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Author post-print (accepted) deposited in CURVE May 2012

**Original citation & hyperlink:**

Petrovic, D. , Xie, Y. , Burnham, K. and Petrovic, R. (2008) Coordinated control of distribution supply chains in the presence of fuzzy customer demand. European Journal of Operational Research, volume 185 (1): 146-158.

<http://dx.doi.org/10.1016/j.ejor.2006.12.020>

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# **Coordinated control of distribution supply chains in the presence of fuzzy customer demand**

Dobрила Petrovic <sup>a 1 \*</sup>, Ying Xie <sup>a 2</sup>, Keith Burnham <sup>a 3</sup>, Radivoj Petrovic <sup>b 4</sup>

<sup>a</sup> Control Theory and Applications Centre (CTAC)

Faculty of Engineering and Computing

Coventry University, Priory Street, Coventry, CV1 5FB, UK

<sup>b</sup> Mihajlo Pupin Institute

Volgina 15, 11000 Belgrade, Serbia & Montenegro

<sup>1</sup> [D.Petrovic@coventry.ac.uk](mailto:D.Petrovic@coventry.ac.uk)

<sup>2</sup> [Y.Xie@gre.ac.uk](mailto:Y.Xie@gre.ac.uk)

<sup>3</sup> [K.Burnham@coventry.ac.uk](mailto:K.Burnham@coventry.ac.uk)

<sup>4</sup> [radivoj@yubc.net](mailto:radivoj@yubc.net)

## **Abstract**

This paper considers a single product inventory control in a Distribution Supply Chain (DSC). The DSC operates in the presence of uncertainty in customer demands. The demands are described by imprecise linguistic expressions that are modelled by discrete fuzzy sets. Inventories at each facility within the DSC are replenished by applying periodic review policies with optimal order up-to-quantities. Fuzzy customer demands imply fuzziness in inventory positions at the end of review intervals and in incurred relevant costs per unit time interval. The determination of the minimum of defuzzified total cost of the DSC is a complex problem which is solved by applying decomposition; the original problem is decomposed into a number of simpler independent optimisation subproblems, where each retailer and the warehouse determine their optimum periodic reviews and order up-to-quantities. An iterative coordination mechanism is proposed for changing the review periods and order up-to-quantities for each retailer and the warehouse in such a way that all parties within the DSC are satisfied with respect to total incurred costs per unit time interval. Coordination is performed by introducing fuzzy constraints on review periods and fuzzy tolerances on retailers and warehouse costs in local optimization subproblems.

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\* Corresponding author

**Key words:** fuzzy sets, fuzzy two-level optimisation, distribution supply chain, inventory control, uncertainty

## **1. Introduction**

Over the past decade, supply chain (SC) management and control has become a strategic focus of leading manufacturing companies. This has been caused by rapid changes in environments in which the companies operate, characterised by high globalisation of markets and ever increasing customer demands for higher levels of service and quality. Research interests in SC management and control have been growing tremendously as well, leading to a wealth of literature devoted to SC problems (e.g., de Kok and Graves, 2003, Geunes, Pardalos and Romeijn, 2002). SC coordinated control, including all parts of SCs, such as buyer-vendor, production-distribution and inventory-distribution has been recognised as one of the key issues in SC problems (Thomas and Griffin, 1996).

A distribution supply chain (DSC) is a particular part of the SC, which has attracted considerable attention in the literature. Both academic researchers and industrial practitioners alike have expended much effort to manage the overall distribution system more efficiently through various coordination and cooperation strategies. The models for DSC coordination developed so far can be categorized into two main groups: a) models that consider integrated DSCs in order to find joint optimal inventory policies that minimise the total cost of the DSC, and b) models that propose some coordination mechanisms which help every member within a DSC to work together harmoniously.

One of the earliest models, developed by Schwarz (1973), considered a continuous review policy for a single-warehouse multi-retailer inventory system, and minimised the average system cost per unit time interval. Goyal (1977) suggested a joint economic lot size model with the objective being to minimize the total cost of a single supplier – single buyer system. Yang and Wee, (2002) used a heuristic procedure to determine the optimal number of deliveries favoured by the buyers in such a way as to reduce the overall costs incurred by a single-vendor multi-buyer system. Lu (1995) developed a one-vendor multi-buyer integrated inventory model,

where the vendor seeks to minimize total annual cost subject to the maximum costs which buyers are prepared to incur. Crowther (1964) proposed a quantity discount as a coordination mechanism and explained the rationale for offering quantity discounts. Monahan (1984) developed a coordination strategy in which discount is offered to buyers so as to induce them to order in the quantity that maximises the vendor's profit. In (Viswanathan and Piplani, 2001), the vendor specifies common replenishment periods and requires all buyers to replenish only at suggested time periods. As compensation, the vendor provides a price discount to entice the buyers to accept the common replenishment periods. A conclusion was made that the proposed coordination mechanism was beneficial only in the case when the order processing cost was larger than a given limit value.

In this paper, a single-warehouse, multi-retailer DSC is considered (see Figure 1). It is supposed that customer demand is uncertain and is represented by discrete fuzzy sets. Retailers are supplied periodically from the warehouse and the warehouse is replenished periodically from an external source. All inventories in the DSC apply an order-up-to level replenishment policy. It is assumed that the DSC operates 'under one roof' in the sense that a single measure of performance of DSC control is defined; it is the total relevant cost per unit time interval, which includes order/set up cost, holding cost and shortage cost. The problem is to determine the review periods and order-up-to levels of all the inventories in the DSC which give a satisfactory small total cost of the DSC.

[Insert Figure 1 here]

In the paper, the complex control problem of the DSC as a whole is decomposed into a number of simpler subproblems of controlling the retailers and the warehouse independently. The solutions obtained in such a way can be improved by coordination actions. An iterative coordination mechanism is developed which coordinates the retailers and the warehouse to get satisfactory control of the DSC as a whole. Coordination is based on a multi-level optimisation technique (Lee and Shih, 2001). It is implemented in the DSC by introducing fuzzy constraints in retailers and warehouse optimisation subproblems.

The paper is organised as follows. The model of the DSC under consideration and the problem statement are given in Section 2 and Section 3, respectively. The

coordination procedure is presented in Section 4. In Section 5, an illustrative example is given to demonstrate the effectiveness of the proposed coordination procedure. Conclusions are outlined in Section 6.

