



AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
<http://ageconsearch.umn.edu>
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*



Fondazione Eni Enrico Mattei

Computational Complexity in Additive Hedonic Games

Shao-Chin Sung and Dinko Dimitrov

NOTA DI LAVORO 98.2008

DECEMBER 2008

CTN – Coalition Theory Network

Shao-Chin Sung, *Department of Industrial and Systems Engineering,
Aoyama Gakuin University*
Dinko Dimitrov, *Department of Economics, University of Munich*

This paper can be downloaded without charge at:

The Fondazione Eni Enrico Mattei Note di Lavoro Series Index:
<http://www.feem.it/Feem/Pub/Publications/WPapers/default.htm>

Social Science Research Network Electronic Paper Collection:
<http://ssrn.com/abstract=1318329>

The opinions expressed in this paper do not necessarily reflect the position of
Fondazione Eni Enrico Mattei
Corso Magenta, 63, 20123 Milano (I), web site: www.feem.it, e-mail: working.papers@feem.it

Computational Complexity in Additive Hedonic Games

Summary

We investigate the computational complexity of several decision problems in hedonic coalition formation games and demonstrate that attaining stability in such games remains NP-hard even when they are additive. Precisely, we prove that when either core stability or strict core stability is under consideration, the existence problem of a stable coalition structure is NP-hard in the strong sense. Furthermore, the corresponding decision problems with respect to the existence of a Nash stable coalition structure and of an individually stable coalition structure turn out to be NP-complete in the strong sense.

Keywords: Additive Preferences, Coalition Formation, Computational Complexity, Hedonic Games, NP-hard, NP-complete

JEL classification: C63, C70, C71, D02, D70, D71

This paper was completed during a visit of Dinko Dimitrov to Aoyama Gakuin University in September 2008. Financial support from the Japan Society for Promotion of Science under contract number S08017 is gratefully acknowledged.

Address for correspondence:

Dinko Dimitrov
Department of Economics
University of Munich
Kaulbachstr. 45
80539 Munich
Germany
Phone: +49 (0) 89 2180 5645
Fax: +49 (0) 89 2180 5650
E-mail: dinko.dimitrov@lrz.uni-muenchen.de

1 Introduction

The study of computational complexity in *hedonic coalition formation* games has a short history, although these issues in cooperative and non-cooperative game theory are being gradually recognized.¹ The formal model of a hedonic game was only recently introduced (cf. Banerjee et al. (2001) and Bogomolnaia and Jackson (2002)). This model consists of a finite set of players and a preference relation for each player defined over the set of all coalitions containing that player. The outcome of a hedonic game is a coalition structure, i.e., a partition of the set of players into coalitions. Thus, in a hedonic game one explicitly takes into account the dependence of an agent's utility on the identity of the members of his or her coalition as recognized in the seminal paper of Drèze and Greenberg (1980), and as it applies to many social and economic situations like the formation of social clubs, groups, societies, etc. The focus in the above mentioned works is on different stability concepts like the (*strict*) *core*, *Nash stability* and *individual stability*, and on conditions guaranteeing the existence of stable coalition structures.

Computational complexity issues related to hedonic games in a general setting are studied by Ballester (2004). As shown by this author, the problems to decide whether for a hedonic game there exists at least one core stable, Nash stable, or individually stable partition are *NP-complete*. In a less general setting (i.e., with some preference restrictions), Cechlarová and Hajduková (2002, 2004) study hedonic games, where the ranking over coalitions for each player is guided either by his most preferred member of the

¹ For complexity considerations in cooperative games the reader is referred, among many others, to Faigle et al. (1997) and Faigle et al. (1998). With respect to non-cooperative games, see Baron et al. (2008), Ben-Porath (1990), Gilboa (1988), Gilboa and Zemel (1989), Koller and Megiddo (1992), and Koller et al. (1996).

group or by his least preferred member of the group, and consider computational complexity issues with respect to core related concepts. Recently, Dimitrov et al. (2006) also study the computational complexity for finding a core element in hedonic games; in particular, preference profiles based on aversion to enemies that constitute a small subdomain of the domain of *additive preferences* (players' preferences are represented by an additive utility function) are considered by these authors and it is shown that finding a core member for such games is NP-hard. In addition, Sung and Dimitrov (2007a) study the problem of core membership testing for hedonic games which is to decide whether a coalition structure belongs to the core of the game and show that this problem is *co-NP complete* when players' preferences are additive; indeed, the co-NP completeness is shown by a reduction to hedonic games in which players' preferences are based on aversion to enemies.

Notice that the existence of an additive utility function defined on the player set allows each player to easily calculate his or her utility from joining a certain coalition. Moreover, each such game can be described by n^2 numbers, where n is the number of the players in the game. Hence, the computational task seems to become less demanding since the input size of the problem is polynomial of n when additivity is imposed. Despite these facts, the above mentioned works show that it is hard to find a coalition structure which is core stable. This let us conjecture a possible referential role the domain of additive hedonic games could play in the study of computational complexity issues. The aim of this paper is to present a detailed analysis of this conjecture by studying the computational complexity of several decision problems with respect to the existence of stable coalition structures in additive hedonic games. Precisely, we prove that when either core stability or strict core stability is under consideration, the existence problem of a stable coalition

structure is NP-hard in the strong sense. Furthermore, the corresponding decision problems with respect to the existence of a Nash stable coalition structure and of an individually stable coalition structure turn out to be NP-complete in the strong sense.

The rest of the paper is structured as follows. Section 2 contains basic definitions and concepts from both the theory of hedonic games and the theory of computational complexity. Sections 3 and 4 are devoted to the computational complexity of the existence problems with respect to the core and the strict core (Section 3) and with respect to Nash stable and individually stable coalition structures (Section 4). We introduce the corresponding stability notions, formulate the decision problems, and prove our results by explicitly providing the intuition when constructing the additive hedonic games in the proofs. Section 5 concludes then with some final remarks.

2 Preliminaries

Throughout this paper we will need the following basic notions and concepts.

2.1 Hedonic games

We denote by $N = \{1, 2, \dots, n\}$ a finite set of players. A *coalition* is a nonempty subset of N . For each player $i \in N$, we denote by $\mathcal{A}^i = \{X \subseteq N \mid i \in X\}$ the collection of all coalitions containing i . A collection Π of coalitions is called a *coalition structure* of N if it is a partition of N , i.e., all coalitions in Π are pairwise disjoint and their union is N . For each coalition structure Π of N and for each player $i \in N$, we denote by $\Pi(i)$ the coalition in Π containing i .

A *hedonic game* is a pair $\langle N, \succeq \rangle$ of a finite set N of players and a preference profile $\succeq = (\succeq_1, \succeq_2, \dots, \succeq_n)$. That is, in a hedonic game $\langle N, \succeq \rangle$, each

player $i \in N$ is endowed with a complete and transitive binary relation $\succeq_i$ over the coalitions in $\mathcal{A}^i$. Moreover, the preference of each player $i \in N$ over all coalition structures is assumed to be *purely hedonic*, i.e., it is completely characterized by $\succeq_i$ in such a way that, for every two coalition structures Π and Π' , each player i weakly prefers Π to Π' if and only if $\Pi(i) \succeq_i \Pi'(i)$.

A preference profile $\succeq = (\succeq_1, \succeq_2, \dots, \succeq_n)$ is called *additive* if, for each $i \in N$, there exists a real-valued function $v_i : N \rightarrow \mathbb{R}$ such that for all $X, Y \in \mathcal{A}^i$,

$$X \succeq_i Y \quad \Leftrightarrow \quad \sum_{j \in X} v_i(j) \geq \sum_{j \in Y} v_i(j).$$

A hedonic game with an additive preference profile is called an *additive hedonic game*. Observe that the preference of each player $i \in N$ can be represented by the vector $(v_i(1), v_i(2), \dots, v_i(n))$, and thus, an additive hedonic game $\langle N, \succeq \rangle$ can be described by n^2 numbers.

In the study of hedonic games one is usually interested in the existence of coalition structures that are *stable* in the sense that they are immune against either group or individual deviations (cf. Sung and Dimitrov (2007b)). We introduce these notions in Section 3 and 4, respectively.

2.2 Computational complexity

In this paper we formulate the problem of the existence of a stable coalition structure in (additive) hedonic games as a specific *decision problem* which can be described by *instances*, which are the inputs, and a *question*, which has either “YES” or “NO” as an answer. Thus, an instance for each of the decision problems we consider will be an additive hedonic game and the question will be whether there exists a stable coalition structure for the game.

Once a decision problem has been formulated, we may next ask how *fast* the problem can be correctly solved by an *algorithm*. Generally, the running

time of an algorithm increases as the size of problem instances (the games) increases. An algorithm is said to be *efficient* if its running time is bound above by a polynomial function of the size of the instance (game).

The class of problems that admit at least one efficient algorithm is denoted by P. NP is the class of all decision problems such that if the answer to a problem instance is “YES”, then there exists a certificate (string of symbols) of polynomial length so that, in polynomial time, an algorithm accepts the certificate as proof for a “YES” answer. The class NP contains P and it is generally accepted that NP and P are different classes of problems.

While many problems have been proved to be in P (generally by explicitly giving an efficient algorithm solving the problem), it is difficult to prove that a problem is *not* in P (and hence, it is hard to be solved). Instead of this, one usually shows that if the problem under consideration can be solved efficiently, then so can every member of a certain class C of problems. Such a problem is said then to be *C-hard* (and *C-complete* if, additionally, the problem has also been shown to lie in C). Furthermore, if a C-hard (C-complete) problem remains C-hard (C-complete) even if each of its instance parameters is bounded by a constant, then the problem is said to be C-hard (C-complete) in the *strong sense*. For a list of problems known to be NP-hard the reader is referred to Garey and Johnson (1979) and Ausiello et al. (1999).

