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Planning the production of a fleet of domestic combined heat and power generators ☆

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ABSTRACT

This paper describes a planning problem, arising in the energy supply chain, that deals with the planning of the production runs of micro combined heat and power (microCHP) appliances installed in houses, cooperating in a fleet. Two types of this problem are described. The first one is the Single House Planning Problem (SHPP), where the focus is on supplying heat in the household. The second one combines many microCHPs into a Fleet Planning Problem (FPP) and focuses on the mutual electricity output, while still considering the local heat demand in the individual households. The problem is modeled as an ILP. For practical use a local search method is developed for the FPP, based on a dynamic programming formulation of the SHPP.

1. Introduction

The classical energy supply chain is changing. Production, distribution, consumption, storage and load management are examples of fields in the energy supply chain that undergo a lot of attention and changes in recent years. In *production* the focus is on renewable technologies and technologies that increase the current energy efficiency. For example, Ayompe et al. (2010) discuss energy models for small-scale Photo Voltaic (PV) systems, Lanzafame and Messina (2010) design a micro-wind turbine and Alanne and Saari (2004) discuss possibilities for small-scale CHP technologies for buildings. Regarding *distribution*, the design and dimensioning of the network is investigated as in Green et al. (1999), but also the subdivision of the electricity network into different voltage layers (Kester et al., 2009 discuss an improved MV/LV station), the distribution of the national gas network (Andre et al., 2009) and the international connection between national electricity/gas networks are important topics. With respect to the latter, Giesbertz and Mulder (2008) present economic aspects of such connected networks. From the *consumption* point of view energy saving appliances are developed and newly developed appliances are more often controllable to some extent. Wemhoff and Frank (2010) show an example of the control of an HVAC system (Heating, Ventilating and Air Conditioning). Also high quality *energy storage* becomes more and more important in the new energy supply chain. The paper of Arsie et al. (2009) is an example of research on Compressed Air Energy Storage (CAES) in combination with a windmill park. *Load management* involves both reducing the energy consumption via focusing on awareness (Mills and Schleich, 2010) and improving the energy efficiency via energy policy (Oikonomou et al., 2009) or scheduling techniques for controllable energy consuming/producing appliances. The papers of Kok et al. (2005) and Molderink et al. (2010) are examples of large-scale energy management systems on domestic level. Summarizing, a lot of current research is ongoing with a focus on energy efficiency in the broadest view.

In this work we consider an energy-related planning problem which occurs when an emerging technique for energy production is introduced in the energy supply chain. In general, an emerging development in one area of the energy supply chain may have implications to other areas and can lead to severe problems in the overall energy infrastructure. For example, a growing share of electricity generated by wind turbines in the total electricity production may lead to more instability in the grid, since the electricity output of a wind turbine park is more dynamic than the steady generation of a coal-fired power plant. To overcome such problems one may have to look at storage (e.g. CAES), other production facilities (e.g. by using a, rather inefficient, peak power plant) or load management.

In this paper we consider the microCHP (combined heat and power) technology, providing generation on a domestic scale. An initial summary of the potentials of this technique is given by the United States Department of Energy (2003). The production of a microCHP in a household is related to the energy production and consumption. If furthermore energy storage is added to this setting, also load management comes into play. Since a large share

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of microCHP in the electricity grid (in this setting we can speak of a 'fleet' of microCHPs) can lead to grid instability problems, also the network side of the problem comes into the picture. As a consequence, the considered problem combines the above mentioned fields of the energy supply chain with multidirectional influences.

More specific, the planning problem for a fleet of microCHPs is twofold. The first problem concerns the supply of the heat demand in the individual households. This demand can be satisfied by planning the production of heat by the microCHP directly at the time of demand, or by planning the supply of heat via a heat buffer, which means that the heat has to be produced by the microCHP at an earlier time. The second problem is to match the total electricity production pattern of a combined fleet of houses with a given total electricity demand pattern. Whereas the first problem is a time related matching problem for a single device, the second has to synchronize the production planning of the individual microCHPs to a global objective (a size related problem).

The paper is organized as follows. The next section gives an overview of microCHP generation and its characteristics. In Section 3 related planning problems, optimization frameworks and the positioning of this work are discussed. Section 4 presents the two planning problems occurring in this context, and methods to solve these problems. Section 5 contains some computational results for the methods proposed in Section 4. In the last section, recommendations for future work are given.

2. Overview of microCHP and its application

MicroCHP appliances consume natural gas and produce both heat and electricity at a certain heat to electricity rate. The electrical output is in the order of 1 kW, which means that it is suitable for use on a household scale. There are several possible technical realizations of a microCHP, such as Stirling engines, rankine cycle generators, reciprocating engines and fuel cells, where Stirling engines are nearest to full market exposure.

MicroCHP is considered as an alternative energy producer, due to its relatively high energy efficiency, compared to that of large power plants. The main advantage is the more efficient use of the heat, since produced heat in a power plant cannot be transported/used as efficiently (if it is not lost already in the production process) as on domestic scale. However, this means that the principle focus of combined heat and power production on a domestic scale should be on the efficient storage/consumption of heat in order not to lose this advantage. Therefore, microCHP mainly can be seen as a replacement for current boiler systems, and secondly as a domestic electricity generator.

The generation characteristics of microCHP depend on the current advances in the generation technology on the one hand, and on house characteristics and grid policy on the other hand. First, the heat production of the microCHP has to fulfill the heat demand of the household. Next, the electricity production of the microCHP is bounded by regulations set by the national government. Also the total energy efficiency of the technology limits the ratio between the heat and electricity generation. Given a certain generation technology, the electricity to heat ratio is known and can be used as given input for the planning problem.

Fig. 1 shows the electricity output profile for an example run of a microCHP based on a Stirling engine. There is no one-to-one relation between the microCHP being switched on and the power output. In general, a run can be roughly divided into three phases:

- a startup phase, in which, after some grid tests, the engine is started and the power output slowly increases to its maximum output value;
- a constant phase, in which the power output balances around the maximum output value;

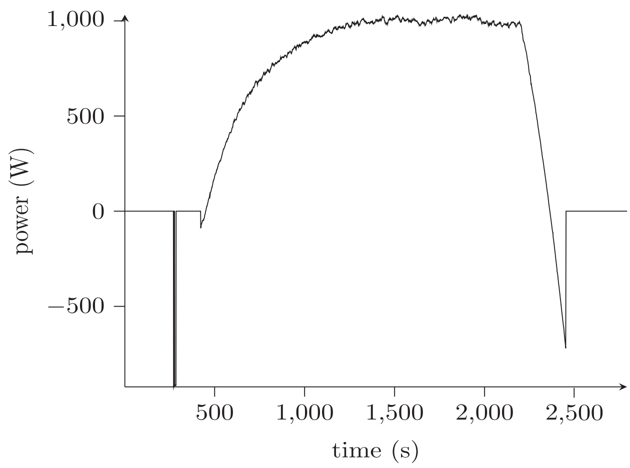


Fig. 1. Electricity output of a microCHP run.

- a shutdown phase, in which the engine is slowed down.

Roughly the same division into phases yields for the heat output. The highest energy efficiency is reached in the constant phase. For this reason, and to prevent wearing of the system, longer runs are preferred over shorter ones. This results in a minimum time that the microCHP has to run, once switched on. For similar reasons the microCHP has to stay off for a minimum amount of time, once switched off.

If the heat consumption is directly supplied by the microCHP, the decisions to run the microCHP are completely determined by the heat demand. As a result often short runs of the microCHP will occur. This is the reason, why microCHPs are in general combined with a heat buffer. An additional heat buffer allows to decouple production from consumption up to a certain degree and, therefore, to make a planning possible. The term 'planning' reflects to the series of decisions to let (*a*/multiple) microCHP(s) run at sequential time periods or not.

Based on the above considerations, the planning for a house with a microCHP and a heat buffer is heat demand driven. Furthermore, decisions in certain time periods have a large impact on possible decisions in future time periods. E.g. switching on a microCHP now leads to a certain minimum amount of heat generation and, therefore, increases the heat level in the heat buffer. This may have as a consequence that in certain future time periods the microCHP cannot run, since it cannot get rid of the produced heat without spoiling heat to the near environment.

