



CEIS Tor Vergata

RESEARCH PAPER SERIES

Vol. 8, Issue 3, No. 166 – May 2010

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by

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JEL codes: D81; L91 ; L50

Keywords: (S) uncertainty modelling, real option, (B) transportation, (P) risk analysis, concession contract

Abstract

In this paper we model concession contracts between a public and a private party, under dynamic uncertainty arising both from the volatility of the cash flow generated by the project and by the strategic behaviour of the two parties. Under these conditions we derive three notions of equilibrium price and apply the model to a case study for one of the most important concession contracts in Italy.

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1. Introduction

A concession is a business operated under a contract associated with a degree of exclusivity within a certain geographical area. In the case of a public service concession, a private company enters into an agreement with the government to have the exclusive right to operate, maintain and carry out investment in a public utility for a given number of years. Very often, public services, such as water supply, railways or toll motorways, are operated as concessions. Several important infrastructure projects have been implemented and are being run through concession contracts. Examples are Eurotunnel, the undersea rail tunnel linking France to the UK, the M6 toll in the UK, the extensions to the Docklands Light Railway system in London, the whole Italian toll motorway net and the Canadian highway system. Latin American countries are perhaps the most frequent users of this type of contract. In this regard, Guash et al (2008), studying the renegotiation of the concession in Latin America, construct a database made up of nearly 1000 concession contracts covering the sectors of telecommunications, energy, transport and water. This is a large number, made even more impressive by the fact that it only concerns a minor proportion of the contracts outstanding and by the large sums of money that are typically at stake in each contract.

The broad diffusion of concession contracts in the past decades can be partly attributed to the tight budget constraints faced by governments throughout the world. However, while concession contracts may allow governments to find adequate funds for infrastructural investment, they also raise problems due to the presence of monopolies. Many of the new private businesses have monopoly power because the market is a natural monopoly, and/or because governments have given exclusive rights to the private companies, thus creating a legal monopoly (Clark and Easaw (2007)). It follows that governments may wish to limit property rights to avoid appropriation of private monopoly power that would hold up system development.

In this article we study the economic relationship between public and private parties in a concession contract. In spite of their relevance, concession contracts have not been extensively studied and modelled in the literature. Our paper aims to contribute to fill this gap, by constructing a theoretical model that can be easily implemented for empirical measurement. The novelty of our contribution, with respect to the studies available, is twofold. Firstly, from an operational point of view, the model presented can be used to calculate a baseline to organize a concession contract and to obtain the basic parameters to establish a tender procedure. Secondly, the model presents a framework to measure the balance of power between the public and the private party, after the concession has been assigned, as well as the critical factors on which the success of the concession depends in the future.

After briefly reviewing the literature on the topic of concessions, Section 2 presents the theoretical model. Section 3 develops this model within a bilateral bargaining setting and derives two equilibria. Section 4 applies the model to the case of the major Italian concessionaire of toll motorways. Section 5 presents some final conclusions.

2. A simplified stylized framework for concession contracts

Real Options theory, henceforth RO, has been broadly applied in operational research to different economic issues. De Reyck et al (2008), Martzoukos (2008), Nagel and Rammerstorfer (2008), Haksoz and Seshadri (2007), Clark and Easaw (2007), Keswani and Shackleton (2006), Carlsson et al (2004)) are some of the more recent contributions. One part of the RO theory has dealt with natural resources. Since public domain exploitation is implemented through concession contracts, most of this strand of literature indirectly examines the concession value of natural resources. For instance, Pindyck (1984) evaluates a renewable resource in the context of property rights, Mork et al (1989) apply RO theory to a concession in Canada, Brennan and Schwartz (1985) analyze interactions of operational options in a copper mine exploited through concession contracts. Schwartz and Trigeorgis (2001) present a number of applications of the RO methodology to natural

resources evaluation. All these contributions develop the idea that the value of the concession is given by the expected Net Present Value (NPV) of future payoffs, plus the option to delay the investment, should the concession design allow it. All of them and many other articles (Rocha et al. (2001), Dias (2004), Dias et al (2004), D'Alpaos and Moretto (2004), D'Alpaos et al (2006)) aim to evaluate the contract and/or the optimal timing to invest under different scenarios. But none of these contributions focuses on the interactions between the public owner and the private party that may lead to the signing of an agreement in the form of a concession contract¹. Modelling the interactions between the principal and the agent(s) is of crucial importance, however, because entering a contract in non optimal conditions (in a sense that we will rigorously define later) may lead to a distortion or simply to failure of the contract to be implemented.

We use the RO paradigm for our modelling, because RO incorporates three crucial characteristics: (i) the strong irreversibility component of the investment, (ii) the uncertainty surrounding the expected returns and, (iii) the term structure (the expiration date) of the concession. The possibility of bargaining between the parties is enhanced by uncertainty, and the additional dimension of risk sharing may be added to seek advantages for both parties.

