# MEASURING THE SPATIAL ACUITY OF VIBROTACTILE STIMULI: A new Approach to determine universal and individual Thresholds

#### A PREPRINT

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#### ABSTRACT

Tactile perception is an increasingly popular gateway in human-machine interaction, yet universal design guidelines for tactile displays are still lacking, largely due to the absence of methods to measure sensibility across skin areas. In this study, we address this gap by developing and evaluating two fully automated vibrotactile tasks that require subjects to discriminate the position of vibrotactile stimuli using a two-interval forced-choice procedure (2IFC). Of the two methodologies, one was initially validated through a preliminary study involving 13 participants. Subsequently, we applied the validated and improved vibrotactile testing procedure to a larger sample of 23 participants, enabling a direct and valid comparison with static perception. Our findings reveal a significantly finer spatial acuity for static stimuli perception compared to vibrotactile stimuli perception from a stimulus separation of 15 mm onwards. This study introduces a novel method for generating both universal thresholds and individual person-specific data for vibratory perception, marking a critical step towards the development of functional vibrotactile displays. The results underline the need for further research in this area and provide a foundation for the development of universal design guidelines for tactile displays.

*Keywords* Tactile perception of vibration  $\cdot$  Sensory testing  $\cdot$  Vibrotactile spatial acuity  $\cdot$  Tactile interface design  $\cdot$  Human-machine interaction  $\cdot$  Tactile displays  $\cdot$  Sensory substitution  $\cdot$  Bayesian adaptive parameter estimation

## 1 Introduction

The human skin, as the body's largest and most accessible sensory organ, presents unique challenges for researchers and professionals in tactile perception and in human-machine interaction. Unlike vision and hearing, standardized test procedures for measuring tactile perception are limited (Wee et al. [2021]). Nevertheless, ongoing research has uncovered insights that hold potential for numerous applications in user-environment communication. The haptic sense could complement or even replace other human senses, greatly benefiting individuals with limitations in other sensory modalities (Lederman and Klatzky [2009], Israr et al. [2012], Johnson and Higgins [2006], Filgueiras et al. [2016]).

Various studies have already explored the use of tactile perception for information transfer. For example, Adame et al. [2013] designed a wearable belt with vibrotactile motors, allowing subjects to successfully navigate a virtual environment using only vibrations. Meers and Ward [2005] developed a substitute vision system employing a stereocamera's depth view to detect object distances in images, transmitting this information to the wearer's fingertips via electrical signals. Additionally, Pamungkas and Ward [2015] focused on enabling tactile sensation of surface texture in the context of prosthetics and immersive technology, devising a feedback system consisting of an artificial finger that converts vibratory input signals into electrical pulses for wearer stimulation.

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Despite these encouraging use cases of the human skin as a gateway for transmitting information, it is crucial to ground practical applications with physiological and neurological research on the human body. Although there already exist several functional devices, targeted measurements need to be carried out in order to optimize the tactile information transfer and minimize the misrecognition rate by the user. Of particular interest are dermal tactile perceptiveness measures, which provide information on perceptual limits and possibilities. One approach to address this issue is to fit psychometric functions of participants with discrimination tasks (cf. Ch. 2.2). The Two-Point-Discrimination (2PD) (Dellon [1978a]) is a widely used procedure for establishing benchmarks of tactile spatial acuity of static (pressure) stimuli, requiring participants to differentiate between the placements of one or two tips at varying distances on a selected skin site. Tong et al. [2013] identified significant weaknesses in the 2PD method and proposed an alternative, the 2-point-orientation-discrimination (2POD), where participants differentiate between the proximal-distal (hereafter referred to as vertical) and medial-lateral (hereafter referred to as horizontal) placement of two tips. By comparing the two methods, they demonstrated the 2PD's susceptibility to non-spatial cues at minimal tip distances on the skin. Furthermore, they utilized Kontsevich and Tyler's (Kontsevich and Tyler [1999]) Bayesian adaptive testing, an algorithm that selects only the most informative intensities or separations, thus minimizing trial costs and duration.

In this paper, two types of stimuli are distinguished: static and vibratory stimuli. Static stimuli, characterized by their very low or non-existent oscillation frequency (< 1 Hz), describe the contact of a stationary object with the skin. For instance, in Tong et al.'s experiments, such stimuli are produced by pressing calipers onto the skin. On the other hand, vibrotactile stimuli are created by objects in contact with the skin, exhibiting higher oscillation frequencies (> 10 Hz, in our case  $\sim 130 Hz$ ). The transducers (stimulus tips) maintain consistent contact with the skin from the onset to the cessation of the vibration stimulus (Schmidt [2007]).

Perception of static stimuli has been extensively researched across various skin areas (Cholewiak and Collins [2003], Jóhannesson et al. [2017], Cholewiak et al. [2004]). However, the practical application of static stimuli in tactile interfaces proves somewhat challenging to implement. The actuation required for static stimuli is bulky and is limited to conveying quasi-binary information. In contrast, vibrotactile stimuli offer a more feasible solution. They allow additional dimensions of information transfer through frequency, pulse patterns, and intensity. Furthermore, vibration motors are easily controllable, compact in size, and therefore integrate well into textiles or similar materials (Kern et al. [2023]).

Vibrations in the range of 20 - 50 Hz are predominantly perceived by Meissner's corpuscles, while those in the range of 100 - 200 Hz are mainly detected by Pacinian corpuscles. Conversely, static pressure stimuli are primarily registered by Merkel's discs and Ruffini corpuscles. These receptors, aside from their distinct sensitivities and receptive fields, also exhibit varied distribution densities across different skin areas (Schmidt [2007]).

Therefore, benchmarks for static stimuli, such as the 2PD and 2POD, cannot readily be applied to vibrotactile stimuli. These require separate investigation. Unlike static stimuli, a comprehensive comparable investigation of various skin areas' response to vibrotactile stimuli is yet to be undertaken.

Significant research has been carried out in the field of vibrotactile stimuli. Cholewiak and Collins [2003] studied the impact of frequency, location on the body, proximity along the loci, and observer age on spatial acuity by introducing an array of tactors on the forearm. They found that stimulus position and separation along sites predominantly influenced spatial acuity. Jóhannesson et al. [2017] utilized eccentric rotating mass (ERM) motors with in-plane vibration, requiring observers to judge which neighboring tactors were active as motor distances decreased. They noted that discrimination accuracy remained well above chance at their smallest motor distance. Cholewiak et al. [2004] also employed a belt equipped with tactors around subjects' abdominal areas, systematically modifying the number and position of tactors to determine the most suitable loci for installing a tactile display.

Within the broader scope of our research efforts, a key goal is the development of expanded design guidelines for vibrotactile interfaces. Our ambition is to strike an optimal balance, maximizing information transfer while minimizing the misrecognition rate. For this purpose, establishing a comprehensive understanding of vibrotactile acuity is indispensable.