Once one problem P1 has been shown to be NP-hard, the task of proving that another problem P2 is also NP-hard becomes much easier: one can do so by *reducing* P1 to P2. Informally, a reduction maps every instance of problem P1 to a corresponding instance of problem P2, in such a way that the answer to the former instance can be easily inferred from the answer to the latter instance. Moreover, the reduction itself should be efficiently

computable. If such an efficient reduction exists, then problem P1 can be seen as computationally *at most* as hard to solve as problem P2. If P1 is NP-hard, then the existence of an efficient reduction tells us that we cannot hope to find an efficient algorithm for P2 without (implicitly) finding such an efficient algorithm for the NP-hard problem P1. Notice finally, that, in order P2 to be NP-hard in the strong sense, the following two requirements have to be satisfied: (1) each of the P2's instance parameters created by the reduction should be bounded by a constant, and (2) P1 has to be NP-hard in the strong sense.

The NP-hardness in the strong sense of all decision problems we consider in the next sections are shown by efficient reductions from a problem known to be NP-complete in the strong sense. This problem is called *Exact Cover by 3 Sets* and it is defined as follows.

Exact Cover by 3 Sets (E3C):

Instance: A pair $(R, \mathcal{S})$, where R is a set and $\mathcal{S}$ is a collection of subsets of R such that $|R| = 3m$ for some positive integer m and $|s| = 3$ for each $s \in \mathcal{S}$.

Question: Is there a sub-collection $\mathcal{S}' \subseteq \mathcal{S}$ which is a partition of R ?

As an example, let us consider an instance $(R, \mathcal{S})$ with $R = \{a, b, c, d, e, f\}$ and $\mathcal{S} = \{\{a, b, c\}, \{c, d, e\}, \{d, e, f\}\}$. Then the answer to **E3C** is “YES” since the sub-collection $\mathcal{S}' = \{\{a, b, c\}, \{d, e, f\}\}$ is a partition of R .

It is known that **E3C** remains NP-complete even if each $r \in R$ occurs in *at most three* members of $\mathcal{S}$. Moreover, in order to exclude some trivial cases, we assume that each $r \in R$ occurs in at least one member of $\mathcal{S}$ (otherwise the answer to **E3C** is “NO”).

3 Group deviations and stability

Let us start by introducing two stability notions for hedonic games that are based on group deviations and consider then the corresponding decision problems.

Let $\langle N, \succeq \rangle$ be a hedonic game, Π be a coalition structure of N , and X be a coalition. We say that

- X is a *strong deviation* from Π in $\langle N, \succeq \rangle$ if $X \succ_i \Pi(i)$ for each $i \in X$;
- X is a *weak deviation* from Π in $\langle N, \succeq \rangle$ if $X \succeq_i \Pi(i)$ for each $i \in X$, and $X \succ_i \Pi(i)$ for some $i \in X$.

Moreover, we say that

- Π is *core stable* if there is no strong deviation from Π ;
- Π is *strictly core stable* if there is no weak deviation from Π .

Thus, Π is core stable if there is no coalition such that each of its members is strictly better off in comparison to his or her corresponding coalition according to Π . For strict core stability, one requires that there is no deviation with at least one member being strictly better off and no one being worse off in comparison to the corresponding coalitions in Π .

The first decision problem we will be concerned with is the following:

Existence of a core stable coalition structure (HC):

Instance: A hedonic game $\langle N, \succeq \rangle$, where N is a set of players and $\succeq$ is an additive preference profile.

Question: Is there a coalition structure which is core stable in $\langle N, \succeq \rangle$?

Analogously, the decision problem of the existence of a strictly core stable partition is as follows.

Existence of a strict core stable coalition structure (HS):

Instance: A hedonic game $\langle N, \succeq \rangle$, where N is a set of players and $\succeq$ is an additive preference profile.

Question: Is there a coalition structure which is strictly core stable in $\langle N, \succeq \rangle$?

The existence problem **HC** in the general setting is considered in Ballester (2004), and is shown to be NP-complete. Observe that, in the general setting, each player i 's preference is given as a binary relation over $\mathcal{A}^i$, where the cardinality of $\mathcal{A}^i$ is 2^{n-1} . Hence, the input size of the problems becomes exponential of n . However, it is not known whether **HC** and **HS** belong to NP when additivity is imposed.

Notice that the NP-hardness of either of the above problems does not imply that the other problem is also NP-hard. The reason is that, since strict core stability implies core stability, a “YES” answer to **HS** implies a “YES” answer to **HC**. However, a “NO” answer to **HS** does not necessary imply a “NO” answer to **HC** since there are core stable coalition structures which are not strictly core stable.

We show in what follows that, when an additive hedonic game is under consideration, *both* **HC** and **HS** are NP-hard in the strong sense. The proofs are based on polynomial time reductions from **E3C**. That is, for a given instance $(R, \mathcal{S})$ of **E3C**, we construct in polynomial time of $|R|$ and $|\mathcal{S}|$ an additive hedonic game $\langle N, \succeq \rangle$, in which all parameters are bounded by a constant.

The constructions of the corresponding games for **HC** and **HS** are slightly different and they are presented in Section 3.1 and 3.2, respectively. In these constructions, for each $i, j \in N$ with $i \neq j$, we define $v_i(j)$ only when it has a positive value. For all other $v_i(j)$ s (that are *not explicitly defined* in the corresponding proofs) we assume that

$$v_i(j) = \begin{cases} 0 & \text{if } j = i, \\ -(M_i + 1) & \text{otherwise,} \end{cases}$$

where $M_i = \sum_{k \in N} \max \{v_i(k), 0\}$. As a consequence, for each $i \in N$ and for each $X \in \mathcal{A}^i$, $\{i\} \succ_i X$ when $v_i(j) < 0$ for some $j \in X$. It follows then that a coalition structure Π cannot be stable in any sense when $v_i(j) < 0$ for some $i \in N$ and $j \in \Pi(i)$. Conversely, if $v_i(j) \geq 0$ for each $i \in N$ and for each $j \in \Pi(i)$, then each $X \subseteq N$ such that $v_i(j) < 0$ for some $i, j \in X$ can be neither a strong nor a weak deviation from Π .

3.1 Core stability

We start by first explaining and exemplifying how we construct an additive hedonic game from an instance $(R, \mathcal{S})$ of **E3C**, and then continue with the formal proof of the NP-hardness of **HC** as to show that the constructed hedonic game has a core stable coalition structure if and only if there is a sub-collection $\mathcal{S}' \subseteq \mathcal{S}$ which is a partition of R .

For simplicity, let us consider again the instance $(R, \mathcal{S})$ of **E3C** with $R = \{a, b, c, d, e, f\}$ and $\mathcal{S} = \{\{a, b, c\}, \{c, d, e\}, \{d, e, f\}\}$. To each element of R we first attach a 5-player hedonic game adapted from one in Bogomolnaia and Jackson (2002, Example 5). If we take $a \in R$, the hedonic game attached to a has the following structure: the player set is

$$\{\alpha_a, \beta_a, \gamma_a, \delta_a, \varepsilon_a\},$$

and players' preferences are

$$v_{\alpha_a}(\beta_a) = v_{\beta_a}(\gamma_a) = v_{\gamma_a}(\delta_a) = v_{\delta_a}(\varepsilon_a) = v_{\varepsilon_a}(\alpha_a) = 17$$

and

$$v_{\alpha_a}(\varepsilon_a) = v_{\varepsilon_a}(\delta_a) = v_{\delta_a}(\gamma_a) = v_{\gamma_a}(\beta_a) = v_{\beta_a}(\alpha_a) = 18.$$

In this game the players form a cycle such that each player likes the following player more than the previous one, and hates each of the other players. Notice that if a coalition structure contains a coalition with three or more players, then it cannot be core stable. Thus, the only possible coalitions are of size one or two, where coalitions of size two must contain consecutive players - for instance $\{\{\alpha_a\}, \{\beta_a, \gamma_a\}, \{\delta_a, \varepsilon_a\}\}$. This coalition structure is not core stable since the coalition $\{\alpha_a, \beta_a\}$ blocks it. In a similar way one can show that there is no core stable coalition structure for this game. Observe that the values 17 and 18 are not essential for the above argument, but these values play an important role in the reduction used in our proof.

Notice additionally, that the players in the coalitions $\{\beta_a, \gamma_a\}$ and $\{\delta_a, \varepsilon_a\}$ would stick together if one provides an incentive for α_a not to be interested in β_a . In order to do this, it is maybe helpful first to interpret $s = \{a, b, c\} \in \mathcal{S}$ as a committee with a being represented by α_a , b being represented by α_b , and c being represented by α_c , where α_b and α_c are the corresponding players (one should provide a similar incentive to as the one for α_a) in the 5-player games attached to $b \in R$ and $c \in R$, respectively.

One possibility to construct these incentives is to first make α_a , α_b , and α_c like each other and then let them stick together (as being representatives of the members in committee s). If we define $v_{\alpha_r}(\alpha_{r'}) = 2$ for $r, r' \in \{a, b, c\}$ with $r \neq r'$, then we accomplish only partially the task: player α_a still has an incentive to form a strong deviation with β_a since the value of the coalition

$\{\alpha_a, \alpha_b, \alpha_c\}$ for α_a is only 4. Thus, we need an additional player who helps the representatives for the members of s stick together. We denote this player by ζ_s and define $v_{\alpha_r}(\zeta_s) = 14$ and $v_{\zeta_s}(\alpha_r) = 2$ for $r \in \{a, b, c\}$ (see Fig. 1). Notice then that, in such a case, player α_a does not have an incentive to form a strong deviation with β_a since his utility from being a member of $\{\alpha_a, \alpha_b, \alpha_c, \zeta_s\}$ equals to 18, while his utility from being together with β_a equals to 17.

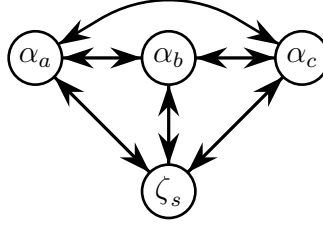


Figure 1:

Observe finally, that α_c is also a representative for c in the committee $s' = \{c, d, e\}$; thus, one has also to guarantee that there is no incentive for α_c to be part of a strong deviation by the coalition $\{\alpha_c, \alpha_d, \alpha_e, \zeta_{s'}\}$. For this, we make α_c being indifferent between representing c in s and representing c in s' by defining $v_{\alpha_r}(\zeta_s) = 14$ and $v_{\zeta_s}(\alpha_r) = 2$ for each $s \in \mathcal{S}$ and for each $r \in s$. The result of this construction is that any representative of an element in a committee belonging to $\mathcal{S}' = \{\{a, b, c\}, \{d, e, f\}\}$ ($\mathcal{S}'$ is a partition of R) has no incentive to strongly deviate together with representatives of some elements in committees belonging to $\mathcal{S} \setminus \mathcal{S}'$.