With respect to the existing literature, our contribution presents several novelties. Firstly, it responds to the need for a theoretical scheme, on which the concession design can be based on an ex-ante evaluation phase. Secondly, it provides an operational tool to understand what indeed happened in some real cases. From this vantage point, the model can answer some practical questions, such as whether the price set was too high/low or the concession time too long/short. It also enables us to determine whether the actual features of a given concession contract are more akin to a Nash game or rather to a Stackelberg leader-follower scheme. It follows that the application of the model can help in inferring ex-post the sort of relationship which occurred between the parties when bargaining. Thirdly, the model suggests the key parameters and variables that may be important in monitoring and evaluating project performance on an ongoing and ex-post basis. Finally, our contribution provides an operational research application of RO theory to contract design, which can be extended to other forms of both bilateral and multilateral bargaining.

We model the case of a natural resource owned by the State in the spirit of the RO literature, by assuming that uncertainty is dynamic, in the sense that it concerns the future and can only be resolved by the passage of time. Moreover, investment is assumed to have irreversible consequences. The natural resource in question can be developed upon the (irreversible) commitment of investment costs concentrated in the first period (the "zero" period). Development yields a net cash flow formed by a systematic part, which is normalized to unity, and a stochastic part, denoted by y , observable in every period, both in the absence and in the presence of development, evolving according to a stochastic process of the geometric Brownian motion variety:

$$dy_t = \alpha y_t dt + \sigma y_t dz \quad (1)$$

where α and σ^2 are respectively the drift and variance parameters and dz is a normally distributed random variable such that $E(dz)=0$ and $E(dz)^2 = dt$. The assumption of the geometric Brownian motion (GBM) under dynamic uncertainty corresponds to the assumption of (log) normality in a static stochastic setting. As is widely discussed in the literature² (see, for example, Dixit and Pindyck (1994, pp. 68-69)), the GBM appears well suited to describe a large class of economic processes that can be considered as the continuous limit of a discrete-time random walk and, by the application of the Central Limit Theorem, to the sum of these processes.

We also assume that, in the absence of development, the resource yields a steady flow of economic benefits (of the public amenity type) net of maintenance cost, and that such a flow would be lost in the event of development. While different hypotheses would be possible in this regard, this particular assumption seems well-suited to the case of many contracts that involve natural

¹ A non RO study facing this problem is presented by Hung et al (2006) and concerns the design of a concession contract under asymmetric information between the State and a potential concessionaires, with production obtained through depletion of a non-renewable resource.

² For a comprehensive treatment of this issue see Cox and Miller (1965).

resources, such as a park, a wildlife area etc. In most of these cases, the concession provides for a period of privatized management of the public space involved, with development consisting of an infrastructure (e.g. a road, a bridge, a building etc.) that permanently reduces the flow of amenities in the area interested. This hypothesis is also interesting because it explores an important dimension of the concession contract, that is, a temporary privatization combined with a development project involving some permanent loss of a public amenity.

Consider the possibility that the resource is developed through a fixed term contract (concession) with a private developer. Let us also assume that the alternative in which the resource is directly and completely developed by the State without a private party is not feasible. In this case the public owner would face the choice between keeping the resource undeveloped and developing it at the cost of foregoing its fruits for a concession period T , to be determined contractually³. The possible agreement includes also a contractual payment and the provision that the developed resource will return to the State after the concession period. From that point onward, the State enjoys the full benefits of the developed resource. The concession thus acts as a substitute for the investment costs.

Under these hypotheses, it is possible to determine the value of the contract for the State as the solution to the following problem:

$$V(y) = \sup E_y \left[e^{-\rho\tau} \left(\int_{\tau+T}^{\infty} e^{-\rho(s-\tau)} y_s ds + \int_{\tau}^{\tau+T} e^{-\rho(s-\tau)} P_M ds - \int_{\tau}^{\infty} x e^{-\rho(s-\tau)} ds \right) \right] \quad (2)$$

In expression (2), under suitable assumptions on market completeness, ρ is the risk-free interest rate, T is the time at which the concession expires, τ is the stochastic time of entry, P_M is, for each admissible value of the cash flow y , the minimum acceptable price to the State and x is the constant flow of public amenity. The contract is thus appraised as a real call option of the American variety, i.e. as the highest value within a family of expectations corresponding to all stopping points of the stochastic process⁴ in (1).

Assuming that the dynamics of the risk contained in the cash flow can be replicated by existing assets, the option in the hands of the State, denoted by $V(y)$, can be evaluated by applying contingent claim evaluation. As Dixit and Pindyck (1994, pp.122-123) show, this evaluation problem has a state dependent solution, contingent on whether the value of the stochastic variable (the cash flow y) is above or below a critical threshold of investment adoption (y_p)⁵:

$$V(y) = \frac{y}{\delta} e^{-\delta t} + \frac{P_M}{\rho} (1 - e^{-\delta t}) - \frac{x}{\rho} \quad \text{if } y \geq y_p \quad (2a)$$

³ The time limit itself is not the unique feature characterizing a concession contract. Licensing, for example, also contemplates a fixed term of validity. From a legal point of view, a concession differs from a licence because in the former case the Government loses the opportunity of using the public domain, while in the latter, the Government simply allows a private concern to run a particular business, without losing any of its rights. Simple examples are the cases of the driving and hunting license on the one hand, and the concession to run a public beach, on the other hand. In spite of the straightforward legal distinction, in practice the two terms are used interchangeably. In the case of spectrum assignment, for instance, the public party definitely loses the possibility to use the portion of spectrum he assigns, hence one should refer to it as a concession, while it is generally referred to as a license, see Maskin (2004), Scandizzo and Ventura (2006).