While the studies mentioned previously have yielded valuable insights for specific skin areas, their methodologies are not easily adaptable to new conditions or settings, and they do not allow a direct comparison of benchmarks due to their divergent principles. These methods are not designed for large-scale studies that span multiple skin areas. Direct comparison with static tactile perception using current studies proves challenging. The comparability of methods for recording vibrotactile and static benchmarks is low. The development of universal design guidelines for tactile interfaces is hindered by the absence of methods for measuring sensibility across skin areas in multiple directions.

## **1.1** Objectives and Hypothesis

Our research aims to address two key objectives, organized in a sequential manner:

- 1. **Method Validation Experiment (MVE):** Our first goal is to develop and validate a method that meets the specific needs of our investigation. This method should embody the following features:
  - It can be applied to various skin areas
  - It yields results that can be directly compared with studies examining the perception of static stimuli.
  - It is capable of quantifying anisotropy effects.
  - It is fully automated to minimize experimenter bias.

Having designed multiple apparatuses, we aim to validate one through a dedicated experiment. This experiment is designed specifically to evaluate the performance of our proposed methods under controlled conditions.

2. Extended Perception Comparison Experiment (EPCE): With the validated method in place, our second goal is to apply this method to investigate our primary research question. This involves conducting a comprehensive experiment using our validated method to gather necessary data.

Regarding our second objective, we have formulated the following hypothesis:

## *H*: *The spatial acuity of human skin for vibrotactile stimuli is significantly different from that for static stimuli.*

This hypothesis embodies our theoretical expectation, rooted in the human physiology concepts discussed earlier, and is the target of our empirical testing using the method we plan to develop and validate as our first objective.

To tackle our main objectives, this paper is structured into two consecutive sections:

**MVE**: To address the absence of suitable methods, we introduce two methods applicable to various body locations that yield comparable results for tactile acuity values. We have adapted Tong et al.'s (Tong et al. [2013]) and Dellon's (Dellon [1978b]) principle of static stimulus presentation and developed vibratory counterparts. Obtaining vibratory perception data requires a significantly more complex apparatus setup. It is essential to avoid effects from static perception, ensuring that detection is based solely on vibratory sensations. We also employ Bayesian adaptive testing for selecting varying distances between vibratory tips, which allows for more representative measurements across different body locations in fewer trials (Kontsevich and Tyler [1999]). We designed the apparatuses with the intention to support extensive studies. Their goals include acquiring vibratory acuity values from various body areas, investigating perception anisotropy, and allowing comparisons with electro-tactile and static perception. The apparatuses can be converted to electro-tactile measuring with minimal effort.

**EPCE**: For this part, we adapt the validated vibrotactile Two-Point-Discrimination (VT-2PD) (cf. Ch. 5.1.1) method to match Tong et al.'s conditions, allowing us to make a direct comparison with static perception.

# 2 Principles

## 2.1 Bayesian Adaptive Parameter Estimation (BAPE)

We determined the psychometric functions of the participants by employing Bayesian Adaptive Parameter Estimation (BAPE) with entropy minimization. This procedure is based on the publication by Kontsevich and Tyler [1999] and has been successfully applied to tactile perception by Tong et al. [2013]. We defined a family of Weibull functions with the following form as the solution set for the sought psychometric function:

$$\psi_{a,b,\gamma}(x) = \gamma + (1 - \delta - \gamma) \left(1 - 2^{-\left(\frac{x}{a}\right)^{b}}\right)$$
(1)

The Weibull function is well-established as a psychometric function (Wichmann and Hill [2001]) as all variable parameters have a directly discernible influence on the curve's shape. An example function of the family is shown in Figure 1, illustrating the properties of the individual function parameters. The parameter  $\gamma$  represents the guess rate, indicating the detection rate at which the participant identifies the stimulus at lower intensities, read off the function as the lower asymptote (e.g.,  $\gamma \approx 0.25$ ). It is opposed to  $(1 - \delta)$ , also called the lapse rate or finger error, which indicates the recognition rate at the highest tested intensity and thus maps the upper asymptote (e.g.,  $(1 - \delta) \approx 0.85$ ). The parameter *a* shifts the curve's inflection point in the x-direction and is often given as a threshold value (e.g.,  $a \approx 15$ ).



Figure 1: Example Weibull function as a possible psychometric function. ( $\gamma$ ) Guess rate. ( $\delta$ ) Lapse rate. (a) Threshold. (b) Slope.

The parameter b is related to the slope in the transition region and stretches or compresses the curve. Using the intervals and step sizes provided below, we created a family of curves with all possible parameter combinations. We fixed  $\delta$  at a constant value, resulting in 90,000 psychometric functions. The priori distribution  $P(\psi_{a,b,\gamma})$  of the psychometric functions was specified as uniformly distributed.

In the test procedure, one response from the participant was required per test query, which was scored as correct or incorrect. After each query, we calculated the probability space  $p_t(a, b, \gamma)$ , which indicates the probability of occurrence of each parameter combination and, consequently, the psychometric function based on the participant's response. We evaluated the entropy  $H_t$  by using  $p_t(a, b, \gamma)$  (Brand [1999]):

$$H_t = -\sum_{a,b,\gamma} p_t(a,b,\gamma) \log(p_t(a,b,\gamma))$$
<sup>(2)</sup>

In this case, smaller entropies indicate higher agreement. The entropy change between two trials indicates the information gain. We determined the subsequent query by minimizing entropy, thus maximizing the information gain. Based on the responses, we determined the posteriori probabilities of the psychometric functions  $P(\psi_{a,b,\gamma}|r_i)$  and expected probabilities for the function parameters  $a, b, and \gamma$ . Proportionally to the probabilities of all possible Weibull functions, we formed the so-called postmean from the set of curves. Thus, we assigned a specific psychometric function to each participant.

The participants underwent 50 trials in the two-interval forced-choice (2IFC) procedure (Ratcliff et al. [2018]). We determined the distances to be tested ( $d_{2PD}$  and  $d_{2POD}$ , see Fig 2) in both procedures using the BAPE algorithm. The interval limits for the guess rate  $\gamma$  were set from 0.01 to 0.99, divided into 100 steps. We fixed the finger error  $\delta$  at 0.02 based on recommendations by Klein [2001]. We set the limits for the threshold *a* at 2.5 to 45 mm in 18 steps and for the slope *b* at 0.01 to 10 in 50 steps.

#### 2.2 Experimental Designs

#### 2.2.1 Experimental Design MVE

In their study, Tong et al. [2013] compared the traditional clinical method of Two-Point-Discrimination (2PD) with their novel method, Two-Point-Orientation-Discrimination (2POD), recording threshold values for tactile perception sensitivity to static stimuli. To establish direct comparability, our experiment for determining threshold values for





Figure 2: Schematic representation of the experimental tasks VT-2PD and VT-2POD for the Method Validation Experiment (MVE). Assessment conducted on the left forearm. Placement of the stimulus tips for VT-2PD ( $\mathbf{A}$ ) and VT-2POD ( $\mathbf{B}$ ) illustrated.