We are ready now to present the formal proof of our first result.

Theorem 1 ***HC** is NP-hard in the strong sense.*

Proof. Let $(R, \mathcal{S})$ be an instance of **E3C** such that each $r \in R$ occurs in at

most three members of $\mathcal{S}$. From $(R, \mathcal{S})$, an instance of **HC**, i.e., an additive hedonic game $\langle N, \succeq \rangle$, is constructed in polynomial time of $|R|$ and $|\mathcal{S}|$.

Let

$$\begin{aligned} N &= \{ \alpha_r, \beta_r, \gamma_r, \delta_r, \varepsilon_r \mid r \in R \} \\ &\cup \{ \zeta_s \mid s \in \mathcal{S} \}. \end{aligned}$$

Observe that $|N| = 5|R| + |\mathcal{S}|$. Players' preferences are defined as follows.

- For each $s \in \mathcal{S}$ and for each $r \in s$, $v_{\alpha_r}(\zeta_s) = 14$ and $v_{\zeta_s}(\alpha_r) = 2$.
- For each $s \in \mathcal{S}$ and for each $r, r' \in s$ with $r \neq r'$, $v_{\alpha_r}(\alpha_{r'}) = v_{\alpha_{r'}}(\alpha_r) = 2$.
- For each $r \in R$, $v_{\alpha_r}(\beta_r) = v_{\beta_r}(\gamma_r) = v_{\gamma_r}(\delta_r) = v_{\delta_r}(\varepsilon_r) = v_{\varepsilon_r}(\alpha_r) = 17$ and $v_{\alpha_r}(\varepsilon_r) = v_{\varepsilon_r}(\delta_r) = v_{\delta_r}(\gamma_r) = v_{\gamma_r}(\beta_r) = v_{\beta_r}(\alpha_r) = 18$.

In the following, we show that there exists a sub-collection $\mathcal{S}' \subseteq \mathcal{S}$ which is a partition of R if and only if there exists a core stable coalition structure Π in the constructed additive hedonic game.

($\Rightarrow$) Suppose there exists a sub-collection $\mathcal{S}' \subseteq \mathcal{S}$ which is a partition of R . Then, consider the following partition of N .

$$\begin{aligned} \Pi &= \{ \{ \zeta_s \} \cup \{ \alpha_r \mid r \in s \} \mid s \in \mathcal{S}' \} \\ &\cup \{ \{ \zeta_s \} \mid s \in \mathcal{S} \setminus \mathcal{S}' \} \\ &\cup \{ \{ \beta_r, \gamma_r \}, \{ \delta_r, \varepsilon_r \} \mid r \in R \}. \end{aligned}$$

Observe that $v_i(j) \geq 0$ for each $i \in N$ and for each $j \in \Pi(i)$. By assumption, $X \subseteq N$ such that $v_i(j) < 0$ for some $i, j \in X$ cannot be a strong (or weak) deviation from Π .

For each $s \in \mathcal{S}'$,

$$\{i \in N \mid v_{\zeta_s}(i) \geq 0\} = \{\zeta_s\} \cup \{\alpha_r \mid r \in s\}.$$

Thus, for each $s \in \mathcal{S}'$, there is no strong (or weak) deviation from Π containing ζ_s .

For each $s \in \mathcal{S} \setminus \mathcal{S}'$, the existence of a strong deviation X from Π containing ζ_s requires the inequality

$$\sum_{i \in X} v_{\zeta_s}(i) > \sum_{i \in \Pi(\zeta_s)} v_{\zeta_s}(i)$$

to be satisfied. Thus, there should exist $r \in s$ with $\alpha_r \in X$. However, we have $\sum_{i \in \Pi(\alpha_r)} v_{\alpha_r}(i) = 18$ for each $r \in R$, and $\sum_{i \in X} v_{\alpha_r}(i) \leq 18$ for each $X \subseteq \{\zeta_s\} \cup \{\alpha_r \mid r \in s\}$. Thus, for each $s \in \mathcal{S} \setminus \mathcal{S}'$, there is no strong deviation from Π containing ζ_s .

We show next that there is no strong deviation from Π containing any of the players β_r , γ_r , δ_r , and ε_r . By definition, we have

$$\sum_{i \in \Pi(\gamma_r)} v_{\gamma_r}(i) = \sum_{i \in \Pi(\varepsilon_r)} v_{\varepsilon_r}(i) = 18$$

and for all $X \subseteq N$, the values $\sum_{i \in X} v_{\gamma_r}(i)$ and $\sum_{i \in X} v_{\varepsilon_r}(i)$ are at most 18. Moreover, the existence of $X \subseteq N$ with

$$\sum_{i \in X} v_{\beta_r}(i) > \sum_{i \in \Pi(\beta_r)} v_{\beta_r}(i)$$

would imply $X = \{\beta_r, \alpha_r\}$, and the existence of $X' \subseteq N$ with

$$\sum_{i \in X'} v_{\delta_r}(i) > \sum_{i \in \Pi(\delta_r)} v_{\delta_r}(i)$$

would imply $X' = \{\delta_r, \gamma_r\}$. However,

$$v_{\alpha_r}(\alpha_r) + v_{\alpha_r}(\beta_r) = 17 < 18 = \sum_{i \in \Pi(\alpha_r)} v_{\alpha_r}(i)$$

and

$$v_{\gamma_r}(\gamma_r) + v_{\gamma_r}(\delta_r) = 17 < 18 = \sum_{i \in \Pi(\gamma_r)} v_{\gamma_r}(i).$$

Thus, there is no strong (or weak) deviation from Π containing anyone of β_r , γ_r , δ_r , and ε_r .

Recall that each $r \in R$ occurs in at most three members of $\mathcal{S}$. Then,

$$|\{\alpha_{r'} \mid r' \in R, v_{\alpha_r}(\alpha_{r'}) > 0\}| \leq 6$$

and thus, for each $X \subseteq \{\alpha_{r'} \mid r' \in R\}$,

$$\sum_{i \in X} v_{\alpha_r}(i) \leq 12 < 18 = \sum_{i \in \Pi(\alpha_r)} v_{\alpha_r}(i).$$

We conclude that $X \subseteq \{\alpha_r \mid r \in R\}$ cannot be a strong (or weak) deviation from Π . Therefore, Π is core stable.

($\Leftarrow$) Suppose there exists a core stable coalition structure Π for the constructed additive hedonic game. First, observe that we have $\Pi(\alpha_r) \neq \{\alpha_r, \beta_r\}$ for each $r \in R$; otherwise, $\Pi(\alpha_r) = \{\alpha_r, \beta_r\}$ implies either

- $\Pi(\varepsilon_r) = \{\varepsilon_r, \delta_r\}$ and $\Pi(\gamma_r) = \{\gamma_r\}$ in which case $\{\delta_r, \gamma_r\}$ becomes a strong (and weak) deviation from Π , or
- $\Pi(\varepsilon_r) = \{\varepsilon_r\}$ in which case $\{\alpha_r, \varepsilon_r\}$ is a strong (and weak) deviation from Π .

Similarly, we have $\Pi(\alpha_r) \neq \{\alpha_r, \varepsilon_r\}$. Moreover, $\sum_{i \in \Pi(\alpha_r)} v_{\alpha_r}(i) \geq 17$ for each $r \in R$; otherwise $\{\alpha_r, \beta_r\}$ becomes a strong (and weak) deviation from Π . It follows that, for each $r \in R$, there exists $s \in \mathcal{S}$ such that $\Pi(\alpha_r) = \{\zeta_s\} \cup \{\alpha_r \mid r \in s\}$. Therefore

$$\{s \in \mathcal{S} \mid \{\alpha_r \mid r \in s\} \subseteq \Pi(\zeta_s)\}$$

is a partition of R . ■

3.2 Strict core stability

Consider again the instance $(R, \mathcal{S})$ of **E3C** with $R = \{a, b, c, d, e, f\}$ and $\mathcal{S} = \{\{a, b, c\}, \{c, d, e\}, \{d, e, f\}\}$, and the game used in the proof of Theorem 1. Let $s^* = \{c, d, e\}$. Since $\mathcal{S}' = \{\{a, b, c\}, \{d, e, f\}\}$ is a partition of R , we already know that the coalition structure

$$\begin{aligned} \Pi = & \{ \{ \zeta_s \} \cup \{ \alpha_r \mid r \in s \} \mid s \in \mathcal{S}' \} \\ & \cup \{ \zeta_{s^*} \} \cup \{ \{ \beta_r, \gamma_r \}, \{ \delta_r, \varepsilon_r \} \mid r \in R \} \end{aligned}$$

is core stable. However, Π is not strictly core stable since the coalition $\{\alpha_c, \alpha_d, \alpha_e, \zeta_{s^*}\}$ is a weak deviation from Π : α_c, α_d , and α_e are all indifferent between $\{\alpha_c, \alpha_d, \alpha_e, \zeta_{s^*}\}$ and their corresponding coalitions according to Π ; and ζ_{s^*} strictly prefers $\{\alpha_c, \alpha_d, \alpha_e, \zeta_{s^*}\}$ over $\Pi(\zeta_{s^*}) = \{\zeta_{s^*}\}$.

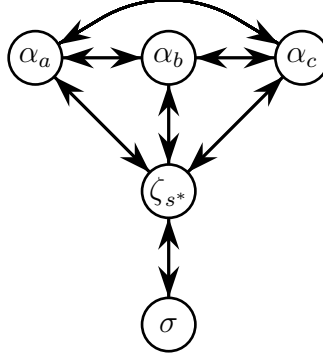


Figure 2:

Hence, when constructing the game in the proof of our next result, we have to eliminate all players' incentive for becoming members of a weak deviation. We do this by introducing $|\mathcal{S} \setminus \mathcal{S}'|$ new players in the game constructed in the previous section. Each of these new players likes and is liked only by the ζ_s -players, where the value of the corresponding utility function is 6.