## 2. Model

### 2.1. Assumptions

- Customer demands are confined to a single type of the product.
- Customer demands are uncertain and described by discrete fuzzy sets.
- All the inventories within the DSC are managed and controlled using a periodic review order-up-to level replenishment policy.
- When a customer demand exceeds the retailer's stock, unmet demand is backordered and delivered to customers as soon as it becomes available on the retailer's stock.
- The warehouse is supplied from an external source with unlimited capacity.
- Orders are placed only at the beginnings of review periods.
- Replenishment quantities for each inventory within the DSC are received with no lead time.
- Measure of performance of the DSC inventories is total relevant cost per unit time interval which includes order cost, holding cost and shortage cost.

### 2.2. Notation

- $N$  - number of retailers within the DSC,
- $n$  - superscript, index of a retailer within the DSC,  $n = 1, \dots, N$ ; superscript 0 denotes the warehouse,
- $t$  - unit time interval,  $t = 1, 2, \dots$ ,
- $\tilde{D}_t^n$  - fuzzy customer demand per  $t$  imposed on retailer  $n$ ,
- $\tilde{D}_t^n = \{(d_t^n, \mu_{\tilde{D}_t^n}(d_t^n)) \mid d_t^n \in \mathcal{D}_t^n\}$ ,  $n = 1, \dots, N$ ,  $\mathcal{D}_t^n \subset \mathbb{Z}^+$  is a discrete support of  $\tilde{D}_t^n$ ,
- $R^n$  - review period of retailer  $n$ ,  $R^n \in \mathbb{Z}^+$ ,

- $R^0$  - review period of the warehouse,  $R^0 \in Z^+$ ,  
 $S^n$  - order-up-to level of retailer  $n$ ,  $S^n \in Z^+$ ,  
 $S^0$  - order-up-to level of the warehouse,  $S^0 \in Z^+$ ,  
 $\tilde{B}^0$  - fuzzy warehouse inventory level after all the retailers have placed their demands,  $\tilde{B}^0 = \{(b, \mu_{\tilde{B}^0}(b)) \mid b \in Z\}$ ,  
 $co^n$  - cost per order of retailer  $n$ ,  
 $cw$  - order processing cost of the warehouse,  
 $ch^n$  - unit holding cost per item per unit time interval at retailer  $n$ ,  
 $ch^0$  - unit holding cost per item per unit time interval at the warehouse,  
 $cs^n$  - unit shortage cost per item at retailer  $n$ ,  
 $cs^0$  - unit shortage cost per item at the warehouse,  
 $F_{proc}^0$  - order processing cost at the warehouse incurred during  $R^0$ ,  
 $\tilde{F}_{hold}_{R^n}^n$  - fuzzy holding cost of retailer  $n$  incurred during  $R^n$ ,  
 $\tilde{F}_{hold}_{R^0}^0$  - fuzzy holding cost of the warehouse incurred during  $R^0$ ,  
 $\tilde{F}_{short}_{R^n}^n$  - fuzzy shortage cost of retailer  $n$  incurred during  $R^n$ ,  
 $\tilde{F}_{short}_{R^0}^0$  - fuzzy shortage cost of the warehouse incurred during  $R^0$ ,  
 $f^n$  - defuzzified total cost of retailer  $n$  incurred per unit time interval,  
 $f^0$  - defuzzified total cost of the warehouse incurred per unit time interval,  
 $f$  - defuzzified total cost of the DSC incurred per unit time interval.

### 2.3. Customer demand

Customer demand per unit time interval is assumed to be uncertain. Traditionally, probability concepts have been used for modelling of uncertainty in demand in inventory control. However, it has been argued in a large body of recent literature that fuzzy sets theory could provide an appropriate framework for dealing with uncertainties in areas where intuition and subjective judgement play an important role (Kosko, 1994). In such cases uncertainty is caused by the imprecision of natural

language description rather than the existence of statistical frequency of the occurrence of events. Appropriateness of modelling uncertainty in customer demand using fuzzy sets has been demonstrated previously in the literature (e.g., Petrovic, Roy, Petrovic, 1998).

In this paper, uncertain customer demand rate is described by imprecise linguistic expressions, such as ‘demand is *about*  $d_m$  products per week, but *not lower* than  $d_l$  and *not higher* than  $d_u$ ’ or ‘demand is *much higher* than  $d_l$  products per week, but *not higher* than  $d_u$ ’, etc. Such expressions can be represented by discrete fuzzy sets  $\tilde{D}_t^n$ , illustrated in Figure 2. The parameters of the membership functions can be subjectively determined by DSC practitioners. Various experimental methods which can be used to determine membership functions have been presented in the literature such as Horizontal method, Vertical method, Pairwise comparison, Inference based on problem specification, Parametric estimation and Fuzzy clustering (Pedrycz and Gomide, 1998, Medaglia, et.al., 2002). One of the conceptually very simple methods that can be easily applied in practice is the Horizontal method of membership estimation. In this method, determination of membership functions’ parameters relies on some experimental findings collected from a group of experts who are asked to answer questions such as: can  $d$  be accepted as compatible with the description of demand being *about*  $d_m$  products per week. The membership degree of the selected value  $d$  is taken as a ratio of the number of positive replies to the total number of responses. The estimated membership degree can be further modified with respect to the statistical relevance, using the standard deviation taken from the results of experiments.

[Insert Figure 2 here]

In a specific case, if fuzzy demand on retailer  $n$  is the same for every unit time interval  $t = 1, 2, \dots$ , there is no need to use index  $t$  to identify them. Fuzzy demand on retailer  $n$  during  $R^n$  is calculated using the standard arithmetic rule for fuzzy number addition (Pedrycz and Gomide, 1998):

$$\begin{aligned}
\tilde{D}_{R^n} &= \underbrace{\tilde{D}^n + \dots + \tilde{D}^n}_{R^n \text{ times}} \\
\mu_{\tilde{D}_{R^n}}(d) &= \sup_{d=d_1+\dots+d_{R^n}} \min(\mu_{\tilde{D}^n}(d_1), \dots, \mu_{\tilde{D}^n}(d_{R^n})) \\
d &\in \mathcal{D}_{R^n}, \quad d_1, \dots, d_{R^n} \in \mathcal{D}^n
\end{aligned} \tag{1}$$

Fuzzy demand  $\tilde{D}_{R^0}$  imposed on the warehouse during  $R^0$  is the sum of fuzzy customer demands imposed on all the retailers during  $R^0$ . In order to calculate  $\tilde{D}_{R^0}$ , the sum of customer demands placed on all the retailers during one unit time interval,  $\tilde{D}_{ret}$ , is calculated, i.e.,

$$\begin{aligned}
\tilde{D}_{ret} &= \underbrace{\tilde{D}^1 + \dots + \tilde{D}^N}_{N \text{ retailers}} \\
\mu_{\tilde{D}_{ret}}(dr) &= \sup_{dr=d^1+\dots+d^N} \min(\mu_{\tilde{D}^1}(d^1), \dots, \mu_{\tilde{D}^N}(d^N)) \\
dr &\in \mathcal{D}_{\mathcal{R}}, \quad d^n \in \mathcal{D}^n, \quad n = 1, \dots, N,
\end{aligned} \tag{2}$$