⁴ For a recent treatment of this specification, see Perskin and Shiryaev (2006, pp. 47-50).

⁵ Assuming dy_t/y_t normally distributed implies y_t lognormally distributed. Given this assumption $E(y_t) = \int_{\Omega} y_t = y e^{\alpha t}$, where y denotes the initial value of y_t (i.e. the current value, if the time perspective of the expectation is from the present moment to the future). The present value of this expectation is obtained by discounting it at rate r yielding: $\int_0^{\infty} \int_{\Omega} y_t e^{-rs} ds d\omega = \int_0^{\infty} y e^{-(r-\alpha)s} ds = y/(r-\alpha)$ with $r < \rho$. See Dixit and Pindyck (1994, p. 71).

$$V(y) = \left(\frac{y}{y_p}\right)^{\beta_1} \left[\frac{y_p}{\delta} e^{-\delta T} + \frac{P_M}{\rho} (1 - e^{-\delta T}) - \frac{x}{\rho} \right] \text{ if } y < y_p \quad (2b)$$

where

$$y_p = \delta e^{\delta T} \frac{\beta_1}{\beta_1 - 1} \left(\frac{x - (1 - e^{-\delta T}) P_M}{\rho} \right) \quad (2c)$$

is the optimal exercise boundary at which the investment opportunity should be exercised⁶, y is the current value of the process and $(y/y_p)^{\beta_1} = Ee^{-\rho\tau}$ is the expected value of a discount factor that depends on the stochastic time of entry τ and finally $\delta = \mu - \alpha > 0$ is the opportunity cost of delaying the construction of the project, with μ is the total expected rate of return, as suggested by the Capital Asset Pricing Model (CAPM) (see Dixit and Pindyck (1994, p.120)).

In (2c) the exercise boundary positively depends on: (i) the foregone amenity x/ρ , (ii) the volatility of the cash flow σ , given that $\frac{\partial y_p}{\partial \sigma} = \frac{\partial y_p}{\partial \beta_1} \frac{\partial \beta_1}{\partial \sigma} > 0$, (iii) the length of the concession T , (iv) the opportunity cost δ .

Equations (2a) and (2c) imply that the following condition has to be satisfied for the contract to be acceptable :

$$\frac{P_M}{\rho} \geq \frac{x}{\rho(1 - e^{-\delta T})} - \frac{y}{\delta(1 - e^{-\delta T})} \frac{\beta_1 - 1}{\beta_1} \quad (2d)$$

Thus, for any given value of the expected cash flow from development, the price received by the State should be at least equal to the amenities lost, minus the cash flow that the State will enjoy after the concession has expired, adjusted for risk. This risk adjustment is greater than 1 (the lower limiting value of β_1) and decreasing with uncertainty (since β_1 is negatively related to the volatility of the cash flow). It follows that an increase in uncertainty renders the contract less attractive for the State, since it increases the lower limit of acceptable prices for the owner of the resource.

On the other side of the contract, for the private party the evaluation problem is

$$V_\pi(y) = \sup E_y \left[e^{-\rho\tau} \left(\int_\tau^T e^{-\rho(s-\tau)} y_s ds - \int_\tau^T e^{-\rho(s-\tau)} P_m ds - I_\pi \right) \right] \quad (3)$$

where P_m is the maximum price the private party is willing to pay and I_π represents the investment cost. Similarly to the problem in (2), the solution to (3) is:

$$V_\pi(y) = \left(\frac{y}{\delta} - \frac{P_m}{\rho}\right)(1 - e^{-\delta T}) - I_\pi \text{ if } y \geq y_\pi \quad (3a)$$

$$V_\pi(y) = \left(\frac{y}{y_\pi}\right)^{\beta_1} \left[\left(\frac{y_\pi}{\delta} - \frac{P_m}{\rho}\right)(1 - e^{-\delta T}) - I_\pi \right] \text{ if } y < y_\pi \quad (3b)$$

where y_π is the threshold value for contract acceptance from the private party.

The minimum value of the cash flow at which the contract becomes acceptable for the private party is now:

$$\frac{y_\pi}{\delta} (1 - e^{-\delta T}) = \frac{\beta_1}{\beta_1 - 1} \left(I_\pi + \frac{P_m}{\rho} (1 - e^{-\delta T}) \right). \quad (3c)$$

⁶ The value of the option to invest (see Dixit and Pindyck (1994, p.140)) is given by expression $F(y) = A_1 y^{\beta_1} + A_2 y^{\beta_2}$ where A_1 and A_2 are constants determined by boundary conditions and β_1 and $-\beta_2$ are, respectively, the positive and the negative root of the characteristic equation: $\rho - \beta(\rho - \delta) - \frac{\beta}{2}(\beta - 1)\sigma^2 = 0$. For the exit option, the relevant root is the negative one.