Once houses are collaborating in a larger grid, the mutual power output of the different houses adds a global electricity driven element to the planning problem. The group of houses can act as a so-called Virtual Power Plant (VPP) by producing a certain electricity output. This output may be partially consumed by the houses themselves, but part of it may also be delivered to the electricity network. This aggregated output has an impact on the way the electricity retailer of the households should act on a short term electricity market in advance (e.g. for 24 h ahead) or on a realtime market. The retailer has to subtract the expected overall production profile of the fleet from its overall need of electricity, and to buy only the reduced amount or, in case of an overproduction, to sell the remaining amount on the market. As the prices of electricity on these markets vary over time, it may be beneficial to steer the fleet to produce more electricity in expensive periods.

The planning problem considered in this paper focuses on using the electricity production of a large fleet of houses on the short term market. As this market is a day ahead market, a retailer has to come up with a possible production plan of the fleet in advance to be able to adapt its acting on the market. This profile somehow will depend on the prices of the market, but for the retailer the most important question is if he is able to reach this profile with the fleet, since a deviation of the realized planning the next day leads to (huge) costs on the balancing market. In this paper we concentrate on the offline planning a day ahead. This planning will be based on predictions for the heat demand of the houses. The online problem occurring the next day is out of the scope of this paper and needs different approaches. Kok (2009) shows some bidding strategies for online decision making. Furthermore, in Molderink et al. (2010) it is shown how the offline planning afterwards can be used as a guideline for online decision making.

3. Related work

In this section we present a summary of related work on the planning problem and the optimization framework in which our planning problem is situated. Also we give an indication of how our model could be used in a real world scenario.

3.1. The unit commitment problem

The planning problem regarding existing generation technologies in the current energy supply chain is known under the general term of the *unit commitment problem* (UCP). For an overview of the basic UCP and literature we refer to Sheble and Fahd (1994) and Padhy (2004). The classic UCP as presented in Kerr et al. (1966) and Hara et al. (1966) combines the economic dispatch problem, which is the problem of scheduling the outputs of up-and-running power generators (see e.g. (Bakirtzis et al., 1994)), with the decision problem of which power generators to use at what time intervals (i.e. the commitment problem). The generators have startup/shutdown generation profiles and minimum periods in which they should run or remain switched off. Typically the generation output can be chosen (between limits). As an objective operation/maintenance costs are minimized or profit is maximized, where the total output of the system should exceed demand (and optionally spinning reserve should exceed a percentage of this demand too). Large thermal power plants are the most common type of generator as in Takriti et al. (2000). In Philpott et al. (2000), Cerisola et al. (2009), Groewe-Kuska and Roemisch (2005), Caroe and Schultz (1998) also hydro power plants are considered, which includes storage in the model. In the latest years, the stochastic UCP is considered (e.g. Takriti et al. (2000), Caroe and Schultz (1998), Philpott et al. (2000), Groewe-Kuska and Roemisch (2005), Cerisola et al. (2009)), which uses load scenarios to take uncertain load profiles into account.

The planning problem presented in this paper is a new variant of the UCP, in the sense that the economic dispatch is completely fixed (the operation of a microCHP is fully determined by the decisions to have it on or off), in combination with the requirement that the total production is also bounded from below (each microCHP is obliged to run due to heat demand) and that the production profile over time is strongly restricted by the heat demand profile of the house. Especially these last requirements differ from the planning of large thermal units, which in general do not focus on the (hard-constrained) storage of heat. Whereas hydro-based power plants also offer the possibility to pump water or have external water inflow, microCHP storage is only based on the generation, which leads to a more strict feasibility problem. Furthermore, the number of generators that is used in this problem also exceeds the common amount in the UCP by a large factor.

3.2. Optimization framework

Our planning problem is part of a three-step optimization methodology (Molderink et al., 2010), that aims at optimizing the control of domestic smart grid technology. Successively, load profiles of relevant devices are predicted one day ahead (e.g. for a microCHP a heat profile for the house is predicted), based on these predicted load profiles a planning of local entities (e.g. a microCHP, a fridge/freezer, an electric vehicle) is made one day ahead, and the local entities are realtime controlled, based on the planning and realtime profiles. A simulator has been developed (Bakker et al., 2010) to verify the behavior of this methodology and to analyze the impact of distributed generation, distributed storage and demand side load management. An alternative approach is the power matcher presented in Kok et al. (2005). It proposes a multi agent system to control supply and demand in electricity networks with a high share of distributed generation. An electronic exchange market is used as a platform for control agents, representing devices in the system. Further approaches are HOMER (National Renewable Energy Laboratory, 2005) and ALEP (International Energy Agency, 2000), which are simulation tools/models to analyze the impact of design choices for distributed power systems. However, the focus in these projects is not on the control of such power systems.

3.3. Positioning of the work

Virtual Power Plants are already in use in practice. An example of a VPP can be found in Hassenmueller (2009), which consists of nine small hydroelectric plants with a total capacity of 8.6 MW. This VPP is planned to be expanded to have at least a capacity of 15 MW (using about 15 generators), in order to be allowed to market electric power on the balancing markets according to the German situation. A distributed energy management system is used to plan the operations of this VPP. Comparing the size to our microCHP use case, this could be translated to a VPP consisting of at least 15000 microCHPs, whose operation has to be planned. For an example of such a type of VPP using generators on micro (domestic) scale we refer to Lichtblick (2009). In their case the electricity supplier is the owner of the local generators and can use them to reach its own objectives. For our general approach we consider a similar ownership construction in which the electricity supplier has control; however, local comfort (i.e. heat demand) is leading. Due to the limited amount of decision freedom for each individual microCHP compared to the hydroelectric generators, in our case the flexibility of the system results mostly from the large number of generators in the VPP. In our planning problem we therefore focus on feasibility aspects. Global constraints on the total electricity output of the VPP result from either demands on network stability or from planned or already offered production patterns on the electricity market. In the latter case, the planning problem can be seen as the problem of finding a detailed planning of an already cleared (and globally optimized) offer. Since we do not consider the market offer itself as in Neame et al. (2003), the electricity prices can be seen as fixed parameters. Note that in this paper the global constraints are also used to verify the dynamics of the problem and might not correspond to realistic market offers. The planning that finally results from this situation is used as a guideline for realtime control of the fleet in the third step of the three-step optimization methodology. Uncertainty of load profiles is thus tackled in the third step and not within the day ahead planning considered in this paper.

4. The microCHP planning problem

In this section we define and treat the microCHP planning problem. The microCHP planning problem can be divided into two problems. The first problem consists of the planning of a single microCHP, subject to local (feasibility) constraints. This problem is called the Single House Planning Problem (SHPP). In the second problem, global constraints on the sum of the production of individual microCHPs are added, as these appliances are considered to cooperate in a so-called fleet. This leads to the Fleet Planning Problem (FPP).

In practice a decision maker is completely free to instantaneously switch on or switch off a microCHP at any moment in time. However in our model, we discretisize the time and allow a decision maker only to switch on or off the microCHP for complete time intervals. The discretization of the time horizon on the one hand leads to a simpler model, but on the other hand, the short term market also works with time intervals, hence a discretization of time is needed. More precisely, we divide the planning horizon $[0,T]$ of the SHPP/FPP into N_T time intervals $[t_i,t_{i+1}]$ of equal length $\frac{T}{N_T}$. The decision to have a microCHP on or off is made for a complete interval $[t_i,t_{i+1}]$. As a consequence of this, we introduce decision variables x_j for the intervals:

$$x_j = \begin{cases} 1 & \text{if the microCHP is on during interval } j \\ 0 & \text{if the microCHP is off during interval } j, \end{cases} \tag{1}$$

where interval j is the interval $[t_{j-1},t_j]$, $j = 1,\ldots,N_T$. A solution to the SHPP is a vector $x = (x_1,\ldots,x_{N_T}) \in X$, where $X = \{0,1\}^{N_T}$ is the N_T-dimensional space of possible binary decision variables.