Now,

$$\begin{aligned}
\tilde{D}_{R^0} &= \underbrace{\tilde{D}_{ret} + \dots + \tilde{D}_{ret}}_{R^0 \text{ times}} \\
\mu_{\tilde{D}_{R^0}}(dr) &= \sup_{dr=dr_1+\dots+dr_{R^0}} \min(\mu_{\tilde{D}_{ret}}(dr_1), \dots, \mu_{\tilde{D}_{ret}}(dr_{R^0})), \\
dr &\in \mathcal{D}_{R^0}, \quad dr_1, \dots, dr_{R^0} \in \mathcal{D}_{\mathcal{R}}
\end{aligned} \tag{3}$$

where  $\mathcal{D}_{\mathcal{R}}$  and  $\mathcal{D}_{R^0}$  are the domains of the fuzzy sets  $\tilde{D}_{ret}$  and  $\tilde{D}_{R^0}$ , respectively.

It might be worth mentioning here a well-known phenomenon of accumulation of fuzziness in computing with fuzzy numbers (Pedrycz and Gomide, 1998). One can notice that in the case of a wide support of a fuzzy set (i.e., a wide range of elements with positive membership degree) or successive additions of a number of fuzzy sets, the support of the resulting fuzzy set expands quickly. This corresponds to situations where there is high uncertainty in customer demand  $\tilde{D}_t^n$ ,  $n = 1, \dots, N$ , or a high



number of retailers  $N$  or long review periods  $R^n$  or  $R^0$ . The accumulation of fuzziness manifests similarly to an error accumulation in numerical computations. In these situations, one should consider to reduce the iterative process of computation to keep the obtained results meaningful.

## 2.4. Costs

Various categories of costs are involved in DSC control: order cost, holding cost and shortage cost. Their precise meaning and quantitative determination are not obvious. They are examined briefly, to the extent required for a proper understanding of the statement that the objective is to minimise costs. It is assumed here that the holding cost is linearly dependent on inventory stock, and the shortage cost is linearly dependent on unsatisfied demand, regardless of the length of the time within which the shortage occurs.

In general, the total cost of the DSC is the sum of the costs of the warehouse and the costs of all the retailers.

$$f(R^0, R^1, \dots, R^N, S^0, S^1, \dots, S^N) = f^0(R^0, R^1, \dots, R^N, S^0) + \sum_{n=1}^N f^n(R^n, S^n) \quad (4)$$

### Warehouse cost

The warehouse cost over  $R^0$ , in the case when all the retailers place their demands at the same time, and  $R^n$ ,  $n = 1, \dots, N$  coincide with  $R^0$  includes (see Figure 3):

[Insert Figure 3 here]

- order processing cost  $F_{proc}^0$  incurred during  $R^0$ , which is the product of  $cw$  and the number of orders over  $R^0$ , i.e.,  $F_{proc}^0 = N \cdot cw$ ,

- fuzzy holding cost  $\tilde{F}hold_{R^0}^0$  incurred during  $R^0$ , where possible holding cost  $Fhold_{R^0}^0$  and the corresponding membership degree  $\mu_{\tilde{F}hold_{R^0}^0}(Fhold_{R^0}^0)$  are

$$\begin{aligned} Fhold_{R^0}^0(S^0, b) &= R^0 \cdot ch^0 \cdot \max(b, 0) \\ \mu_{\tilde{F}hold_{R^0}^0}(Fhold_{R^0}^0(S^0, b)) &= \mu_{\tilde{B}^0}(b) \end{aligned} \quad (5)$$

- fuzzy shortage cost  $\tilde{F}short_{R^0}^0$  incurred during  $R^0$ , where possible shortage cost  $Fshort_{R^0}^0$  and the corresponding membership degree  $\mu_{\tilde{F}short_{R^0}^0}(Fshort_{R^0}^0)$  are

$$\begin{aligned} Fshort_{R^0}^0(S^0, b) &= cs^0 \cdot \max(-b, 0) \\ \mu_{\tilde{F}short_{R^0}^0}(Fshort_{R^0}^0(S^0, b)) &= \mu_{\tilde{B}^0}(b) \end{aligned} \quad (6)$$

### Retailers costs

The cost of retailer  $n$  over  $R^n$  includes (see Figure 4):

[Insert Figure 4 here]

- ordering cost  $co^n$ ,

- fuzzy holding cost  $\tilde{F}hold_{R^n}^n$  incurred during  $R^n$ , where possible holding cost

$Fhold_{R^n}^n$  and the corresponding membership degree  $\mu_{\tilde{F}hold_{R^n}^n}(Fhold_{R^n}^n)$  are

$$\begin{aligned} Fhold_{R^n}^n(S^n, d) &= ch^n \cdot \left\{ \max(S^n - d'_1, 0) + \max(S^n - d'_1 - d'_2, 0) + \dots \right. \\ &\quad \left. + \max(S^n - d'_1 - d'_2 - \dots - d'_{R^n}, 0) \right\} \end{aligned}$$

where  $d = d'_1 + d'_2 + \dots + d'_{R^n}$  and  $\mu_{\tilde{D}_{R^n}}(d) = \min(\mu_{\tilde{D}^n}(d'_1), \mu_{\tilde{D}^n}(d'_2), \dots, \mu_{\tilde{D}^n}(d'_{R^n}))$ ,

$$\mu_{\tilde{F}^{hold}_{R^n}}(F^{hold}_{R^n}(S^n, b)) = \mu_{\tilde{D}_{R^n}}(d) \quad (7)$$

- fuzzy shortage cost  $\tilde{F}^{short}_{R^n}$  incurred during  $R^n$ , where possible shortage cost

$F^{short}_{R^n}$  and the corresponding membership degree  $\mu_{\tilde{F}^{short}_{R^n}}(F^{short}_{R^n})$  are

$$F^{short}_{R^n}(S^n, d) = cs^n \cdot \max(d - S^n, 0)$$

$$\mu_{\tilde{F}^{short}_{R^n}}(F^{short}_{R^n}(S^n, d)) = \mu_{\tilde{D}_{R^n}}(d) \quad (8)$$

### 3. Problem statement

In the context of the DSC considered in this paper several optimisation control problems can be stated as follows.