Figure 3: Schematic representation of the experimental task bidirectional VT-2PD for the Extended Perception Comparison Experiment (EPCE). Assessment conducted on the left forearm. Consecutive placement of the stimulus tips as ( $\mathbf{A}$ ) and ( $\mathbf{B}$ ). Order is randomized in the experiment.

vibratory stimuli is based on Tong et al.'s work. We focused on the skin area of the left forearm and adapted our first method from the 2PD task (Dellon [1978a]). As illustrated in Figure 2 (A), our design employs two stimulus generators,  $S_a$  and  $S_b$ , placed on the skin at varying distances ( $d_{2PD}$ ). For each specified distance, a query is conducted by randomly emitting a vibration pulse from first one generator, then the other (2IFC)(Schlauch and Rose [1990]). The participant must identify which side was stimulated first. We named this method vibrotactile Two-Point-Discrimination (VT-2PD). It is important to note that, unlike the static 2PD model procedure, there is no intensity bias in this method. In each query, both sides are activated, with only the order being randomized. Both stimulus tips have full contact pressure while the query is executed.

We based the principle of the second experiment on the 2POD task (Tong et al. [2013]). As depicted in Figure 2 (**B**), three signal generators are arranged in an isosceles triangle with adjustable varying distances ( $d_{2POD}$ ). Participants must distinguish between horizontal and vertical stimulus inputs. This is achieved by activating signal generator  $S_c$  in each stimulus pair along with either randomly selected signal generator  $S_v$  or signal generator  $S_h$ . We named this method vibrotactile Two-Point-Orientation-Discrimination (VT-2POD). Here also all three stimulus tips have full contact pressure while the query is executed.

#### 2.2.2 Experimental Design EPCE

Upon the successful validation of our vibrotactile Two-Point-Discrimination (VT-2PD) method, as detailed in chapter (cf. Ch. 5.1.1), our goal was to juxtapose our findings of vibrotactile sensibility with those of Tong et al.'s work on the forearm's sensibility to static stimuli. To facilitate this comparison in the Extended Perception Comparison Experiment (EPCE), we took measures to ensure the compatibility of our method with Tong et al.'s Two-Point-Orientation-Discrimination (2POD) procedure.

Tong et al.'s approach involved discerning between horizontal and vertical stimuli, a process that inherently neutralized potential anisotropic effects. On the other hand, our VT-2PD task was constructed with anisotropy in mind, examining only one directional axis on the skin. To ensure a meaningful comparison with Tong et al.'s 2POD task, we expanded our procedure to probe two axes on the skin, subsequently offsetting anisotropic effects by generating the psychometric functions including both directions.

While the fundamental design of the VT-2PD remained consistent with that outlined in Chapter 2.2.1, our approach involved the consecutive measurement in two directions. The stimuli were introduced sequentially in two orientations - (A) horizontal and (B) vertical (see Fig. 3). The sequence of these orientations was randomized for each participant. Both orientations shared the same center point, creating a '+'-formation on the skin. This approach allowed us to retain the primary structure of the VT-2PD method while accommodating the requirements for a fair comparison with Tong et al.'s 2POD task.



Figure 4: Calipers for VT-2PD and VT-2POD trial. Both mounted with axis (s) to the lifter in order to be able to move in height. (a) Stimulus generator. (b) Linear guide. (c) Spindle drive for distance adjustment. (d) Stepper motor. (e) Caliper attachment. (f) Four-bar linkage. (g) Tension spring for calibration of contact pressure. (h) Pushbutton for detecting the contact pressure.

# 3 Material and Methods

This chapter outlines the materials and methodologies used in both our studies MVE and EPCE. We detail the apparatus, experimental procedures, and participant selection, providing a basis for understanding our results and findings.

## 3.1 Apparatus

The technical setup consisted of two main elements: the calipers, which guide and position the stimulus generators, and the lifter, which moves the calipers vertically and establishes a consistent contact pressure of the stimulus tips on the subject's skin.

## 3.1.1 Calipers

In both the 2PD and the 2POD configurations, two six-bar linkages guide two stimulus generators ( $\mathbf{a}$ ; see Fig. 4), each consisting of a stimulus tip attached to a vibration motor. They can be adjusted in a common plane without rotational motion or tilting. The head of the coupling gear is linearly guided ( $\mathbf{b}$ ) and positioned via a spindle drive ( $\mathbf{c}$ ) with a stepper motor ( $\mathbf{d}$ ) located in the center.

## 3.1.2 Lifter

The lifter allows the caliper to move linearly in height. The guide rail is attached to a stand with a ball joint (see Fig. 5), enabling flexible alignment of the entire apparatus. The caliper's clamping ( $\mathbf{s}$  in  $\mathbf{e}$ , see Fig. 4) is supported by a four-bar linkage ( $\mathbf{f}$ ) and hangs with its self-weight on an adjustable tension spring ( $\mathbf{g}$ ). This suspension permits minimal up and down movements with very low breakaway torque. The spring preload can be used to calibrate the contact pressure of the stimulus tips on the skin area. When the desired contact pressure is reached during lowering, the pushbutton ( $\mathbf{h}$ ) is triggered, and the spindle drive stops.

## 3.1.3 Potential for Adaptation to Electrostimulation

In addition to its current configuration for vibrotactile experiments, the design of our apparatuses allows for easy adaptation for future research involving electrostimulation. With minimal modifications, the stimulus generators could be replaced or supplemented with electrodes, expanding the range of sensory modalities that can be investigated with our method.



Figure 5: Full experimental setup, featuring the VT-2PD calipers currently installed. Participant's response remote is located on the right.

#### 3.1.4 Components and Values

The stimulus is applied using eccentric rotating mass (ERM) vibrators with a diameter of 10 mm and a weight of 0.9 g (Stronks et al. [2015]). We operate these at a voltage of 5 V and adjust the RPM using pulse width modulation (PWM). The average frequency with no load is 131 Hz. A NEMA 11 stepper motor is used in the caliper, and a NEMA 17 stepper motor in the lifter. All flexible pivot points are double-sided with radial ceramic bearings to ensure the lowest possible friction and precise guidance. The ERM vibrators are connected to the guide with a damper to decouple vibration. Stimulation is transmitted from the ERM vibrator to the skin through a rounded spring steel tip of 1.5 mm in diameter. The stimulus tips' center points can be moved in an interval of 2.5 mm to 60 mm with a tolerance of < 0.2 mm. We set the contact pressure to 0.5 N via the tension spring's position, which is achieved with a tolerance of < 4 %. This ensures sufficient contact pressure of the stimulus tips and does not cause discomfort for the participants. The length of the vibration pulses is set to 200 ms. We use an NVIDIA Jetson Nano developer board to drive the components, providing sufficient processing capacity to perform the more computationally intensive entropy minimization in a reasonable amount of time. We programmed the application exclusively in Python 3 (Van Rossum and Drake [2009]). All structural elements of the experimental apparatus are 3D printed using fused deposition modeling (FDM). Furthermore, only off-the-shelf components were used to allow other researchers to quickly reproduce the apparatus.