Equations (3a) and (3c) imply that in order to be acceptable to the private developer, the price should respect the condition:

$$\frac{P_m}{\rho} \leq \frac{y}{\delta} \frac{\beta_1 - 1}{\beta_1} - \frac{I_\pi}{(1 - e^{-\delta T})} \quad (3d)$$

This price is larger, the larger the cash flow, the longer the concession period and the lower uncertainty. As for the case of the State, an increase in uncertainty tends to decrease the attractiveness of the contract for the private party.

For given time span, a feasible deal can be reached if $P_m \geq P_M$, i.e., from (2d) and (3d):

$$\frac{x}{\rho(1 - e^{-\delta T})} - \frac{y}{\delta} \frac{e^{-\delta T}}{(1 - e^{-\delta T})} \frac{\beta_1 - 1}{\beta_1} \leq \frac{y}{\delta} \frac{\beta_1 - 1}{\beta_1} - \frac{I_\pi}{(1 - e^{-\delta T})}$$

Solving for y , we obtain:

$$\frac{\bar{y}}{\delta} \geq \frac{\beta_1}{\beta_1 - 1} \left[\frac{x}{\rho} + I_\pi \right] \quad (4)$$

this expression indicates the minimum entry point, a threshold for the cash flow above which both parties find it optimal to enter a concession contract of length T . This threshold is such that the expected present value of the cash flow has to cover the sum of the public loss of amenities and private investment costs.

3. Equilibrium solutions

In order to analyze the equilibrium solutions, we have to analyze further the two parties' objective functions. In the first instance, we can specify them as their extended NPV from the concession, i.e. as the difference between the expected NPV and the value of the option to wait⁷ (extended NPV in the sense used by the RO literature, e.g. Pennisi and Scandizzo (2006)). For the State, the payoff function can thus be defined as follows:

$$\Pi_p = \left[\frac{y}{\delta} e^{-\delta T} + P^* - \frac{x}{\rho} \right] - \left(\frac{y}{y_p} \right)^{\beta_1} \left[\frac{y_p}{\delta} e^{-\delta T} + P^* - \frac{x}{\rho} \right] \quad (5)$$

where $P^* \equiv \frac{P}{\rho}(1 - e^{-\delta T})$, and y_p is the threshold value of expected cash flow at which the

contract becomes attractive to the State as specified in (2c).

The first term on the right hand side represents the gain of the State if the contract is stipulated successfully, and a price is accepted such that condition (2d) is respected. The second term, with the minus in front, on the other hand, represents the gain under the no agreement condition, under which the State would not sign the contract for the price proposed, but would nevertheless keep the option of doing so at a later date. An increase in the price to be paid by the concessionaire thus increases the expected payoff from the concession, but it also increases the option value, i.e. the cost of exercising the option today rather than in the future. It is easy to verify, by equating to zero the first derivative of (5) with respect to P^* , that the solution is identical to (2d). As inspection of the second derivative of (5) at this value shows, however, this solution is not a maximum, but a minimum. Therefore the following proposition holds:

Proposition 1.

⁷ For more details on the value of the option to wait see Dixit and Pindyck (1994, pp. 140, 258, 260).

The principal's extended NPV is minimized when the (positive) concession price equals the minimum acceptable price. The latter price is defined as the minimum price that the State is willing to accept to enter the contract, for a given expected cash flow level. Recalling (2d), this implies:

$$\arg \min_{P^*} \Pi_p = \frac{P_M}{\rho} = \frac{x}{\rho(1-e^{-\delta T})} - \frac{y}{\delta} \frac{e^{-\delta T}}{(1-e^{-\delta T})} \frac{\beta_1 - 1}{\beta_1}.$$

Consider now the private party's payoff function, Π_π . Similarly to the State's payoff in (5), we define it as :

$$\Pi_\pi = \left[\frac{y}{\delta} (1 - e^{-\delta T}) - I_\pi - P^* \right] - \left(\frac{y}{y_\pi} \right)^{\beta_1} \left[\frac{y_\pi}{\delta} (1 - e^{-\delta T}) - I_\pi - P^* \right] \quad (6)$$

where y_π is the threshold value of expected cash flow at which the contract becomes attractive for the private party, as specified in (3d). Again, this function exhibits a minimum point for P^* equal to the value indicated in (3d), i.e.:

Proposition 2.

The agent's extended NPV is minimized when the (positive) concession price equals the maximum affordable price. The latter price is defined as the maximum price the agent is willing to pay for a given expected cash flow level: $\arg \min_{P^*} \Pi_\pi = \frac{P_m}{\rho} = \frac{y}{\delta} \frac{\beta_1 - 1}{\beta_1} - \frac{I_\pi}{(1 - e^{-\delta T})}$.

Since the price represents a monetary compensation that can be paid to accommodate both parties' desires to optimize, a cooperative solution is possible. In such a solution, the two parties can join their payoff functions and find:

$$P_c^* = \arg \max_{P^*} (\Pi_p + \Pi_\pi) \quad (7)$$

where P^c represents the equilibrium price in a cooperative solution.