#### 3.1. Problem 1: Optimisation of the DSC as a whole

The problem is to find the minimum of  $f$ , where  $f$  is the sum of the defuzzified cost of the warehouse and the defuzzified costs of all retailers per unit time interval,

$$\min_{R^0, R^1, \dots, R^N, S^0, S^1, \dots, S^N} f(R^0, R^1, \dots, R^N, S^0, S^1, \dots, S^N) \quad (9)$$

s.t.  $R^n \in \mathcal{R}^n$ , where  $\mathcal{R}^n \subset \mathbb{Z}^+$ ,  $n = 0, 1, \dots, N$ ,

$$S^n \in \mathbb{Z}^+, n = 0, 1, \dots, N,$$

where  $\mathcal{R}^n = \{ R \mid R_{\min}^n \leq R \leq R_{\max}^n \}$ ,  $R_{\min}^n$  and  $R_{\max}^n$ , denote the minimum and

maximum possible review periods of the warehouse ( $n = 0$ ) and retailers

( $n = 1, \dots, N$ ), respectively, and  $\mathbb{Z}^+$  denotes the set of positive integers.

The optimisation problem in (9) is a very complex problem with  $2(N+1)$  variables and a complicated calculation of  $f$ .

### 3.2. Problem 2: Independent optimisation of retailers

If the retailers are viewed as independent parts of the DSC, then  $N$  optimisation subproblems can be stated as: find the minimum of  $f^n$ , where  $f^n$  is the defuzzified retailer cost per unit time interval,

$$\min_{R^n, S^n} f^n(R^n, S^n), \quad n = 1, \dots, N \quad (10)$$

$$\text{s.t.} \quad R^n \in \mathcal{R}^n, \text{ where } \mathcal{R}^n \subset Z^+,$$

$$\mathcal{R}^n = \{ R \mid R_{\min}^n \leq R \leq R_{\max}^n \},$$

$$S^n \in Z^+, n = 1, \dots, N$$

$N$  subproblems in (10) can be solved by applying a two-dimensional search over  $R^n$  and  $S^n$  (see Algorithm 1 in Appendix). The solutions obtained independently are designated as  $R^{n*}, S^{n*}, f^{n*}, n = 1, \dots, N$ .

### 3.3. Problem 3: Optimisation of warehouse subject to constraints on all periodic reviews

Under assumption that the warehouse and all the retailers have the same review periods, i.e.,  $R^0 = R^1 = \dots = R^N$ , an independent optimisation subproblem for the warehouse is: find the minimum of  $f^0$  - the defuzzified warehouse cost per unit time interval,

$$\min_{R^0, R^1, \dots, R^N, S^0} f^0(R^0, R^1, \dots, R^N, S^0) \quad (11)$$

$$\text{s.t.} \quad R^0 = R^1 = \dots = R^N$$

$$R^0 \in \mathcal{R}^0, \text{ where } \mathcal{R}^0 \subset Z^+,$$

$$\mathcal{R}^0 = \{ R \mid R_{\min}^0 \leq R \leq R_{\max}^0 \},$$

$$S^0 \in Z^+$$

The subproblem in (11) can be solved by applying a two-dimensional search over  $R^0$  and  $S^0$  (see Algorithm 2 in Appendix). The solution of (11) is designated as  $R^{0*}$ ,  $S^{0*}$ ,  $f^{0*}$ .

If the optimal review periods of all the retailers and the warehouse, obtained by solving Problem 2 and Problem 3, independently, coincide, i.e.,  $R^0 = R^1 = \dots = R^N$ , the satisfactory control of the DSC as a whole is obtained. Otherwise, if the review periods obtained independently are different, it is necessary to coordinate control of the DSC. Two-level coordination is proposed: on the warehouse level and on the retailers level. The coordination mechanism is performed by introducing fuzzy constraints into the control problems which represent compromises between the different control levels, namely the warehouse and the retailer control levels.

#### Fuzzy constraint on the review periods of the retailers

The warehouse holding cost would be smaller if all the retailers replenish themselves with the same review period equal to  $R^{0*}$ . However, this may result in larger stocks at the retailers or higher risks of shortage. In order to balance between the cost of the warehouse and the cost of the retailers, some coordination instructions from the warehouse to the retailers are sent. They are given in a fuzzy form: ' $R^n$  has to be *around*  $R^{0*}$ '. This vague expression is modelled by a discrete triangular fuzzy set, where the support consists of integers in the interval  $[R^{0*} - r_l, R^{0*} + r_u]$ , where  $R^{0*}$  represents the most preferred review period,  $r_l$  and  $r_u$  define the lower and upper bounds of acceptable review periods from the warehouse point of view, respectively (Figure 5). Triangular types of membership functions are used as the simplest models of uncertain numerical quantities. Three basic parameters, including a modal value, the lower and upper bounds can be obtained using the Horizontal method mentioned previously. The membership function  $\mu_{\tilde{R}^0}(R^n) = \alpha^n$ ,  $n = 1, \dots, N$  defines the degree of satisfaction on the warehouse level with  $R^n$  determined by retailer  $n$ . The minimum acceptable degree of satisfaction is  $\underline{\alpha}^n$ ,  $n = 1, \dots, N$ . The choice of all  $\underline{\alpha}^n$  is subjective and arbitrary.

[Insert Figure 5 here]

#### Fuzzy constraint on the objective function of the warehouse

On the warehouse level, the objective function is fuzzified by introducing a tolerance regarding the incurred cost. The tolerance with respect to cost is vaguely expressed as ‘warehouse cost  $f^0$  has to be *near to*  $f^{0*}$ , but definitely not higher than  $f_u^0$ ’. This vague expression is modelled by a linear fuzzy set where  $f_u^0$  is an upper bound of the warehouse cost (Figure 6). The membership function  $\mu_{\tilde{f}^0}(f^0) = \beta$  defines the degree of satisfaction with warehouse cost  $f^0$ . The minimum acceptable degree of satisfaction related to the cost incurred at the warehouse is  $\underline{\beta}$ . The choice of  $\underline{\beta}$  is subjective and arbitrary.

[Insert Figure 6 here]

#### Fuzzy constraint on the objective functions of the retailers

On the retailers’ level, the tolerances related to the retailers’ costs are introduced and defined by vague linguistic expressions ‘ $f^n$  is *near to*  $f^{n*}$ , but definitely not higher than  $f_u^n$ ’. These vague expressions are modelled by linear fuzzy sets, where  $f_u^n$  is an upper bound of the cost of retailer  $n$  (Figure 7). The membership function  $\mu_{\tilde{f}^n}(f^n) = \gamma^n, n = 1, \dots, N$ , describe the degree of satisfaction with respect to the retailers’ costs. The minimum acceptable degree of satisfaction with the cost of retailer  $n$  is  $\underline{\gamma}^n, n = 1, \dots, N$ . The choice of all  $\underline{\gamma}^n$  is subjective and arbitrary.