The reason for selecting this value is that we are going to bring together in a coalition a player as ζ_{s^*} above with a newly introduced player σ ; for $(R, \mathcal{S})$ as above, σ is the only new player added to the game (see Fig. 2).

Then, both coalitions $\{\zeta_{s^*}, \sigma\}$ and $\{\alpha_c, \alpha_d, \alpha_e, \zeta_{s^*}\}$ would have value 6 for ζ_{s^*} (recall that for each $s \in \mathcal{S}$ and for each $r \in s$, $v_{\zeta_s}(\alpha_r) = 2$). That is, the strong incentive for player ζ_{s^*} has been eliminated.

Theorem 2 *HS is NP-hard in the strong sense.*

Proof. In addition to the additive hedonic game constructed in Theorem 1, $\ell = |\mathcal{S}| - |R|/3 = |\mathcal{S}| - m$ new players $\sigma_1, \sigma_2, \dots, \sigma_\ell$ are included in the game. The preferences related to these newly added players are as follows.

- For each $s \in \mathcal{S}$ and for each $1 \leq k \leq \ell$, $v_{\zeta_s}(\sigma_k) = v_{\sigma_k}(\zeta_s) = 6$.

Observe that the number of players remains polynomial of $|R|$ and $|\mathcal{S}|$, and all parameters are bounded above by a constant. Again, we show that there exists a sub-collection $\mathcal{S}' \subseteq \mathcal{S}$ which is a partition of R if and only if there exists a core stable coalition structure Π .

($\Rightarrow$) Suppose there exists a sub-collection $\mathcal{S}' \subseteq \mathcal{S}$ which is a partition of R . Let $\{s_1, s_2, \dots, s_\ell\} = \mathcal{S} \setminus \mathcal{S}'$. Then, consider the following partition of N .

$$\begin{aligned} \Pi &= \{ \{\zeta_s\} \cup \{\alpha_r \mid r \in s\} \mid s \in \mathcal{S}' \} \\ &\quad \cup \{ \{\zeta_{s_1}, \sigma_1\}, \{\zeta_{s_2}, \sigma_2\}, \dots, \{\zeta_{s_\ell}, \sigma_\ell\} \} \\ &\quad \cup \{ \{\beta_r, \gamma_r\}, \{\delta_r, \varepsilon_r\} \mid r \in R \}. \end{aligned}$$

As shown in the proof of Theorem 1, there is no weak deviation from Π containing anyone of $\alpha_r, \beta_r, \gamma_r, \delta_r$, and ε_r for all $r \in R$. Moreover, for each $s \in \mathcal{S}$ and for each $X \subseteq N$,

$$\sum_{i \in X} v_{\zeta_s}(i) > 6 \text{ only if } v_i(j) < 0 \text{ for some } i, j \in X,$$

and $\sum_{i \in X} v_{\zeta_s}(i) = 6$ only if either

- $X = \{\zeta_s\} \cup \{\alpha_r \mid r \in s\}$, or
- $X = \{\zeta_s, \sigma_k\}$ for some $1 \leq k \leq \ell$.

Notice that, for each $X \subseteq N$,

$$\sum_{i \in \Pi(\alpha_r)} v_{\alpha_r}(i) \geq \sum_{i \in X} v_{\alpha_r}(i)$$

and

$$\sum_{i \in \Pi(\sigma_k)} v_{\sigma_k}(i) \geq \sum_{i \in X} v_{\sigma_k}(i).$$

Thus, for each $s \in S$, there is no weak deviation from Π containing ζ_s . Therefore Π is strictly core stable.

($\Leftarrow$) Suppose there exists a core stable coalition structure Π for the constructed additive hedonic game. By the same argument as in the proof of Theorem 1,

$$\{s \in \mathcal{S} \mid \{\alpha_r \mid r \in s\} \subseteq \Pi(\zeta_s)\}$$

is a partition of R . ■

4 Individual deviations and stability

Now let us introduce the stability concepts for hedonic games, which are based on individual deviations. Let $\langle N, \succeq \rangle$ be a hedonic game, and Π be a coalition structure of N . We say that,

- Π is a *Nash stable* in $\langle N, \succeq \rangle$ if, for each $i \in N$ and for each $X \in \Pi \cup \{\emptyset\}$,
 - $\Pi(i) \succeq_i X \cup \{i\}$

- Π is a *individually stable* in $\langle N, \succeq \rangle$ if, for each $i \in N$ and for each $X \in \Pi \cup \{\emptyset\}$,
 - $\Pi(i) \succeq_i X \cup \{i\}$, or
 - there exists $j \in X$ such that $X \cup \{i\} \prec_j X$.

In other words, Π is Nash stable if no player is strictly better off by either staying alone or by moving to another coalition in Π . Individual stability additionally requires that the coalitions in Π do not accept entering members who make some player worse off.

The first decision problem we consider in this section is the following:

Existence of a Nash stable coalition structure (HN):

Instance: A hedonic game $\langle N, \succeq \rangle$, where N is a set of players and $\succeq$ is an additive preference profile.

Question: Is there a coalition structure which is Nash stable in $\langle N, \succeq \rangle$?

Analogously, the decision problem of the existence of an individually stable coalition structure is as follows.

Existence of an individually stable coalition structure (HI):

Instance: A hedonic game $\langle N, \succeq \rangle$, where N is a set of players and $\succeq$ is an additive preference profile.

Question: Is there a coalition structure which is individually stable in $\langle N, \succeq \rangle$?

The existence problem **HN** and **HI** in the general setting is considered in Ballester (2004), and are shown to be NP-complete. Notice again that the

NP-hardness of either of the above problems does not imply that the other problem is also NP-hard. The reason is similar to the one about **HC** and **HS** from the previous section and it is based on the fact that Nash stability implies individual stability but the reverse implication does not hold.

We show in what follows that, when an additive hedonic game is under consideration, *both* **HN** and **HI** are NP-complete in the strong sense². For this, we first show that these problems lie in NP.

Lemma 1 ***HN** and **HI** belong to NP.*

Proof. In order to show that **HN** belongs to NP, it suffices to provide a polynomial time algorithm for the following test.

- For a given hedonic game $\langle N, \succeq \rangle$ and a given coalition structure Π , test whether Π is Nash stable in $\langle N, \succeq \rangle$.

This test can be done in an obvious way, i.e., test whether $\Pi(i) \succeq_i X \cup \{i\}$ for each $i \in N$ and for each $X \in \Pi \cup \{\emptyset\}$. Observe that the test whether $\Pi(i) \succeq_i X \cup \{i\}$ can be in $O(n)$ time³, because players' preferences are additive. From $|\Pi| \leq n$, the test, whether Π is Nash stable in $\langle N, \succeq \rangle$, can be done in $O(n^3)$ time.

Similarly, **HI** belongs to NP, because for a given hedonic game $\langle N, \succeq \rangle$ and a given coalition structure Π , the test, whether Π is individually stable in $\langle N, \succeq \rangle$, can be done in $O(n^4)$ time. ■

In order to show the NP-hardness of these two existence problems we use polynomial time reductions again from **E3C**.

² Thus, our result is stronger than the corresponding result in Olsen (2007). In that work, the NP-completeness of **HN** is shown by reduction from a problem (PARTITION) which is known to be NP-hard but not in the strong sense.

³ That is, the running time of the test is bounded from above by n , up to a constant factor.

4.1 Nash stability

More precisely, for the problem **HN**, we use in our reduction a similar trick to the one used in the construction of the games for **HC** and **HS**. That is, given an instance $(R, \mathcal{S})$ of **E3C**, we attach a game with no Nash stable coalition structure to each element of R and add an additional player for each $s \in \mathcal{S}$.

The game we attach to each $r \in R$ is very simple: the player set is $\{\alpha_r, \beta_r\}$ with $v_{\alpha_r}(\beta_r) < 0$ and $v_{\beta_r}(\alpha_r) > 0$. Observe that there are only two coalition structures $\{\{\alpha_r\}, \{\beta_r\}\}$ and $\{\{\alpha_r, \beta_r\}\}$, and neither of them is Nash stable, because player α_r prefers to be alone (i.e., $\{\alpha_r\} \succ_{\alpha_r} \{\alpha_r, \beta_r\}$) but player β_r prefers to be with player α_r (i.e., $\{\alpha_r, \beta_r\} \succ_{\beta_r} \{\alpha_r\}$). Thus, when defining the corresponding utilities with respect to the additional player ζ_s with $r \in s$, the task will be to provide a good incentive for β_r to stick together with ζ_s . Notice that such an incentive should be given to each $\beta_{r'}$ with $r' \in s$ and hence, any two players β_r and $\beta_{r'}$ with $r, r' \in s$ and $r \neq r'$ should also like each other; otherwise, one of these players would be strictly better off by staying single no matter how much he likes ζ_s .

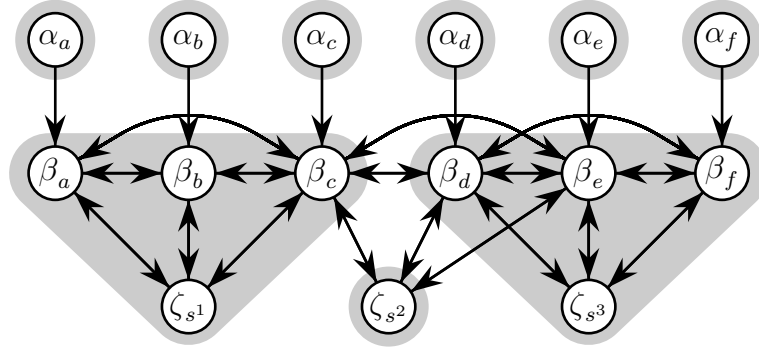


Figure 3:

Having done this, we can consider the coalition structure shown in Fig. 3. It consists of

- all α_r players being single,
- the coalitions, each of which consisting of all β_r players together with ζ_s , where $r \in s$ and $s \in \mathcal{S}'$, and
- all $\zeta_{s'}$ players for $s' \in \mathcal{S} \setminus \mathcal{S}'$ being single.

As we show next, we can use the above construction as to prove the NP-hardness of **HN**.