Before describing the microCHP planning problem for a fleet of houses $1,\ldots,N$, we first focus on the constraints on the choice for the decision vectors for individual microCHPs. A solution x^i for house i needs to consider the technical constraints of the microCHP of house i (hard constraints) and should respect the heat demand of house i at all times (semi-hard constraints). These constraints lead to subspaces of feasible decision vectors, where by $X_1^i \subseteq X$ we denote the set of all decision vectors respecting the hard technical constraints for house i and by $X_2^i \subseteq X$ the set of vectors which satisfy the semi-hard heat demand constraints. In Fig. 2 a sketch of the construction of the solution spaces for the fully constrained FPP is given. Based on these notations the set $X_{1,2}^i := X_1^i \cap X_2^i$ denotes the set of solutions for house i which respect the hard and semi-hard constraints. Furthermore, the sets $Y_1 := X_1^1 \times \cdots \times X_1^N$ and $Y_{1,2} := X_{1,2}^1 \times \cdots \times X_{1,2}^N$ form the feasible spaces in the FPP, when each individual house has to respect the given hard or hard and semi-hard constraints. However, next to the local constraints for each house, we may add semi-hard constraints on a global production pattern, i.e. on the set of all individual decision vectors. This leads to sets $\widetilde{Y}_1 \subseteq Y_1$ and $\widetilde{Y}_{1,2} \subseteq Y_{1,2}$ which denote the solution spaces for the fully constrained FPP, if next to the hard constraints or the hard and semi-hard constraints for each house also the semi-hard constraints on the global production pattern have to be taken into account.

Besides the constraints on the decisions also an objective is added to the FPP. Here multiple directions of choosing this objective are possible. Variable electricity prices influence the planning decisions, but also energy efficiency, heat buffer levels (for sequential planning problems) and network capacity can be taken into account. In general, the objective function is a function z on the space $S = X^1 \times \cdots \times X^N$. Different versions of the FPP may now be formed by the selection of an objective function $z(S)$ and a subspace $S^* \subseteq S$, where $S^* \in \{Y_1, Y_{1,2}, \widetilde{Y}_1, \widetilde{Y}_{1,2}\}$. Furthermore, the Single House Planning Problem is a special case achieved by setting $N = 1$. By the choice of the space S^*, the decision maker may allow decision vectors which do not fulfill all the semi-hard constraints. In such cases, penalty costs for violating these semi-hard constraints are used and the problem becomes more flexible in the sense of finding a feasible solution. In this paper we focus on electricity prices as objective and the problem variant where all semi-hard constraints have to be taken into account.

In the following section we give, next to a more detailed definition of the problem, an overview of the solution approaches to the Single House Planning Problem and the Fleet Planning Problem. First, we present a general Integer Linear Programming (ILP) formulation in Section 4.1 that can be applied to both problems. This ILP formulation has mainly as goal to give a more detailed explanation of the constraints on the decision vectors. Next, Section 4.2 presents a dynamic programming approach to solve the SHPP. Starting from this single house approach, in Section 4.3 a heuristic method is derived to tackle the FPP, which is able to solve the fleet problem in reasonable time for use in practice.

4.1. ILP formulation

To model the FPP as an ILP we start with a general description of the input parameters. Next we describe the constraints, using these input parameters and the decision variables x_j^i, as specified in (1). Finally the objective function is presented.

The input of the FPP consists of numbers specifying the dimensions of the problem and data specifying characteristic behavior within the problem. The size of the problem is determined by the planning horizon, specified by the number of intervals N_T, and the number of houses forming the fleet, denoted by N. Behavioral parameters can be divided into three categories:

- generation of the microCHPs;
- heat demand of the houses;
- electricity demand of the fleet.

The generation of the microCHP of house i, $i \in \{1,\ldots,N\}$, is characterized by a minimum runtime MR^i, a minimum offtime MO^i, the heat generation G^i_{max} for a time interval if the microCHP is running at full power, and a value α^i specifying the ratio between electricity and heat generation. Furthermore, each house has two vectors: $\widehat{G}^i = \left(\widehat{G}^i_1,\ldots,\widehat{G}^i_{N^i_{up}}\right)$, giving the loss of the heat generation during so-called start up intervals and $\check{G}^i = \left(\check{G}^i_1,\ldots,\check{G}^i_{N^i_{down}}\right)$, giving the extra heat generation during shutdown intervals. For the length N^i_{up} of the start up period it is assumed that $N^i_{up} \leqslant MR^i$ and for the length N^i_{down} it is assumed that $N^i_{down} \leqslant MO^i$ (which is valid in the current available microCHPs). Next to the above data, specifying the behavior of the microCHP, it has to be known in which state the microCHP is at the begin of the planning period. To specify the state of the microCHP, it would be sufficient to specify if it is on or off and for how long it is in this state. However, for ease of

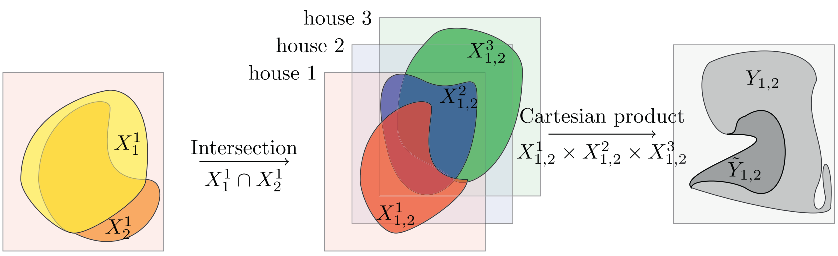


Fig. 2. Solution space for the microCHP planning problem.

notation we represent the initial state of the microCHP of house i by a vector $\bar{x}^i = \left(\bar{x}^i_{1-M^i}, \ldots, \bar{x}^i_0\right)$ of length $M^i = \max\{MR^i, MO^i\}$, specifying for some periods in the past the behavior of the microCHP.

The heat demand of house i is characterized by a heat demand vector $H^i = \left(H^i_1, \ldots, H^i_{N_T}\right)$. Since this heat demand is supplied by a heat buffer we use a value BL^i to describe the initial heat level in the buffer, a value BC^i to describe the buffer capacity and a value K^i to describe the heat loss parameters for the buffer. This heat loss is assumed to be constant for all intervals.

The electricity demand of the FPP is specified by lower and upper bound vectors $P^{lower} = \left(P^{lower}_1, \ldots, P^{lower}_{N_T}\right)$ and $P^{upper} = \left(P^{upper}_1, \ldots, P^{upper}_{N_T}\right)$ for the production pattern of the fleet. Furthermore, the electricity price is given by a price vector $\hat{\pi} = (\hat{\pi}_1, \ldots, \hat{\pi}_{N_T})$.

As mentioned before, the only decision variables in the problem are the binary variables x^i_j, for $j = 1,\ldots,N_T$ and $i = 1,\ldots,N$. Next to these variables we introduce variables g^i_j and e^i_j to represent the generation of heat and electricity, respectively. These variables depend on the decision variables x and the generation characteristics. To formalize this dependence we need to know in which state the microCHP is. For this, it is helpful to introduce binary variables $start^i_j$ and $stop^i_j$, where $start^i_j$ is a binary variable for interval j and house i, indicating whether the microCHP is started at the beginning of interval j or not:

$$start^i_j = \begin{cases} 1 & \text{if the microCHP starts in interval } j \text{ in house } i\left(x^i_j = 1 \text{ and } x^i_{j-1} = 0\right) \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Likewise, $stop^i_j$ is defined as:

$$stop^i_j = \begin{cases} 1 & \text{if the microCHP stops in interval } j \text{ in house } i\left(x^i_j = 0 \text{ and } x^i_{j-1} = 1\right) \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

To ensure that the binary variables $start_j$ and $stop_j$ are consistent with the x-variables, constraints (4)–(9) are added. If necessary, the run history $\bar{x}$ is used in these equations.

$$start^i_j \geqslant x^i_j - x^i_{j-1} \qquad j = 2 - MR^i, \ldots, N_T, \tag{4}$$
$$start^i_j \leqslant x^i_j \qquad j = 2 - MR^i, \ldots, N_T, \tag{5}$$
$$start^i_j \leqslant 1 - x^i_{j-1} \qquad j = 2 - MR^i, \ldots, N_T, \tag{6}$$
$$stop^i_j \geqslant x^i_{j-1} - x^i_j \qquad j = 2 - MR^i, \ldots, N_T, \tag{7}$$
$$stop^i_j \leqslant x^i_{j-1} \qquad j = 2 - MR^i, \ldots, N_T, \tag{8}$$
$$stop^i_j \leqslant 1 - x^i_j \qquad j = 2 - MR^i, \ldots, N_T. \tag{9}$$