#### 3.2 Participants and Procedure

## 3.2.1 Participants and Procedure MVE

To evaluate our test procedures, a preliminary study was conducted on 13 participants, consisting of 8 men and 5 women aged 21 to 33, with an average age of 28 years. All participants were right-handed and had no medical history of restrictive diseases affecting tactile perception (e.g., diabetes or carpal tunnel syndrome) or cognitive perception disorders (central nervous system disorders, learning disabilities, attention deficit disorder, or dyslexia) (Grant et al. [1999]). The tested skin areas were free of skin irritation, tattoos, and scar tissue. All subjects were instructed and signed informed consent and privacy statements.

After the participant had completed the self-report form, she\*he was seated and her\*his left forearm was comfortably placed on a foam pad on the table with the palm facing up. The experimental apparatus was aligned perpendicularly on

the skin area on the forearm, ensuring uniform contact pressure of the stimulus tips at all distances. The participant was given the remote control for response input in the right hand and performed a first run with ten trials to understand the experiment and was then provided with noise-canceling headphones playing white noise. The experimenter then started the experiment.

We implemented the following steps fully automated in the program code to eliminate bias or influence by the experimenter. The BAPE algorithm determines the distance to be selected for the stimulus tips with the highest information content. This is set, and then the calipers are lowered onto the subject until the desired contact pressure is achieved. For VT-2PD, a random burst is first delivered on one side, and then on the other side (cf. Ch 2.2). For VT-2POD, a random burst orientation is chosen. The participant enters their response via the remote control, and the lifter raises the calipers again. The BAPE algorithm determines the new distance based on the user response, and the loop runs again until the required number of trials is reached. Then the determined postmean function is returned (cf. Ch 2.1), and the data is saved under an anonymous participant identification (test subject identification, TSID).

## 3.2.2 Participants and Procedure EPCE

In the Extended Perception Comparison Experiments (EPCE), we engaged a group of 23 participants (18 males, 5 females) aged between 18 and 38 years to determine the quantitatively meaningful vibrotactile perception acuity. The participants were financially compensated for their time commitment. It is important to note that none of the participants from the prestudy were re-examined in this phase of the study.

To compare the results of our validated method with those of static stimuli, we conducted the VT-2PD experiment again in a bidirectional form. The procedure was generally the same as in the MVE. The orientation axis (horizontal or vertical) was randomly determined at the beginning, and trials were conducted. After 50 queries, the participant remained seated while the VT-2PD apparatus was reoriented. The apparatus was rotated by 90 degrees, and another 50 queries were conducted.

The orientation of the buttons on the remote control could also be rotated accordingly to avoid confusion. The psychometric function was then formed from all 100 queries, allowing for a direct comparison with Tong et al.'s method.

#### 3.3 Preventive Steps for valid Results

#### 3.3.1 Correct below-Threshold Detection

In both experiments, we need to control two main sources of correct below-threshold detection: 1) the subject identifying the stimulus with the help of other senses, and 2) the subject recognizing one-sided tactile motor characteristics. Other senses can be easily blocked by using a blindfold and noise-canceling headphones playing white noise. Even precisely manufactured vibration motors have deviating characteristics, especially in amplitude and frequency. We have examined identical motors in advance and selected those with the highest agreement. Driving the motors via PWM allows us to calibrate their amplitude and frequency. To further refine the control and prevent participants from learning specific motor characteristics, we introduce a random variation to the intensity of the motors. This variation is based on a Gaussian distribution, with the duty cycle ratio serving as the expected value and a standard deviation of 3. By selecting a value within this distribution for each trial, we ensure a slight random fluctuation in motor intensity, effectively masking the motor characteristics and mitigating learning effects.

#### **3.3.2** One-sided Application Pressure Bias

In the application of the VT-2PD methodology, it is crucial to mitigate the risk of one-sided bias that could compromise the validity of the results. To this end, we have implemented several precautionary measures, starting with the EPCE, in addition to employing the 2IFC procedure, which is inherently designed to reduce bias Macmillan and Creelman [2004].

**Alignment Process** A meticulous alignment process is conducted to ensure uniform pressure application across different stimulus separations. The experimenter initially aligns the apparatus manually on the skin area, which is then verified with a short automated program. This program sequentially moves through stimulus separations in five steps, from the largest to the smallest, and lowers the stimulus tips onto the skin. Following the methodology outlined in Tong et al. [2013], the experimenter monitors the penetration depth of the tips to ensure consistent applied pressure across all stimulus separations.

**Monitoring Parameters** While we believe that the variations in applied pressure are not as significant as those encountered in manual application of stimuli, it remains important to acknowledge that achieving perfectly uniform pressure cannot be fully guaranteed. To address this issue, the experimenter carefully monitors three key parameters:

- Visual inspection of output data is conducted after each test run to identify frequent one-sided biased answers, particularly at recurring stimulus separations.
- A binomial test ( $\alpha = 0.05$ ) is conducted to determine if one side is preferentially chosen in general Howell [2012].
- Anomaly detection in response times, flagged by a t-test, serves as an additional indicator for the experimenter Howell [2012].

**Exclusion Criteria** Test runs exhibiting indications of bias are systematically excluded from the final analysis to preserve the integrity of the study.

By implementing these preventive measures, we aim to enhance the robustness and validity of our findings, thereby contributing to the reliability of the VT-2PD methodology in tactile perception research.

# 4 **Results**

## 4.1 Results MVE

## 4.1.1 Exemplary individual Test Runs

Figure 6 ( $\mathbf{A}$  and  $\mathbf{B}$ ) illustrates the automated BAPE algorithm procedure based on the participants' responses. If the participants' responses become increasingly accurate between iterations, the algorithm tends to investigate smaller distances and vice versa.

In one execution of the VT-2PD ( $\mathbf{A}$ ,  $\mathbf{C}$ ), the exemplary search pattern is evident. The query stabilizes in the transition region of the psychometric function ( $\mathbf{C}$ ), near the mean threshold. The participant's guess rate stands at 0.62, and at a 23 mm stimulus separation, the 0.95 level of the recognition rate is surpassed. The standard deviation is substantial for small stimulus separations and diminishes towards the top, attributable to the low polling frequency of small separations.

The search pattern in the displayed VT-2POD run (**B**, **D**) exhibits bipolar behavior, starting at Trial 18. The BAPE algorithm conducts queries exclusively at the maximum (45 mm) and minimum (2.5 mm) of the query interval. Consequently, the participant demonstrates a high error rate even at the maximum distance. The guess rate is at 0.65, and the recognition rate does not exceed 0.75 with increasing stimulus separation.

#### 4.1.2 Individual and mean Psychometric Functions

Figure 7 presents individual and mean psychometric functions for both the VT-2PD and VT-2POD experiments, derived from the Weibull function. Participant-specific psychometric functions were computed iteratively using the BAPE algorithm across 50 queries during each experimental run. The individual curves (light teal **A**, light orange **B**) were averaged (teal **A**, orange **B**) in both trials.