[Insert Figure 7 here]

Now, the satisfaction degree regarding all the objectives and constraints in DSC control as a whole is

$$\lambda = \min\{\alpha^1, \dots, \alpha^N, \beta, \gamma^1, \dots, \gamma^N\}, \quad (12)$$

$\lambda$  takes values between 0 and 1; the nearer the value of  $\lambda$  to 1, the better acceptable compromised DSC control as a whole is achieved.

In this approach, the coordination of different levels of control is achieved by fuzzifying the constraint and objectives of both the warehouse and retailer optimisation tasks. The fuzzy sets are used to represent satisfactions of the corresponding control levels. It is worth noting that, generally, it is easy to manipulate membership functions in the fuzzy models. Suppose a decision maker wishes to allow higher tolerances for the cost incurred. It can be described by a similar linguistic expression to that mentioned above, ‘the warehouse cost has to be *near to*  $f^{0*}$ , but definitely *not higher* than  $f_m^0$ ’, where  $f_m^0$  is a new boundary such that  $f_m^0 > f_u^0$ . The corresponding membership function can be modified by simply moving the upper bound of the fuzzy set support to the point  $f_m^0$ . It is worth noticing that conventional concepts of probability theory cannot be applied straightforwardly to model uncertainty related to the tolerances of objectives and constraint satisfaction degrees.

#### 4. Coordination Procedure

First, the concept of the satisfactory control of DSC is introduced. For the warehouse and the retailers, the satisfactory control is the one that gives acceptable degrees of satisfaction related to the cost incurred at the warehouse and the cost of each of the retailers.

In the following, the coordination procedure for determining coordinated  $R^n$  and  $S^n$ ,  $n = 0, 1, \dots, N$  is presented. The procedure involves two-level coordination: the warehouse is responsible for determining a review policy for the warehouse and for the overall coordination, whilst the retailers determine their review policies taking into consideration coordination instructions given by the warehouse.

Step 1: The retailers and the warehouse solve independently their isolated subproblems given as Problem 2 and Problem 3 in Section 3, respectively. If the solutions obtained, i.e.,  $R^{0*}$  and  $R^{1*}, \dots, R^{N*}$  coincide, the satisfactory control of the DSC as a whole is obtained. Stop.

Step 2: Otherwise, the warehouse defines: fuzzy constraint on the review periods ( $r_l$ ,

$r_u$ ,  $\mu_{\tilde{R}}^0$ ,  $\underline{\alpha}^n$ ),  $n = 1, \dots, N$ , and fuzzy constraint on the cost of the warehouse

( $f_u^0$ ,  $\mu_{\tilde{f}}^0$ ,  $\underline{\beta}$ ); the retailers define fuzzy constraints on the cost of the

retailers ( $f_u^n$ ,  $\mu_{\tilde{f}}^n$ ,  $\underline{\gamma}^n$ ),  $n = 1, \dots, N$ .

Step 3: Apply the following recursive algorithm that includes five substeps:

3.1: Set  $R^n = R^{0*}$ ,  $n = 1, \dots, N$ .

3.2: Calculate  $f^0(R^{0*}, R^1, \dots, R^N, S^{0*})$  and  $f^n(R^n, S^n)$ , and  $\alpha^n$ ,  $\beta$  and  $\gamma^n$ ,  
 $n = 1, \dots, N$ , and determine  $\lambda$  in (12).

3.3: If  $\lambda$  is equal to any one of  $\alpha^n$ ,  $n = 1, \dots, N$ , or  $\beta$ , the maximum satisfaction degree of the whole DSC is achieved. (Indeed, any other change of the retailers' review periods would decrease either some among  $\alpha^n$  or  $\beta$ , and it would lead to a lower  $\lambda$ ). Stop.

3.4: Otherwise, select the retailer  $n'$  for which the lowest  $\gamma^{n'}$  is achieved, and change the corresponding  $R^{n'}$  to a point from interval  $[R^{0*} - r_l, R^{0*} + r_u]$  which has the next highest membership degree  $\mu_{\tilde{R}}^0(\cdot)$ .

3.5: Repeat the steps 3.2 to 3.4 until the satisfactory control for the warehouse and the retailers is obtained, i.e.,  $\alpha^n \geq \underline{\alpha}^n$ ,  $\beta \geq \underline{\beta}$  and  $\gamma^n \geq \underline{\gamma}^n$ .

Step 4: If no control obtained is satisfactory to the warehouse and all the retailers,

adjust the acceptable tolerances  $r_l$ ,  $r_u$ ,  $f_u^0$  and  $f_u^n$ ,  $n = 1, \dots, N$  in the fuzzy constraints. The process of changing the membership functions in fuzzy constraints can continue until review policies in the DSC are reached which are satisfactory to the warehouse and all the retailers.

The coordination procedure iteratively generates coordinated controls. We would like to point out that the coordination procedure does not necessarily navigate to the minimum total cost of the DSC incurred per unit time interval. Instead, the best



acceptable compromised control among the generated controls which gives the maximum satisfaction degree  $\lambda$  is searched for.

## 5. Illustrative example

A DSC with one warehouse and three retailers is considered. The time is homogeneously divided into one-week periods.

Customer demands imposed on the retailers on a weekly basis are:

$$D_t^1 = \{(11,0.5), (12,1), (13,0.75), (14,0.5)\},$$

$$D_t^2 = \{(9,0.8), (10,1), (11,0.8)\}, \text{ and}$$

$$D_t^3 = \{(6,0.5), (7,0.75), (8,1), (9,0.75), (10,0.5)\}, t=1, 2, \dots$$

The cost parameters of each facility are:

$$ch^0 = 1.5, cs^0 = 6.5, cw = 30,$$

$$ch^1 = 2.0, cs^1 = 6.0, co^1 = 30,$$

$$ch^2 = 3.0, cs^2 = 4.0, co^2 = 20,$$

$$ch^3 = 2.5, cs^3 = 5.0, co^3 = 50.$$

First, both the retailers and the warehouse solve independently the Problem 2 and Problem 3, respectively. The results are given in Table 1 and Table 2.