Table 1 shows how these constraints force the variables $start_j$ and $stop_j$ to take their correct values. Using the start and stop variables, the generation of heat g^i_j can be expressed as:

$$g^i_j = G^i_{\max} x^i_j - \sum_{k=0}^{N^i_{up}-1} \widehat{G}^i_{k+1} start^i_{j-k} + \sum_{k=0}^{N^i_{down}-1} \check{G}^i_{k+1} stop^i_{j-k} \quad j = 1, \ldots, N_T, \tag{10}$$

and the generation of electricity as:

$$e^i_j = \alpha^i g^i_j \quad j = 1, \ldots, N_T. \tag{11}$$

Table 1
The construction of *start* and *stop* variables from consecutive x variables.

x^i_{j-1}	x^i_j	Eq. (4)	Eq. (5)	Eq. (6)	$Start^i_j$	Eq. (7)	Eq. (8)	Eq. (9)	$Stop^i_j$
0	0	$\geqslant 0$	$\leqslant 0$	$\leqslant 1$	0	$\geqslant 0$	$\leqslant 0$	$\leqslant 1$	0
0	1	$\geqslant 1$	$\leqslant 1$	$\leqslant 1$	1	$\geqslant -1$	$\leqslant 0$	$\leqslant 0$	0
1	0	$\geqslant -1$	$\leqslant 0$	$\leqslant 0$	0	$\geqslant 1$	$\leqslant 1$	$\leqslant 1$	1
1	1	$\geqslant 0$	$\leqslant 1$	$\leqslant 0$	0	$\geqslant 0$	$\leqslant 1$	$\leqslant 0$	0

The consequences of starting/stopping on the generation a microCHP are specified by (10) and (11). Yet it still remains to guarantee a correct functioning of the microCHP with respect to the minimum runtime and offtime requirements. The minimum runtime constraint demands that the microCHP has to run for at least MR^i consecutive intervals, once a choice is made to switch it on. The minimum offtime constraint demands that the microCHP has to stay off for at least MO^i consecutive intervals, once a choice is made to switch it off. Eq. (12) forces the decision variable x^i_j to be one if one start occurs in the previous $MR^i - 1$ intervals, since x^i_j is only allowed to take the values zero and one. Likewise, Eq. (13) forces the decision variable x^i_j to be zero if one stop occurs in the previous $MO^i - 1$ intervals. Again, if needed the given *start* and *stop* variables from the past (following from the given $\bar{x}$ values) are used.

$$x^i_j \geqslant \sum_{k=j-MR^i+1}^{j-1} start^i_k \quad j = 1, \ldots, N_T, \tag{12}$$
$$x^i_j \leqslant 1 - \sum_{k=j-MO^i+1}^{j-1} stop^i_k \quad j = 1, \ldots, N_T. \tag{13}$$

Note, that after a start of the microCHP, it takes at least MR^i intervals before a stop may occur. Since furthermore between two consecutive starts one stop occurs, we never can have more than one start in MR^i consecutive intervals. Similar reasoning learns that we never can have more than one stop in MO^i consecutive intervals.

To specify the constraints resulting from the heat demand, we introduce variables hl^i_j specifying the heat level in the buffer of house i at the beginning of interval j. For the first interval, this level is given by the initial heat level BL^i (Eq. (14)). The change of the heat level in interval j is given by the amount of generated heat $\left(g^i_j\right)$ minus the heat demand $\left(H^i_j\right)$ and the loss parameter (K^i) (see Eq. (15)). Finally, the capacity of the heat buffer has to be respected (Eq. (16)).

$$hl^i_1 = BL^i, \tag{14}$$
$$hl^i_j = hl^i_{j-1} + g^i_{j-1} - H^i_{j-1} - K^i \quad j = 2, \ldots, N_T + 1, \tag{15}$$
$$0 \leqslant hl^i_j \leqslant BC^i \quad j = 1, \ldots, N_T + 1. \tag{16}$$

The semi-hard constraints on the global production pattern can be formulated as follows:

$$\sum_{i=1}^{N} e^i_j \leqslant P^{upper}_j \quad j = 1, \ldots, N_T, \tag{17}$$
$$\sum_{i=1}^{N} e^i_j \geqslant P^{lower}_j \quad j = 1, \ldots, N_T. \tag{18}$$

The most natural objective function for the FPP is to maximize the profit on the electricity market:

$$z = \max \sum_{j=1}^{N_T} \sum_{i=1}^{N} \hat{\pi}_j e^i_j. \tag{19}$$

The ILP model of the FPP now consists of all Eqs. (1)–(19). This ILP problem has $N \times N_T$ binary decision variables and $O(N \times N_T)$ constraints. Even for a single house, we still have N_T decision variables. If we choose for 5 min intervals, this results in an ILP with 288 binary decision variables for a planning horizon of one day. In the next subsection we show, that for a single house a faster approach can be developed.

4.2. Dynamic programming formulation

In this section we propose dynamic programming (DP) formulations for the planning problem for a single house and for a fleet of houses. We first introduce a DP for a single house. Next the individual house formulations are combined into a larger DP for the fleet.

The DPs use the same input parameters and decision variables x as in the previous subsection.

SHPP– The DP for the single house i is based on state variables $\sigma_j^i := \left(A_j^i, B_j^i, C_j^i\right)$, describing the possible states of the microCHP of house i at the *begin* of interval j. More precisely, we have:

- A_j^i, expressing the number of consecutive intervals that the on/off state of the microCHP is unchanged looking back from the start of the current interval j (positive values indicate that the microCHP is running and negative values indicate that the microCHP is off);
- B_j^i, expressing the total number of intervals the microCHP has been running from the beginning of the planning period until the start of the current interval j;
- C_j^i, expressing the number of runs of the microCHP which have already been finished.

Note, that based on these three characteristics the current state of the microCHP and the produced heat can be completely specified. Furthermore, the number of possible states for a given house i is bounded by N_T^4. In the DP we use $N_T + 1$ phases corresponding to the start of the intervals $j = 1,\ldots,N_T + 1$, where the final phase corresponds to the state at the *end* of the planning horizon (after interval N_T). For each state σ_j^i in phase j a cost function $F_j^i\left(\sigma_j^i\right)$ is introduced, which expresses the maximal profit which can be achieved in the intervals $j,\ldots,N_T$ if the microCHP is in state σ_j^i at the begin of interval j. The calculation of $F_j^i\left(\sigma_j^i\right)$ depends on the possible actions in state σ_j^i and the values of the cost function for some states in phase $j + 1$. The possible actions are to either have the on/off state unchanged or to switch it. If we leave the state unchanged (no start or stop) we get as new state in interval $j + 1$:

$$\hat{\sigma}_j^i := \begin{cases} \left(A_j^i + 1, B_j^i + 1, C_j^i\right) & \text{if } A_j^i > 0 \\ \left(A_j^i - 1, B_j^i, C_j^i\right) & \text{if } A_j^i < 0. \end{cases}$$

If we change the on/off state, we have:

$$\check{\sigma}_j^i := \begin{cases} \left(-1, B_j^i, C_j^i + 1\right) & \text{if } A_j^i > 0 \\ \left(1, B_j^i + 1, C_j^i\right) & \text{if } A_j^i < 0. \end{cases}$$

This leads to the following recursive expression for $F_j^i\left(\sigma_j^i\right)$:

$$F_j^i\left(\sigma_j^i\right) := \max\left\{c_j^i\left(\sigma_j^i, \hat{\sigma}_j^i\right) + F_{j+1}^i\left(\hat{\sigma}_j^i\right), c_j^i\left(\sigma_j^i, \check{\sigma}_j^i\right) + F_{j+1}^i\left(\check{\sigma}_j^i\right)\right\},$$

where $c_j^i(\sigma, \sigma')$ denotes the cost associated with the choice corresponding to the transition from σ to σ'. The calculation of these costs is similar to the calculation of the values e_j^i used in the previous subsection plus some feasibility checks on the state transitions and can be done in constant time.