For the VT-2PD experiment (**A**), both the individual participant curves and the mean exhibit the common s-shape for psychometric functions (cf. Ch. 2.1). As such, we can divide the graph areas into three distinct segments. Segment I represents the participants' sub-threshold level, falling within the random guessing (0.5) region and indicating minimal or negligible stimulus recognition. Segment II includes the transition region, where the steepest slope signifies a rapid increase in recognition as distance expands. In Segment III, participants exceed the 0.95 recognition rate threshold. When defining physiological processes, this threshold is frequently employed as the recognition level (Crawford et al. [2009]). Four individual participant curves closely cluster together and approximate the mean, while two curves diverge more significantly. The mean guess rate  $\gamma$  (cf. Ch. 2.1) for the collected data is 0.54. The mean value surpasses the 0.95 recognition rate level at 32.5 mm.

In contrast, the VT-2POD test (**B**) displays neither discernible psychometric functions in the mean nor for individual participants. As a result, we cannot subdivide the graph areas as we did for the VT-2PD (**A**). Although some stimulus detection occurs, as the curves at greater distances notably exceed random guessing, no participant surpasses the 0.95 recognition rate level. The dispersion of the curves is considerably high, preventing any direct correlation.

We exclusively employ the results for functional verification of the experimental procedures. Owing to the small number of participants, quantitative statements regarding physiological perception are not feasible at this stage (Green and



Figure 6: **MVE**: Exemplary diagrams of an individual VT-2PD experiment (**A**) and an individual VT-2POD experiment (**B**). Participant response at each respective trial is plotted over stimulus separation. Correct responses are marked with crosses, while incorrect responses are marked with crosses. Point accumulation illustrates the search focus area of the BAPE algorithm. Lower plots (**C**, **D**) display the resulting psychometric function (**C**: teal, **D**: orange) with  $\pm 1$  SE (**C**: light teal, **D**: light orange) for individual participants' test runs. Plus symbols (+) denote the separations at which measurements were taken.

Swets [1966]). Furthermore, we have chosen not to report standard deviations as they would not be representative given the limited participant pool. It is important to note that these results were not included in the EPCE, primarily due to enhancements in our methodology and to prevent a potential bias from training effects. This decision aligns with our commitment to ensure the robustness and validity of our findings.

#### 4.2 Results EPCE

Figure 8 presents the averaged psychometric function of the bidirectional VT-2PD experiment. The guess rate starts at 0.59 at a distance of 2.5 mm, and the maximum recognition rate escalates to 0.93 at 45 mm. However, the 0.95 threshold was not reached, potentially reflecting the impact of the added complexity of vertical measurements.

The curve ascends notably more slowly than the curve from the purely horizontal VT-2PD experiment (cf. Fig. 7), indicating that the additional vertical measurements may have impacted the detection sensitivity or threshold. This divergence in gradient suggests a measurable discrepancy between the horizontal and vertical measurements, emphasizing the challenge posed by multi-directional vibrotactile perception.



Figure 7: **MVE**: Plot of psychometric functions for individual participants and mean for (**A**) VT-2PD and (**B**) VT-2POD. The 0.5 threshold (random guessing) of recognition rate is marked with a dotted line in both plots. Clear s-shaped psychometric functions are observed for VT-2PD but not for VT-2POD. The VT-2PD diagram is divided into three intervals, while no separation is possible for the VT-2POD diagram. Quantitative results are not significant for physiology due to the small number of participants. Plus symbols (+) denote the separations at which measurements were taken.



Figure 8: **EPCE**: Plot of mean psychometric function for bidirectionally (horizontally and vertically) applied VT-2PD.  $\pm 1$  SE in light blue. The 0.5 threshold (random guessing) of recognition rate is marked with a dotted line. Plus symbols (+) denote the separations at which measurements were taken.

Three distinct segments can be observed in the curve, as also shown in Figure 7, though they are not as clearly defined as in the purely horizontal test condition. The transition region is noticeably more pronounced, which could be interpreted as an indicator of the added intricacy of combining vertical and horizontal measurements.

The standard deviation remains relatively high throughout the measurements, which might be indicative of the inherent differences between horizontal and vertical measurements. This high variability underscores the complexity of accurately measuring and interpreting vibrotactile perception when multiple directional stimuli are involved.

## 5 Discussion

## 5.1 Discussion MVE

## 5.1.1 Quality of the Experiments

Despite the relatively small sample size, the results of the Method Validation Experiment (MVE) primarily support the conceptual validity of the VT-2PD experimental setup over the VT-2POD.

**VT-2PD** The VT-2PD experiment demonstrates a clear tendency towards a 50 % detection rate below the perception level, which can be considered a goodness-of-fit measure with an argument against strong bias. The measures against unintentional correct below-threshold detection prove to be effective. Furthermore, reproducible psychometric functions are obtained in the VT-2PD trial, clearly displaying the three perceptual intervals: random, transitional, and detection sections. Multiple experimental runs with the same participant exhibit minor differences, supporting the experiment's reproducibility.

A weakness of the experiment lies in the precise alignment of the experimental apparatus with the participant, as any misalignment can lead to a one-sided bias caused by increased contact pressure. After several runs, the experimenter became adept at alignment, and the error did not affect the results. False responses from the participant at higher distances can cause significant distortion of the psychometric function. One potential solution is to increase the fixed delta value (cf. Ch. 2.1) of the Weibull parameters, thereby generating more tolerance for so-called finger errors.

The individual psychometric functions exhibit a high degree of variability in the guess rate, leading to a substantial standard deviation in the derived mean. This variability can be attributed to the limited number of queries conducted at lower stimulus separations. To address this issue in the EPCE, we adjusted the hyperparameters of the BAPE algorithm. This adjustment ensures more frequent querying of the peripheral areas, thereby providing a more comprehensive understanding of the perception thresholds across the entire range of stimulus separations.

There is a known bias for the static 2PD experiment (Tong et al. [2013]) due to an intensity difference between the two possible queries in the task, which the participant can recognize. However, this bias does not apply to the VT-2PD experiment. Using the 2IFC method, both stimulus generators are activated in every query, and both stimulus tips maintain constant contact with the skin, ensuring no intensity bias arises.

**VT-2POD** In contrast, the VT-2POD experiment yields unreliable results. Participants report that they do not perceive any horizontal or vertical orientation of the stimulus, but only focus on the two outer stimulus generators. This results in the principle of the VT-2POD trial being given with an increased minimum distance by a factor of  $\sqrt{2}$ . Detection at greater distances exhibits relatively low accuracy, and the improvement in detection is not substantial compared to that at smaller stimulus separations. The psychometric functions are not reproducible in the same participant and do not display the expected basic s-shape.