Taking the derivative of (7) with respect to P^* , using (5), (6), (2c) and (3c), we find that the cooperative solution requires:

$$P_c^* = (1 - e^{-\delta T}) \frac{x}{\rho} - e^{-\delta T} I_\pi = \frac{x}{\rho} - e^{-\delta T} \left(\frac{x}{\rho} + I_\pi \right) \quad (8)$$

or, equivalently

$$P_c = x - \frac{\rho e^{-\delta T}}{(1 - e^{-\delta T})} I_\pi$$

As inspection of the second derivative of $(\Pi_p + \Pi_\pi)$ with respect to P^* at P_c^* readily shows, the expression in (8) defines a maximizing value for the concession price, so that the following proposition can be stated:

Proposition 3.

The price that maximizes the sum of the extended NPVs of the two parties is independent of current cash flow and of uncertainty. Its present value (PV) equals the PV of the public amenities lost minus the PV of public and private costs for the period after the concession.

Basically, the cooperative solution yields a price whereby the private party pays a compensation for the loss of public amenities equal to the PV of such a loss, net of the PV of the

costs that would have been incurred for development beyond the concession horizon. As can be easily checked, this price is such that the two parties would both agree to enter at the same threshold level of the cash flow ($y_p=y_\pi$).

If the two parties reject the cooperative solution, their payoffs in the bargaining space are defined by the two limiting conditions (2d) and (3d), which correspond, as we have seen, to the minimum values of their payoffs. Condition (2d), in fact, defines the minimum value of P that is acceptable by the State (the value that, by rendering its NPV equal to the option to wait, minimizes its extended NPV). Any value above this will yield a surplus equal to

$$\Pi_p(P) = P^* - P_M^* = P^* - \left(\frac{x}{\rho} - \frac{y}{\delta} e^{-\delta T} \frac{\beta_1 - 1}{\beta_1} \right).$$

Expression (3d) defines the maximum value acceptable to the concessionaire, such that any value below this will yield a surplus for her⁸ of:

$$\Pi_\pi(P) = P_m^* - P^* = \left[\frac{y}{\delta} (1 - e^{-\delta T}) \frac{\beta_1 - 1}{\beta_1} - I_\pi \right] - P^*.$$

Assuming that in the case of disagreement both parties have no alternative except the *status quo*, we compute the Nash equilibrium, by maximizing the product of the players' net payoffs, defined as the difference between the price agreed upon and the two limiting minimizing values (Nash (1951), Harsanyi (1967), Harsanyi (1968)).

The Nash equilibrium price can be calculated as:

$$P_R^* = \arg \max_p \left\{ \left[P^* + \frac{y}{\delta} e^{-\delta T} \frac{\beta_1 - 1}{\beta_1} - \frac{x}{\rho} \right]^w \left[\frac{y}{\delta} (1 - e^{-\delta T}) \frac{\beta_1 - 1}{\beta_1} - I_\pi - P^* \right]^{1-w} \right\} \quad (9)$$

i.e.:

$$P_R^* = \frac{P_R}{\rho} (1 - e^{-\delta T}) = w \left[\frac{y}{\delta} \frac{\beta_1 - 1}{\beta_1} (1 - e^{-\delta T}) - I_\pi \right] + (1 - w) \left[\frac{x}{\rho} - \frac{y}{\delta} e^{-\delta T} \frac{\beta_1 - 1}{\beta_1} \right] = w P_m^* + (1 - w) P_M^* \quad (10)$$

where $0 \leq w \leq 1$ is a weight quantifying the State bargaining power and P_R^* represents the Nash equilibrium price.

Proposition 4.

The Nash equilibrium is a weighted average of the limiting acceptable prices for the two parties. The weights represent the other party's bargaining power.

The bargaining power of the State, w , can be defined as the degree to which the public sector is able to influence price setting. This degree may be larger or smaller, depending on the fact that the seller can exploit competition among the bidders to drive up the price. In general, however, the State will not be able to drive the price up so far as to equal the valuation of the bidder who values the item the most, because it does not know what this valuation is.

It can be shown that a Rubinstein game of repeated offers and counter-offers yields the equivalent price value that maximizes (9) and that gives the solution in (10). This solution is a Nash equilibrium, in the sense that it is the best price that either party could choose, given their knowledge of the alternative possible choices (but not of the actual choice) of the other party.

A final type of equilibrium solution is a non cooperative Stackelberg equilibrium (Shubik (1991, pp. 85-86)) in which the State acts as a leader and the concessionaire as a follower. This solution derives from the fact that the price can be used as a device to renegotiate the terms of the contract and keep the objectives of the principal and the agent properly aligned under changing

⁸ Recall that $P^* \equiv \frac{P}{\rho} (1 - e^{-\delta T})$ and similar specifications hold for P_M^* , P_m^* and P_R^* .

circumstances. Without loss of generality, we assume that the price is re-negotiated in every period, given the expectations on the future cash flows and once the contract has already been signed by the two parties. Although the State has the interest of extracting the highest possible price in every period from the private contractor, it has to consider the possibility that too a high price, though acceptable on the basis of expected values, may become nonetheless unsustainable if future states of nature prove to be sufficiently unfavourable.