If we define $F_{N_T+1}^i\left(\sigma_{N_T+1}^i\right) = 0$ for all possible states $\sigma_{N_T+1}^i$ in phase $N_T + 1$ we can recursively calculate $F_1^i(\sigma_1^i)$ and deduce a corresponding optimal decision vector x^i.

In case we optimize for the electricity market, the DP of a single house can be seen as a function f^i on a price vector π:

$$f^i(\pi) \to x^i.$$

The function $f^i(\hat{\pi})$ gives an optimal local planning for the SHPP and can be calculated in runtime $O(N_T^4)$, given a certain price vector (and of course the data of house i).

FPP– The optimal decision vector x^i for house i is represented by a path in the network representing the DP of house i. If we want to extend this DP to a DP for the FPP, the resulting N decision vectors $x^1,\ldots,x^N$ need to fulfill also the constraints on the production pattern (17) and (18). The consequence for the DP formulation is that the number of states in each phase increases exponentially. A state in the DP for the fleet has to be specified by a vector of states for the individual houses; $\sigma_j := \left(\sigma_j^1, \ldots, \sigma_j^N\right)$. From each state σ_j we have 2^N possible actions that can be taken (existing of N binary choices to leave the state unchanged or not in each house). Note that a state transition is only feasible if, next to the individual feasibility checks on the house states, the state vector (of the combined houses) also fulfills the production pattern constraints of the given interval.

To formalize the DP for the fleet of houses, we denote by $D_j(\sigma_j)$ the maximal fleet profit that can be achieved in the intervals $j,\ldots,N_T$ if, at the begin of interval j, the state of house i is given by σ_j^i, for $i = 1,\ldots,N$. Due to the semi-hard fleet production constraints a state transition from σ_j to σ_j' may not be allowed even if all individual state transitions $\left(\sigma_j^i, \sigma_j'^i\right)$ are allowed for the individual houses. Therefore we cannot simplify the DP by calculating the individual house DPs independently and merging the results. So we need to calculate the complete fleet DP, which has an exponential runtime of $O(N_T^{3N+1})$, since the state space explodes by the possible combinations of houses in each phase of the DP ($O(N_T^{3N})$).

Note that in Bosman et al. (2010b) it has been shown that (a restricted version of) the FPP is NP-complete in the strong sense. Thus, it is very unlikely to get a fast exact approach for the FPP. Therefore, in the next subsection we present a heuristic method to solve the FPP in a faster way.

4.3. Local search using single house DPs for the FPP

The DP for the SHPP presented in the previous subsection can be seen as a function $f^i(\pi)$ which maps a given price vector π to a decision vector x^i for house i. Furthermore, the DP for the FPP can be seen as a function d on a price vector π:

$$d(\pi) \to (x^1, \ldots, x^N).$$

This function $d(\pi)$ maximizes a certain objective function and outputs N vectors consisting of the planning in N corresponding houses. Whereas $f^i(\pi)$ finds a solution in polynomial time, $d(\pi)$ needs exponential time to be evaluated. Since this is not feasible in practice, a heuristic is developed to find a solution to the FPP that is both feasible and, hopefully, close to the optimum solution, and can be found in reasonable time.

Structure– The presented heuristic method is based on two principles:

1. As long as the semi-hard fleet production constraints are not considered, the fleet optimum is a combination of the individual optima, i.e. we may solve the individual house DPs separately: $d(\pi):=(f^1(\pi),\ldots,f^N(\pi))$;
2. The individual function $f^i(\pi)$ for house i only depends on the price vector π.

The idea of the approximation method is the following. If we discard the semi-hard fleet constraints in first instance, we can calculate the fleet DP by separating it into N single house DPs. This reduces the runtime to $O(N \times N_T{}^4)$. Now we reintroduce the fleet constraints as a feasibility check on the output of this calculation. This combination of calculating separate DPs and performing a feasibility check results in a new structure: a certain fleet production and a yes/no answer whether this production is allowed by the fleet constraints. The basis of the heuristic method now is to use this structure of separately calculated DPs and a feasibility check on the sum of these individual DPs, by iteratively searching the set of price vectors in an effective way until a price vector π is found, where the feasibility check leads to a positive answer and where π is not too different from the real price vector $\hat{\pi}$. In this way we may expect that the resulting solution is close to the optimum.

Iterative search– In Subsection 4.2 we proposed the fleet DP, where all possible combinations of production vectors in individual houses are coded by the state space. In the heuristic we need a way to search through these possible combinations, since the dependence between different house productions is lost when calculating separate house DPs. Since we do not want to change the state definition in the house DP (this would lead to the original fleet DP or similar state expansions), the only way of applying a search can go via the input of the DPs. Since $f^i(\pi)$ depends on the price vector π we change the price vector of the house DPs in our search. Of course the objective function for the fleet production is still calculated with the original price vector $\hat{\pi}$.

Starting with a price vector $\pi^i = \hat{\pi}$ for each house i, we iteratively adjust the price vectors based on the result of the DPs for the individual houses using their current price vectors. We try to remain as close as possible to the original price vector, in the hope to stay close to the optimal value for the objective function. In each iteration the price π^i_j of interval j is locally adjusted if:

- P^{upper}_j is violated and the microCHP of house i is decided to be on in the current solution;
- P^{lower}_j is violated and the microCHP of house i is decided to be off in the current solution.

In the first case π^i_j is multiplied with a factor a, where $0 < a < 1$. In the second case π^i_j is multiplied with a factor $2 - a$. All other prices remain unchanged.

Stop criteria– The method stops when a feasible solution is found or when a maximum number of iterations *MaxIt* is reached. If the maximum iteration count *MaxIt* is reached and we did not find a feasible solution, the solution with the smallest error value *err* is given as a best approximation to the fleet constraints. This error *err* is the absolute sum of the mismatch to the upper and/or lower bounds P^{upper}_j and P^{lower}_j:

$$err := \sum_{j=1}^{N_T}\left(\max\left(\sum_{i=1}^{N} e^i_j - P^{upper}_j, 0\right) + \max\left(P^{lower}_j - \sum_{i=1}^{N} e^i_j, 0\right)\right).$$

In Algorithm 1 a summary of the algorithm is given.

Algorithm 1. Local search on the FPP

```
Input: price vector π̂, lower and upper bounds P^lower and
   P^upper, π^i := π̂ for all houses i
repeat
  solve f^i(π^i) for all i resulting in solution x = (x^1, ..., x^N);
  calculate total production (Σ_{i=1}^N e^i_1, ..., Σ_{i=1}^N e^i_{N_T}) of solution
  x;
  for all j do
    if  Σ_{i=1}^N e^i_j > P^upper_j then
        for all i with x^i_j = 1 do
            π^i_j ⇐ a π^i_j
        end for
    end if
    if Σ_{i=1}^N e^i_j < P^lower_j then
        for all i with x^i_j = 0 do
            π^i_j ⇐ (2 − a) π^i_j
        end for
    end if
  end for
until solution x is feasible or MaxIt is reached
```

Note that the basic structure of this heuristic may also be applied to other dynamic programming formulations which allow a decomposition of the state, leading to a simplified version, consisting of a set of individual DPs.

5. Results

In this section we report on some computational experiments to test the possibilities and limitations of the proposed solution approaches of the previous section. For this, we first describe in Subsection 5.1 a set of scenarios used for the computational tests. These scenarios are solved and compared in two ways:

- using the ILP formulation of the FPP, implemented in AIMMS modeling software and solved by CPLEX 11.1;
- using the local search approach, implemented in C++.

Both solution methods are executed on a desktop computer (2.40 GHz and 2.00 GB RAM).

The corresponding results are presented in Subsection 5.2. Since the computation time to solve the ILP grows rather fast, only the scenarios consisting of relatively small instances are solved by both methods and compared for their objective value and computation time. To the larger instances only the local search method is applied. Some special attention is given to the ability of the local search method to find feasible solutions for narrowing fleet constraints.