The primary issue with the experiment is the uneven contact pressure on the curved forearm surface. An alignment that ensures all three stimulus tips have sufficient contact pressure at all distances of the interval is difficult or, depending on the participant's physiology, impossible. At higher distances, a stimulus tip has insufficient contact with the skin, leading to an inaccurate detection rate. Trial runs of the VT-2PD with orientation vertical instead of horizontal on the arm reveal significantly worsened recognition rates for identical participants. This suggests an anisotropic perception of vibratory stimuli in the examined skin area. If further studies confirm this finding, there will be a directional bias, limiting the applicability of the VT-2POD method.

The VT-2POD isosceles triangle configuration may be inappropriate due to differing centers of mass between the horizontal and vertical stimulus configurations. A cross (+) configuration could ensure a consistent center of mass, maintaining the validity of two-point spatial acuity measurements as point separation approaches zero. However, the cross configuration with four stimulus tips introduces even more challenges in maintaining uniform contact pressure.

#### 5.1.2 Comparison with existing Experiments

Our developed VT-2PD experiment has demonstrated potential validity for generating information on vibrotactile perception in future studies. Existing study designs typically allow for the investigation of one specific target skin area (Jóhannesson et al. [2017], Hoffmann et al. [2018], Jouybari et al. [2021]). Although we have currently tested our experiment only on the forearm, our experimental apparatus enables versatile application to different skin areas, thereby facilitating an overall comparison of skin sensitivities. Test runs in other skin areas have shown promising results and require formal validation in larger studies.



Figure 9: **EPCE**: Mean psychometric functions of bidirectional VT-2PD compared with 2POD by Tong et al. Recognition rate scale starts at 0.5 for detailed view. Bonferroni corrected p-values of one-sample t-test are displayed on a logarithmic scale. Plus symbols (+) indicate the separations where measurements were taken.

The use of Bayesian adaptive parameter estimation (BAPE) in combination with entropy minimization accelerates the procedure. The participant's response is used to search for distances with the highest information content, saving redundant queries and time. Consequently, more participants and skin areas can be examined within a single test session.

#### 5.2 Discussion EPCE

In our investigation of the Extended Perception Comparison Experiments (EPCE), we found that the elevated standard deviation could be attributed to the amalgamation of horizontal and vertical measurements into a single psychometric function. In particular, the inclusion of vertical measurements appears to significantly deteriorate the perception curve. This suggests that, especially in the case of vibration, horizontal and vertical measurements should be examined separately. The results hint at a strong anisotropy of perception, which could be a crucial factor in future studies. While an S-shaped curve is discernible, it exhibits a very flat progression.

## 5.2.1 Comparison bidirectional VT-2PD and 2POD

In order to compare our results with those of Tong et al.'s, we applied the VT-2PD method bidirectionally, aligning our approach with theirs. The psychometric functions of Tong et al. and ours (cf. Ch. 4.2) are superimposed in Figure 9 (**A**). To validate the divergence between the curves, we conducted a one-sample t-test at each interval step (+), with Bonferroni corrected p-values, plotted logarithmically in the lower diagram (**B**).

The curves exhibit pronounced differences from a distance of 15 mm onwards. The p-value (log-scale) robustly confirms the significance of these differences. The recognition rate in Tong et al.'s transition area ascends significantly steeper, indicating a clear distinction in the perception of static and vibrotactile stimuli. For the direction-independent



Stimulus Separations for different Threshold Levels

Figure 10: **EPCE**: Stimulus separation required to reach five different Threshold levels for vibrotactile stimuli (VT-2PD) and static stimuli (2POD, Tong et al. [2013]). Error bars for VT-2PD:  $\pm 1$  SE

secure recognition of vibrotactile stimuli, a larger distance is necessitated compared to static stimuli. This observation substantiates our hypothesis H.

In Figure 10, we compare the different stimulus separations required for vibrotactile stimuli (VT-2PD) and static stimuli (2POD, Tong et al. [2013]) to reach five distinct threshold levels (0.75, 0.8, 0.85, 0.9, 0.95) for both VT-2PD and 2POD. This comparison further underscores the need for a significantly smaller stimulus separation in the 2POD experiment to achieve the same threshold levels, ranging from 0.75 to 0.9, while the 0.95 level is not reached with VT-2PD within 45 mm.

In aligning our study with that of Tong et al., we intentionally diverged in the type of stimulus used, yet ensured a high degree of similarity in other aspects of the methodology. Like Tong et al., we applied the stimuli bidirectionally, thereby neglecting any anisotropy effects. Additionally, in our method, two stimulus tips are always in contact with the skin, paralleling the conditions in Tong et al.'s study. The use of the Bayesian Adaptive Parameter Estimation (BAPE) algorithm in a manner akin to Tong et al.'s resulted in a comparable question-answer paradigm. Moreover, our study was fully automated, which we believe could further minimize any potential experimenter bias.

In terms of participant responses, we observed some notable differences between our study and Tong et al.'s. These differences could provide valuable insights into the perception of static and vibrotactile stimuli, and warrant further investigation. For instance, Tong et al.'s study surpassed the 0.9 recognition rate threshold at 20.7 mm, while ours did so at 36.6 mm. Interestingly, the 0.95 threshold was not reached within our maximum distance of 45 mm with vibrotactile stimuli, indicating potential limitations in vibrotactile perception. This could be due to the propagation of vibrations in the skin leading to reduced spatial resolution, as suggested by Shah et al. [2019].

This observation raises questions about the transferability of spatial acuity values between static and vibrotactile stimuli, suggesting that vibrotactile perception requires a completely new investigation. Whether a reliable 0.95 level can be achieved with vibrotactile stimuli remains an open question.

Finally, our findings have important implications for the existing body of knowledge. By directly comparing our results with those of Tong et al.'s, we are able to provide new insights into the perception of vibrotactile stimuli. This not only confirms our hypothesis but also contributes to a deeper understanding of this complex phenomenon. The potential correlation between vibrotactile and static perception, possibly represented by a transformation function, can only be confirmed in further studies.

## 5.3 Limitations

While our study has made significant strides in the field of tactile perception research, it is not without its limitations, which we acknowledge as opportunities for future research and refinement of our methodology.

Firstly, to achieve the smallest possible stimulus separation and the highest possible application accuracy, the stimulus application via the stimulus tips is very small-scale, almost point-like. In tactile displays, larger application areas have mostly been used so far (Cholewiak and Collins [2003], Jóhannesson et al. [2017], Cholewiak et al. [2004]). This might have an increased influence on spatial acuity and requires further investigation.

Secondly, the automation of the apparatus eliminates human bias in the stimulus application during the experiment. However, inadequate alignment of the apparatus can render a run unusable. The alignment on the skin area before the start of the experiment must be carried out with great care and monitored during the experiment. Test runs exhibiting indications of bias should be rigorously evaluated and, if necessary, excluded from the analysis (cf. Ch. 3.3.2).