Theorem 3 ***HN** is NP-Complete.*

Proof. Let $(R, \mathcal{S})$ be an instance of **E3C**. From $(R, \mathcal{S})$, an instance of **HN**, i.e., an additive hedonic game $\langle N, \succeq \rangle$, is constructed in polynomial time of $|R|$ and $|\mathcal{S}|$.

Let $N = \{\alpha_r, \beta_r \mid r \in R\} \cup \{\zeta_s \mid s \in \mathcal{S}\}$. Players' preference are defined as follows.

- $v_{\beta_r}(\beta_{r'}) = 2$ for all $r, r' \in R$ with $r \neq r'$,
- $v_{\beta_r}(\zeta_s) = 2|R|$ and $v_{\zeta_s}(\beta_r) = 1$ if $r \in s$,
- $v_{\beta_r}(\alpha_r) = 2|R| + 3$ for all $r \in R$.

The remaining $v_i(j)$ s are negative, and each of which is defined by

$$v_i(j) = -(6|R| + 2).$$

Observe that $\sum_{j \in N} \max\{v_i(j), 0\} \leq 6|R| + 1$ for each $i \in N$. Thus, $\{i\} \succ_i X$ if $v_i(j) < 0$ for some $j \in X \subseteq N$, i.e., X is not individually rational for i . In other words,

- a coalition X is individually rational if and only if $v_i(j) > 0$ for all $i, j \in X$.

Notice that a coalition structure Π is Nash stable only if every coalition in Π is individually rational. Hence, only individually rational coalitions are considered in the following.

Now, we have the following observations.

- for each $X \in \mathcal{A}^{\alpha_r}$ with $X \neq \{\alpha_r\}$, X is not individually rational since $\{\alpha_r\} \succ_{\alpha_r} X$.

For each $s \in \mathcal{S}$, let $P_s = \{\beta_r \mid r \in s\} \cup \{\zeta_s\}$. Then,

- for each $s \in \mathcal{S}$, $X \in \mathcal{A}^{\zeta_s}$ is individually rational only if $X \subseteq P_s$, since $\{i \in N \mid v_{\zeta_s}(i) > 0\} = P_s$; and
- for each $r \in s$ and for each $X \in \mathcal{A}^{\beta_r}$, X is individually rational and $X \succeq_{\beta_r} P_s$ if and only if $X = P_s$.

Suppose now that there exists a sub-collection $\mathcal{S}' \subseteq \mathcal{S}$ which is a partition of R . Then, consider the following coalition structure.

$$\{\{\alpha_r\} \mid r \in R\} \cup \{P_s \mid s \in \mathcal{S}'\} \cup \{\{\zeta_s\} \mid s \in \mathcal{S} \setminus \mathcal{S}'\}.$$

Observe that all α_r s and all β_r s have no incentive to deviate. Moreover, for each $s \in \mathcal{S}$, each $X \in \Pi \setminus \{\Pi(\zeta_s)\}$ contains a member i such that $v_{\zeta_s}(i) < 0$, and thus, each ζ_s has no incentive to deviate. Hence, Π is Nash stable.

Suppose next that there exists a coalition structure Π which is Nash stable in the above additive hedonic game. From the above observations, we have $\Pi(\alpha_r) = \{\alpha_r\}$ for each $r \in R$. It follows that, for each $r \in R$, we have $\Pi(\beta_r) \subseteq \{\beta_{r'} \mid r' \in R\}$ or $\Pi(\beta_r) \subseteq P_s$ for some $s \in \mathcal{S}$; otherwise $\Pi(\beta_r)$ is not individually rational. Moreover, if $\Pi(\beta_r) \neq P_s$ for some $s \in \mathcal{S}$, we have

$$\sum_{i \in \Pi(\beta_r)} v_{\beta_r}(i) \leq 2|R| + 2 < v_{\beta_r}(\alpha_r)$$

and thus, $\Pi(\alpha_r) \cup \{\beta_r\} \succ_{\beta_r} \Pi(\beta_r)$, which implies that Π is not Nash stable. Therefore, for each $r \in R$, there exists $s \in \mathcal{S}$ such that $\Pi(\beta_r) = P_s$. In other words, we have

$$\bigcup_{s \in \mathcal{S}'} s = R,$$

where

$$\mathcal{S}' = \{s \in \mathcal{S} \mid P_s \in \Pi\}.$$

Since Π is a partition of N , $\mathcal{S}'$ is a partition of R . Hence, from Lemma 1, we can conclude that **HN** is NP-complete. ■

Notice that in the proof of the above theorem we did not use the fact that **E3C** remains NP-complete even when restricted to instances in which each $r \in R$ occurs in at most three members of $\mathcal{S}$. By considering those instances satisfying such a restriction, we are able to show a stronger result.

Theorem 4 ***HN** is NP-complete in the strong sense.*

Proof. Let $(R, \mathcal{S})$ be an instance of **E3C** such that each $r \in R$ occurs in at most three members of $\mathcal{S}$. From $(R, \mathcal{S})$, an instance of **HN** is constructed as follows.

Let $N = \{\alpha_r, \beta_r \mid r \in R\} \cup \{\zeta_s \mid s \in \mathcal{S}\}$. Players' preferences are defined as follows.

- $v_{\beta_r}(\beta_{r'}) = 2$ if $r \neq r'$ and $r, r' \in s$ for some $s \in \mathcal{S}$.
- $v_{\beta_r}(\zeta_s) = 13$ and $v_{\zeta_s}(\beta_r) = 1$ if $r \in s$,
- $v_{\beta_r}(\alpha_r) = 16$ for all $r \in R$.

All the remaining $v_i(j)$ s are negative, and each of them is defined by

$$v_i(j) = -42.$$

By a similar argument to the one in the proof of Theorem 3, it can be verified that such an additive hedonic game has a Nash stable coalition structure if and only if there exists a sub-collection $\mathcal{S}' \subseteq \mathcal{S}$ which is a partition of R . Observe that each $v_i(j)$ is bounded above and below by constants. Therefore, **HN** is NP-complete in the strong sense. ■

4.2 Individual stability

In the reduction of **HI** from **E3C**, we use the following trick. Let w and z be real numbers satisfying $w > z > 0$ and let us consider the following additive hedonic game with 5 players.

- $N = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$,
- $v_{\alpha_1}(\alpha_2) = v_{\alpha_2}(\alpha_3) = v_{\alpha_3}(\alpha_4) = v_{\alpha_4}(\alpha_5) = v_{\alpha_5}(\alpha_1) = w$,
- $v_{\alpha_1}(\alpha_5) = v_{\alpha_2}(\alpha_1) = v_{\alpha_3}(\alpha_2) = v_{\alpha_4}(\alpha_3) = v_{\alpha_5}(\alpha_4) = z$, and
- all the remaining $v_{\alpha_i}(\alpha_j)$ s are defined by $v_{\alpha_i}(\alpha_j) = -(w + z + 1)$.

It is shown that this game does not have an individually stable coalition structure (cf. Example 5 in Bogomolnaia and Jackson (2002)). Now suppose a player β is introduced such that

- $v_{\beta}(\alpha_1) = v_{\alpha_1}(\beta) = z$,
- $v_{\alpha_j}(\beta) = v_{\beta}(\alpha_j) = -(w + z + 1)$ for $j \in \{2, 3, 4, 5\}$.

Then, this hedonic game has an individually stable coalition structure, namely $\{\{\alpha_1, \beta\}, \{\alpha_2, \alpha_3\}, \{\alpha_4, \alpha_5\}\}$.

The structure of the above game can be generalized as follows. Let ℓ be a positive integer with $\ell > 2$. Define the player set to be $N = A \cup B$ with

- $A = \{\alpha_j^k \mid j \in \{1, 2, 3, 4, 5\}, k \in \{1, \dots, \ell - 1\}\}$ and $B = \{\beta_k \mid k \in \{1, \dots, \ell\}\}$,

and players' preferences be as follows. For each $k \in \{1, 2, \dots, \ell - 1\}$,

- $v_{\alpha_1^k}(\alpha_2^k) = v_{\alpha_2^k}(\alpha_3^k) = v_{\alpha_3^k}(\alpha_4^k) = v_{\alpha_4^k}(\alpha_5^k) = v_{\alpha_5^k}(\alpha_1^k) = w$,
- $v_{\alpha_1^k}(\alpha_5^k) = v_{\alpha_2^k}(\alpha_1^k) = v_{\alpha_3^k}(\alpha_2^k) = v_{\alpha_4^k}(\alpha_3^k) = v_{\alpha_5^k}(\alpha_4^k) = z$, and

For each $k \in \{1, 2, \dots, \ell - 1\}$ and $k' \in \{1, 2, \dots, \ell\}$,

- $v_{\beta_{k'}}(\alpha_1^k) = v_{\alpha_1^k}(\beta_{k'}) = z$,
- $v_{\alpha_j^k}(\beta_{k'}) = v_{\beta_{k'}}(\alpha_j^k) = -(w + z + 1)$ for $j \in \{2, 3, 4, 5\}$.

Moreover, each of the remaining $v_i(j)$ s is defined as $v_i(j) = -(w + z + 1)$. For this hedonic game, a coalition structure Π is individually stable only if, for each $k \in \{1, 2, \dots, \ell - 1\}$, there exists $k' \in \{1, 2, \dots, \ell\}$ such that $\Pi(\alpha_1^k) = \{\alpha_1^k, \beta_{k'}\}$.

Let us now explain how we use the above facts in the game defined in the proof of our last result. We start by attaching a β -player β_{rs} to each $r \in R$ and $s \in \mathcal{S}$ with $r \in s$, and a ζ_s -player to each $s \in \mathcal{S}$. The corresponding preferences are defined in such a way that there is a unique individually stable partition and each of its elements is of the form $P_s = \{\beta_{rs} \mid r \in s\} \cup \{\zeta_s\}$. Notice however, that this specification of the game and the corresponding individually stable coalition structure do not always imply that there is a subcollection $\mathcal{S}' \subseteq \mathcal{S}$ which is a partition of R ; the reason is that there might exist two (different) players β_{rs} and $\beta_{r's'}$ with $r = r'$ and $s \neq s'$ since an element of R may belong to more than one element of $\mathcal{S}$. Thus, we have to redefine the game such that for each $r \in R$, there exists *at most one* $s \in \mathcal{S}$ with $r \in s$ and P_s being an element of an individually stable partition. Roughly speaking,

we have to get rid of all but one of the players from $\{\beta_{rs} \mid s \in \mathcal{S} \text{ s.t. } r \in s\}$ as being members of a coalition in an individually stable partition. For this, we use the 5-player game defined above and illustrate the procedure in Fig. 4 for $\{\beta_{r's'} \mid s \in \mathcal{S} \text{ s.t. } r \in s\} = \{\beta_{rs^1}, \beta_{rs^2}, \beta_{rs^3}\}$.