In particular, assume that the contractor has the option to withdraw from the concession in every period by recovering the difference D between the salvage value and the exit costs. This option to default can be framed in the following mathematical form:

$$\left(\frac{y}{\delta} - \frac{P}{\rho}\right)(1 - e^{-\delta T}) + By^{-\beta_2} = D \quad (11)$$

where now T indicates the time to expiration of the contract. Equation (11) says that, once the concessionaire has been granted the contract, if the state of nature is sufficiently unfavourable, she can exit the concession and recover a net salvage value D . For example, a simple assumption for the evolution of such a value is that salvage equals a fraction of the investment cost, while exit costs, C , are a constant and that both decline linearly to zero as the end of the concession approaches:

$$D = \theta I_{\pi}(T - t) - C(T - t).$$

In (11) $By^{-\beta_2}$ is the value of the option to exit and $-\beta_2$ is the negative root of the characteristic equation. The concessionaire will choose the time of exit by solving the (put) RO evaluation problem⁹:

$$V(y) = \sup E_y \left[e^{-\rho \tau} \left(D - \int_{\tau}^T e^{-\rho(s-\tau)} (y_s) ds + \int_{\tau}^T e^{-\rho(s-\tau)} P ds \right) \right] \quad (12)$$

whose solution is:

$$V_{\pi}(y) = D - \left(\frac{y}{\delta} - \frac{P}{\rho}\right)(1 - e^{-\delta T}) \quad \text{if } y \leq y_e \quad (12a)$$

and

$$V_{\pi}(y) = \left(\frac{y}{y_e}\right)^{-\beta_2} \left[D - \left(\frac{y}{\delta} - \frac{P}{\rho}\right)(1 - e^{-\delta T}) \right] \quad \text{if } y > y_e \quad (12b)$$

and y_e is the critical threshold at which the concessionaire will decide to exit:

$$\frac{y_e}{\delta} = \frac{\beta_2}{\beta_2 + 1} \left(\frac{D}{(1 - e^{-\delta T})} + \frac{P}{\rho} \right). \quad (12c)$$

The value of the option to default can be obtained by exploiting the smooth pasting condition:

$$By^{-\beta_2} = \frac{(1 - e^{-\delta T})}{\delta \beta_2} \left(\frac{\beta_2 \delta}{\beta_2 + 1} \right)^{1+\beta_2} \left(\frac{D}{(1 - e^{-\delta T})} + \frac{P}{\rho} \right)^{1+\beta_2} y^{-\beta_2} \quad (13)$$

This value represents a threat for the State's payoff as the counterpart of the concession contract. The full payoff incorporating this threat, W_p , in fact, can be written as follows:

⁹ This is equivalent to the evaluation of an American put option. See Perskin and Shiryaev (2006, pp. 47-50)

$$W_p = \frac{y}{\delta} e^{-\delta t} + \frac{P}{\rho} (1 - e^{-\delta t}) - \frac{x}{\rho} - B y^{-\beta_2} \quad (14)$$

By substituting (13) into (14) and differentiating with respect to P :

$$\frac{dW_p}{dP} = \frac{1 - e^{-\delta t}}{\rho} - \frac{1 - e^{-\delta t}}{\rho} \frac{(\beta_2 + 1)}{\beta_2 \delta} \left(\frac{\beta_2 \delta}{\beta_2 + 1} \right)^{1 + \beta_2} \left[\frac{D}{(1 - e^{-\delta t})} + \frac{P}{\rho} \right]^{\beta_2} y^{-\beta_2}$$

Equating to zero and solving for P , we obtain P_s , the Stackleberg equilibrium price¹⁰

$$\frac{P_s}{\rho} = \frac{\beta_2 + 1}{\beta_2} \frac{y}{\delta} - \frac{D}{1 - e^{-\delta t}} \quad (15)$$

Proposition 5.

In a concession contract in which the price can be renegotiated periodically, the Stackleberg equilibrium is reached when the price PV equals the expected NPV of the cash flow during the remainder of the concession time, adjusted for risk, minus net salvage value for the concessionaire before expiration. In mathematical terms, recalling expression (12), we can express this result as follows:

$$\frac{P_s}{\rho} = \arg \max_P \left\{ W_p; V(y) = \sup E_y \left[e^{-\rho \tau} \left(D - \int_{\tau}^T e^{-\rho(s-\tau)} (y_s) ds + \int_{\tau}^T e^{-\rho(s-\tau)} P_s ds \right) \right] \right\} \quad (16)$$

i.e. as the result of the maximization of the State payoff in (14) under the constraint of the reaction function of the concessionaire according to (12).

The price in (15) and (16) thus maximizes the payoff of the State, but it is also optimal for the concession holder, since it is derived from the dynamic optimization process underlying the valuation of the option to default and the setting of the corresponding (optimal) threshold value to end unilaterally the contract. This price is higher, the higher the expected cash flow, the lower the uncertainty, the longer the remaining duration of the contract and the lower the salvage value for the concession holder. The higher the salvage value, in fact, the higher the value of the threat for the State that the concessionaire may abandon the contract if the cash flow falls below a sufficiently low level. Thus, the higher the net salvage value, the lower the price that the State will be willing to accept to prevent such an exit. Note that if the salvage value is negative, because of contractual or *de facto* exit costs, the price will be higher, the higher such costs net of the salvage value. The role played by uncertainty is also notable. An increase in the volatility of the cash flow, in fact, also increases the value of the option to default of the concession holder, thus allowing her to negotiate a lower price to continue to manage the concession.