5.1. Scenario

The set of scenarios is divided into two subsets, one which contains relatively small sized instances and one which contains larger instances.

5.1.1. Small instances

The small instances consist of one up to ten houses, all equipped with a microCHP and a heat buffer, where a planning is required for 24 time intervals. These instances are solved by both the ILP and the local search method, in order to compare the quality of the solutions achieved by these methods. In comparing the ILP with the local search method, both quality and computational speed play an important role. The quality Q is defined as the quotient of the local search objective value and the ILP objective value:

$$Q := \frac{z_{ls}}{z_{ILP}}.$$

The relative speed S is the quotient of the local search' computation time and the ILP computation time:

$$S := \frac{time_{ls}}{time_{ILP}}.$$

In more detail, the following parameters are used for the small instances. The houses have a heat buffer with initial heat level $BL^i = 5$ kWh and capacity $BC^i = 10$ kWh. The electricity to heat ratio α^i is $\frac{1}{8}$ and the heat loss is $K^i = 50$ Wh for each house i. These values correspond to the characteristics of a nowadays microCHP using a Stirling engine and a heat buffer of around 150 l. Fig. 3 shows the average heat demand in the 24 intervals for the ten houses. More detailed information about the used heat demand profiles can be found in Bosman et al. (2010a). The generation characteristics of the microCHP are summarized in Table 2. We assume that the microCHPs in house 1, 2, 3 and 10 are on at the start of the planning horizon; the other microCHPs are initially off. As price vector $\hat{\pi}$ we use the prices stated in Table 3.

To verify the quality of a solution dependent on the total electricity output, we also vary the constraints on the total electricity production. In this context not only the quality of the

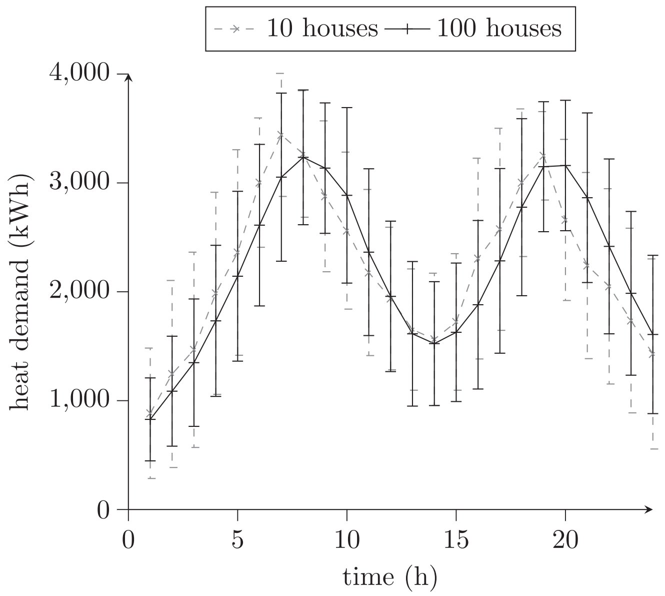


Fig. 3. The average heat demand and the standard deviation in the ten houses of the small instances and the 100 houses of the large instances.

Table 2
MicroCHP characteristics (in case of 24 intervals).

Parameter	N^i_{up}	N^i_{down}	MR^i	MO^i	$\widehat{G}^i$	$\check{G}^i$	$G^i_{\max}$
value	1	1	1	1	800	400	8000

achieved solutions is of interest, but also the question whether or not the local search method is able to find feasible solutions in situations where the bounds on the production pattern get more tight. We use ten production pattern variants, where we set the lower and upper bounds P^{lower} and P^{upper} and specify them as constant percentages of the total maximally possible electricity output of the group of houses. These percentages are given in Table 4. The last variant gives the tightest combination of lower and upper bounds: the highest lower bound for which a feasible solution is found (variant 1, 8 or 9) is combined with the lowest upper bound for which a feasible solution is found (variants 1–7). We denote by $I(k,l)$ an instance of the k houses $\{1,\ldots,k\}$, where we use variant l for the electricity production bounds.

5.1.2. Large instances

The larger instances are used to test the local search method more thoroughly on more realistically sized fleets of houses. These instances contain up to 100 houses and require a planning for up to 96 intervals, which comes down to intervals of 15 min length in a horizon of 24 h. For these instances the setting for the parameter a, which defines the changes in the price vector, is tested against varying fleet constraints and interval length to see how many iterations are needed to find a feasible solution. In case a feasible solution cannot be found the error from the fleet constraints should be minimized.

We use fleets of sizes 25, 50, 75 and 100 and intervals of 60, 30 and 15 min length, which gives 24, 48 and 96 intervals respectively in total. The generation/household characteristics are similar to the settings in the small instances (accounted for the interval length). Minimum run-and offtimes are set to half an hour. Also, the heat demand is similar to that for the small instances, as shown in Fig. 3. Electricity prices are equal to those used in the small instances. The production pattern variants for the large instances are given by Table 4.

5.2. Solutions

Below we present the results for both types of instances. In the local search method for the small instances we set the parameters $a = 0.9$ and $MaxIt = 100$. In the large instances we have again $MaxIt = 100$. As multiplication factor a we now use the values 0.9, 0.7, 0.5, 0.3 and 0.1.

5.2.1. Small instances

The normalized objective values for the instances $I(k,l)$ (i.e. the objective values divided by the number of houses), calculated by the ILP approach, are given in Table 5. If an instance does not have a feasible solution this is denoted by a dash (–).

Some of the instances with a large number of houses and tight production pattern constraints were terminated by the ILP solver, due to slow convergence towards the best found solution. For these instances, where the ILP solver did not find the optimal solution, the upper bound on the objective, given by the solver, is also presented below the table. This gives an indication that the 'large' small instances are close to the largest ones that can be solved to optimality by the current approach. Note that the upper bound of $I(10,4)$ can be lowered by looking at the objective value of $I(10,3)$. The average objective values show that the tighter the fleet constraints are, the less money can be earned.

In Table 6 the computational times are given, where we only show the times corresponding to the feasible instances. A star (*)

Table 3
Electricity prices (APX market 29-10-2007).

Hour	P_j(€/MWh)	Hour	P_j(€/MWh)	Hour	P_j(€/MWh)	Hour	P_j(€ /MWh)
1	37.00	7	63.01	13	124.96	19	275.00
2	29.65	8	91.06	14	135.00	20	187.57
3	22.38	9	103.97	15	111.61	21	92.50
4	19.01	10	179.89	16	103.96	22	66.50
5	28.07	11	150.44	17	171.04	23	51.50
6	37.04	12	242.80	18	500.00	24	47.00

Table 4
Electricity production bounds, based on percentages of possible electricity production.

Production pattern variant	Small instances										Large instances			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Lower bound (%)	0	0	0	0	0	0	0	10	20	*tight*	0	10	20	25
Upper bound (%)	100	90	80	70	60	50	40	100	100	*tight*	75	50	40	35

Table 5
Objective value for instances $I(k,l)$.

k	l = 1	2	3	4	5	6	7	8	9	10	μ
1	1.147	1.092	–	–	–	–	–	–	–	1.092	1.110
2	1.236	1.208	1.016	1.016	1.016	1.016	–	–	–	1.016	1.075
3	1.197	1.197	1.106	1.106	1.002	–	–	–	–	1.002	1.102
4	1.183	1.164	1.128	1.114	1.021	1.021	–	1.009	1.009	0.949	1.066
5	1.164	1.149	1.120	1.060	1.060	–	–	1.118	–	1.023	1.099
6	1.163	1.150	1.130	1.092	1.048	1.027	–	1.139	–	1.021	1.096
7	1.156	1.145	1.137	1.109	1.069	0.972	0.925	1.150	–	0.924	1.065
8	1.156	1.145	1.130	1.114	1.080	1.032	0.919	1.152	1.069	0.902	1.070
9	1.153	1.143	1.121	1.098	1.072	0.993[b]	–	1.150	1.113	0.976[e]	1.091
10	1.176	1.162	1.143	1.122[a]	1.095	1.037[c]	0.948[d]	1.173	1.028	0.945	1.083
μ	1.173	1.156	1.115	1.092	1.051	1.014	0.931	1.127	1.055	0.985	

[a] terminated by solver, upper bound 1.144.
[b] terminated by solver, upper bound 1.058.
[c] terminated by solver, upper bound 1.075.
[d] terminated by solver, upper bound 0.989.
[e] terminated by solver, upper bound 0.999.

denotes an instance that is terminated by the solver premature, without determining the optimality of the solution. The computational times grow extremely fast if the number of houses grows or the production pattern bounds get more tight. Also note the large variance in these times under a fixed number of houses or a fixed production pattern variant.

