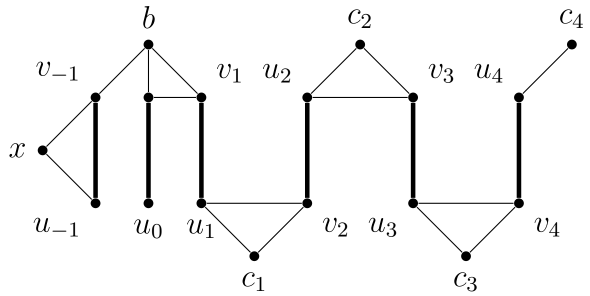

Figure 7: A subgraph of G with vertex set X, where $k = 4$.

Let v_{k+1} be the neighbor of u_k distinct from v_k and c_k. Since G is claw-free, the vertex v_{k+1} is adjacent to c_k. Since $V(G) \setminus V(M)$ is independent, we have $u_{k+1}v_{k+1} \in M$ for some vertex u_{k+1}. Since $c_k \in C$ and $u_k \in D$, we obtain $v_{k+1} \notin D$ and $u_{k+1} \in D$, which implies that the vertex v_{k+1} does not belong to X.

If u_{k+1} belongs to X, then $u_{k+1} = x$, and replacing D with

$$D' = (D \setminus \{u_1, u_2, \ldots, u_{k+1}\}) \cup \{v_1, v_2, \ldots, v_{k+1}\}$$

reduces $|B|$, which is a contradiction. Hence, the vertex u_{k+1} does not belong to X. If u_{k+1} has a neighbor c_{k+1} in C, then, by the structural conditions, the vertex c_{k+1} does not belong to X, and the sequence σ can be extended by appending $v_{k+1}, u_{k+1}, c_{k+1}$, contradicting its choice. Hence, the vertex u_{k+1} has no neighbor in C, and replacing D with the set D' as above again reduces $|B|$. This final contradiction completes the proof. $\square$

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