4. A case study: Autostrade per l'Italia

Atlantia S.p.A., formerly Autostrade S.p.A., is an Italy-based company that, together with its subsidiaries, is primarily engaged in the development, construction and management of toll motorways. The Company, together with its subsidiaries, manages a European motorway network covering over 3,400 km of motorway concessions, which equals 61% of the national motorway network and 19% of European toll roads, and services national and international motorists, as well as freight haulers. The Company is also involved in other related activities, including the provision of road engineering and maintenance services, the provision of road safety solutions, the provision of logistic services, the operation of parking facilities and other activities. The Company's

¹⁰ The second order condition for a maximum is:

$$\frac{d^2 W_p}{dP^2} = - \frac{(1 - e^{-\delta t})}{\rho} \left(\frac{\beta_2 \delta}{\beta_2 + 1} \right)^{\beta_2} \left(\frac{D}{(1 - e^{-\delta t})} + \frac{P}{\rho} \right)^{\beta_2 - 1} \beta_2 y^{-\beta_2}, \text{ which is always } \leq 0 \text{ for } D \geq 0.$$

subsidiaries include Autostrade per l'Italia S.p.A. (100% owned by Atlantia), the concession holder. The concessionaire and ANAS, the assignor, signed a contract expiring on 31st December 2038, as established in 1997.

At the time it was privatized, in 1999, the company paid a price for the concession contract of €8,100 million, which was generally considered (see Atlantia (2007a, p.15)) too high. The model presented in the previous section may help in tackling the issue of whether such a general conviction was justified. In terms of the model, the question can be restated as follows: was the price of the concession paid in 1999 compatible with the Nash equilibrium in eq. (10)?

To answer the question one must estimate the parameters of the process in (1), α and σ from historical data of the operating cash flow. Unfortunately, accounting criteria have changed throughout the years making the budget figures not comparable over time and therefore useless to our aim. In order to overcome this problem we may define an interval for the parameters consistently with the RO theory. The drift parameter α lies in a narrow interval, $\alpha \in (0; \mu)$, therefore $\delta = \mu - \alpha$ still belongs to the interval $\delta \in (0; \mu)$, where μ is the total expected rate of return, as suggested by the CAPM¹¹. μ can be reasonably approximated by using the “ b ” as provided by a financial web site (For instance Yahoo Finance provides an estimate of “ b ” at 0.688: <http://it.finance.yahoo.com/q/tt?s=ATL.MI>). Combining this piece of information with data reported in the appendix, δ is numerically located in the interval $\delta \in (0; 4.14\%)$.

Consistency with the estimate of δ enables us to work out an approximate lower bound for the variance of the process. We know that $b = \frac{\sigma_{my}}{\sigma_m^2}$ where m and y respectively represent market

and project returns, so that $\sigma_m b = \frac{\sigma_{my}}{\sigma_m}$ and by Schwarz inequality¹² $\sigma_m b = \frac{\sigma_{my}}{\sigma_m} \leq \sigma_y$, where σ_y represents, for the return to the project, the approximation to our parameter of interest, σ . The approximation is based on the fact that “ b ” provided by Yahoo Finance is estimated using the returns on the S&P index and the returns on equity, while a correct practice should have correlated the former to a different measure of project returns, such as the operating cash flow. However, this approximation does not alter the results of the analysis in any way and is perfectly in line with the empirical literature, as, for example, shown by Schwartz (2004).

To derive the upper bound of σ , let us consider that the volatility of returns on equity is greater than that of the cash flow, because the former is affected by speculative trading, especially for a listed company. Therefore, we can consider the estimate of the volatility of equity prices as an upper bound of the volatility of the cash flow: $\sigma \in [b\sigma_m; \sigma_e)$ where σ_e stands for the volatility of the equity, numerically $\sigma \in [10.23\%; 17.06\%)$.

In addition to the price, the company agreed to commit itself to invest 4,424 million Euros over the period. By the 30th of September 2007 the company had invested only 31% of that figure. Assuming that these costs were equally distributed along the first seven years and nine months of the concession, they correspond to a present value of 1,038 millions Euro. Coupled with the estimate of 651.59 million Euros for the cash flow at the time of the contract, the two figures can be used to estimate P_m^* , P_M^* , x/ρ and \bar{y} according to (3d), (10), (2d) and (4) respectively.

Therefore, a test to check the Nash equilibrium nature of the actual price paid is given by the consistency of the figures of the agreement signed in 1999 with the feasibility region, i.e. $P_m \geq P_M$, and with eq. (10).

¹¹ Recall that $\mu = \rho + b(r_m - \rho)$ where r_m is the expected rate of return of the market and b is given by the covariance between market returns and the cash flow of the project divided by the variance of market returns. Differently from the usual notation we use the symbol “ b ”, instead of β for the covariance to avoid confusion with β , the roots of the characteristic equation in footnote 6.

¹² The Schwartz inequality claims that $\sigma_{x,y} \leq \sigma_x \sigma_y$.

The key variables for the test are worked out under different scenarios for σ and α (and therefore δ), but the only admissible values are found for α and σ at their lower bound¹³, regardless of the strength of the bargaining power, w .