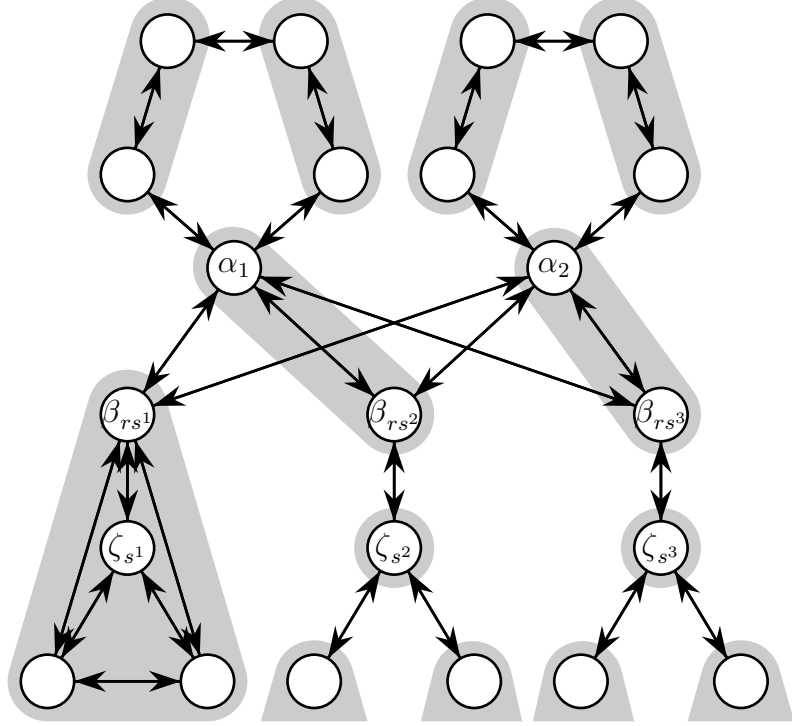


Figure 4:

Let the players we would like to get rid off be β_{rs^2} and β_{rs^3} . Let us then add β_{rs^2} to the above 5-player game and do the same operation for β_{rs^3} with respect to an analogous 5-player game. Then, in an individually stable partition, β_{rs^2} and β_{rs^3} will be attracted by the corresponding α -players. The final result of this construction, together with the specification of players' preferences, is that it singles out, for each $r \in R$, only one $s \in \mathcal{S}$ ($s = s^1$) with $r \in s$ that guarantees the existence of a sub-collection of $\mathcal{S}$

which partitions R .

Theorem 5 ***HI** is NP-complete in the strong sense.*

Proof. Let $(R, \mathcal{S})$ be an instance of **E3C**. In order to avoid trivial cases, we assume that each $r \in R$ is included in at least one member of $\mathcal{S}$. From $(R, \mathcal{S})$, an instance of **HI** is constructed as follows.

Let $M = \{\beta_{rs} \mid s \in \mathcal{S}, r \in s\} \cup \{\zeta_s \mid s \in \mathcal{S}\}$ be a set of $4|\mathcal{S}|$ players. Players' preferences are defined as follows.

- $v_{\beta_{rs}}(\zeta_s) = 1$ and $v_{\zeta_s}(\beta_{rs}) = 1$.
- $v_{\beta_{rs}}(\beta_{r's}) = 1$ if $r \neq r'$, and
- $v_{\beta_{rs}}(\beta_{rs'}) = 0$ if $s \neq s'$.

All the remaining $v_i(j)$ s with $i, j \in M$ is defined by $v_i(j) = -4$. Let $P_s = \{\beta_{rs} \mid r \in s\} \cup \{\zeta_s\}$ for each $s \in \mathcal{S}$. Observe that among players belonging to M , individually stable coalition structure exists and is unique, namely,

$$\Pi = \{P_s \mid s \in \mathcal{S}\}.$$

Now we introduce more players in order to have an additive hedonic game such that a coalition structure Π is individually stable if and only if $\{s \in \mathcal{S} \mid P_s \in \Pi\}$ is a partition of R . In other words, an additive hedonic game is constructed in such a way that a coalition structure Π is not individually stable if

- there exist $s, s' \in \mathcal{S}$ such that $s \neq s'$, $s \cap s' \neq \emptyset$, and $P_s, P_{s'} \in \Pi$, or
- there exists $r \in R$ such that $\Pi(\beta_{rs}) \neq P_s$ for each $s \in \mathcal{S}$.

Suppose $r \in R$ is included in ℓ members of $\mathcal{S}$, where $\ell > 1$. Then, by using the trick mentioned above, and introducing $5(\ell - 1)$ new players with $w = 2$ and $z = 1$, a coalition structure Π' is individually stable only if

- for each $r \in R$, there exists at most one $s \in \mathcal{S}$ such that $r \in s$ and $P_s \in \Pi'$.

Moreover, we have $\{\beta_{rs}, \zeta_s\} \succ_{\beta_{rs}} \{\beta_{rs}\}$, and for each $X \in \mathcal{A}^{\zeta_s}$, $X \succeq_{\zeta_s} \{\zeta_s\}$ if and only if $X \subseteq P_s$. Hence, a coalition structure Π' is individually stable only if $\Pi'(\beta_{rs}) = \Pi'(\zeta_s)$ for some $s \in \mathcal{S}$ with $r \in s$. Moreover, if $\Pi'(\beta_{rs}) = \Pi'(\zeta_s) \subseteq P_s$ but $\Pi'(\zeta_s) \neq P_s$, then $\Pi'(\zeta_s) \cup \{\beta_{r's}\} \succ_{\beta_{r's}} \Pi'(\beta_{r's})$ for some $r' \in s$. It follows that a coalition structure Π' is individually stable only if

- for each $r \in R$, there exists $s \in \mathcal{S}$ such that $r \in s$ and $P_s \in \Pi'$.

Conversely, it can be verified that an individually stable coalition structure Π' exists among all coalition structures satisfying the above conditions.

Finally, the hedonic game we have constructed has $9|\mathcal{S}| - |R|$ players and each $v_i(j)$ is bounded below and above by constants. Therefore, **HI** is NP-complete in the strong sense. ■

5 Concluding remarks

We provided reductions from the NP-complete problem Exact Cover by 3 Sets that demonstrate that in additive hedonic games:

- it is NP-hard in the strong sense to determine (1) whether a core partition exists, and (2) whether a strict core stable partition exists. Moreover,

- the problem of deciding (1) whether a Nash stable partition exists, and (2) whether an individually stable partition exists are NP-complete in the strong sense.

In all reductions we used procedures with some common properties we would like to stress now. Given an instance $(R, \mathcal{S})$ of **E3C**, all additive hedonic games were constructed by respecting the following pattern. A set of players was first attached to each element of R ; these players were involved in a basic additive game with no stable coalition structure. Each of the two types of basic games (the 5-player game and the 2-player game) were selected in such a way as to have a boundary property in the sense that when adding or removing a player we were able to construct games for which a stable coalition structure do exist. Then, the operations of adding or removing a player from the basic games were done by attaching a player to each element of $\mathcal{S}$. This common pattern allowed us to derive a sub-collection of $\mathcal{S}$ which is a partition of R in order to complete the corresponding reductions.

References

- [1] Ausiello, G., P. Crescenzi, G. Gambosi, V. Kann, A. Marchetti-Spaccamela, and M. Protasi (1999): Complexity and Approximation, Springer.
- [2] Ballester, C. (2004): NP-completeness in hedonic games, Games and Economic Behavior 49, 1-30.

- [3] Banerjee, S., H. Konishi, and T. Sönmez (2001): Core in a simple coalition formation game, *Social Choice and Welfare* 18, 135-153.
- [4] Baron, R., J. Durieu, H. Haller, R. Savani, and Ph. Solal (2008): Good neighbors are hard to find: computational complexity of network formation, *Review of Economic Design* 12, 1-19.
- [5] Ben-Porath, E. (1990): The complexity of computing a best response automation in repeated games with mixed strategies, *Games and Economic Behavior* 2, 1-12.
- [6] Bogomolnaia, A. and M. O. Jackson (2002): The stability of hedonic coalition structures, *Games and Economic Behavior* 38, 201-230.
- [7] Cechlarová, K. and J. Hajduková (2004): Stable partitions with $\mathcal{W}$ -preferences, *Discrete Applied Mathematics* 138, 333-347.
- [8] Cechlarová, K. and J. Hajduková (2002): Computational complexity of stable partitions with $\mathcal{B}$ -preferences, *International Journal of Game Theory* 31, 353-364.
- [9] Dimitrov, D., P. Borm, R. Hendrickx, and S.-C. Sung (2006): Simple priorities and core stability in hedonic games, *Social Choice and Welfare* 26, 421-433.
- [10] Faigle, U., W. Kern, S.P. Feteke, and W. Hochstätter (1997): On the complexity of testing membership in the core of min-cost spanning tree games, *International Journal of Game Theory* 26, 361-366.
- [11] Faigle, U., W. Kern, and J. Kuipers (1998): Computing the nucleolus of min-cost spanning trees is NP-hard, *International Journal of Game Theory* 27, 443-450.