[Insert Table 1 here]

[Insert Table 2 here]

The solutions determined independently do not coincide. Therefore, a constraint on the review periods imposed from the warehouse to the retailers is fuzzified as follows. The boundaries that define the range of acceptable  $R^n$ ,  $n = 1, 2, 3$ , are  $r_l = r_u = 6$ . In addition, the following two requirements are imposed: 1) the new domain of  $R^n$ ,  $n = 1, 2, 3$ , is defined as integer multiplies of  $R^{0*}$ , or fractions of  $R^{0*}$ , i.e., it is  $\{k \cdot R^{0*}\}$ , where  $k \in \{1, 2, 3, \dots\} \cup \{1/2, 1/3, 1/4, \dots\}$ , and 2)  $k$  is selected in such a way that  $R^n$  is an integer. Consequently, the domain of  $R^n$  is  $\{1, 2, 3, 6, 12\}$ .

The discrete fuzzy set  $\tilde{R}^0$ , defined by the warehouse, is  $\{(0,0), (1,0.5), (2,0.6), (3,0.7), (6,1), (12,0)\}$ , as given in Figure 8.

[Insert Figure 8 here]

In addition, the warehouse allows a tolerance for its cost  $f^0(R^{0*}, R^1, \dots, R^N, S^{0*})$  with boundary  $f_u^0 = 2.5 \cdot f^{0*}$ , i.e.,  $f_u^0 = 115.88$ .

The three retailers also determine their tolerances for  $f^n(R^n, S^n)$ ,  $n = 1, 2, 3$ , where the upper bounds for retailers 1, 2, and 3 are  $1.5 \cdot f^{1*}$ ,  $2 \cdot f^{2*}$ ,  $1.5 \cdot f^{3*}$ , respectively. The corresponding fuzzy sets are given in Figure 9.

[Insert Figure 9 here]

In the first iteration, two retailers, retailer 1 and retailer 2, for which the lowest satisfaction is achieved, ( $\gamma^1 = \gamma^2 = 0$ ), change their review periods,  $R^1$  and  $R^2$ , from 6 to 3 weeks (Table 3), as the membership degree  $\mu_{\tilde{R}^0}(3) = 0.7$  is the next highest degree in fuzzy set  $\tilde{R}^0$ . The maximum satisfaction degree  $\lambda = 0.49$  is achieved in the second iteration. As more iterations are run, the satisfaction degrees of the warehouse decrease, whilst the retailers satisfaction degrees increase. If, for example,  $\underline{\alpha}^n = 0.6$ ,  $n = 1, 2, 3$ ,  $\underline{\beta} = 0.5$  and  $\underline{\gamma}^n = 0.4$ ,  $n = 1, 2, 3$ , both the warehouse and the retailers are satisfied with the review periods suggested and the costs incurred, in this case, the sub-optimal compromised solution becomes  $R^{0^c} = 6$ ,  $R^{1^c} = 3$ ,  $R^{2^c} = 2$ ,  $R^{3^c} = 6$ . One should note that the lowest cost  $f$  of the whole DSC is achieved in the second iteration, but at the expense of the higher cost of the warehouse. It is worth noting that if both the warehouse and the retailers solve their own problems independently,  $f$  is higher than in the case when coordination is applied.

[Insert Table 3 here]

## 6. Conclusions

It was shown that determination of optimum periodic review policies in a DSC consisting of one-warehouse and  $N$ -retailers with uncertain customer demand modelled by fuzzy sets is a complex optimisation problem. The problem is decomposed into simpler subproblems, where each retailer and the warehouse solve local optimisation tasks and determine their review policies independently. An iterative coordination procedure is developed for changing locally determined review periods and the stock levels for each facility in the DSC, which are satisfactory with respect to the costs incurred at all the facilities. Coordination is performed by introducing fuzzy constraints in local optimum control subproblems of each retailer and the warehouse. It is shown that fuzzy sets can be used in the coordination procedure to model imprecisely specified constraints and objective functions of different control levels, namely the warehouse and retailers. In such a way, the optimisation problem of the DSC as a whole with  $2N+2$  control variables is transformed into a series of  $N+1$  simpler optimisation problems with only two control variables each, which are solved by a systematic two-dimensional search. The proposed approach constitutes a novel step forward in the optimisation of complex real-world problems where traditional analytical models may be difficult to formulate.

## Appendix

In this Appendix two fuzzy optimisation algorithms are given.

### Algorithm 1. Fuzzy optimisation of retailers

This algorithm determines the local optimum  $R^n$  and  $S^n$ ,  $n = 1, \dots, N$ , with respect to  $f^n$  under the assumption that the characteristics of uncertain customer demand do not change during unit time intervals  $t=1, 2, \dots$ . The algorithm is based on a two-dimensional search in a part of the  $R^n - S^n$  plane, for each  $n$  independently.

Step 1. Set a value  $R^n \in \mathcal{R}^n$

Step 2. Calculate  $\tilde{D}_{R^n}$  (see (1))

Step 3. Calculate  $\tilde{F}_{hold}^n(S^n, d)$  for each  $S^n$ ,  $d \in \mathcal{D}_{R^n}$  (see (7))

Step 4. Calculate  $\tilde{F}_{short}^n(S^n, d)$  for each  $S^n$ ,  $d \in \mathcal{D}_{R^n}$  (see (8))

Step 5. Calculate

$$f^n(R^n, S^n) = (1/R^n) \cdot [co^n + defuzz(\tilde{F}_{hold}^n(S^n)) + defuzz(\tilde{F}_{short}^n(S^n))]$$

for each  $S^n$ , where *defuzz* denotes arithmetic defuzzification based on the Centre of gravity method. The result of defuzzification is a scalar  $\bar{x}$  which approximately represents the fuzzy set. The centre of gravity of  $\tilde{F}(x)$  is:

$$\bar{x} = \frac{\int x \tilde{F}(x) dx}{\int \tilde{F}(x) dx}$$

It is worth noting that the following can be proved:

$$defuzz(\tilde{F}_{hold}) + defuzz(\tilde{F}_{short}) = defuzz(\tilde{F}_{hold} + \tilde{F}_{short})$$

Review period  $R^n$  and stock  $S^n$  which minimise  $f^n(R^n, S^n)$  is determined by a simple search over  $R^n \in \mathcal{R}^n$  and repeating steps 1 to 5, within which steps 3 to 5 are repeated for each value  $S^n \in \mathcal{D}_{R^n}$ . The results of the two-dimensional search within  $R^n \in \mathcal{R}^n$  and  $S^n \in \mathcal{D}_{R^n}$  are  $R^{n*}$  and  $S^{n*}$  that give the minimum  $f^{n*}$ .