Thirdly, although our apparatus offers high flexibility in its application, it has limitations in examining certain skin areas. Uneven areas and regions with variable skin thickness (subcutaneous fat) present challenges in ensuring uniform pressure application across all stimulus separations, potentially introducing bias. Areas with limited space, such as the hands and fingers, are also less amenable to examination. For these smaller regions, however, a more compact version of the experimental apparatus can be utilized.

In conclusion, while our study has provided valuable insights into the perception of vibrotactile stimuli, these limitations highlight the need for further research to refine our methodologies and expand the scope of our investigations.

## 5.4 Future Work

Following the development of the experimental apparatus in this study, a large-scale study with a minimum of 30 participants will be conducted. The processes for the VT-2PD trial will be optimized based on the insights gained from the preliminary study, and potential sources of error will be eliminated. For the implementation of this study, we propose two hypotheses:

 $H_1$ : Sensitivity to vibrotactile stimuli varies significantly across different skin areas of the body.

 $H_2$ : Perception of vibrotactile stimuli on the skin is anisotropic - exhibiting significant differences in various orientations at the same skin area.

We aim to establish direct comparability of vibrotactile sensitivity across different skin areas by investigating  $H_1$ . To this end, we will conduct our study on the forearm, lower back, abdomen, and thigh areas, and evaluate the results using statistical analyses.

We plan to investigate  $H_2$  by conducting the experiment in both horizontal and vertical directions for each skin area, thereby establishing separate psychometric functions, this time.

In addition to the large-scale study and the exploration of the hypotheses mentioned, the apparatus developed in this research presents potential for further adaptations and applications. One such possibility, without committing to a definite plan, is the adaptation of the apparatus for electrostimulation studies. The design of our apparatus allows for such modifications with minimal effort, opening up new avenues for research in tactile perception. However, the feasibility and implications of this adaptation would need to be thoroughly evaluated in the context of future work.

The primary objective of the subsequent investigation is to contribute to the design principles of tactile displays. This includes focusing on the dimensioning of tactile interfaces, determining the appropriate spacing of actuators with respect to the skin area, and considering the spacing of actuators in relation to the directional vector on the skin (horizontal/vertical).

# 6 Conclusion

This study has made substantial advancements in the field of tactile perception research by 1) introducing a verified novel and versatile approach for measuring the spatial acuity of vibrotactile stimuli and 2) enabling a direct comparison between vibrotactile and static stimuli on the forearm. Our findings have confirmed the hypothesis that the spatial acuity of human skin for vibrotactile stimuli is significantly different from that for static stimuli. This discovery has profound

implications for the design and application of tactile interfaces, particularly in the context of assistive technologies for visually impaired individuals.

In part 1) of our research, we developed and evaluated two distinct methods for conducting vibrotactile experiments: the vibrotactile Two-Point-Discrimination (VT-2PD) and the vibrotactile Two-Point-Orientation-Discrimination (VT-2POD). The VT-2PD emerged as a robust and reliable method for conducting initial experiments, demonstrating its effectiveness and potential for future studies. This method allows for the investigation of anisotropy effects in future studies and can be applied to uneven skin areas, offering a broad spectrum of investigation possibilities. Furthermore, the use of the Bayesian Adaptive Parameter Estimation (BAPE) algorithm in conjunction with the VT-2PD method allows for significant results with fewer trials by actively searching for threshold values rather than relying on fixed interval steps. In contrast, the VT-2POD did not yield satisfactory results and will therefore be discontinued.

In part 2) of our research, we have successfully conducted a comprehensive study with 23 participants, enabling us to directly compare our findings with those of Tong et al. This confirmed our hypothesis of significant differences in stimulus separation from a distance of 15 mm upwards. Our results indicate that vibrotactile perception exhibits lower acuity at higher separations, potentially due to the propagation of vibrations through the skin, thereby penetrating additional receptive fields. This observation underscores the fact that guidelines for the perception of static stimuli (2PD, 2POD) cannot be directly applied to vibrotactile perception.

Our findings provide a compelling argument for the need to consider the unique characteristics of vibrotactile stimuli when designing and interpreting tactile perception experiments. The potential influence of vibration propagation on spatial acuity is a complex issue that warrants further investigation.

Looking forward, we plan to conduct a large-scale study with a minimum of 30 participants to further investigate the variability of vibrotactile sensitivity across different skin areas and the potential anisotropy of vibrotactile perception. The insights gained from this future work will be invaluable in informing the design principles of tactile displays, including the dimensioning of tactile interfaces, the appropriate spacing of actuators with respect to the skin area, and the orientation of actuators on the skin.

In conclusion, our research has provided a new perspective on the perception of vibrotactile stimuli, offering valuable insights that will guide future research and the development of tactile interfaces. We look forward to the continued exploration of this fascinating aspect of human sensory perception.

# **Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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# **Author Contributions**

*vom Stein*: Conceived and led the research topic, developed the concept, constructed and built the test apparatus, programmed the experimental setup, conducted the central part of the research and played a leading role in the overall investigation, and wrote the majority of the manuscript along with creating the figures.

*Hoppe*: Conducted research, primarily contributed to the writing of the mathematical section of the paper, consulted on technical difficulties with the apparatus, collaborated on initial experiments, and participated in discussions for the development of the study design.

*Sommer*: Conducted research, contributed to the writing of the introduction section of the manuscript, collaborated on initial experiments, and participated in discussions for the development of the study design.

Wolf: Provided funding through the university, offered guidance and consultation, and reviewed the manuscript.