- [12] Garey, M. and D. Johnson (1979): Computers and Intractability. A Guide to the Theory of NP-Completeness, Freeman, New York.
- [13] Gilboa, I. (1988): The complexity of computing best-response automata in repeated games, *Journal of Economic Theory* 45, 342-352.
- [14] Gilboa, I. and E. Zemel (1989): Nash and correlated equilibria: some complexity considerations, *Games and Economic Behavior* 1, 80-93.
- [15] Koller, D. and N. Megiddo (1992): The complexity of two-person zero-sum games in extensive form, *Games and Economic Behavior* 4, 528-552.
- [16] Koller, D., N. Megiddo, and B. von Stengel (1996): Efficient computation of equilibria for extensive two-person games, *Games and Economic Behavior* 14, 247-259.
- [17] Olsen, M. (2007): Nash stability in additive separable hedonic games is NP-hard, *Lecture Notes in Computer Science* 4497, 598-605.
- [18] Sung, S.-C. and D. Dimitrov (2007a): On core membership testing for hedonic coalition formation games, *Operations Research Letters* 35, 155-158.
- [19] Sung, S.-C. and D. Dimitrov (2007b): On myopic stability concepts for hedonic games, *Theory and Decision* 62, 31-45.

NOTE DI LAVORO DELLA FONDAZIONE ENI ENRICO MATTEI

Fondazione Eni Enrico Mattei Working Paper Series

Our Note di Lavoro are available on the Internet at the following addresses:

<http://www.feem.it/Feem/Pub/Publications/WPapers/default.htm>

<http://www.ssrn.com/link/feem.html>

<http://www.repec.org>

<http://agecon.lib.umn.edu>

<http://www.bepress.com/feem/>

NOTE DI LAVORO PUBLISHED IN 2008

CCMP	1.2008	<i>Valentina Bosetti, Carlo Carraro and Emanuele Massetti: <u>Banking Permits: Economic Efficiency and Distributional Effects</u></i>
CCMP	2.2008	<i>Ruslana Palatnik and Mordechai Shechter: <u>Can Climate Change Mitigation Policy Benefit the Israeli Economy? A Computable General Equilibrium Analysis</u></i>
KTHC	3.2008	<i>Lorenzo Casaburi, Valeria Gattai and G. Alfredo Minerva: <u>Firms' International Status and Heterogeneity in Performance: Evidence From Italy</u></i>
KTHC	4.2008	<i>Fabio Sabatini: <u>Does Social Capital Mitigate Precariousness?</u></i>
SIEV	5.2008	<i>Wisdom Akpalu: <u>On the Economics of Rational Self-Medication</u></i>
CCMP	6.2008	<i>Carlo Carraro and Alessandra Sgobbi: <u>Climate Change Impacts and Adaptation Strategies In Italy. An Economic Assessment</u></i>
ETA	7.2008	<i>Elodie Rouvière and Raphaël Soubeyran: <u>Collective Reputation, Entry and Minimum Quality Standard</u></i>
IEM	8.2008	<i>Cristina Cattaneo, Matteo Manera and Elisa Scarpa: <u>Industrial Coal Demand in China: A Provincial Analysis</u></i>
IEM	9.2008	<i>Massimiliano Serati, Matteo Manera and Michele Plotegher: <u>Modeling Electricity Prices: From the State of the Art to a Draft of a New Proposal</u></i>
CCMP	10.2008	<i>Bob van der Zwaan and Reyer Gerlagh: <u>The Economics of Geological CO₂ Storage and Leakage</u></i>
KTHC	11.2008	<i>Maria Francesca Cracolici and Teodora Erika Uberti: <u>Geographical Distribution of Crime in Italian Provinces: A Spatial Econometric Analysis</u></i>
KTHC	12.2008	<i>Victor Ginsburgh, Shlomo Weber and Sheila Weyers: <u>Economics of Literary Translation. A Simple Theory and Evidence</u></i>
NRM	13.2008	<i>Carlo Giupponi, Jaroslav Mysiak and Alessandra Sgobbi: <u>Participatory Modelling and Decision Support for Natural Resources Management in Climate Change Research</u></i>
NRM	14.2008	<i>Yaella Depietri and Carlo Giupponi: <u>Science-Policy Communication for Improved Water Resources Management: Contributions of the Nostrum-DSS Project</u></i>
CCMP	15.2008	<i>Valentina Bosetti, Alexander Golub, Anil Markandya, Emanuele Massetti and Massimo Tavoni: <u>Abatement Cost Uncertainty and Policy Instrument Selection under a Stringent Climate Policy. A Dynamic Analysis</u></i>
KTHC	16.2008	<i>Francesco D'Amuri, Gianmarco I.P. Ottaviano and Giovanni Peri: <u>The Labor Market Impact of Immigration in Western Germany in the 1990's</u></i>
KTHC	17.2008	<i>Jean Gabszewicz, Victor Ginsburgh and Shlomo Weber: <u>Bilingualism and Communicative Benefits</u></i>
CCMP	18.2008	<i>Benno Torgler, María A.GarcíaValiñas and Alison Macintyre: <u>Differences in Preferences Towards the Environment: The Impact of a Gender, Age and Parental Effect</u></i>
PRCG	19.2008	<i>Gian Luigi Albano and Berardino Cesi: <u>Past Performance Evaluation in Repeated Procurement: A Simple Model of Handicapping</u></i>
CTN	20.2008	<i>Pedro Pintassilgo, Michael Finus, Marko Lindroos and Gordon Munro (lxxxiv): <u>Stability and Success of Regional Fisheries Management Organizations</u></i>
CTN	21.2008	<i>Hubert Kempf and Leopold von Thadden (lxxxiv): <u>On Policy Interactions Among Nations: When Do Cooperation and Commitment Matter?</u></i>
CTN	22.2008	<i>Markus Kinateder (lxxxiv): <u>Repeated Games Played in a Network</u></i>
CTN	23.2008	<i>Taiji Furusawa and Hideo Konishi (lxxxiv): <u>Contributing or Free-Riding? A Theory of Endogenous Lobby Formation</u></i>
CTN	24.2008	<i>Paolo Pin, Silvio Franz and Matteo Marsili (lxxxiv): <u>Opportunity and Choice in Social Networks</u></i>
CTN	25.2008	<i>Vasileios Zikos (lxxxiv): <u>R&D Collaboration Networks in Mixed Oligopoly</u></i>
CTN	26.2008	<i>Hans-Peter Weikard and Rob Dellink (lxxxiv): <u>Sticks and Carrots for the Design of International Climate Agreements with Renegotiations</u></i>
CTN	27.2008	<i>Jingang Zhao (lxxxiv): <u>The Maximal Payoff and Coalition Formation in Coalitional Games</u></i>
CTN	28.2008	<i>Giacomo Pasini, Paolo Pin and Simon Weidenholzer (lxxxiv): <u>A Network Model of Price Dispersion</u></i>
CTN	29.2008	<i>Ana Mauleon, Vincent Vannetelbosch and Wouter Vergote (lxxxiv): <u>Von Neumann-Morgenstern Farsightedly Stable Sets in Two-Sided Matching</u></i>
CTN	30.2008	<i>Rahmi İlkiliç (lxxxiv): <u>Network of Commons</u></i>
CTN	31.2008	<i>Marco J. van der Leij and I. Sebastian Buhai (lxxxiv): <u>A Social Network Analysis of Occupational Segregation</u></i>
CTN	32.2008	<i>Billand Pascal, Frachisse David and Massard Nadine (lxxxiv): <u>The Sixth Framework Program as an Affiliation Network: Representation and Analysis</u></i>
CTN	33.2008	<i>Michèle Breton, Lucia Sbragia and Georges Zaccour (lxxxiv): <u>Dynamic Models for International Environmental Agreements</u></i>