#### Algorithm 2. Fuzzy optimisation of warehouse

This algorithm determines the local optimum  $R^0$  and  $S^0$ , with respect to  $f^0$  under the assumption that the characteristics of uncertain customer demand do not change during unit time intervals  $t=1, 2, \dots$ , and that all the retailers replenish their stock at the same time as the warehouse. The algorithm is based on a two-dimensional search in a part of the  $R^0 - S^0$  plane.

Step 1. Set a value  $R^0 \in \mathcal{R}^0$

Step 2. Calculate  $\tilde{D}_{R^0}$  (see (3))

Step 3. Calculate  $Fproc^0 = N \cdot cw$

Step 4. Calculate  $\tilde{B}^0 = S^0 - \tilde{D}_{R^0}$  for each  $S^0 \in \mathcal{D}_{R^0}$

Step 5. Calculate  $\tilde{F}^{hold}_{R^0}(S^0, b)$  for each  $S^0 \in \mathcal{D}_{R^0}$  (see (5))

Step 6. Calculate  $\tilde{F}^{short}_{R^0}(S^0, b)$  for each  $S^0 \in \mathcal{D}_{R^0}$  (see (6))

Step 7. Calculate

$$f^0(R^0, \underbrace{R^0, \dots, R^0}_{N \text{ times}}, S^0) = (1/R^0) \cdot [Fproc^0 + defuzz(\tilde{F}^{hold}_{R^0}(S^0, b)) + defuzz(\tilde{F}^{short}_{R^0}(S^0, b))]$$

for each  $S^0 \in \mathcal{D}_{R^0}$ , where *defuzz* denotes arithmetic defuzzification obtained using the Centre of gravity method.

Review period  $R^0$  and stock  $S^0$  which minimise  $f^0$  in Step 7 are determined by a search over  $R^0 \in \mathcal{R}^0$ , repeating steps 1 to 7, within which steps 4 to 7 are repeated for each  $S^0 \in \mathcal{D}_{R^0}$ . The results of the two-dimensional search are  $R^{0*}$  and  $S^{0*}$  that give the minimum of the total defuzzified warehouse cost per unit time interval, on the condition that the warehouse operates independently.

These two algorithms are used in Step 1 of the coordination procedure, given in Section 4. Let us note that two additional similar fuzzy algorithms were developed, which are used in Step 3.2. of the coordination procedure: (1) a fuzzy algorithm for optimisation of warehouse control under the assumption that the retailers replenish their stocks at different times, and (2) a fuzzy algorithm for optimisation of retailers under the assumption that the warehouse imposes the review periods on the retailer.

## **Acknowledgment**

This research is supported by Engineering and Physical Sciences Research Council (EPSRC), grant no. GR/N11841. This support is gratefully acknowledged.

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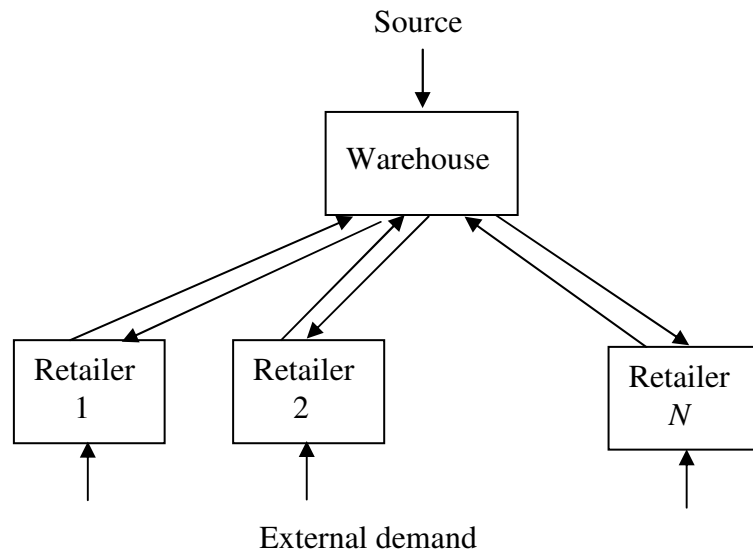


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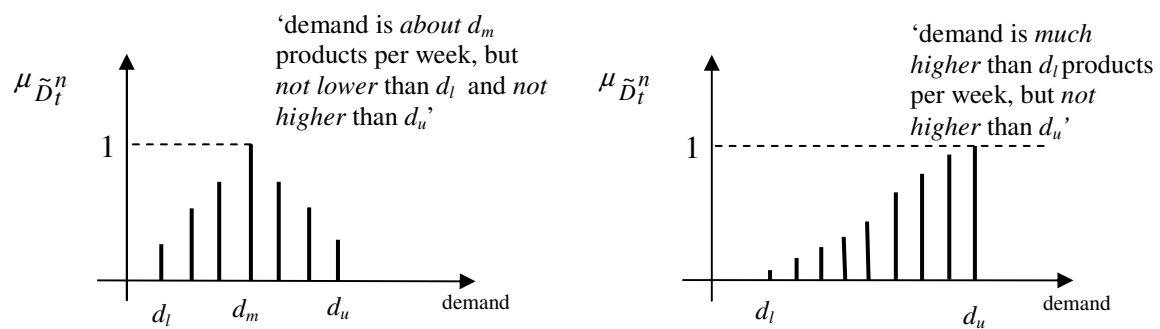


Figure 2. Fuzzy sets that describe uncertain customer demand

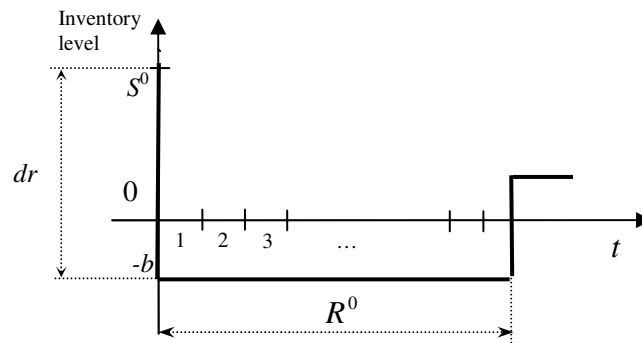


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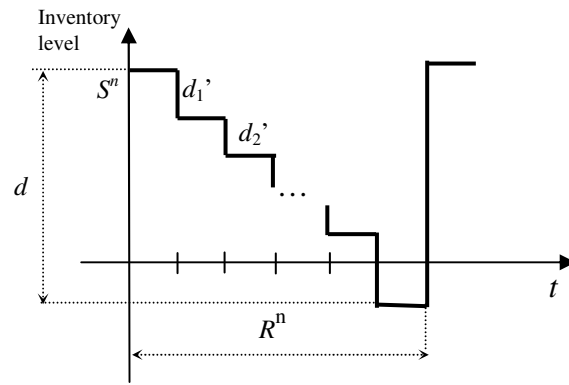