## References

- Chyanna Wee, Kian Meng Yap, and Woan Ning Lim. Haptic interfaces for virtual reality: Challenges and research directions. *IEEE Access*, 9:112145–112162, 2021. doi:10.1109/ACCESS.2021.3103598.
- S. J. Lederman and R. L. Klatzky. Haptic perception: a tutorial. *Attention, perception & psychophysics*, 71(7): 1439–1459, 2009. doi:10.3758/APP.71.7.1439.
- Ali Israr, Olivier Bau, Seung-Chan Kim, and Ivan Poupyrev. Tactile feedback on flat surfaces for the visually impaired. In Joseph A. Konstan, Ed H. Chi, and Kristina Höök, editors, CHI '12 Extended Abstracts on Human Factors in Computing Systems, pages 1571–1576, New York, NY, USA, 2012. ACM. ISBN 9781450310161. doi:10.1145/2212776.2223674.
- Lise A. Johnson and Charles M. Higgins. A navigation aid for the blind using tactile-visual sensory substitution. Conference proceedings : ... Annual International Conference of the IEEE Engineering in Medicine and Biology Society. IEEE Engineering in Medicine and Biology Society. Annual Conference, 2006:6289–6292, 2006. ISSN 1557-170X. doi:10.1109/IEMBS.2006.259473.
- Thiago Silva Filgueiras, Ana Carolina Oliveira Lima, Renan Lima Baima, Gabriel Tadayoshi Rodrigues Oka, Luiz Alberto Queiroz Cordovil, and Moises Pereira Bastos. Vibrotactile sensory substitution on personal navigation: Remotely controlled vibrotactile feedback wearable system to aid visually impaired. In 2016 IEEE International Symposium on Medical Measurements and Applications (MeMeA), pages 1–5. IEEE, 2016. ISBN 978-1-4673-9172-6. doi:10.1109/MeMeA.2016.7533768.
- M. R. Adame, Jing Yu, K. Moller, and E. Seemann. A wearable navigation aid for blind people using a vibrotactile information transfer system. In 2013 ICME International Conference on Complex Medical Engineering, pages 13–18. IEEE, 2013. ISBN 978-1-4673-2971-2. doi:10.1109/ICCME.2013.6548203.
- Simon Meers and Koren Ward. A substitute vision system for providing 3d perception and gps navigation via electro-tactile stimulation. 2005.
- Daniel Pamungkas and Koren Ward. Tactile sensing system using electro-tactile feedback. In 2015 6th International Conference on Automation, Robotics and Applications (ICARA), pages 295–299. IEEE, 2015. ISBN 978-1-4799-6466-6. doi:10.1109/ICARA.2015.7081163.
- A. L. Dellon. The moving two-point discrimination test: clinical evaluation of the quickly adapting fiber/receptor system. *The Journal of hand surgery*, 3(5):474–481, 1978a. ISSN 0363-5023. doi:10.1016/s0363-5023(78)80143-9.
- Jonathan Tong, Oliver Mao, and Daniel Goldreich. Two-point orientation discrimination versus the traditional twopoint test for tactile spatial acuity assessment. *Frontiers in human neuroscience*, 7:579, 2013. ISSN 1662-5161. doi:10.3389/fnhum.2013.00579.
- L. L. Kontsevich and C. W. Tyler. Bayesian adaptive estimation of psychometric slope and threshold. *Vision research*, 39(16):2729–2737, 1999. ISSN 0042-6989. doi:10.1016/s0042-6989(98)00285-5.
- Robert F. Schmidt, editor. *Physiologie des Menschen: Mit Pathophysiologie*. Springer-Lehrbuch. Springer-Medizin-Verl., Heidelberg, 30., neu bearb. und aktualisierte aufl. edition, 2007. ISBN 3540329080.
- Roger W. Cholewiak and Amy A. Collins. Vibrotactile localization on the arm: effects of place, space, and age. *Perception & psychophysics*, 65(7):1058–1077, 2003. ISSN 0031-5117. doi:10.3758/bf03194834.
- Ómar I. Jóhannesson, Rebekka Hoffmann, Vigdís Vala Valgeirsdóttir, Rúnar Unnthórsson, Alin Moldoveanu, and Árni Kristjánsson. Relative vibrotactile spatial acuity of the torso. *Experimental brain research*, 235(11):3505–3515, 2017. doi:10.1007/s00221-017-5073-6.
- Roger W. Cholewiak, J. Christopher Brill, and Anja Schwab. Vibrotactile localization on the abdomen: effects of place and space. *Perception & psychophysics*, 66(6):970–987, 2004. ISSN 0031-5117. doi:10.3758/bf03194989.
- Thorsten A. Kern, Christian Hatzfeld, and Alireza Abbasimoshaei. Engineering haptic devices. 2023. doi:10.1007/978-3-031-04536-3.
- A. L. Dellon. The moving two-point discrimination test: clinical evaluation of the quickly adapting fiber/receptor system. *The Journal of hand surgery*, 3(5):474–481, 1978b. ISSN 0363-5023. doi:10.1016/s0363-5023(78)80143-9.
- F. A. Wichmann and N. J. Hill. The psychometric function: I. fitting, sampling, and goodness of fit. *Perception & psychophysics*, 63(8):1293–1313, 2001. ISSN 0031-5117. doi:10.3758/bf03194544.
- Matthew Brand. Pattern discovery via entropy minimization. In Seventh International Workshop on Artificial Intelligence and Statistics, pages 10–17. PMLR, 1999.
- Roger Ratcliff, Chelsea Voskuilen, and Andrei Teodorescu. Modeling 2-alternative forced-choice tasks: Accounting for both magnitude and difference effects. *Cognitive psychology*, 103:1–22, 2018. doi:10.1016/j.cogpsych.2018.02.002.

- S. A. Klein. Measuring, estimating, and understanding the psychometric function: a commentary. *Perception & psychophysics*, 63(8):1421–1455, 2001. ISSN 0031-5117. doi:10.3758/bf03194552.
- R. S. Schlauch and R. M. Rose. Two-, three-, and four-interval forced-choice staircase procedures: estimator bias and efficiency. *The Journal of the Acoustical Society of America*, 88(2):732–740, 1990. ISSN 0001-4966. doi:10.1121/1.399776.
- H. Christiaan Stronks, Daniel J. Parker, Janine Walker, Paulette Lieby, and Nick Barnes. The feasibility of coin motors for use in a vibrotactile display for the blind. *Artificial organs*, 39(6):480–491, 2015. doi:10.1111/aor.12414.
- Guido Van Rossum and Fred L. Drake. *Python 3 Reference Manual*. CreateSpace, Scotts Valley, CA, 2009. ISBN 1441412697. doi:10.5555/1593511.
- A. C. Grant, A. Zangaladze, M. C. Thiagarajah, and K. Sathian. Tactile perception in developmental dyslexia: a psychophysical study using gratings. *Neuropsychologia*, 37(10):1201–1211, 1999. ISSN 0028-3932. doi:10.1016/s0028-3932(99)00013-5.
- Neil A Macmillan and C Douglas Creelman. Detection theory: A user's guide. Psychology press, 2004.
- David C Howell. Statistical methods for psychology. Cengage Learning, 2012.
- John R. Crawford, Paul H. Garthwaite, and Daniel J. Slick. On percentile norms in neuropsychology: proposed reporting standards and methods for quantifying the uncertainty over the percentile ranks of test scores. *The Clinical neuropsychologist*, 23(7):1173–1195, 2009. doi:10.1080/13854040902795018.

David M. Green and John A. Swets. Signal detection theory and psychophysics. 1966.

- Rebekka Hoffmann, Vigdís Vala Valgeirsdóttir, Ómar I. Jóhannesson, Runar Unnthorsson, and Árni Kristjánsson. Measuring relative vibrotactile spatial acuity: effects of tactor type, anchor points and tactile anisotropy. *Experimental brain research*, 236(12):3405–3416, 2018. doi:10.1007/s00221-018-5387-z.
- Atena Fadaei Jouybari, Matteo Franza, Oliver Alan Kannape, Masayuki Hara, and Olaf Blanke. Tactile spatial discrimination on the torso using vibrotactile and force stimulation. *Experimental brain research*, 239(11):3175– 3188, 2021. doi:10.1007/s00221-021-06181-x.
- Valay A. Shah, Maura Casadio, Robert A. Scheidt, and Leigh A. Mrotek. Vibration propagation on the skin of the arm. *Applied sciences (Basel, Switzerland)*, 9(20), 2019. ISSN 2076-3417. doi:10.3390/app9204329.