PRCG	34.2008	<i>Carmine Guerriero: <u>The Political Economy of Incentive Regulation: Theory and Evidence from US States</u></i>
IEM	35.2008	<i>Irene Valsecchi: <u>Learning from Experts</u></i>
PRCG	36.2008	<i>P. A. Ferrari and S. Salini: <u>Measuring Service Quality: The Opinion of Europeans about Utilities</u></i>
ETA	37.2008	<i>Michele Moretto and Gianpaolo Rossini: <u>Vertical Integration and Operational Flexibility</u></i>
CCMP	38.2008	<i>William K. Jaeger and Van Kolpin: <u>The Environmental Kuznets Curve from Multiple Perspectives</u></i>
PRCG	39.2008	<i>Benno Torgler and Bin Dong: <u>Corruption and Political Interest: Empirical Evidence at the Micro Level</u></i>
KTHC	40.2008	<i>Laura Onofri, Paulo A.L.D. Nunes, Jasone Cenoz and Durk Gorter: <u>Language Diversity in Urban Landscapes: An econometric study</u></i>
CTN	41.2008	<i>Michel Le Breton, Valery Makarov, Alexei Savvateev and Shlomo Weber (lxxxiv): <u>Multiple Membership and Federal Structures</u></i>
NRM	42.2008	<i>Gideon Kruseman and Lorenzo Pellegrini: <u>Institutions and Forest Management: A Case Study from Swat, Pakistan</u></i>
SIEV	43.2008	<i>Pietro Caratti and Ludovico Ferraguto: <u>Analysing Regional Sustainability Through a Systemic Approach: The Lombardy Case Study</u></i>
KTHC	44.2008	<i>Barbara Del Corpo, Ugo Gasparino, Elena Bellini and William Malizia: <u>Effects of Tourism Upon the Economy of Small and Medium-Sized European Cities. Cultural Tourists and "The Others"</u></i>
CTN	45.2008	<i>Dinko Dimitrov and Emiliya Lazarova: <u>Coalitional Matchings</u></i>
ETA	46.2008	<i>Joan Canton, Maia David and Bernard Sinclair-Desgagné: <u>Environmental Regulation and Horizontal Mergers in the Eco-industry</u></i>
ETA	47.2008	<i>Stéphane Hallegatte: <u>A Proposal for a New Prescriptive Discounting Scheme: The Intergenerational Discount Rate</u></i>
KTHC	48.2008	<i>Angelo Antoci, Paolo Russu and Elisa Ticci: <u>Structural Change, Environment and Well-being: Interactions Between Production and Consumption Choices of the Rich and the Poor in Developing Countries</u></i>
PRCG	49.2008	<i>Gian Luigi Albano, Federico Dini Roberto Zampino and Marta Fana: <u>The Determinants of Suppliers' Performance in E-Procurement: Evidence from the Italian Government's E-Procurement Platform</u></i>
CCMP	50.2008	<i>Inmaculada Martínez-Zarzoso: <u>The Impact of Urbanization on CO2 Emissions: Evidence from Developing Countries</u></i>
KTHC	51.2008	<i>Michele Moretto and Sergio Vergalli: <u>Managing Migration through Quotas: an Option-theory Perspective</u></i>
KTHC	52.2008	<i>Ugo Gasparino, Elena Bellini, Barbara Del Corpo and William Malizia: <u>Measuring the Impact of Tourism Upon Urban Economies: A Review of Literature</u></i>
ETA	53.2008	<i>Reyer Gerlagh, Snorre Kverndokk and Knut Einar Rosendahl: <u>Linking Environmental and Innovation Policy</u></i>
KTHC	54.2008	<i>Oguzhan C. Dincer and Burak Gunalp: <u>Corruption, Income Inequality, and Poverty in the United States</u></i>
PRCG	55.2008	<i>Carmine Guerriero: <u>Accountability in Government and Regulatory Policies: Theory and Evidence</u></i>
KTHC	56.2008	<i>Tanmoyee Banerjee (Chatterjee) and Nilanjana Mitra: <u>Export, Assembly-line FDI or FDI with the Possibility of Technology Diffusion: Optimal Entry Mode for Multinationals</u></i>
ETA	57.2008	<i>Xavier Pautrel: <u>Environmental Policy, Education and Growth: A Reappraisal when Lifetime Is Finite</u></i>
CCMP	58.2008	<i>Natalia Zugravu, Katrin Millock and Gérard Duchene: <u>The Factors Behind CO2 Emission Reduction in Transition Economies</u></i>
NRM	59.2008	<i>Benno Torgler, María A.García-Valiñas and Alison Macintyre: <u>Justifiability of Littering: An Empirical Investigation</u></i>
SIEV	60.2008	<i>Paolo Rosato, Anna Alberini, Valentina Zanatta and Margaretha Breil: <u>Redeveloping Derelict and Underused Historic City Areas: Evidence from a Survey of Real Estate Developers</u></i>
CTN	61.2008	<i>Ricardo Nieva: <u>Networks with Group Counterproposals</u></i>
CTN	62.2008	<i>Michael Finus and Dirk T.G. Rübbelke: <u>Coalition Formation and the Ancillary Benefits of Climate Policy</u></i>
SIEV	63.2008	<i>Elisabetta Strazzera, Elisabetta Cerchi and Silvia Ferrini: <u>A Choice Modelling Approach for Assessment of Use and Quasi-Option Values in Urban Planning for Areas of Environmental Interest</u></i>
SIEV	64.2008	<i>Paolo Rosato, Lucia Rotaris, Margaretha Breil and Valentina Zanatta: <u>Do We Care about Built Cultural Heritage? The Empirical Evidence Based on the Veneto House Market</u></i>
KTHC	65.2008	<i>Luca Petruzzellis and Antonia Rosa Guerrieri: <u>Does Network Matter in International Expansion? Evidence from Italian SMEs</u></i>
NRM	66.2008	<i>Sheila M. Olmstead and Robert N. Stavins: <u>Comparing Price and Non-price Approaches to Urban Water Conservation</u></i>
CCMP	67.2008	<i>Robert N. Stavins: <u>Addressing Climate Change with a Comprehensive U.S. Cap-and-Trade System</u></i>
CCMP	68.2008	<i>Geoffrey J. Blanford, Richard G. Richels and Thomas F. Rutherford: <u>Impact of Revised CO₂ Growth Projections for China on Global Stabilization Goals</u></i>
CCMP	69.2008	<i>Valentina Bosetti, Carlo Carraro, Alessandra Sgobbi and Massimo Tavoni: <u>Delayed Action and Uncertain Targets. How Much Will Climate Policy Cost?</u></i>
CCMP	70.2008	<i>Valentina Bosetti, Carlo Carraro and Massimo Tavoni: <u>Delayed Participation of Developing Countries to Climate Agreements: Should Action in the EU and US be Postponed?</u></i>
SIEV	71.2008	<i>Massimiliano Mazzanti, Anna Montini and Francesco Nicoli: <u>Embedding Landfill Diversion in Economic, Geographical and Policy Settings Panel based evidence from Italy</u></i>
ETA	72.2008	<i>Reyer Gerlagh and Matti Liski: <u>Strategic Resource Dependence</u></i>
CCMP	73.2008	<i>Sonia Ben Kheder and Natalia Zugravu: <u>The Pollution Haven Hypothesis: A Geographic Economy Model in a Comparative Study</u></i>
SIEV	74.2008	<i>Jérôme Massiani and Paolo Rosato: <u>The Preferences of Trieste Inhabitants for the Re-use of the Old Port: A Conjoint Choice Experiment</u></i>
SIEV	75.2008	<i>Martin F. Quaas and Sjak Smulders: <u>Pollution and the Efficiency of Urban Growth</u></i>
CCMP	76.2008	<i>Anil Markandya and Dirk T.G. Rübbelke: <u>Impure Public Technologies and Environmental Policy</u></i>

KTHC	77.2008	<i>Gianmarco I P Ottaviano and Giovanni Peri: <u>Immigration and National Wages: Clarifying the Theory and the Empirics</u></i>
CCMP	78.2008	<i>Vivekananda Mukherjee, Dirk T.G. Rübbelke and Tilak Sanyal: <u>Technology Transfer in the Non-traded Sector as a Means to Combat Global Warming</u></i>
SIEV	79.2008	<i>A. Ghermandi, J.C.J.M. van den Bergh, L.M. Brander, H.L.F. de Groot, and P.A.L.D. Nunes: <u>The Economic Value of Wetland Conservation and Creation: A Meta-Analysis</u></i>
CCMP	80.2008	<i>Snorre Kverndokk and Adam Rose: <u>Equity and Justice in Global Warming Policy</u></i>
ETA	81.2008	<i>Sonia Oreffice: <u>Sexual Orientation and Household Decision Making. Same-Sex Couples' Balance of Power and Labor Supply Choices</u></i>
CCMP	82.2008	<i>Robert N. Stavins: <u>A Meaningful U.S. Cap-and-Trade System to Address Climate Change</u></i>
NRM	83.2008	<i>Ruben N. Lubowski, Andrew J. Plantinga and Robert N. Stavins: <u>What Drives Land-Use Change in the United States? A National Analysis of Landowner Decisions</u></i>
CSRM	84.2008	<i>Forest L. Reinhardt, Robert N. Stavins, and Richard H. K. Vietor: <u>Corporate Social Responsibility Through an Economic Lens</u></i>
CCMP	85.2008	<i>Valentina Bosetti, Carlo Carraro, Alessandra Sgobbi and Massimo Tavoni: <u>Modelling Economic Impacts of Alternative International Climate Policy Architectures. A Quantitative and Comparative Assessment of Architectures for Agreement</u></i>
PRCG	86.2008	<i>Gian Luigi Albano, Federico Dini and Roberto Zampino: <u>Bidding for Complex Projects: Evidence From the Acquisitions of IT Services</u></i>
SIEV	87.2008	<i>Dennis Guignet and Anna Alberini: <u>Voluntary Cleanups and Redevelopment Potential: Lessons from Baltimore, Maryland</u></i>
IEM	88.2008	<i>Louis-Gaëtan Giraudet and Philippe Quirion: <u>Efficiency and Distributional Impacts of Tradable White Certificates Compared to Taxes, Subsidies and Regulations</u></i>
CCMP	89.2008	<i>Damien Demailly and Philippe Quirion: <u>Changing the Allocation Rules in the EU ETS: Impact on Competitiveness and Economic Efficiency</u></i>
CCMP	90.2008	<i>Judson Jaffe and Robert N. Stavins: <u>Linkage of Tradable Permit Systems in International Climate Policy Architecture</u></i>
SIEV	91.2008	<i>Silvio Giove, Paolo Rosato and Margaretha Breil: <u>A Multicriteria Approach for the Evaluation of the Sustainability of Re-use of Historic Buildings in Venice</u></i>
CCMP	92.2008	<i>Corrado Di Maria, Sjak Smulders and Edwin van derWerf: <u>Absolute Abundance and Relative Scarcity: Announced Policy, Resource Extraction, and Carbon Emissions</u></i>
CCMP	93.2008	<i>Dirk T.G. Rübbelke and Nathan Rive: <u>Effects of the CDM on Poverty Eradication and Global Climate Protection</u></i>
SIEV	94.2008	<i>Massimiliano Mazzanti and Roberto Zoboli: <u>Waste Generation, Incineration and Landfill Diversion. Decoupling Trends, Socio-Economic Drivers and Policy Effectiveness in the EU</u></i>
CCMP	95.2008	<i>Benno Torgler, Maria A. García Valiñas and Alison Macintyre: <u>Environmental Participation and Environmental Motivation</u></i>
CCMP	96.2008	<i>Geoffrey Heal and Nori Tarui: <u>Technology Diffusion, Abatement Cost, and Transboundary Pollution</u></i>
CCMP	97.2008	<i>Alessio D'Amato and Edilio Valentini: <u>Enforcement and Environmental Quality in a Decentralized Emission Trading System</u></i>
CTN	98.2008	<i>Shao-Chin Sung and Dinko Dimitrov: <u>Computational Complexity in Additive Hedonic Games</u></i>

(lxxxiv) This paper was presented at the 13th Coalition Theory Network Workshop organised by the Fondazione Eni Enrico Mattei (FEEM), held in Venice, Italy on 24-25 January 2008.

2008 SERIES	
CCMP	<i>Climate Change Modelling and Policy</i> (Editor: Carlo Carraro)
SIEV	<i>Sustainability Indicators and Environmental Valuation</i> (Editor: Carlo Carraro)
NRM	<i>Natural Resources Management</i> (Editor: Carlo Giupponi)
KTHC	<i>Knowledge, Technology, Human Capital</i> (Editor: Gianmarco Ottaviano)
IEM	<i>International Energy Markets</i> (Editor: Matteo Manera)
CSRM	<i>Corporate Social Responsibility and Sustainable Management</i> (Editor: Giulio Sapelli)
PRCG	<i>Privatisation Regulation Corporate Governance</i> (Editor: Bernardo Bortolotti)
ETA	<i>Economic Theory and Applications</i> (Editor: Carlo Carraro)
CTN	<i>Coalition Theory Network</i>