The quality of the local search method is verified by comparing its objective values and computational times to the ones given in Tables 5 and 6. The local search method is only applied to the feasible instances as found by solving the ILP. The results for Q and the computation times $time_{ls}$ and $time_{ILP}$ are summarized in Table 7, where average values are categorized by number of houses (left side of the table) and by production pattern variant (right side of the table). On the left hand side averages are taken over all (feasible) production pattern variants and on the right hand side averages over all (feasible) numbers of houses. As a first verification, the local search method produces optimal results for all instances $I(k,1)$ as should be the case, since independent DPs can be used in case of no network restrictions. The same yields for the single house, since the available network sizes (1, 2 and 10) put no practical restrictions on the electricity output. The average quality $\overline{Q}$ of all instances is 0.95 and the average relative speed $\overline{S}$ is 0.0098. No trend can be identified between the number of houses and the quality of the local search method. It would be

Table 6
Computational time (in seconds) for instances $I(k,l)$.

k	l = 1	2	3	4	5	6	7	8	9	10	μ	σ
1	0.08	0.06	–	–	–	–	–	–	–	0.08	0.07	0.01
2	0.22	0.31	0.70	0.48	0.51	0.95	–	–	–	1.00	0.60	0.28
3	1.33	1.41	1.75	2.50	1.17	–	–	–	–	1.22	1.56	0.46
4	1.34	1.81	2.45	2.05	8.27	7.36	–	5.98	2.78	9.86	4.66	3.05
5	5.00	6.06	9.33	45.28	57.41	–	–	28.19	–	467.30	88.37	155.84
6	6.78	4.88	20.06	38.64	221.17	254.84	–	25.52	–	2326.13	362.25	748.13
7	7.89	16.77	25.11	47.64	839.84	6373.31	3052.55	9.75	–	1745.06	1346.44	2037.84
8	27.36	43.89	60.72	109.48	396.69	3302.30	9918.91	39.03	97.66	17999.19	3199.52	5753.94
9	129.39	200.31	332.52	2382.53	2858.05	7143.67*	–	130.49	1270.13	16265.91*	3412.56	5016.03
10	461.98	1174.94	873.14	6879.08*	17285.17	6704.20*	8648.84*	79.89	1765.80	5757.88	4963.09	5080.93
μ	64.14	145.04	147.31	1056.41	2407.59	3398.09	7206.77	45.55	784.09	4457.36		
σ	137.81	348.21	275.37	2185.10	5330.13	3087.73	2982.89	41.38	755.25	6570.37		

Table 7
Results for small instances.

Houses	*Q* μ	σ	*Time (s)* $\mu(ILP)$	$\mu(ls)$	*Infeas.%*	*Production pattern*	*Q* μ	σ	*Time (s)* $\mu(ILP)$	$\mu(ls)$	*Infeas.%*
1	1.000	0.000	0.07	0.015	0.00	1	1.000	0.000	64.14	0.015	0.00
2	0.967	0.038	0.60	0.042	42.86	2	0.979	0.028	145.04	0.017	0.00
3	0.951	0.063	1.56	0.021	0.00	3	0.962	0.024	147.31	0.023	11.11
4	0.916	0.062	4.66	0.066	33.33	4	0.980	0.022	1056.41	0.029	11.11
5	0.942	0.059	88.37	0.051	14.29	5	0.955	0.047	2407.59	0.043	11.11
6	0.937	0.057	362.25	0.066	12.50	6	0.892	0.068	3398.09	0.065	0.00
7	0.969	0.032	1346.44	0.096	22.22	7	0.932	0.024	7206.77	0.182	33.33
8	0.958	0.047	3199.52	0.088	20.00	8	0.936	0.067	45.55	0.118	42.86
9	0.947	0.063	3412.56	0.073	11.11	9	0.912	0.030	784.09	0.239	100.00
10	0.962	0.036	4963.09	0.094	20.00	10	0.913	0.057	4457.36	0.131	40.00

interesting to see whether this still holds for large numbers of houses. The production pattern variant has an effect on the quality. An explanation for this behavior is that the local search method has more difficulty in finding a feasible solution under tighter network constraints, resulting in larger deviations from the original price vector. This original vector is used in the objective value, which results in worse results. This is also shown in the percentages of infeasible solutions (violating the electricity constraints) that are found by the local search method. The network variant has more influence on this percentage than the number of houses. If we look at the deviation from the electricity bounds (given by the error *err*), the solutions are relatively close to these bounds. Therefore we included these infeasible solutions in all calculations and comparisons.

Table 8
Results for large instances with $a = 0.9$.

Houses	z_{ls}/N	*Time(s)*	*Iterations*	*Error*	*Infeas.(%)*
25	1.007	1048	80.3	20588	75.0
50	1.026	1982	79.1	36063	75.0
75	1.040	2869	78.7	59734	75.0
100	1.031	3831	78.8	71050	75.0
Production pattern					
11	1.165	859	16.8	0	0.0
12	0.971	2951	100.0	9340	100.0
13	0.984	2962	100.0	65654	100.0
14	0.984	2958	100.0	112440	100.0
Intervals					
24	0.953	1	75.3	27163	75.0
48	1.023	243	80.5	39252	75.0
96	1.103	7053	81.9	74162	75.0

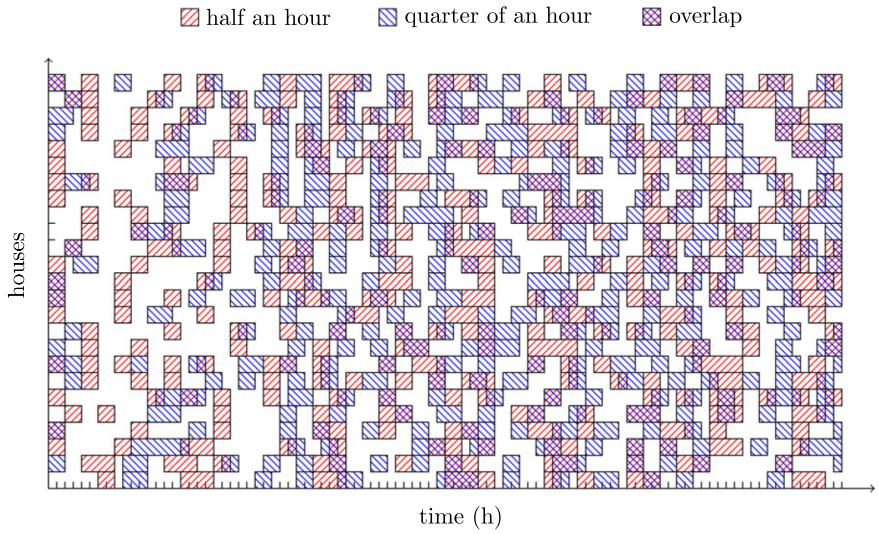
5.2.2. Large instances

For the large instances, we are interested in the behavior of the local search method dependent on the following three instance parameters: the size of the group of houses, the production pattern variant, and the number of intervals in a planning for 24 h. The criteria we use to evaluate the behavior are the objective value, the computational time, the number of iterations the local search method needs, the error and the percentage of infeasible solutions. The results in Tables 8 and 9 are, for a given value of one of the parameters, the averages over all combinations which are derived from the two other parameters. The results achieved with the value $a = 0.9$ (as applied to the small instances) are given in Table 8.