		P_m^*	P_M^*	x/ρ	\bar{y}
w	0.1	8,287	8,079	10,407	640
	0.5	8,287	7,913	10,240	631
	0.7	8,287	7,663	9,991	617
	0.9	8,287	6,414	8,742	547

Table 1: Values of P_m^* , P_M^* , x/ρ and \bar{y} for different of w , with α and σ at their lower bound. (million Euro)

Table 1 shows that for any admissible value of w $P_R^*=8,100$ straddles the two limiting prices, P_m^* and P_M^* . It follows that the options was exercised in the money, $\bar{y} < 651.59$. Besides, the perpetual value of the amenity turns out to be quite stable.

Another interesting question concerns the entity of D , the salvage value for the contractor in a Stackelberg equilibrium in case she exits before T . By setting $P_s^*=8,100$, the actual price, and y equal to the actual value occurred in 1999, we can determine D from (15) and compute the ratio P_s^*/D according to the possible combinations of α and σ

		Delta		
		<i>Lower bound</i>	<i>Mid Point</i>	<i>Upper bound</i>
Sigma	<i>Lower bound</i>	0.39	0.58	0.70
	<i>Mid point</i>	0.33	0.47	0.56
	<i>Upper bound</i>	0.29	0.40	0.48

Table 2: Ratio P_s^*/D in a Stackelberg equilibrium

Table 2 shows that the salvage value D may be a non negligible component of the contract, in a Stackelberg equilibrium in which the contractor has the option to withdraw from the concession in every period. For any admissible value of the process parameters, the price is always smaller than the salvage value, ranging from a minimum of 29% to a maximum of 70%. In the case examined, the results may reveal the tendency of the State to lose bargaining power in the course of the contract and accept smaller and smaller prices, or smaller and smaller degrees of compliance in the agreed program of investment, because of the small value that Autostrade can expect to recover in the case of the concession being terminated.

5. Concluding remarks

In this paper we have examined the possible negotiating conditions of a concession contract between a public and a private party under dynamic uncertainty. The uncertainty arises because of

¹³ The results obtained for the other values of α and σ are not reported in the text because uninteresting and for sake of room. However, evidence can be given to the interested reader upon request.

the volatility of the cash flow produced by the underlying resource, and, in both contract stipulation and monitoring, by the strategic behaviour of the two parties.

Because the contract has both uncertain and irreversible consequences, its adoption presents a real option problem for the two parties, which can be solved if an agreement is reached on the length of the contract and on the price to be paid by the concessionaire. For any given contract length, in particular, the bargaining space is non empty if, by pooling resources, the parties can cover public and private costs, including allowances for risk. This suggests that a cooperative solution is in order, and this is found to be such that the price paid by the concessionaire perfectly compensates the State for the external costs of developing the resource, taking however into account the investment costs borne by the private party to develop it, but not compensating either party for the risk taken.

We have also examined the case where the price is used as an incentive to re-align the objectives of the two parties and can thus be renegotiated periodically. With the State in the natural position of a Stackelberg leader, this price is found to be increasing in the expected cash flow, and the remaining duration of the contract and decreasing in salvage value for the concession holder and uncertainty.

Finally, we have applied the two notions of equilibrium price developed to a case study taken from one of the most important concession contracts in Italy, stipulated for the main national toll motorways by the Italian Government and the private company Autostrade S.p.A.. Under certain conditions of income growth we have found that the price paid in the agreement signed in 1999 is compatible with a Nash equilibrium price, as derived from the model. We have also found that if the State and the private parties behave respectively as a Stackelberg leader and follower in renegotiating the price during the concession, lower and lower prices should be expected to follow. This result depends on the low salvage values of infrastructure investment and the need to discourage the tendency of the concessionaire to react to unfavourable circumstances by foregoing the contract as the concession nears expiration.

Acknowledgement

The authors gratefully thank D. Dicuonzo for having provided them with useful information on the interpretation of the financial statements of Autostrade per l'Italia. The opinions expressed by the authors do not necessarily reflect the position of their institutions.

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Appendix: data and estimates of model parameters

Parameter	Value	Source
μ total expected rate of return, or expected rate of return from owning the completed project	4.14%	Our estimates on Datastream, S&P 500 and Yahoo Finance data
r_m market rate of return	4.51%	Datastream, average return of S&P 500 composite index over the period 1998-2006
ρ risk-free rate	3.59%	Datastream, average Italian zero coupon bond over the period 1998-06
b from the CAPM provided by yahoo finance	0.688	Yahoo Finance http://it.finance.yahoo.com/q/tt?s=ATL.MI
σ_m historical volatility of market rate of return	14.86%	Datastream, sd of S&P 500 composite index returns over the period 1998-2006
σ_e historical volatility of the returns on equity	17.06%	Bloomberg, sd of returns on equity.
Y_{99} operating CF in 1999	€651.59 Millions	Balance sheet data available at http://www.atlantia.it/en/bilanci/
P_R concession price	€8,100 Millions	Atlantia (2007a) available at http://www.atlantia.it/it/presentazioni/
I_π	€4,424 Millions	Atlantia (2007b) available at http://www.atlantia.it/it/presentazioni/
T time to expiration of the contract	39 years	Atlantia (2007a) available at http://www.atlantia.it/it/presentazioni/