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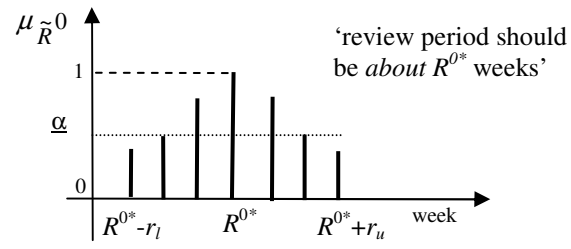


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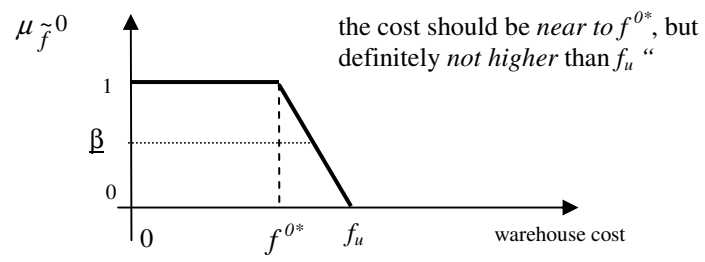


Figure 6. Fuzzy set that represents subjective cost tolerance of the warehouse cost

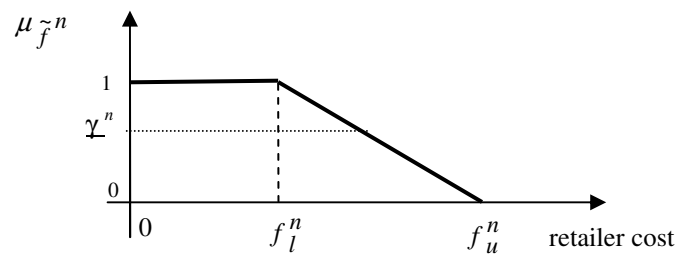


Figure 7. Fuzzy set that represents tolerance on the retailer's cost

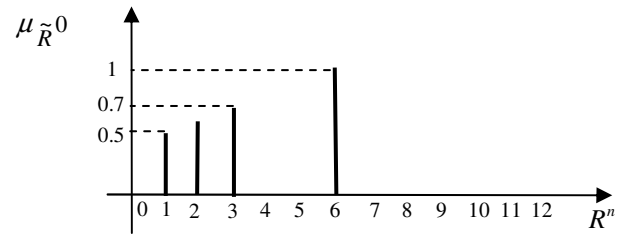
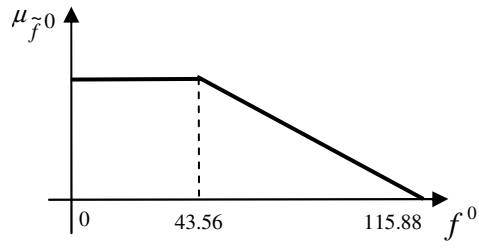
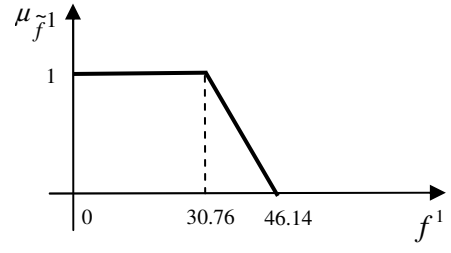


Figure 8. Discrete fuzzy set that represents fuzzy constraint on the review periods

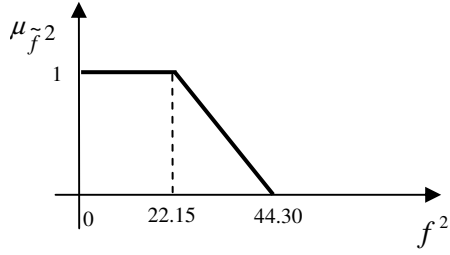




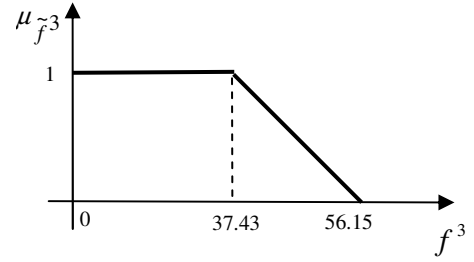
‘Acceptable cost for the warehouse’



‘Acceptable cost for retailer 1’



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Table 1. Results of solving Problem 2; local optimal solutions of the retailers

$R^1 / S^1$	$R^2 / S^2$	$R^3 / S^3$	$f^1$	$f^2$	$f^3$
2 / 25	1 / 10	3 / 18	30.76	22.15	37.43

Table 2. Results of solving Problem 3;

local optimal solutions of the warehouse under the constraint  
that all the inventories have the same review periods  
and the corresponding costs of the retailers

$R^0 / S^0$	$R^1 / S^1$	$R^2 / S^2$	$R^3 / S^3$	$f^0$	$f^1$	$f^2$	$f^3$	$f$
6 / 198	6 / 66	6 / 54	6 / 36	46.35	62.92	69.45	46.97	225.69

$\alpha^1$	$\alpha^2$	$\alpha^3$	$\beta$	$\gamma^1$	$\gamma^2$	$\gamma^3$	$\lambda$
1	1	1	1	0	0	0.49	0

Table 3. Results of coordinated control

Itr	$R^0/S^0$	$R^1/S^1$	$R^2/S^2$	$R^3/S^3$	$f^0$	$f^1$	$f^2$	$f^3$	$F$	$\alpha^1$	$\alpha^2$	$\alpha^3$	$\beta$	$\gamma^1$	$\gamma^2$	$\gamma^3$	$\lambda$
1	6/168	3/35	3/27	6/36	66.06	37.78	35.36	46.97	186.17	0.70	0.70	1.00	0.72	0.54	<b><u>0.40</u></b>	0.49	0.40
2	6/165	3/35	2/18	6/36	74.53	37.78	26.29	46.97	185.57	0.70	0.60	1.00	0.59	0.54	0.81	<b><u>0.49</u></b>	0.49
3	6/165	3/35	2/18	3/18	97.38	37.78	26.29	37.43	197.10	0.70	0.60	0.70	0.26	0.54	0.81	1.00	0.28