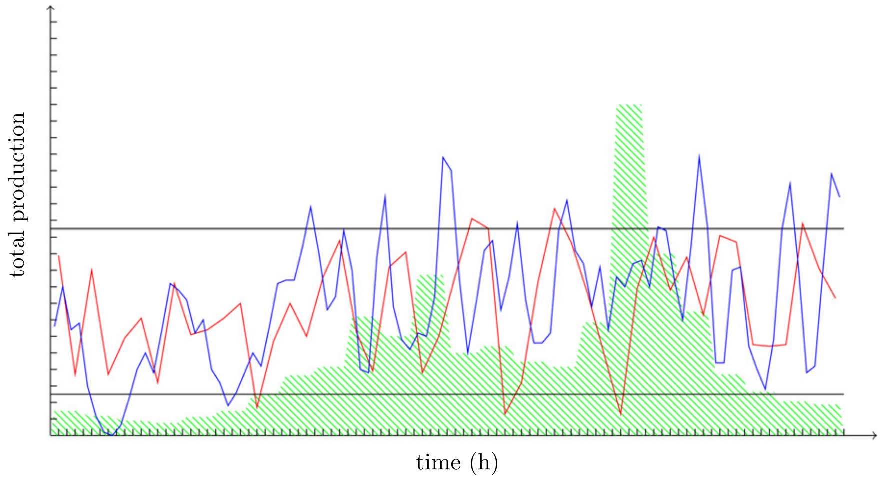
The computational time per house and the error per house decrease slightly when the number of houses increases. For 100 houses the error corresponds to 0.7 kWh over/underproduction per house. For production pattern variant 11 the method always finds a feasible solution (in a few iterations), while for the variants 12, 13 and 14 no feasible solution is found (and the method stops after 100 iterations). However, note that these production constraints are more tight than in the small instances and, thus, there is quite a chance that no feasible solution may exist. Regarding the number of intervals the computational times grow fast. The objective value increases as the number of intervals increases; however, the error increases accordingly, so the convergence is slower for a larger number of intervals.

Table 9
Results for large instances and varying a.

Houses	z_{ls}/N					*Time(s)*				
	0.9	0.7	0.5	0.3	0.1	0.9	0.7	0.5	0.3	0.1
25	1.007	0.994	0.996	0.977	0.979	1048	1007	1011	997	1012
50	1.026	0.981	0.984	0.976	0.949	1982	1915	1912	1898	1925
75	1.040	1.001	0.975	0.966	0.970	2869	2719	2741	2746	2737
100	1.031	0.981	0.962	0.976	1.026	3831	3628	3569	3686	3647
	Iterations					*Error*				
	0.9	0.7	0.5	0.3	0.1	0.9	0.7	0.5	0.3	0.1
25	80.3	76.9	76.0	70.4	71.2	20588	21498	16103	17807	16377
50	79.1	77.0	72.8	71.8	72.3	36063	41865	33840	33758	28492
75	78.7	76.6	71.4	72.3	69.8	59734	60839	44525	47031	41800
100	78.8	76.6	72.1	72.4	70.8	71050	78146	68117	63392	56813
Production pattern	z_{ls}/N					*Time(s)*				
	0.9	0.7	0.5	0.3	0.1	0.9	0.7	0.5	0.3	0.1
11	1.165	1.090	1.033	1.005	1.005	859	405	417	519	535
12	0.971	0.964	0.966	0.964	0.971	2951	2949	2900	2886	2867
13	0.984	0.942	0.951	0.966	0.961	2962	2956	2957	2958	2956
14	0.984	0.961	0.967	0.959	0.987	2958	2959	2957	2963	2963
	Iterations					*Error*				
	0.9	0.7	0.5	0.3	0.1	0.9	0.7	0.5	0.3	0.1
11	16.8	7.1	8.7	9.9	9.6	0	0	0	0	0
12	100.0	100.0	83.7	77.0	74.6	9340	13148	12204	13381	10079
13	100.0	100.0	100.0	100.0	100.0	65654	69146	58121	48221	38450
14	100.0	100.0	100.0	100.0	100.0	112440	120054	92259	100386	94951
Intervals	z_{ls}/N					*Time(s)*				
	0.9	0.7	0.5	0.3	0.1	0.9	0.7	0.5	0.3	0.1
24	0.953	0.955	0.947	0.952	0.946	1	1	1	1	1
48	1.023	0.953	0.939	0.926	0.939	243	235	197	191	180
96	1.103	1.060	1.052	1.043	1.058	7053	6716	6726	6803	6810
	Iterations					*Error*				
	0.9	0.7	0.5	0.3	0.1	0.9	0.7	0.5	0.3	0.1
24	75.3	75.1	75.1	75.2	75.2	27163	16775	16588	17969	17041
48	80.5	77.0	65.8	61.3	58.8	39252	44048	26431	23150	25548
96	81.9	78.2	78.4	78.8	79.2	74162	90937	78919	80372	65021



(a) Two planning results using $a = 0.9$ for production pattern 12 and 25 houses



(b) Fleet behavior

Fig. 4. The detailed planning of a case with a different number of intervals.

Next, since optimal objective values are unknown for these instances, the solutions of different updating schemes of the price vector are compared to each other. In this comparison, the focus is in first instance on the ability to find a feasible solution and the objective value is only of secondary interest. The results for using the values 0.9, 0.7, 0.5, 0.3 and 0.1 for the parameter a are given in Table 9. The different updating schemes perform similar. If the focus is more on minimizing the error, the values 0.5, 0.3 and 0.1 are advantageous. For these values of a for some instances with production pattern variant 12 the local search method could find feasible solutions. If the focus is on the objective value, $a = 0.9$ gives better results against a slightly higher number of iterations and computational time.

Fig. 4 shows a comparison of the detailed planning of a fleet of 25 houses and production pattern variant 12. A planning based on half an hour intervals (red) is compared to a planning based on intervals of a quarter of an hour (blue). 202.5 run hours are planned for the half an hour based planning and 210.75 run hours for the quarter of an hour planning. Fig. 4(a) shows that only 74.5 of these run hours of the two plannings do overlap. In Fig. 4(b) the total generation is plotted against the background of the original price vector (shaded area in green). This example emphasizes that making a planning for 15 min intervals clearly leads to different results compared to a planning for 30 min intervals (both in total as for individual houses), although the minimum runtime and offtime stay fixed on 30 min.

In general we can state that an increase in the number of houses leads to a better fit for the fleet to the given production bounds (i.e. the amount of electricity per house outside the bounds decreases). Concerning the amount of iterations, the largest improvement in objective value is reached within the first 25 iterations. Remaining iterations only lead to slightly better objective values. As a general comment, it is hard to flatten the total output profile over the whole day, when the aggregated heat demand profile deviates too much from the desired production bounds for a too long period.

6. Concluding remarks and recommendations

In this paper the Single House Planning Problem (SHPP) and the Fleet Planning Problem (FPP) are introduced. For the FPP an ILP model is given and a local search method is developed to cope with large instances. Small instances are tested to verify the quality of this heuristic method in comparison to the (optimal) solutions by solving the ILP; the local search method results in a 5% loss in objective value and a 99% gain in computation time. Furthermore, the local search method is tested for larger instances, to see whether it is applicable in practice. Considering the fact that, in practice, we can unfold one calculating entity per house, a planning for 100 houses, 96 intervals and using 100 iterations can be made within 2.3 min. In our experience we find that the maximum number of iterations *MaxIt* can easily be reduced with a factor four, since most best solutions are found within the first 25 iterations. Using this reduction a planning can be made in about half a minute. Regarding feasibility, a feasible solution for the small instances is not found in 19% of the cases, where the ILP formulation did find a solution. Depending on the value of a, 67%–75% of the large instances are infeasible (note that the production bounds for the large instances are more tight).

Looking at possibilities to improve the heuristic method, the updating scheme can be adjusted. In the current local search method the adjustments are based on the performance of the complete group of houses. This could be changed by introducing sub groups, each of which gets its own goal production pattern. Also opposite adjustments in sequential iterations could be prohibited, to prevent a 'flipping' effect in the used steering signals. As an alternative price updating scheme, Lagrangian relaxation can be used to derive the new prices. Finally, it may be worth to investigate if the (re)calculation of the dynamic program can be done more efficiently by using the results of the previous iteration.

To accommodate improvements in the design of a heuristic method different problem formulations may be investigated. A possible direction may be to formulate the problem in such a way that a column generation technique can be used to separate the local constraints on the production profiles from the global constraints on the total production to simplify the planning problem.

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